Determination of 7-dimensional indecomposable nilpotent complex Lie algebras by adjoining a derivation: all 30 cases detailed *

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Abstract

For any complex 6-dimensional nilpotent Lie algebra $\mathfrak{g}$, we compute (up to isomorphism) the strain of all indecomposable 7-dimensional nilpotent Lie algebras which contain $\mathfrak{g}$ by the adjoining a derivation to nilpotent 6-dimensional Lie algebras method. We present the results in tables for each of the 30 non isomorphic cases for $\mathfrak{g}$, along with a cross-table intertwining nilpotent 6- and 7-dimensional Lie algebras.

1 Introduction.

In a preceding paper [4], we applied, for any given 6-dimensional complex nilpotent Lie algebra $\mathfrak{g}$, the adjoining a derivation method along with the computer algebra system *Reduce* to determine (up to isomorphism) the set of all complex indecomposable nilpotent 7-dimensional Lie algebras containing $\mathfrak{g}$, which we called the strain of $\mathfrak{g}$. There, we presented background and method, along with a cross table summarizing all strains for the 30 6-dimensional complex nilpotent Lie algebra, yet provided detailed tables only for 4 selected exemples out of the 30 cases. The present paper is a companion paper to [4], to which we refer for motivations, backdrop and notations. Here, we present detailed tables for each of the 30 cases, along with the cross table.

Let us simply recall that the method of adjoining a derivation introduces for any nilpotent complex Lie algebra $\mathfrak{g}$ and any nilpotent derivation $\delta \in \mathrm{Der}\,(\mathfrak{g})$ the semidirect product $\tilde{\mathfrak{g}}_\delta = \mathbb{C}\delta \oplus \mathfrak{g}$ i.e. the extension $0 \to \mathfrak{g} \to \tilde{\mathfrak{g}}_\delta \to \mathbb{C} \to 0$ with $\mathbb{C}$ acting on $\mathfrak{g}$ via δ. The nilpotent $\mathrm{Aut}\,\mathfrak{g}$-orbits in the projective adjoint cohomology space $PH^1(\mathfrak{g})$ classify all $\tilde{\mathfrak{g}}_\delta$'s which give *quasi-equivalent extensions*. However, a major unsolved question would be to define an object classifying all $\tilde{\mathfrak{g}}_\delta$'s that are isomorphic as *Lie algebras*, i.e. to find conditions on δ, δ' for $\tilde{\mathfrak{g}}_\delta, \tilde{\mathfrak{g}}_{\delta'}$ to be isomorphic Lie algebras.

2 Preliminaries.

Recall that we use historic notations for 6-dimensional Lie algebras and Carles' notations for 7 dimensional Lie algebras as in [4],[2] and [6]. For any 6-dimensional complex nilpotent Lie algebra

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$\mathfrak{g}$ and any nilpotent derivation $\delta \in \mathrm{Der}\,(\mathfrak{g})$ we denote by $\tilde{\mathfrak{g}}_\delta$ the semi-direct product $\tilde{\mathfrak{g}}_\delta = \mathbb{C}\delta \oplus \mathfrak{g}$. We say that δ, δ' are projectively equivalent if and only if there exists $\varphi \in \mathrm{Aut}\,\mathfrak{g}$ and $k \in \mathbb{C} \setminus \{0\}$ such that $\delta' = k\,\varphi^{-1}\,\delta\,\varphi \bmod \mathrm{ad}(\mathfrak{g})$. This simply means that there exists an isomorphism $\psi : \tilde{\mathfrak{g}}_{\delta'} \to \tilde{\mathfrak{g}}_\delta$ leaving $\mathfrak{g}$ invariant [4].

For each of the 30 6-dimensional complex nilpotent Lie algebra $\mathfrak{g}$ (including direct products), we first give the generic nilpotent derivation δ mod ad $\mathfrak{g}$, and the elements of Aut $\mathfrak{g}$. Then we present in tables the successive reductions of the parameters in δ by projective equivalence, and the identification of the Lie algebra $\tilde{\mathfrak{g}}_\delta$. There, we refer to the appropriate file of the companion archive [5] (typically, case1N1 in the table for $\mathfrak{g}_{6,1}$ stands for the file `reducparautommodg6_1case1N1.red` in the directory `6_1strain`; in the same way, case1I stands for `calculderivgtildedelta6_1case1I.red` and test1case2N1 stands for `test1case2N1.red`). We mention here that the essential guides for the identification of the Lie algebras and the construction of an appropriate isomorphism were the weight system and, occasionally inside the weight system, the adjoint cohomology.

Unless otherwise stated in the footnotes, when for a given $\mathfrak{g}$ two distinct δ's in the last column of the tables give rise to the same algebra $\tilde{\mathfrak{g}}_\delta$, they *are not* projectively equivalent (that includes the δ's corresponding to different values of some continuous parameter a or $\varepsilon = 0, 1$). We dismiss all 7-dimensional direct products. Unless otherwise specified, the notation $\sqrt{z}$ will denote any fixed square root of the complex number z. The abbreviation toms in the tables stands for *then one may suppose*. Inside each strain, the first occurrence of an algebra is underlined, for an easier reading of the tables. We count each continuous series for one.

3 $\mathfrak{g}_{6,1}$

Commutation relations for $\mathfrak{g}_{6,1}$: $[x_1, x_2] = x_5$; $[x_1, x_4] = x_6$; $[x_2, x_3] = x_6$.
Weight system: $\beta_1, \beta_2, \beta_3, -\beta_1 + \beta_2 + \beta_3, \beta_1 + \beta_2, \beta_2 + \beta_3$.
Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & -\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 & -\xi_2^1 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & -\xi_1^1 & 0 & 0 \\ 0 & 0 & \xi_3^5 & \xi_4^5 & 0 & 0 \\ 0 & 0 & \xi_3^6 & \xi_4^6 & \xi_2^4 - \xi_1^3 & 0 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_1^1 & \xi_2^1 \\ \xi_1^2 & -\xi_1^1 \end{pmatrix} \text{ nilpotent.}$$

Aut $\mathfrak{g}_{6,1}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_1^1 u & -b_2^1 u & 0 & 0 \\ b_1^4 & b_2^4 & -b_1^2 u & b_2^2 u & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_4^5 & b_2^2 b_1^1 - b_1^2 b_2^1 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & b_2^3 b_1^2 - b_1^3 b_2^2 - b_1^4 b_2^1 + b_2^4 b_1^1 & (b_2^2 b_1^1 - b_1^2 b_2^1) u \end{pmatrix}$$

with $u \in \mathbb{C}, u \neq 0, b_2^2 b_1^1 - b_1^2 b_2^1 \neq 0$. We then get 2 cases: case 1. $A \neq 0$, case2. $A = 0$. In the case 1 $A \neq 0$ we may suppose $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. We denote δ in the case 1 (resp. 2) by $(\xi_1^3, \xi_2^3; \xi_1^4, \xi_2^4; \xi_3^5, \xi_4^5; \xi_3^6, \xi_4^6)$ (resp. $[\xi_1^3, \xi_2^3; \xi_1^4, \xi_2^4; \xi_3^5, \xi_4^5; \xi_3^6, \xi_4^6]$). We get finally for $\tilde{\mathfrak{g}}_\delta$ 27 nonisomorphic algebras:

$\mathfrak{g}_{6,1}$. **Table 1. Case 1:** $A \neq 0$.

Then one may suppose $\xi_1^1 = \xi_1^2 = 0, \xi_2^1 = 1$ **and moreover** $\xi_2^3 = \xi_2^4 = \xi_4^5 = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^4 \neq 0$ (case1N1)	$\xi_1^4 = 1$, $\xi_4^6 = 0$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_3^6 = 0$, $\xi_1^3 = 0, 1$	$(\varepsilon, 0; 1, 0; 1, 0; 0, 0)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^3)$	$\underline{\mathfrak{g}_{7,1.5}}$ $(\varepsilon = 0)$ (case1I) $\underline{\mathfrak{g}_{7,0.1}}$ $(\varepsilon = 1)$ (case1I2)
		$\xi_3^5 = 0$ (case1N2)	$\xi_1^3 = 0, 1$ $\xi_3^6 = 0$	$(\varepsilon, 0; 1, 0; 0, 0; 0, 0)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^3)$	$\underline{\mathfrak{g}_{7,2.6}}$ $(\varepsilon = 0)$ (case1II) $\underline{\mathfrak{g}_{7,1.03}}$ $(\varepsilon = 1)$ (case1II3)
$\xi_1^4 = 0$ (case1N3)	$\xi_4^6 = 0$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_1^3 = 0, 1$ $\xi_3^6 = 0$	$(\varepsilon, 0; 0, 0; 1, 0; 0, 0)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^3)$	$\underline{\mathfrak{g}_{7,2.35}}$ $(\varepsilon = 0)$ (case1III) $\underline{\mathfrak{g}_{7,1.11}}$ $(\varepsilon = 1)$ (case1III3)
		$\xi_3^5 = 0$ (case1N4)	$\xi_3^6 = 0, 1$ $\xi_1^3 = 0, 1$	$(\varepsilon, 0; 0, 0; 0, 0; \eta, 0)$ $\varepsilon, \eta = 0, 1$ $(\varepsilon = \xi_1^3,$ $\eta = \xi_3^6)$	$\underline{\mathfrak{g}_{7,3.5}}$ $(\varepsilon = \eta = 0)$ (case1IV) $\underline{\mathfrak{g}_{7,2.26}}$ $(\varepsilon = 0, \eta = 1)$ (case1IV2) $\underline{\mathfrak{g}_{7,2.37}}$ $(\varepsilon = 1, \eta = 0)$ (case1IV3) $\underline{\mathfrak{g}_{7,1.2(ii)}}$ $(\varepsilon = 1, \eta = 1)$ (case1IV4)

$\mathfrak{g}_{6,1}$. **Table 2. Case 2:** $A = 0$. **Subcase 2.1:** $\begin{pmatrix} \xi_3^5 \\ \xi_4^5 \end{pmatrix} \neq 0$ **(case2N1).**

Then one may suppose $\xi_3^5 = 1,\ \xi_4^5 = 0,\ \xi_3^6 = 0,\ \xi_4^6 = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^3 \neq 0$	$\xi_2^3 = 1$ $\xi_1^3 = 0$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$[0, 1; a, 1; 1, 0; 0, 0]$ $a \in \mathbb{C}$ $(a = \xi_1^4)$	$\underline{\mathfrak{g}_{7,1.3(i_\lambda)}}$ $(\lambda = a)$ $(a \neq 0)$ (case2I) $\underline{\mathfrak{g}_{7,1.3(v)}}$ $(a = 0)$ (case2I2)
		$\xi_2^4 = 0$	$\xi_1^4 = 0, 1$	$[0, 1; \varepsilon, 0; 1, 0; 0, 0]$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^4)$	$\underline{\mathfrak{g}_{7,2.29}}$ $(\varepsilon = 0)$ (case2II) $\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon = 1)$ (case2II2)
$\xi_2^3 = 0$, $\xi_1^3 \neq 0$ (case2N2)	$\xi_1^3 = 1$	$\xi_2^4 \neq -1$	$\xi_1^4 = 0$	$[1, 0; 0, a; 1, 0; 0, 0]$ $a \in \mathbb{C}, a \neq -1$ $(a = \xi_2^4)$	$\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = \frac{a}{a-1})$ $(a \neq 0, \pm 1)$ (case2III) $\underline{\mathfrak{g}_{7,2.1(iii)}}$ $(a = 0)$ (case2III2) $\underline{\mathfrak{g}_{7,2.1(v)}}$ $(a = 1)$ (case2III3)
		$\xi_2^4 = -1$	$\xi_1^4 = 0, 1$	$[1, 0; \varepsilon, -1; 1, 0; 0, 0]$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^4)$	$\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = \frac{1}{2})$ $(\varepsilon = 0)$ (case2IV) $\underline{\mathfrak{g}_{7,1.3(ii)}}$ $(\varepsilon = 1)$ (case2IV2)
$\xi_2^3 = 0$, $\xi_1^3 = 0$ (case2N3)	$\longrightarrow\longrightarrow$	$\xi_2^4 \neq 0$	$\xi_1^4 = 0$, $\xi_2^4 = 1$	$[0, 0; 0, 1; 1, 0; 0, 0]$	$\underline{\mathfrak{g}_{7,2.1(iv)}}$ (case2V)
		$\xi_2^4 = 0$	$\xi_1^4 = 0, 1$	$[0, 0; \varepsilon, 0; 1, 0; 0, 0]$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^4)$	$\underline{\mathfrak{g}_{7,3.19}}$ $(\varepsilon = 0)$ (case2VI) $\underline{\mathfrak{g}_{7,2.36}}$ $(\varepsilon = 1)$ (case2VI2)

$\mathfrak{g}_{6,1}$. **Table 2. (continued) Case 2:** $A = 0$.

Subcase 2.2: $\begin{pmatrix} \xi_3^5 \\ \xi_4^5 \end{pmatrix} = 0$, $\begin{pmatrix} \xi_3^6 \\ \xi_4^6 \end{pmatrix} \neq 0$ **(case2N4). Then one may suppose** $\xi_3^6 = 1$, $\xi_4^6 = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^3 \neq 0$	$\xi_2^3 = 1$, $\xi_1^3 = 0$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$[0,1;a,1;0,0;1,0]$ $a \in \mathbb{C}$ ($a = \xi_1^4$)	$\mathfrak{g}_{7,3.1(i_\lambda)}$ [*] ($a \neq -2, 0, \frac{1}{4}$) (case2VII) $\mathfrak{g}_{7,3.1(iii)}$ ($a = 0$) (case2VII2) $\mathfrak{g}_{7,2.2}$ ($a = \frac{1}{4}, -2$) (case2VII3,4)
		$\xi_2^4 = 0$	$\xi_1^4 = 0,1$	$[0,1;\varepsilon,0;0,0;1,0]$ $\varepsilon = 0,1$ ($\varepsilon = \xi_1^4$)	$\mathfrak{g}_{7,3.15}$ ($\varepsilon = 0$) (case2VIII) $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = 0$) ($\varepsilon = 1$) (case2VIII2)
$\xi_2^3 = 0$ (case2N5)	$\longrightarrow\longrightarrow$	$\xi_2^4 \neq -\xi_1^3$	$\xi_1^4 = 0$, $\xi_1^3 = 0,1$	$[\varepsilon,0;0,a;0,0;1,0]$ $a \in \mathbb{C}$, $\varepsilon = 0,1$ ($a \neq -\varepsilon$) ($\varepsilon = \xi_1^3$, $a = \xi_2^4$)	$\mathfrak{g}_{7,3.1(iii)}$ ($\varepsilon = 0$, $a \neq 0$) or ($\varepsilon = 1$, $a = 0$) (case2IX,IX5)[†] $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = a$) ($\varepsilon = 1$, $a \neq 0, -1, \frac{1}{2}, 2$) (case2IX3)[†] $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = -1$) ($\varepsilon = 1$, $a = 1/2$) (case2IX8) $\mathfrak{g}_{7,2.2}$ ($\varepsilon = 1, a = 2$) (case2IX4)
		$\xi_2^4 = -\xi_1^3$	$\xi_1^4 = 0,1$ $\xi_1^3 = 0,1$	$[\varepsilon,0;\eta,-\varepsilon;0,0;1,0]$ $\varepsilon, \eta = 0,1$ ($\varepsilon = \xi_1^3$, $\eta = \xi_1^4$)	$\mathfrak{g}_{7,4.3}$ ($\varepsilon = \eta = 0$) (case2X) $\mathfrak{g}_{7,3.14}$ ($\varepsilon = 0$, $\eta = 1$) (case2X2) $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = -1$) ($\varepsilon = 1$, $\eta = 0$) (case2X3) $\mathfrak{g}_{7,2.2}$ ($\varepsilon = \eta = 1$) (case2X4)[‡]

$\mathfrak{g}_{6,1}$. **Table 2. (continued) Case 2:** $A = 0$.

Subcase 2.3: $\begin{pmatrix} \xi_3^5 \\ \xi_4^5 \end{pmatrix} = 0$, $\begin{pmatrix} \xi_3^6 \\ \xi_4^6 \end{pmatrix} = 0$ **(case2N6). Then one may suppose** $\begin{pmatrix} \xi_1^3 \\ \xi_2^3 \end{pmatrix} \neq 0$. **Then** [§]

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^3 \neq 0$	$\xi_2^3 = 1$ $\xi_1^3 = 0$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$[0,1;a,1;0,0;0,0]$ $a \in \mathbb{C}$ ($a = \xi_1^4$)	$\mathfrak{g}_{7,3.1(i_\lambda)}$[¶] ($a \neq 0, \frac{1}{4}$) (case2XI, case2XI4) $\mathfrak{g}_{7,3.1(iii)}$ ($a = 0$) (case2XI2) $\mathfrak{g}_{7,2.2}$ ($a = \frac{1}{4}$) (case2XI3)
		$\xi_2^4 = 0$	$\xi_1^4 = 0,1$	$[0,1;\varepsilon,0;0,0;0,0]$ $\varepsilon = 0,1$ ($\varepsilon = \xi_1^4$)	$\mathfrak{g}_{7,3.14}$ ($\varepsilon = 0$) (case2XII) $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = 0$) ($\varepsilon = 1$) (case2XII2)
$\xi_2^3 = 0$ (case2N7)	$\xi_1^3 = 1$ $\xi_1^4 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$[1,0;\varepsilon,a;0,0;0,0]$ $a \in \mathbb{C}$, $\varepsilon = 0,1$ ($a = \xi_2^4$, $\varepsilon = \xi_1^4$)	$\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = \frac{a}{a-1}$) ($\varepsilon = 0$, $a \neq 0, 1$) (case2XIII,XIII4,5,6) or ($\varepsilon = 1$, $a \neq 0, \pm 1$) (case2XIII7,10) $\mathfrak{g}_{7,3.1(i_\lambda)}$ ($\lambda = 0$) ($a = 1$) (case2XIII3,9) $\mathfrak{g}_{7,3.1(iii)}$ ($a = 0$) (case2XIII2,8) $\mathfrak{g}_{7,2.2}$ ($\varepsilon = 1$, $a = -1$) (case2XIII11)

4 $\mathfrak{g}_{6,2}$

Commutation relations for $\mathfrak{g}_{6,2}$: $[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_3, x_4] = x_6$.

[*] $\lambda = \frac{(3a-2)\sqrt{1-4a} - 2a^2 + 7a - 2}{(a-2)\sqrt{1-4a} + 5a - 2}$. The denominator is $\neq 0$; changing the sign of the square root changes λ to $1 - \lambda$. The range of λ is $\mathbb{C} \setminus (\{0,1\} \cup \{-1, \frac{1}{2}, 2\})$.

[†] proj. equiv. to some cases $\xi_2^3 \neq 0$ of subcase 2.2 (test1case2N5); e.g. for $a = 1$, case2IX3 $\sim$ case2VIII2 (test1case2N52).

[‡] proj. equiv. to case2VII3 ($a = \frac{1}{4}$).

[§] All cases in this continued table are proj. equiv. to cases from the preceding continued table (testkcase2N6, k=1,2,3).

[¶] $\lambda = \frac{1+\sqrt{1-4a}}{2}$. Changing the sign of the sq. root changes λ to $1 - \lambda$. Range of λ: $\mathbb{C} \setminus \{0, 1, \frac{1}{2}\}$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 \\ \xi_1^4 & 0 & \xi_3^4 & -\xi_3^3 & 0 & 0 \\ 0 & 0 & \xi_1^4 & -\xi_1^3 & 0 & 0 \\ 0 & \xi_2^6 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_3^3 & \xi_4^3 \\ \xi_3^4 & -\xi_3^3 \end{pmatrix} \text{ nilpotent.}$$

Aut $\mathfrak{g}_{6,2}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & \frac{\Delta}{{b_1^1}^2} & 0 & 0 & 0 & 0 \\ b_1^3 & 0 & b_3^3 & b_4^3 & 0 & 0 \\ b_1^4 & 0 & b_3^4 & b_4^4 & 0 & 0 \\ b_1^5 & b_2^5 & \frac{b_1^4 b_3^3 - b_3^4 b_1^3}{b_1^1} & \frac{b_1^4 b_4^3 - b_4^4 b_1^3}{b_1^1} & \frac{\Delta}{b_1^1} & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & b_2^5 b_1^1 & \Delta \end{pmatrix}$$

with $\Delta = b_3^3 b_4^4 - b_4^3 b_3^4 \neq 0$ and $b_1^1 \neq 0$. We then get 2 cases: in the case 1 where $A \neq 0$ we may suppose $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$; in the case 2, $A = 0$. We denote δ in the case 1 (resp. 2) by $(\xi_1^2;\xi_1^3;\xi_1^4;\xi_2^6)$ (resp. $[\xi_1^2;\xi_1^3;\xi_1^4;\xi_2^6]$). We get finally for $\tilde{\mathfrak{g}}_\delta$ 15 nonisomorphic algebras :

$\mathfrak{g}_{6,2}$. **Table 1. Case 1 :** $A \neq 0$. **Then one may suppose** $\xi_3^3 = \xi_3^4 = 0, \xi_4^3 = 1$ **and moreover** $\xi_1^3 = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^4 \neq 0$ (case1N1)	$\xi_1^4 = 1$ $\xi_1^2 = 0,1$ $\xi_2^6 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(\varepsilon;0;1;\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^2,\ \eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.13}}$ $(\varepsilon = 0,\ \eta = 0)$ (case1I) $\underline{\mathfrak{g}_{7,1.20}}$ $(\varepsilon = 0,\ \eta = 1)$ (case1I2) $\underline{\mathfrak{g}_{7,1.21}}$ $(\varepsilon = 1,\ \eta = 0)$ (case1I3) $\underline{\mathfrak{g}_{7,0.8}}$ $(\varepsilon = 1,\ \eta = 1)$ (case1I4)
$\xi_1^4 = 0$ (case1N2)	$\xi_1^2 = 0,1$ $\xi_2^6 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(\varepsilon;0;0;\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^2,\ \eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,3.16}}$ $(\varepsilon = 0,\ \eta = 0)$ (case1II) $\underline{\mathfrak{g}_{7,2.23}}$ $(\varepsilon = 0,\ \eta = 1)$ (case1II2) $\underline{\mathfrak{g}_{7,2.33}}$ $(\varepsilon = 1,\ \eta = 0)$ (case1II3) $\underline{\mathfrak{g}_{7,1.8}}$ $(\varepsilon = 1,\ \eta = 1)$ (case1II4)

$\mathfrak{g}_{6,2}$. **Table 2. Case 2 :** $A = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\begin{pmatrix} \xi_1^3 \\ \xi_1^4 \end{pmatrix} \neq 0$ (case2N1)	$\xi_1^3 = 1,$ $\xi_1^4 = 0,$ $\xi_1^2 = 0,1$ $\xi_2^6 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$[\varepsilon;1;0;\eta]$ $(\varepsilon,\eta = 0,1)$ $(\varepsilon = \xi_1^2,\ \eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,3.10}}$ $(\varepsilon = 0,\ \eta = 0)$ (case2I) $\underline{\mathfrak{g}_{7,2.1(iv)}}$ $(\varepsilon = 0,\ \eta = 1)$ (case2I2) $\underline{\mathfrak{g}_{7,2.25}}$ $(\varepsilon = 1,\ \eta = 0)$ (case2I3) $\underline{\mathfrak{g}_{7,1.3(v)}}$ $(\varepsilon = 1,\ \eta = 1)$ (case2I4)
$\begin{pmatrix} \xi_1^3 \\ \xi_1^4 \end{pmatrix} = 0$	$\longrightarrow\!\!\longrightarrow$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1,$ $\xi_1^2 = 0,1$	$[\varepsilon;0;0;1]$ $(\varepsilon = 0,1)$ $(\varepsilon = \xi_1^2)$	$\underline{\mathfrak{g}_{7,3.18}}$ $(\varepsilon = 0)$ (case2II) $\underline{\mathfrak{g}_{7,2.30}}$ $(\varepsilon = 1)$ (case2II2)
		$\xi_2^6 = 0$	$\xi_1^2 = 1$	$[1;0;0;0]$	$\underline{\mathfrak{g}_{7,3.17}}$ (case2III)

5 $\mathfrak{g}_{6,3}$

Commutation relations for $\mathfrak{g}_{6,3}$: $[x_1,x_2] = x_4$; $[x_1,x_3] = x_5$; $[x_2,x_3] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & \xi_1^1 + \xi_2^2 & \xi_3^2 & -\xi_3^1 \\ 0 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_1^1 + \xi_3^3 & \xi_2^1 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & \xi_1^2 & \xi_2^2 + \xi_3^3 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 \end{pmatrix} \text{ nilpotent.}$$

Aut $\mathfrak{g}_{6,3}$ is comprised of the matrices $\Phi = \begin{pmatrix} U & 0 \\ T & V \end{pmatrix}$ where $U = (b^i_j)_{1\leqslant i,j\leqslant 3}$, $V = \begin{pmatrix} b^1_1b^2_2 - b^2_1b^1_2 & b^1_1b^2_3 - b^2_1b^1_3 & b^1_2b^2_3 - b^2_2b^1_3 \\ b^1_1b^3_2 - b^3_1b^1_2 & b^1_1b^3_3 - b^3_1b^1_3 & b^1_2b^3_3 - b^3_2b^1_3 \\ b^2_1b^3_2 - b^3_1b^2_2 & b^2_1b^3_3 - b^3_1b^2_3 & b^2_2b^3_3 - b^3_2b^2_3 \end{pmatrix}$, $T = (b^i_j)_{4\leqslant i\leqslant 6, 1\leqslant j\leqslant 3}$, and the $b^i_j \in \mathbb{C}$ are arbitrary with the condition $\det U \neq 0$. We then get 3 cases according to the nilpotency order k of A. In the case 1, $k = 3$ we may suppose $A = \left(\begin{smallmatrix} 0&1&0\\0&0&1\\0&0&0 \end{smallmatrix}\right)$; in the case 2, $k = 2$ we may suppose $A = \left(\begin{smallmatrix} 0&1&0\\0&0&0\\0&0&0 \end{smallmatrix}\right)$; in the case 3, $k = 1$ i.e. $A = 0$. We denote δ in the case 1 (resp. 2,3) by $(\xi^4_3;\xi^5_2,\xi^5_3;\xi^6_1,\xi^6_2,\xi^6_3)$ (resp. $[\xi^4_3;\xi^5_2,\xi^5_3;\xi^6_1,\xi^6_2,\xi^6_3]$, $\langle\xi^4_3;\xi^5_2,\xi^5_3;\xi^6_1,\xi^6_2,\xi^6_3\rangle$). We get finally for $\tilde{\mathfrak{g}}_\delta$ 11 nonisomorphic algebras :

$\mathfrak{g}_{6,3}$. **Table 1. Case 1 : Then one may suppose** $\xi^4_3 = \xi^5_2 = \xi^5_3 = \xi^6_2 = 0$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi^6_1 \neq 0$ (case1N1)	$\xi^6_1 = 1$, $\xi^6_3 = 0$	$\longrightarrow\dashrightarrow$	$\longrightarrow\dashrightarrow$	$(0;0,0;1,0,0)$	$\underline{\mathfrak{g}_{7,1.1(i_\lambda)}}$ $(\lambda = 0)$ (case1I)
$\xi^6_1 = 0$	$\xi^6_3 = 0,1$	$\longrightarrow\dashrightarrow$	$\longrightarrow\dashrightarrow$	$(0;0,0;0,0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_3)$	$\underline{\mathfrak{g}_{7,2.5}}$ $(\varepsilon = 0)$ (case1II) $\underline{\mathfrak{g}_{7,1.02}}$ $(\varepsilon = 1)$ (case1II2)

$\mathfrak{g}_{6,3}$. **Table 2. Case 2 : Then one may suppose** $\xi^5_2 = \xi^5_3 = \xi^6_2 = 0$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi^6_1 \neq 0$ (case2N1)	$\xi^6_1 = 1$, $\xi^6_3 = 0$, $\xi^4_3 = 0,1$	$\longrightarrow\dashrightarrow$	$\longrightarrow\dashrightarrow$	$[\varepsilon;0,0;1,0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi^4_3)$	$\underline{\mathfrak{g}_{7,2.17}}$ $(\varepsilon = 0)$ (case2I) $\underline{\mathfrak{g}_{7,1.2(iii)}}$ $(\varepsilon = 1)$ (case2I2)
$\xi^6_1 = 0$	$\longrightarrow\dashrightarrow$	$\xi^6_3 \neq 0$	$\xi^6_3 = 1$, $\xi^4_3 = 0$	$[0;0,0;0,0,1]$	$\underline{\mathfrak{g}_{7,2.26}}$ (case2II)
		$\xi^6_3 = 0$	$\xi^4_3 = 0,1$	$[\varepsilon;0,0;0,0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi^4_3)$	$\underline{\mathfrak{g}_{7,3.22}}$ $(\varepsilon = 0)$ (case2III) $\underline{\mathfrak{g}_{7,2.1(v)}}$ $(\varepsilon = 1)$ (case2III2)

$\mathfrak{g}_{6,3}$. **Table 3. Case 3 : $A=0$. Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_3^4 \neq 0$ (case3N1)	$\xi_3^4 = 1$, $\xi_3^5 = 0$, $\xi_3^6 = 0$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0$, $\xi_1^6 = 0,1$	$\langle 1;1,0;\varepsilon,0,0\rangle$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,4.1}}$ $(\varepsilon = 0)$ (case3I) * $\underline{\mathfrak{g}_{7,3.12}}$ $(\varepsilon = 1)$ (case3I2) †
		$\xi_2^5 = 0$, $\xi_2^6 \neq 0$ (case3N2)	$\xi_2^6 = 1$, $\xi_1^6 = 0$	$\langle 1;0,0;0,1,0\rangle$	$\mathfrak{g}_{7,3.12}$ (case3II) †
		$\xi_2^5 = 0$, $\xi_2^6 = 0$	$\xi_1^6 = 0,1$	$\langle 1;0,0;\varepsilon,0,0\rangle$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,3.24}}$ $(\varepsilon = 0)$ (case3III) ‡ $\mathfrak{g}_{7,4.1}$ $(\varepsilon = 1)$ (case3III2) *
$\xi_3^4 = 0$, $\xi_3^5 \neq 0$ (case3N3)	$\xi_3^5 = 1$, $\xi_2^5 = 0$, $\xi_2^6 = 0$, $\xi_3^6 = 0$, $\xi_1^6 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$\langle 0;0,1;\varepsilon,0,0\rangle$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\mathfrak{g}_{7,4.1}$ $(\varepsilon = 0)$ (case3IV) * $\mathfrak{g}_{7,3.12}$ $(\varepsilon = 1)$ (case3IV2) †
$\xi_3^4 = 0$, $\xi_3^5 = 0$	$\longrightarrow\longrightarrow$	$\xi_2^5 \neq 0$, $\xi_3^6 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0$, $\xi_3^6 = 1$, $\xi_1^6 = 0$	$\langle 0;1,0;0,0,1\rangle$	$\mathfrak{g}_{7,3.12}$ (case3V) †
		$\xi_2^5 \neq 0$, $\xi_3^6 = 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0$, $\xi_1^6 = 0,1$	$\langle 0;1,0;\varepsilon,0,0\rangle$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\mathfrak{g}_{7,3.24}$ $(\varepsilon = 0)$ (case3VI) ‡ $\mathfrak{g}_{7,4.1}$ $(\varepsilon = 1)$ (case3VI2) *
		$\xi_2^5 = 0$, $\xi_3^6 \neq 0$ (case3N4)	$\xi_3^6 = 1$, $\xi_1^6 = 0$, $\xi_2^6 = 0$	$\langle 0;0,0;0,0,1\rangle$	$\mathfrak{g}_{7,4.1}$ (case3VII) *
		$\xi_2^5 = 0$, $\xi_3^6 = 0$, $\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_1^6 = 0$	$\langle 0;0,0;0,1,0\rangle$	$\mathfrak{g}_{7,4.1}$ (case3VIII) *
		$\xi_2^5 = 0$, $\xi_3^6 = 0$, $\xi_2^6 = 0$	$\xi_1^6 = 1$	$\langle 0;0,0;1,0,0\rangle$	$\mathfrak{g}_{7,3.24}$ (case3IX) ‡

6 $\mathfrak{g}_{6,4}$

Commutation relations for $\mathfrak{g}_{6,4}$: $[x_1,x_2] = x_4$; $[x_1,x_3] = x_6$; $[x_2,x_4] = x_5$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & 0 & 0 \\ 0 & \xi_2^6 & \xi_3^6 & \xi_2^3 & 0 & 0 \end{pmatrix}.$$

Aut $\mathfrak{g}_{6,4}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_3^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & 0 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & -b_1^4 b_2^2 & {b_2^2}^2 b_1^1 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_2^3 b_1^1 & 0 & b_3^3 b_1^1 \end{pmatrix}$$

where $b_3^3 b_2^2 b_1^1 \neq 0$. We denote δ by $(\xi_1^3,\xi_2^3;\xi_1^5,\xi_3^5;\xi_2^6,\xi_3^6)$. We get for $\tilde{\mathfrak{g}}_\delta$ 17 nonisomorphic algebras :

*all these appear to be projectively equivalent

†all these appear to be projectively equivalent

‡all these appear to be projectively equivalent

$\mathfrak{g}_{6,4}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^3 \neq 0$ (N1)	$\xi_1^3 = 1$, $\xi_1^5 = 0$	$\xi_2^3 \neq 0$	$\xi_2^3 = 1$, $\xi_2^6 = 0$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$	$(1,1;0,1;0,a)$ $a \in \mathbb{C}$ $(a = \xi_3^6)$ *	$\underline{\mathfrak{g}_{7,1.19}}$ $(a \neq \pm 2)$ (I) $\underline{\mathfrak{g}_{7,2.35}}$ $(a = \pm 2)$ (I2,I3)
				$\xi_3^5 = 0$	$\xi_3^6 = 0,1$	$(1,1;0,0;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,2.34}}$ $(\varepsilon = 1)$ (II2) $\underline{\mathfrak{g}_{7,2.26}}$ $(\varepsilon = 0)$ (II)
		$\xi_2^3 = 0$		$\xi_3^6 \neq 0$	$\xi_2^6 = 0$, $\xi_3^6 = 1$, $\xi_3^5 = 0,1$	$(1,0;0,\varepsilon;0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\mathfrak{g}_{7,2.34}$ $(\varepsilon = 1)$ (III) $\underline{\mathfrak{g}_{7,3.4}}$ $(\varepsilon = 0)$ (III2)
				$\xi_3^6 = 0$	$\xi_3^5 = 0,1$, $\xi_2^6 = 0,1$	$(1,0;0,\varepsilon;\eta,0)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_3^5,$ $\eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon,\eta = 1)$ (IV) $\underline{\mathfrak{g}_{7,3.21}}$ $(\varepsilon,\eta = 0)$ (IV4) $\underline{\mathfrak{g}_{7,2.12}}$ $(\varepsilon = 1, \eta = 0)$ (IV2) $\underline{\mathfrak{g}_{7,2.42}}$ $(\varepsilon = 0, \eta = 1)$ (IV3)
$\xi_1^3 = 0$, $\xi_2^3 \neq 0$ (N2)	$\xi_2^3 = 1$, $\xi_2^6 = 0$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_1^5 = 0$, $\xi_3^6 = 0,1$			$(0,1;0,1;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\mathfrak{g}_{7,2.35}$ (V)
		$\xi_3^5 = 0$	$\xi_1^5 = 0,1$, $\xi_3^6 = 0,1$			$(0,1;\varepsilon,0;0,\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^5,$ $\eta = \xi_3^6)$	$\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon,\eta = 1)$ (VI) $\underline{\mathfrak{g}_{7,3.22}}$ $(\varepsilon,\eta = 0)$ (VI4) $\underline{\mathfrak{g}_{7,2.1(v)}}$ $(\varepsilon = 1, \eta = 0)$ (VI2) $\mathfrak{g}_{7,2.12}$ $(\varepsilon = 0, \eta = 1)$ (VI3)

$\mathfrak{g}_{6,4}$. **continued**

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^3 = 0$, $\xi_2^3 = 0$ (N3)		$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_1^5 = 0$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_2^6 = 0$	$(0,0;0,1;0,1)$	$\underline{\mathfrak{g}_{7,3.7}}$ (VII)
				$\xi_3^6 = 0$	$\xi_2^6 = 0,1$	$(0,0;0,1;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.36}}$ $(\varepsilon = 1)$ (VIII) $\underline{\mathfrak{g}_{7,3.15}}$ $(\varepsilon = 0)$ (VIII2)
		$\xi_3^5 = 0$ (N4)	$\xi_1^5 = 0,1$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_2^6 = 0$	$(0,0;\varepsilon,0;0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^5)$	$\mathfrak{g}_{7,3.7}$ $(\varepsilon = 1)$ (IX) direct product $(\mathfrak{g}_4 \times \mathfrak{n})$ $(\varepsilon = 0)$ (IX2)
				$\xi_3^6 = 0$	$\xi_2^6 = 0,1$	$(0,0;\varepsilon,0;\eta,0)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^5,$ $\eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.45}}$ $(\varepsilon,\eta = 1)$ (X) $\underline{\mathfrak{g}_{7,3.9}}$ $(\varepsilon = 1, \eta = 0)$ (X3) $\underline{\mathfrak{g}_{7,3.11}}$ $(\varepsilon = 0, \eta = 1)$ (X4)

*2 cases with parameters a, a' are proj. equiv. if and only if $a' = \pm a$.

7 $\mathfrak{g}_{6,5}$

Commut. relations for $\mathfrak{g}_{6,5}$: $[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_6$.
Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^5 & \xi_3^5 & 0 & 0 & 0\\ \xi_1^6 & 0 & \xi_3^6 & -\xi_1^3 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^3,\xi_2^3;\xi_2^5,\xi_3^5;\xi_1^6,\xi_3^6)$. The automorphisms of $\mathfrak{g}_{6,5}$ fall into 2 kinds. The first kind is comprised of the matrices

$$\Phi=\begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & b_2^2 & 0 & 0 & 0 & 0\\ b_1^3 & b_2^3 & b_2^2 b_1^1 & 0 & 0 & 0\\ b_1^4 & b_2^4 & 0 & b_2^2 b_1^1 & 0 & 0\\ b_1^5 & b_2^5 & b_3^5 & b_2^4 b_1^1 & b_2^2 {b_1^1}^2 & 0\\ b_1^6 & b_2^6 & b_3^6 & -(b_1^4+b_1^3)b_2^2 & 0 & {b_2^2}^2 b_1^1 \end{pmatrix} \quad (1)$$

where $b_1^1 b_2^2 \neq 0$. The second kind is comprised of the matrices

$$\Psi=\begin{pmatrix} 0 & b_2^1 & 0 & 0 & 0 & 0\\ b_1^2 & 0 & 0 & 0 & 0 & 0\\ b_1^3 & b_2^3 & b_1^2 b_2^1 & 0 & 0 & 0\\ b_1^4 & b_2^4 & -b_1^2 b_2^1 & -b_1^2 b_2^1 & 0 & 0\\ b_1^5 & b_2^5 & b_3^5 & -b_1^4 b_2^1 & 0 & -b_1^2 {b_2^1}^2\\ b_1^6 & b_2^6 & b_3^6 & b_1^2(b_2^4+b_2^3) & -{b_1^2}^2 b_2^1 & 0 \end{pmatrix} \quad (2)$$

where $b_2^1 b_1^2 \neq 0$. We then get 3 cases (which are not proj. equiv.) according to whether ξ_1^3,ξ_2^3 are both nonzero or exactly one of them is 0 or are both 0. Then taking suitable values for the b_j^i's, $(1\leqslant i,j\leqslant 2)$ in (1) or (2), one may suppose in the case 1 that $\xi_1^3=\xi_2^3=1$ and in the case 2 that $\xi_1^3=1,\xi_2^3=0$. We get finally for $\tilde{\mathfrak{g}}_\delta$ 15 nonisomorphic algebras:

$\mathfrak{g}_{6,5}$. **Table continues next.**

if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^3\xi_2^3\neq 0$ (case1)	$\xi_1^3=1$, $\xi_2^3=1$, $\xi_2^5=0$, $\xi_1^6=0$	$(1,1;0,a;0,b)$ * $a,b\in\mathbb{C}$ $(a=\xi_3^5,\ b=\xi_3^6)$	$\underline{\mathfrak{g}_{7,1.19}}$ $(b\neq 0,-1;\ a\neq 0;$ $\underline{(a+b)^2+4a\neq 0}$) (case1I) or $(b=0, a=-1)$ (case1I2) $\underline{\mathfrak{g}_{7,2.26}}$ $(a=b=0)$ (case1I3) $\underline{\mathfrak{g}_{7,2.35}}$ $((a+b)^2+4a=0,\ ab\neq 0)$ (case1I7) $\underline{\mathfrak{g}_{7,2.34}}$ $(a\neq 0,-1,\ b=-1)$ (case1I5) $\underline{\mathfrak{g}_{7,3.4}}$ $(a=0,\ b=-1)$ (case1I6)

*$(1,1;0,a;0,b)$ and $(1,1;0,a';0,b')$ are projectively equivalent under some first kind automorphism if and only if $a=a',b=b'$; they are projectively equivalent under some second kind automorphism if and only if $a'=-(1+b),b'=-(1+a)$ (`testcase1`). $(a+b)^2+4a$ is invariant under that substitution. In the last column discussion, we take advantage of 2d kind proj. equiv. to reduce the cases. The case $b=0,a\neq 0,-4$ is 2d kind proj. equiv. to $a'=-1,b'\neq -1,3$ which is included in case1I except for $a=-1$, hence we consider as case1I2 only $a=-1,b=0$ which is 2d kind proj. equiv. to itself. The case $a=0,b=0$ gives in the same way $a'=-1,b'=-1$ which we hence do not list; the case $a=-4,b=0$ gives $a'=-1,b'=3$ which is included in case1I7; the case $a\neq 0,-1,b=-1$ gives $a'=0,b'\neq 0,-1$ which is complementary to case1I; the case $a=0,b=-1$ is 2d kind proj. equiv. to itself.

$\mathfrak{g}_{6,5}$. Continued.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
exactly one of $\xi_1^3, \xi_2^3 \neq 0$ (case2)	$\xi_1^3 = 1$, $\xi_2^3 = 0$, $\xi_1^6 = 0$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_2^5 = 0$	$(1,0;0,1;0,a)$ $a \in \mathbb{C}$ $(a = \xi_3^6)$	$\mathfrak{g}_{7,1.19}$ $(a \neq -1)$ (case2I) $\mathfrak{g}_{7,2.34}$ $(a = -1)$ (case2I2)
		$\xi_3^5 = 0$	$\xi_2^5 = 0, 1$	$(1,0;\varepsilon,0;0,a)$ $a \in \mathbb{C}$, $\varepsilon = 0, 1$ $(a = \xi_3^6$, $\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.12}}$ $(\varepsilon = 0,\ a \neq 0, -1)$ (case2II) $\underline{\mathfrak{g}_{7,3.22}}$ $(\varepsilon = 0,\ a = 0)$ (case2II2) $\underline{\mathfrak{g}_{7,3.21}}$ $(\varepsilon = 0,\ a = -1)$ (case2II3) $\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon = 1,\ a \neq 0, -1)$ (case2II4) $\underline{\mathfrak{g}_{7,2.1(v)}}$ $(\varepsilon = 1,\ a = 0)$ (case2II5) $\underline{\mathfrak{g}_{7,2.42}}$ $(\varepsilon = 1,\ a = -1)$ (case2II6)
$\xi_1^3 = 0$, $\xi_2^3 = 0$ (case3)		$\xi_3^5 \xi_3^6 \neq 0$	$\xi_3^5 = 1$, $\xi_3^6 = 1$, $\xi_2^5 = 0$, $\xi_1^6 = 0$	$(0,0;0,1;0,1)$	$\underline{\mathfrak{g}_{7,2.29}}$ (case3I)
		exactly one of $\xi_3^5, \xi_3^6 \neq 0$	$\xi_3^5 = 0$, $\xi_3^6 = 1$, $\xi_2^5 = 0, 1$ $\xi_1^6 = 0$	$(0,0;\varepsilon,0;0,1)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,3.13}}$ $(\varepsilon = 0)$ (case3II) $\mathfrak{g}_{7,2.29}$ $(\varepsilon = 1)$ (case3II2)
		$\xi_3^5 = 0$, $\xi_3^6 = 0$	$\xi_2^5 = 0, 1$ $\xi_1^6 = 1$	$(0,0;\varepsilon,0;1,0)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.27}}$ $(\varepsilon = 0)$ (case3III) $\underline{\mathfrak{g}_{7,2.38}}$ $(\varepsilon = 0)$ (case3III2)

8 $\mathfrak{g}_{6,6}$

Commut. relations for $\mathfrak{g}_{6,6}$: $[x_1, x_2] = x_4$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_5$.
Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & \xi_2^1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & 0 & 0 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & 0 & \xi_3^4 \\ \xi_1^6 & 0 & \xi_3^6 & -\xi_1^3 & 0 & 0 \end{pmatrix}.$$

Aut $\mathfrak{g}_{6,6}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_3^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & -b_1^4 b_2^2 & {b_2^2}^2 b_1^1 & b_3^4 b_2^2 \\ b_1^6 & b_2^6 & b_3^6 & -b_1^3 b_2^2 & 0 & b_3^3 b_2^2 \end{pmatrix}$$

where $b_1^1 b_2^2 b_3^3 \neq 0$. We denote δ by $(\xi_2^1; \xi_1^3, \xi_2^3; \xi_3^4; \xi_1^5, \xi_3^5; \xi_1^6, \xi_3^6)$. We get for $\tilde{\mathfrak{g}}_\delta$ 35 nonisomorphic algebras:

$\mathfrak{g}_{6,6}$. **Table 1: Suppose first $\xi_2^1 \neq 0$. Then one may suppose $\xi_2^1 = 1$, $\xi_2^3 = 0$. Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_3^4 \neq 0$ (N1)	$\xi_3^4 = 1$, $\xi_3^5 = 0$, $\xi_1^5 = 0$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_3^6 = 0$, $\xi_1^6 = 0, 1$	$(1;1,0;1;0,0;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,0.2}}$ $(\varepsilon = 1)$ (II) $\underline{\mathfrak{g}_{7,1.1(i_\lambda)}}$ $(\varepsilon = 0)$ $\underline{(\lambda = 1)}$ (II2)
		$\xi_1^3 = 0$	$\xi_1^6 = 0, 1$ $\xi_3^6 = 0, 1$	$(1;0,0;1;0,0;\varepsilon,\eta)$ $\varepsilon, \eta = 0,1$ $(\varepsilon = \xi_1^6$, $\eta = \xi_3^6)$	$\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = 1)$ $\underline{(\varepsilon = \eta = 0)}$ (III) $\underline{\mathfrak{g}_{7,2.31}}$ $(\varepsilon = 0, \eta = 1)$ (III2) $\underline{\mathfrak{g}_{7,1.2(i_\lambda)}}$ $(\lambda = 1)$ $(\varepsilon = 1, \eta = 0)$ (III3) or $(\varepsilon = 1, \eta = 1)$ (III4)
$\xi_3^4 = 0$, $\xi_1^3 \neq 0$ (N2)	$\xi_1^3 = 1$, $\xi_1^5 = 0$, $\xi_1^6 = 0$,	$\longrightarrow\!\!-\!\!\!\rightarrow$	$\xi_3^6 = 0$, $\xi_3^5 = 0, 1$	$(1;1,0;0;0,\varepsilon;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,2.9}}$ $(\varepsilon = 1)$ (V) $\underline{\mathfrak{g}_{7,2.8}}$ $(\varepsilon = 0)$ (V2)
$\xi_3^4 = 0$, $\xi_1^3 = 0$	$\longrightarrow\!\!-\!\!\!\rightarrow$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_3^5 = 0$, $\xi_1^5 = 0, 1$ $\xi_1^6 = 0, 1$	$(1;0,0;0;\varepsilon,0;\eta,1)$ $\varepsilon, \eta = 0,1$ $(\varepsilon = \xi_1^5, \eta = \xi_1^6)$	$\underline{\mathfrak{g}_{7,3.3}}$ $(\varepsilon = \eta = 0)$ (VI) $\underline{\mathfrak{g}_{7,2.11}}$ $(\varepsilon = 0, \eta = 1)$ (VI2) $\underline{\mathfrak{g}_{7,2.22}}$ $(\varepsilon = 1, \eta = 0)$ (VI3) $\underline{\mathfrak{g}_{7,1.18}}$ $(\varepsilon = 1, \eta = 1)$ (VI4)
		$\xi_3^6 = 0$, $\xi_1^6 \neq 0$	$\xi_1^6 = 1$, $\xi_1^5 = 0$, $\xi_3^5 = 0, 1$	$(1;0,0;0;0,\varepsilon;1,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,1.19}}$ $(\varepsilon = 0)$ (VII) $\underline{\mathfrak{g}_{7,1.9}}$ $(\varepsilon = 1)$ (VII2)
		$\xi_3^6 = 0$, $\xi_1^6 = 0$	$\xi_1^5 = 0, 1$ $\xi_3^5 = 0, 1$	$(1;0,0;0;\varepsilon,\eta;0,0)$ $\varepsilon, \eta = 0,1$ $(\varepsilon = \xi_1^5, \eta = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.2}}$ $(\varepsilon = \eta = 0)$ (VIII) $\underline{\mathfrak{g}_{7,2.21}}$ $(\varepsilon = 0, \eta = 1)$ (VIII2) $\underline{\mathfrak{g}_{7,2.20}}$ $(\varepsilon = 1, \eta = 0)$ (VIII3) $\underline{\mathfrak{g}_{7,1.16}}$ $(\varepsilon = 1, \eta = 1)$ (VIII4)

$\mathfrak{g}_{6,6}$. **Table 2: Suppose now $\xi_2^1 = 0$. Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_3^4 \neq 0$ (N3)	$\xi_3^4 = 1$, $\xi_1^5 = 0$, $\xi_3^5 = 0$, $\xi_3^6 = 0$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_2^3 = 0$, $\xi_1^6 = 0, 1$	$(0;1,0;1;0,0;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,2.14}}$ $(\varepsilon = 0)$ (IX2) $\underline{\mathfrak{g}_{7,1.01(i)}}$ $(\varepsilon = 1)$ (IX3)
		$\xi_1^3 = 0$	$\xi_2^3 = 0, 1$ $\xi_1^6 = 0, 1$	$(0;0,\varepsilon;1;0,0;\eta,0)$ $\varepsilon, \eta = 0,1$ $(\eta = \xi_1^6, \varepsilon = \xi_2^3)$	$\underline{\mathfrak{g}_{7,3.10}}$ $(\varepsilon = \eta = 0)$ (XI) $\underline{\mathfrak{g}_{7,2.41}}$ $(\varepsilon = 1, \eta = 0)$ (XI3) $\underline{\mathfrak{g}_{7,3.16}}$ $(\varepsilon = 0, \eta = 1)$ (X) $\mathfrak{g}_{7,1.2(i_\lambda)}$ $(\lambda = 1)$ $(\varepsilon = 1, \eta = 1)$ (X3)
$\xi_3^4 = 0$, $\xi_1^3 \neq 0$ (N4)	$\xi_1^3 = 1$, $\xi_2^3 = 0$, $\xi_1^5 = 0$	$\longrightarrow\!\!-\!\!\!\rightarrow$	$\xi_1^6 = 0$, $\xi_3^6 = 0$, $\xi_3^5 = 0, 1$	$(0;1,0;0;0,\varepsilon;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.5}}$ $(\varepsilon = 0)$ (XIII) $\underline{\mathfrak{g}_{7,3.4}}$ $(\varepsilon = 1)$ (XII)
$\xi_3^4 = 0$, $\xi_1^3 = 0$	$\longrightarrow\!\!-\!\!\!\rightarrow$	$\xi_3^6 \neq 0$ (N5)	$\xi_3^6 = 1$, $\xi_3^5 = 0$, $\xi_1^6 = 0$, $\xi_2^3 = 0, 1$, $\xi_1^5 = 0, 1$	$(0;0,\varepsilon;0;\eta,0;0,1)$ $\varepsilon, \eta = 0,1$ $(\varepsilon = \xi_2^3, \eta = \xi_1^5)$	direct product $\mathfrak{g}_4 \times \mathfrak{n}$ $(\varepsilon = \eta = 0)$ (XV) $\underline{\mathfrak{g}_{7,3.21}}$ $(\varepsilon = 1, \eta = 0)$ (XIVbis) $\underline{\mathfrak{g}_{7,3.7}}$ $(\varepsilon = 0, \eta = 1)$ (XV3) $\underline{\mathfrak{g}_{7,2.42}}$ $(\varepsilon = 1, \eta = 1)$ (XIVbis2)

$\mathfrak{g}_{6,6}$. **Table 2 continued**

	$\xi_3^6=0$, $\xi_1^6\neq 0$ (N6)	$\xi_1^6=1$, $\xi_1^5=0$, $\xi_2^3=0,1$ $\xi_3^5=0,1$	$(0;0,\varepsilon;0;0,\eta;1,0)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^3,\eta=\xi_3^5)$	$\underline{\mathfrak{g}_{7,3.11}}$ $(\varepsilon=\eta=0)$ (XVI) $\underline{\mathfrak{g}_{7,2.36}}$ $(\varepsilon=0,\eta=1)$ (XVI2) $\underline{\mathfrak{g}_{7,2.39}}$ $(\varepsilon=1,\eta=0)$ (XVI3) $\underline{\mathfrak{g}_{7,1.7}}$ $(\varepsilon=1,\eta=1)$ (XVI4)
	$\xi_3^6=0$, $\xi_1^6=0$, $\xi_3^5\neq 0$	$\xi_3^5=1$, $\xi_1^5=0$, $\xi_2^3=0,1$	$(0;0,\varepsilon;0;0,1;0,0)$ $\varepsilon=0,1$ $(\varepsilon=\xi_2^3)$	$\underline{\mathfrak{g}_{7,3.14}}$ $(\varepsilon=0)$ (XVII) $\underline{\mathfrak{g}_{7,2.43}}$ $(\varepsilon=1)$ (XVII2)
	$\xi_3^6=0$, $\xi_1^6=0$, $\xi_3^5=0$, $\xi_2^3\neq 0$	$\xi_2^3=1$, $\xi_1^5=0,1$	$(0;0,1;0;\varepsilon,0;0,0)$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^5)$	$\underline{\mathfrak{g}_{7,3.20}}$ $(\varepsilon=0)$ (XVIII) $\mathfrak{g}_{7,2.39}$ $(\varepsilon=1)$ (XVIII2)
	$\xi_3^6=0$, $\xi_1^6=0$, $\xi_3^5=0$, $\xi_2^3=0$	$\xi_1^5=1$	$(0;0,0;0;1,0;0,0)$	$\underline{\mathfrak{g}_{7,3.8}}$ (XIX)

9 $\mathfrak{g}_{6,7}$

Commut. rel. for $\mathfrak{g}_{6,7}$: $[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_2,x_3]=-x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_3^4 & 0 & 0 & 0\\ 0 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 0 & 0\\ 0 & \xi_2^6 & \xi_3^6 & \xi_1^3 & -\xi_1^2+\xi_3^4 & 0 \end{pmatrix}.$$

We denote δ by $(\xi_1^2;\xi_1^3,\xi_2^3;\xi_3^4;\xi_2^5,\xi_3^5;\xi_2^6,\xi_3^6)$.

Aut $\mathfrak{g}_{6,7}$ is comprised of the matrices

$$\Phi=\begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0\\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0\\ b_1^3 & b_2^3 & {b_1^1}^2 & 0 & 0 & 0\\ b_1^4 & b_2^4 & b_3^4 & b_2^2b_1^1 & 0 & 0\\ b_1^5 & b_2^5 & b_3^5 & b_2^3b_1^1 & {b_1^1}^3 & 0\\ b_1^6 & b_2^6 & b_3^6 & b_2^4b_1^1-b_2^3b_1^2+b_1^3b_2^2 & b_1^1(b_3^4-b_1^2b_1^1) & b_2^2{b_1^1}^2 \end{pmatrix}$$

where $b_2^2b_1^1\neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 28 nonisomorphic algebras :

$\mathfrak{g}_{6,7}$. **Table 1 : Suppose first $\xi_1^2 \neq 0$. Then one may suppose $\xi_1^2 = 1, \xi_1^3 = 0$. Then**

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^3 \neq 0$ (N1)	$\xi_2^3 = 1$, $\xi_2^5 = 0$, $\xi_3^5 = 0$, $\xi_2^6 = 0$	$\xi_3^4 \neq 0$	$\xi_3^6 = 0$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;0,1;a;0,0;0,0)$ $a \neq 0$ $(a = \xi_3^4)$	$\mathfrak{g}_{7,1.1(i_\lambda)}$ $\lambda = \frac{a}{a-1}$ $(a \neq 0,1)$ (I) $\mathfrak{g}_{7,1.1(iii)}$ $(a = 1)$ (II2)
		$\xi_3^4 = 0$	$\xi_3^6 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;0,1;0;0,0;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\mathfrak{g}_{7,0.6}$ $(\varepsilon = 1)$ (IV) $\mathfrak{g}_{7,1.1(iv)}$ $(\varepsilon = 0)$ (IV1)
$\xi_2^3 = 0$ (N2)	$\longrightarrow\longrightarrow$	$\xi_3^4 = 0$, $\xi_3^5 = 0$	$\xi_2^6 = 0$, $\xi_3^6 = 0$ $\xi_2^5 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;0,0;0;\varepsilon,0;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\mathfrak{g}_{7,2.1(ii)}$ $(\varepsilon = 0)$ (V) $\mathfrak{g}_{7,1.2(i_\lambda)}$, $\lambda = 0$, $(\varepsilon = 1)$ (VI)
		$\xi_3^4 = 0$, $\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_2^6 = 0$, $\xi_3^6 = 0$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;0,0;0;a,1;0,0)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\mathfrak{g}_{7,1.2(i_\lambda)}$ $(a \neq 0,1)$ $\lambda = \frac{1}{1-2a+2\sqrt{a(a-1)}}$ * (VII) $\mathfrak{g}_{7,1.2(ii)}$ $(a = 1)$(VII2) $\mathfrak{g}_{7,1.2(iv)}$ $(a = 0)$ (VII1)
		$\xi_3^4 \neq 0$, $\xi_3^5 = 0$	$\xi_3^6 = 0$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0$	$(1;0,0;a;1,0;0,0)$ $a \neq 0$ $(a = \xi_3^4)$	$\mathfrak{g}_{7,1.2(i_\lambda)}$ $(a \neq \frac{1}{2},0,1)$ $\lambda = \frac{a}{a-1}$ (VIII) $\mathfrak{g}_{7,2.37}$ $(a = \frac{1}{2})$ (VIIIbis) $\mathfrak{g}_{7,1.2(iii)}$ $(a = 1)$ (VIII2)
				$\xi_2^5 = 0$	$\xi_2^6 = 0,1$	$(1;0,0;a;0,0;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$ $a \neq 0$ $(a = \xi_3^4)$	$\mathfrak{g}_{7,2.1(i_\lambda)}$ $(\varepsilon = 0)$ (IX,IX1) or $(\varepsilon = 1, a \neq 2)$ (IX2,IX4) $\lambda = \frac{a-1}{a}$ † $\mathfrak{g}_{7,1.3(ii)}$ $(\varepsilon = 1, a = 2)$ (IX3)
		$\xi_3^4 \neq 0$, $\xi_3^5 \neq 0$ (N3)	$\xi_3^5 = 1$, $\xi_2^6 = 0$, $\xi_3^6 = 0$	$\xi_2^5 \neq 0$	$\longrightarrow\longrightarrow$	$(1;0,0;a;b,1;0,0)$ $a,b \neq 0$ $(a = \xi_3^4$, $b = \xi_2^5)$	$\mathfrak{g}_{7,1.2(i_\lambda)}$ $(a \neq 1$, $b(2a-1)^2 \neq 1-a)$ ‡ (Xa) $\mathfrak{g}_{7,1.2(ii)}$ $(a \neq 1, \frac{1}{2}$; $b(2a-1)^2 = 1-a)$ (Xa) $\mathfrak{g}_{7,1.2(iii)}$ $(a = 1, b \neq -\frac{1}{4})$ (Xa2) $\mathfrak{g}_{7,2.17}$ $(a = 1, b = -\frac{1}{4})$ (Xa3)
				$\xi_2^5 = 0$	$\longrightarrow\longrightarrow$	$(1;0,0;a;0,1;0,0)$ $a \neq 0$ $(a = \xi_3^4)$	$\mathfrak{g}_{7,1.2(iv)}$ $(a \neq 0,1)$ (XI) $\mathfrak{g}_{7,2.24}$ (XII) $(a = 1)$

*$\lambda \neq -1, 1$ from $a \neq 0, 1$. Changing the sign of the square root changes λ to $\frac{1}{\lambda}$ which is in line with the fact that $\mathfrak{g}_{7,1.2(i_{\lambda'})} \cong \mathfrak{g}_{7,1.2(i_\lambda)} \Leftrightarrow \lambda' = \lambda, \frac{1}{\lambda}$.

†For $a \neq 2$, $(1;0,0;a;0,0;1,0)$ is proj.equiv. to $(1;0,0;a;0,0;0,0)$ (test1N2)

‡$\lambda = -\frac{1}{\beta}\frac{\beta b(3a-1+\beta^2)+\sqrt{b}\sqrt{a-1}}{ab-\beta^2 b(a-1)-\beta\sqrt{b}\sqrt{a-1}}$ where $\beta^2 - 2\beta\sqrt{b}\frac{2a-1}{\sqrt{a-1}} - 1 = 0$ and $\beta^2 + \frac{\beta}{\sqrt{b}\sqrt{a-1}} - \frac{a}{a-1} \neq 0$.

$\mathfrak{g}_{6,7}$. **Table 2: Suppose now $\xi_1^2 = 0$. Then**

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^3 \neq 0$ (N4)	$\xi_2^3 = 1$, $\xi_1^3 = 0$, $\xi_2^5 = 0$, $\xi_3^5 = 0$, $\xi_2^6 = 0$	$\xi_3^4 \neq 0$	$\xi_3^4 = 1$, $\xi_3^6 = 0$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;0,1;1;0,0;0,0)$	$\underline{\mathfrak{g}_{7,1.1(v)}}$ (XVI)
		$\xi_3^4 = 0$	$\xi_3^6 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;0,1;0;0,0;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,1.19}}$ $(\varepsilon = 1)$ (XIV) $\underline{\mathfrak{g}_{7,2.35}}$ $(\varepsilon = 0)$ (XIV2)
$\xi_2^3 = 0$, $\xi_3^4 \neq 0$ (N5)	$\xi_3^4 = 1$, $\xi_3^5 = 0$, $\xi_2^6 = 0$, $\xi_3^6 = 0$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;1,0;1;a,0;0,0)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\mathfrak{g}_{7,1.2(i_\lambda)}$ $(4a \neq 1)$ (XIX) $(\lambda = \frac{\sqrt{1-4a}-1}{\sqrt{1-4a}+1}$ $(\sqrt{1} = 1))$ $\mathfrak{g}_{7,1.2(ii)}$ $(4a = 1)$ (XIX2)
		$\xi_1^3 = 0$	$\xi_2^5 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;0,0;1;\varepsilon,0;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.23}}$ $(\varepsilon = 1)$ (XXI) $\underline{\mathfrak{g}_{7,2.1(iv)}}$ $(\varepsilon = 0)$ (XXI2)
$\xi_2^3 = 0$, $\xi_3^4 = 0$ (N6)	$\longrightarrow\!\!\longrightarrow$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_2^6 = 0$	$\xi_3^5 \neq 0$	$\xi_2^5 = 0$, $\xi_3^6 = 0$	$(0;1,0;0;0,a;0,0)$ $a \neq 0$ $(a = \xi_3^5)$	$\underline{\mathfrak{g}_{7,2.26}}$ $(a \neq 1)$ (XVII3) $\underline{\mathfrak{g}_{7,3.5}}$ $(a = 1)$ (XVII4)
				$\xi_3^5 = 0$	$\xi_3^6 = 0$, $\xi_2^5 = 0,1$	$(0;1,0;0;\epsilon,0;0,0)$ ε $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.1(v)}}$ $(\varepsilon = 1)$ (XVIII) $\underline{\mathfrak{g}_{7,3.22}}$ $(\varepsilon = 0)$ (XVIII2)
		$\xi_1^3 = 0$ (N7)	$\longrightarrow\!\!\longrightarrow$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_2^5 = 0$, $\xi_2^6 = 0$, $\xi_3^6 = 0$	$(0;0,0;0;0,1;0,0)$	$\underline{\mathfrak{g}_{7,3.7}}$ (XXII)
				$\xi_3^5 = 0$ $\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0$	$(0;0,0;0;1,0;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,2.36}}$ $(\varepsilon = 1)$ (XXIII) $\underline{\mathfrak{g}_{7,2.45}}$ $(\varepsilon = 0)$ (XXIII2)
				$\xi_3^5 = 0$, $\xi_2^5 = 0$, $\xi_3^6 \neq 0$	$\xi_2^6 = 0$, $\xi_3^6 = 1$	$(0;0,0;0;0,0;0,1)$	$\underline{\mathfrak{g}_{7,3.14}}$ (XXIV)
				$\xi_3^5 = 0$, $\xi_2^5 = 0$, $\xi_3^6 = 0$	$\xi_2^6 = 1$	$(0;0,0;0;0,0;1,0)$	$\underline{\mathfrak{g}_{7,3.8}}$ (XXV)

10 $\mathfrak{g}_{6,8}$

Commut. relations for $\mathfrak{g}_{6,8}$: $[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & \xi_2^1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^5 & -\xi_1^3 & 0 & \xi_2^1 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & 0 \end{pmatrix}.$$

We denote δ by $(\xi_2^1; \xi_1^3, \xi_2^3; \xi_3^5; \xi_1^6, \xi_2^6, \xi_3^6)$.

Aut $\mathfrak{g}_{6,8}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & 0 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & -(b_1^4 b_2^1 + b_1^3 b_2^2 - b_2^4 b_1^1) & b_2^2 {b_1^1}^2 & b_2^2 b_2^1 b_1^1 \\ b_1^6 & b_2^6 & b_3^6 & -b_1^4 b_2^2 & 0 & {b_2^2}^2 b_1^1 \end{pmatrix}$$

where $b_2^2 b_1^1 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 19 nonisomorphic algebras :

$\mathfrak{g}_{6,8}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^1=0$ (N1)	$\longrightarrow\longrightarrow$	$\xi_1^3\neq 0$	$\xi_1^3=1$, $\xi_2^3=0$, $\xi_1^6=0$, $\xi_2^6=0$	$\xi_3^6\neq 0$	$\xi_3^6=1$	$(0;1,0;a;0,0,1)$ $a\in\mathbb{C}$ $(a=\xi_3^5)$ *	$\underline{\mathfrak{g}_{7,1.19}}$ $(a\neq\pm 2)$ (I) $\underline{\mathfrak{g}_{7,2.35}}$ $(a=\pm 2)$ (I2)
				$\xi_3^6=0$	$\xi_3^5=0,1$	$(0;1,0;\varepsilon;0,0,0)$ $\varepsilon=0,1$ $(\varepsilon=\xi_3^5)$	$\underline{\mathfrak{g}_{7,3.5}}$ $(\varepsilon=0)$ (II) $\underline{\mathfrak{g}_{7,2.34}}$ $(\varepsilon=1)$ (II2)
$\xi_2^1=0$, $\xi_1^3=0$, $\xi_2^3=0$ (N2)	$\longrightarrow\longrightarrow$	$\xi_3^6\neq 0$	$\xi_3^5=0$, $\xi_1^6=0$, $\xi_2^6=0$, $\xi_3^6=1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(0;0,0;0;0,0,1)$	$\underline{\mathfrak{g}_{7,2.29}}$ (VII)
		$\xi_3^6=0$ (N3)	$\longrightarrow\longrightarrow$	$\xi_3^5\neq 0$, $\xi_1^6\neq 0$	$\xi_3^5=1$, $\xi_1^6=1$, $\xi_2^6=0$	$(0;0,0;1;1,0,0)$	$\mathfrak{g}_{7,2.29}$ (XI)
				$\xi_3^5\neq 0$, $\xi_1^6=0$	$\xi_3^5=1$, $\xi_2^6=0$	$(0;0,0;1;0,0,0)$	$\underline{\mathfrak{g}_{7,3.13}}$ (XII)
				$\xi_3^5=0$, $\xi_1^6\neq 0$	$\xi_1^6=1$, $\xi_2^6=0$	$(0;0,0;0;1,0,0)$	$\underline{\mathfrak{g}_{7,2.38}}$ (VIII)
				$\xi_3^5=0$, $\xi_1^6=0$	$\xi_2^6=1$	$(0;0,0;0;0,1,0)$	$\underline{\mathfrak{g}_{7,2.27}}$ (X)
$\xi_2^1=0$, $\xi_1^3=0$, $\xi_2^3\neq 0$ (N4)	$\xi_2^3=1$, $\xi_2^6=0$	$\xi_3^6\neq 0$	$\xi_3^6=1$, $\xi_3^5=0$, $\xi_1^6=0$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(0;0,1;0;0,0,1)$	$\mathfrak{g}_{7,1.19}$ (XIV)
		$\xi_3^6=0$	$\xi_3^5=0,1$ $\xi_3^6=0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(0;0,1;\varepsilon;\eta,0,0)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_3^5$, $\eta=\xi_1^6)$	$\underline{\mathfrak{g}_{7,2.43}}$ $(\varepsilon=0,\eta=0)$ (XVII) $\underline{\mathfrak{g}_{7,2.12}}$ $(\varepsilon=1,\eta=0)$ (XV) $\underline{\mathfrak{g}_{7,1.7}}$ $(\varepsilon=0,\eta=1)$ (XVIII) $\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon=1,\eta=1)$ (XVI)
$\xi_2^1\neq 0$ (N5)	$\xi_2^1=1$, $\xi_2^3=0$, $\xi_3^5=0$, $\xi_2^6=0$,	$\xi_1^3\neq 0$	$\xi_1^3=1$, $\xi_3^6=0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;1,0;0;a,0,\varepsilon)$ $\varepsilon=0,1$, $a\in\mathbb{C}$ † $(a=\xi_1^6,\varepsilon=\xi_3^6)$	$\underline{\mathfrak{g}_{7,0.4(\lambda)}}$ $\lambda=-ia$ $(\varepsilon=1)$ (XIXbis) $\underline{\mathfrak{g}_{7,0.7}}$ $(\varepsilon=0,a\neq 0)$ (XX) $\underline{\mathfrak{g}_{7,1.03}}$ $(\varepsilon=0,a=0)$ (XXI)
		$\xi_1^3=0$, $\xi_3^6=0$	$\longrightarrow\longrightarrow$	$\xi_1^6\neq 0$	$\xi_1^6=1$, $\xi_2^6=0$	$(1;0,0;0;1,0,0)$	$\underline{\mathfrak{g}_{7,1.1(vi)}}$ (XXII1)
				$\xi_1^6=0$	$\xi_2^6=0$	$(1;0,0;0;0,0,0)$	$\underline{\mathfrak{g}_{7,2.10}}$ (XXII2)
		$\xi_1^3=0$, $\xi_3^6\neq 0$	$\xi_2^6=0$, $\xi_3^6=1$, $\xi_1^6=0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(1;0,0;0;\varepsilon,0,1)$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^6)$	$\underline{\mathfrak{g}_{7,1.20}}$ $(\varepsilon=0)$ (XXII3) $\underline{\mathfrak{g}_{7,0.8}}$ $(\varepsilon=1)$ (XXII4)

11 $\mathfrak{g}_{6,9}$

Commut. relations for $\mathfrak{g}_{6,9}$: $[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_2,x_5]=x_6$; $[x_3,x_4]=x_6$.

*The cases corresponding to $(0;1,0;a;0,0,1)$ and $(1;1,0;a';0,0,1)$ are proj. equiv. if and only if $a'=\pm a$.

†The cases corresponding to $(1;1,0;0;a,0,\varepsilon)$ and $(1;1,0;0;a',0,\varepsilon')$ are proj. equiv. if and only if $a'=\pm a,\varepsilon'=\varepsilon$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & 0 & 0 & 0 \\ 0 & \xi_2^5 & 0 & 0 & 0 & 0 \\ \xi_1^6 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_3^4;\xi_2^5;\xi_1^6)$.

The automorphisms of $\mathfrak{g}_{6,9}$ fall into 2 kinds. The first kind is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & \frac{b_2^4 b_3^3}{b_2^2} & 0 & b_3^3 b_1^1 & 0 \\ b_1^6 & b_2^6 & b_3^6 & -b_1^5 b_2^2 & -b_1^4 b_3^3 & b_3^3 b_2^2 b_1^1 \end{pmatrix} \tag{3}$$

where $b_1^1 b_2^2 b_3^3 \neq 0$. The second kind is comprised of the matrices

$$\Psi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3^2 & 0 & 0 & 0 \\ 0 & b_2^3 & 0 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & 0 & b_3^2 b_1^1 & 0 \\ b_1^5 & \frac{b_3^4 b_2^3}{b_3^2} & b_3^5 & b_2^3 b_1^1 & 0 & 0 \\ b_1^6 & b_2^6 & b_3^6 & -b_1^4 b_2^3 & -b_1^5 b_3^2 & b_2^3 b_3^2 b_1^1 \end{pmatrix} \tag{4}$$

where $b_2^3 b_3^2 b_1^1 \neq 0$.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 5 nonisomorphic algebras :

$\mathfrak{g}_{6,9}$ ·

if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_3^4\xi_2^5 \neq 0$ (N1)	$\xi_3^4 = 1$, $\xi_2^5 = 1$, $\xi_1^6 = 0,1$	$(1;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,3.16}}$ $(\varepsilon = 0)$ (I) $\underline{\mathfrak{g}_{7,2.23}}$ $(\varepsilon = 1)$ (I2)
$\xi_3^4 \neq 0$, $\xi_2^5 = 0$ or $\xi_3^4 = 0$, $\xi_2^5 \neq 0$ (N2)	$\xi_3^4 = 1$, $\xi_2^5 = 0$, $\xi_1^6 = 0,1$	$(1;0;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,3.10}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,2.1(iv)}}$ $(\varepsilon = 1)$ (II2)
$\xi_3^4 = 0$, $\xi_2^5 = 0$ (N3)	$\xi_1^6 = 1$	$(0;0;1)$	$\underline{\mathfrak{g}_{7,3.1(iii)}}$ (III)

12 $\mathfrak{g}_{6,10}$

Commut. relations for $\mathfrak{g}_{6,10}$: $[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_3,x_5]=x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & \xi_3^2 & 0 & 0 & 0 \\ -\xi_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & 0 & \xi_3^2 & 0 \\ 0 & 0 & \xi_3^5 & 0 & 0 & 0 \\ 0 & \xi_2^6 & 0 & 0 & \xi_3^4 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2,\xi_3^2;\xi_3^4;\xi_3^5;\xi_2^6)$.

Aut $\mathfrak{g}_{6,10}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & \frac{{b_3^3}^2}{b_1^1} & b_3^2 & 0 & 0 & 0 \\ -\frac{b_3^2 b_1^1}{b_3^3} & 0 & b_3^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & {b_3^3}^2 & b_3^2 b_1^1 & 0 \\ b_1^5 & 0 & b_3^5 & 0 & b_3^3 b_1^1 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_2^4 b_1^1 & b_3^4 b_1^1 - b_1^5 b_3^3 - \frac{b_3^5 b_3^2 b_1^1}{b_3^3} & {b_3^3}^2 b_1^1 \end{pmatrix} \tag{5}$$

where $b_1^1 b_3^3 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 17 nonisomorphic algebras :

$\mathfrak{g}_{6,10}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_3^2 \neq 0$ (N1)	$\xi_3^2 = 1$, $\xi_3^4 = 0$	$\xi_1^2 \neq 0$	$\xi_1^2 = 1$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$	$(1,1;0;1;a)$ $a \in \mathbb{C}$ $(a = \xi_2^6)$	$\mathfrak{g}_{7,0.4(\lambda)}$ * $(a \neq 0)$ (I) $\mathfrak{g}_{7,0.5}$ $(a = 0)$ (I2)
				$\xi_3^5 = 0$	$\xi_2^6 = 0,1$	$(1,1;0;0;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\mathfrak{g}_{7,1.02}$ $(\varepsilon = 0)$ (II) $\mathfrak{g}_{7,0.6}$ $(\varepsilon = 1)$ (II2)
		$\xi_1^2 = 0$ (N2)	$\longrightarrow$	$\xi_3^5 \neq 0$	$\xi_3^5 = 1$, $\xi_2^6 = 0,1$	$(0,1;0;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\mathfrak{g}_{7,1.17}$ $(\varepsilon = 0)$ (III) $\mathfrak{g}_{7,0.4(\lambda)}$ $(\lambda = 2i)$ $(\varepsilon = 1)$ (III2)
				$\xi_3^5 = 0$	$\xi_2^6 = 0,1$	$(0,1;0;0;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\mathfrak{g}_{7,2.5}$ $(\varepsilon = 0)$ (IV) $\mathfrak{g}_{7,1.1(iv)}$ $(\varepsilon = 1)$ (IV2)
$\xi_3^2 = 0$, $\xi_3^5 \neq 0$ (N3)	$\xi_3^5 = 1$, $\xi_3^4 = 0$, $\xi_1^2 = 0,1$ $\xi_2^6 = 0,1$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$(\varepsilon,0;0;1;\eta)$ $(\varepsilon,\eta = 0,1)$ $(\varepsilon = \xi_1^2, \eta = \xi_2^6)$	$\mathfrak{g}_{7,3.16}$ $(\varepsilon = 0, \eta = 0)$ (V) $\mathfrak{g}_{7,2.33}$ $(\varepsilon = 1, \eta = 0)$ (V2) $\mathfrak{g}_{7,2.23}$ $(\varepsilon = 0, \eta = 1)$ (V3) $\mathfrak{g}_{7,1.8}$ $(\varepsilon = 1, \eta = 1)$ (V4)
$\xi_3^2 = 0$, $\xi_3^5 = 0$ (N3)	$\longrightarrow$	$\xi_3^4 \neq 0$	$\xi_3^4 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$	$(a,0;1;0;1)$ $a \in \mathbb{C}$ $(a = \xi_1^2)$	$\mathfrak{g}_{7,1.3(i_\lambda)}$ $(\lambda = -a)$ $(a \neq 0)$ (VI) $\mathfrak{g}_{7,2.1(iv)}$ $(a = 0)$ (VI2)
				$\xi_2^6 = 0$	$\xi_1^2 = 0,1$	$(\varepsilon,0;1;0;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^2)$	$\mathfrak{g}_{7,3.10}$ $(\varepsilon = 0)$ (VII) $\mathfrak{g}_{7,1.3(i_\lambda)}$ $(\lambda = 0)$ $(\varepsilon = 1)$ (VII2)
		$\xi_3^4 = 0$	$\longrightarrow$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_1^2 = 0,1$	$(\varepsilon,0;0;0;1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^2)$	$\mathfrak{g}_{7,2.28}$ $(\varepsilon = 0)$ (VIII) $\mathfrak{g}_{7,1.3(iii)}$ $(\varepsilon = 1)$ (VIII2)
				$\xi_2^6 = 0$	$\xi_1^2 = 1$	$(1,0;0;0;0)$	$\mathfrak{g}_{7,2.32}$ (IX)

13 $\mathfrak{g}_{6,11}$

Commut. relations for $\mathfrak{g}_{6,11}$: $[x_1,x_2] = x_4$; $[x_1,x_4] = x_5$; $[x_1,x_5] = x_6$; $[x_2,x_3] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & -\xi_1^2 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^6 & -\xi_1^3 + \xi_2^5 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_1^3, \xi_2^3; \xi_2^5; \xi_3^6)$.

* $\lambda = \frac{i}{a}(1 - 2a)$. Note that $\lambda \neq -2i$. Now since $\mathfrak{g}_{7,0.4(\lambda')} \cong \mathfrak{g}_{7,0.4(\lambda)} \Leftrightarrow \lambda' = \pm\lambda$, we got in fact all possibilities.

Aut $\mathfrak{g}_{6,11}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & 0 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & -b_1^2 {b_1^1}^2 & b_2^4 b_1^1 & b_2^2 {b_1^1}^2 & 0 \\ b_1^6 & b_2^6 & b_3^6 & -b_1^3 b_2^2 + b_2^3 b_1^2 + b_2^5 b_1^1 & b_2^4 {b_1^1}^2 & b_2^2 {b_1^1}^3 \end{pmatrix} \tag{6}$$

where $b_1^1 b_2^2 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 17 nonisomorphic algebras :

$\mathfrak{g}_{6,11}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_1^3 = 0$	$\xi_2^3 \neq 0$	$\xi_2^3 = 1$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$	$(1;0,1;a;1)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\mathfrak{g}_{7,0.4(\lambda)}$ $(\lambda = \frac{i}{a})$ $(a \neq 0)$ (I) $\mathfrak{g}_{7,0.5}$ $(a = 0)$ (I2)
				$\xi_3^6 = 0$	$\xi_2^5 = 0,1$	$(1;0,1;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\mathfrak{g}_{7,1.17}$ $(\varepsilon = 0)$ (II) $\mathfrak{g}_{7,0.4(\lambda)}$ $(\lambda = 0)$ $(\varepsilon = 1)$ (II2)
		$\xi_2^3 = 0$ (N2)	$\longrightarrow\!\!\longrightarrow$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$	$(1;0,0;a;1)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\mathfrak{g}_{7,1.11}$ $(a \neq 0)$ (III) $\mathfrak{g}_{7,1.01(ii)}$ $(a = 0)$ (III2)
				$\xi_3^6 = 0$	$\xi_2^5 = 0,1$	$(1;0,0;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\mathfrak{g}_{7,2.15}$ $(\varepsilon = 0)$ (IV) $\mathfrak{g}_{7,1.11}$ $(\varepsilon = 1)$ (IV2)
$\xi_1^2 = 0$ (N3)	$\longrightarrow\!\!\longrightarrow$	$\xi_2^3 \neq 0$	$\xi_2^3 = 1$, $\xi_1^3 = 0$, $\xi_2^5 = 0,1$, $\xi_3^6 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;0,1;\varepsilon;\eta)$ $(\varepsilon, \eta = 0,1)$ $(\varepsilon = \xi_2^5,\ \eta = \xi_3^6)$	$\mathfrak{g}_{7,2.31}$ $(\varepsilon = 0,\ \eta = 0)$ (V) $\mathfrak{g}_{7,1.2(iv)}$ $(\varepsilon = 1,\ \eta = 0)$ (V3) $\mathfrak{g}_{7,2.33}$ $(\varepsilon = 0,\ \eta = 1)$ (V2) $\mathfrak{g}_{7,1.8}$ $(\varepsilon = 1,\ \eta = 1)$ (V4)
		$\xi_2^3 = 0$	$\longrightarrow\!\!\longrightarrow$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_3^6 = 0,1$	$(0;1,0;a;\varepsilon)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\mathfrak{g}_{7,2.1(i_\lambda)}$ $(\lambda = \frac{1}{a})$ $(a \neq 0,\ \varepsilon = 0)$ (VI,VI3) $\mathfrak{g}_{7,2.1(ii)}$ $(a = 0, \varepsilon = 0)$ (VI2) $\mathfrak{g}_{7,1.3(i_\lambda)}$ $(\lambda = \frac{a-1}{a^2})$ $(a \neq 0,\ \varepsilon = 1)$ (VI4) $\mathfrak{g}_{7,1.3(iii)}$ $(a = 0,\ \varepsilon = 1)$ (VI5)
				$\xi_1^3 = 0$, $\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_3^6 = 0,1$	$(0;0,0;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\mathfrak{g}_{7,2.1(iii)}$ $(\varepsilon = 0)$ (VII) $\mathfrak{g}_{7,1.3(v)}$ $(\varepsilon = 1)$ (VII2)
				$\xi_1^3 = 0$, $\xi_2^5 = 0$	$\xi_3^6 = 1$	$(0;0,0;0;1)$	$\mathfrak{g}_{7,3.17}$ (VIII)

14 $\mathfrak{g}_{6,12}$

Commut. relations for $\mathfrak{g}_{6,12}$: $[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & -\xi_1^2 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^6 & \xi_2^5 - \xi_1^3 & \xi_1^2 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_1^3, \xi_2^3; \xi_2^5; \xi_3^6)$.

Aut $\mathfrak{g}_{6,12}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & {b_1^1}^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & 0 & {b_1^1}^3 & 0 & 0 \\ b_1^5 & b_2^5 & -b_1^2 b_1^1 & b_2^4 b_1^1 & b_1^1 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_2^3 b_1^2 - b_1^3 {b_1^1}^2 - b_1^4 {b_1^1}^2 + b_2^4 b_1^2 + b_2^5 b_1^1 & (b_2^4 + b_1^2){b_1^1}^2 & {b_1^1}^5 \end{pmatrix}$$

We get for $\tilde{\mathfrak{g}}_\delta$ 21 nonisomorphic algebras :

$\mathfrak{g}_{6,12}$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_1^3 = 0$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$	$(1;0,a;b;1)$ $a,b \in \mathbb{C}$ $(a = \xi_2^3,\ b = \xi_2^5)$	$\underline{\mathfrak{g}_{7,0.4(\lambda)}}$ * $(a \neq 0,1,\ b \neq 1)$ (I) $\underline{\mathfrak{g}_{7,0.6}}$ $(a = 1,\ b \neq 1, -\frac{1}{3})$ (I2) $\underline{\mathfrak{g}_{7,0.8}}$ $(a = 0,\ b \neq 0,1)$ (I3) $\underline{\mathfrak{g}_{7,1.02}}$ $(a = 1,\ b = 1)$ (I4) $\underline{\mathfrak{g}_{7,1.1(iv)}}$ $(a = 1,\ b = -\frac{1}{3})$ (I5) $\underline{\mathfrak{g}_{7,1.20}}$ $(a = 0,\ b = 0)$ (I6) $\underline{\mathfrak{g}_{7,1.21}}$ $(a = 0,\ b = 1)$ (I7)
		$\xi_3^6 = 0$	$\xi_2^5 = 0,1$	$(1;0,a;\varepsilon;0)$ $a \in \mathbb{C},\ \varepsilon = 0,1$ $(a = \xi_2^3,\ \varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.17}}$ $(a \neq 0,\ \varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,2.13}}$ $(a = \varepsilon = 0)$ (II2) $\mathfrak{g}_{7,0.4(\lambda)}$ † $(a \neq 0,1,\ \varepsilon = 1)$ (II3) $\mathfrak{g}_{7,0.8}$ $(a = 0,\ \varepsilon = 1)$ (II4) $\mathfrak{g}_{7,0.6}$ $(a = \varepsilon = 1)$ (II5)
$\xi_1^2 = 0$, $\xi_2^3 \neq 0$ (N2)	$\xi_2^3 = 1$, $\xi_1^3 = 0$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$	$(0;0,1;a;1)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.8}}$ $(a \neq 1)$ (III) $\underline{\mathfrak{g}_{7,2.33}}$ $(a = 1)$ (III2)
		$\xi_3^6 = 0$	$\xi_2^5 = 0,1$	$(0;0,1;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.31}}$ $(\varepsilon = 0)$ (IV) $\underline{\mathfrak{g}_{7,1.2(iv)}}$ $(\varepsilon = 1)$ (IV2)
$\xi_1^2 = 0$, $\xi_2^3 = 0$	$\longrightarrow\longrightarrow$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$	$(0;1,0;a;b)$ $a,b \in \mathbb{C}$ $(a = \xi_2^5,\ b = \xi_3^6)$	$\underline{\mathfrak{g}_{7,1.3(i_\lambda)}}$ ‡ $(a \neq 0,\ b \neq 0)$ (V) $\underline{\mathfrak{g}_{7,1.3(iii)}}$ $(a = 0,\ b \neq 0,-1)$ (V2) $\underline{\mathfrak{g}_{7,2.32}}$ $(a = 0,\ b = -1)$ (V3) $\underline{\mathfrak{g}_{7,2.1(ii)}}$ $(a = 0,\ b = 0)$ (V4)
		$\xi_1^3 = 0$, $\xi_2^5 \neq 0$	$\xi_2^5 = 1$	$(0;0,0;1;a)$ $a \in \mathbb{C}$ $(a = \xi_3^6)$	$\underline{\mathfrak{g}_{7,1.3(v)}}$ $(a \neq 0,1)$ (VI) $\underline{\mathfrak{g}_{7,2.1(iii)}}$ $(a = 0)$ (VI2) $\underline{\mathfrak{g}_{7,2.25}}$ $(a = 1)$ (VI3)
		$\xi_1^3 = 0$, $\xi_2^5 = 0$	$\xi_3^6 = 1$	$(0;0,0;0;1)$	$\underline{\mathfrak{g}_{7,2.30}}$ (VII)

15 $\mathfrak{g}_{6,13}$

Commut. rel. for $\mathfrak{g}_{6,13}$: $[x_1,x_2] = x_4$; $[x_1,x_4] = x_5$; $[x_1,x_5] = x_6$; $[x_2,x_3] = x_5$; $[x_3,x_4] = -x_6$.

$^*\lambda = -i\frac{a^2-3ab+a+b}{a\sqrt{a}(b-1)}$

$^\dagger\lambda = -i\frac{3a-1}{a\sqrt{a}}$

$^\ddagger\lambda = \frac{a-b-1}{a^2}$

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^6 & \xi_3^6 & 0 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_2^6, \xi_3^6)$.

Aut $\mathfrak{g}_{6,13}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & {b_1^1}^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & -b_1^2 b_1^1 & b_2^2 b_1^1 & 0 & 0 \\ b_1^5 & b_2^5 & -b_1^4 b_1^1 & b_2^4 b_1^1 & b_2^2 {b_1^1}^2 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_2^5 b_1^1 & b_2^4 {b_1^1}^2 & b_2^2 {b_1^1}^3 \end{pmatrix} \tag{7}$$

where $b_1^1 b_2^2 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 6 nonisomorphic algebras :

$\mathfrak{g}_{6,13}$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$ $\xi_3^6 = 0,1$	$(1;1,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,1.1(v)}}$ (I,I2)
		$\xi_2^6 = 0$	$\xi_3^6 = 0,1$	$(1;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,2.14}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,1.01(i)}}$ $(\varepsilon = 1)$ (II2)
$\xi_1^2 = 0$ (N2)	$\longrightarrow\!-\!\!\longrightarrow$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$ $\xi_3^6 = 0,1$	$(0;1,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\underline{\mathfrak{g}_{7,2.1(iii)}}$ $(\varepsilon = 0)$ (III) $\underline{\mathfrak{g}_{7,1.3(v)}}$ $(\varepsilon = 1)$ (III2)
		$\xi_2^6 = 0$	$\xi_3^6 = 1$	$(0;0,1)$	$\underline{\mathfrak{g}_{7,2.25}}$ (IV)

16 $\mathfrak{g}_{6,14}$

Commut. relations for $\mathfrak{g}_{6,14}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & \xi_2^4 & 0 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & 0 & \xi_1^2 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_2^4; \xi_2^5; \xi_1^6, \xi_2^6)$.

Aut $\mathfrak{g}_{6,14}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_2^2 b_1^1 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_2^3 b_1^1 & b_2^2 {b_1^1}^2 & 0 & 0 \\ b_1^5 & b_2^5 & b_2^4 b_1^1 & b_2^3 {b_1^1}^2 & b_2^2 {b_1^1}^3 & 0 \\ b_1^6 & b_2^6 & -b_1^3 b_2^2 + b_2^3 b_1^2 & b_2^2 b_1^2 b_1^1 & 0 & {b_2^2}^2 b_1^1 \end{pmatrix} \tag{8}$$

where $b_1^1 b_2^2 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 10 nonisomorphic algebras :

$\mathfrak{g}_{6,14}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_1^6 = 0$, $\xi_2^6 = 0$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$, $\xi_2^5 = 0,1$	$\longrightarrow\!\!\!\longrightarrow$	$\longrightarrow\!\!\!\longrightarrow$	$(1;1;\varepsilon;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.1(iii)}}$ (I,I2)
		$\xi_2^4 = 0$ (N2)	$\xi_2^5 = 0,1$	$\longrightarrow\!\!\!\longrightarrow$	$\longrightarrow\!\!\!\longrightarrow$	$(1;0;\varepsilon;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.4}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,1.5}}$ $(\varepsilon = 1)$ (II2)
$\xi_1^2 = 0$ (N3)	$\longrightarrow\!\!\!\longrightarrow$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$, $\xi_2^6 = 0$, $\xi_2^5 = 0,1$, $\xi_2^6 = 0,1$	$\longrightarrow\!\!\!\longrightarrow$	$\longrightarrow\!\!\!\longrightarrow$	$(0;1;\varepsilon;\eta,0)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_2^5,\ \eta = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.17}}$ $(\eta = 0)$ (III,III2) $\underline{\mathfrak{g}_{7,1.2(iii)}}$ $(\eta = 1)$ (III3,III4)
		$\xi_2^4 = 0$ (N4)	$\longrightarrow\!\!\!\longrightarrow$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_1^6 = 0$, $\xi_2^5 = 0,1$	$(0;0;\varepsilon;0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.11}}$ $(\varepsilon = 0)$ (IV) $\underline{\mathfrak{g}_{7,1.18}}$ $(\varepsilon = 1)$ (IV2)
				$\xi_2^6 = 0$, $\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_1^6 = 0,1$	$(0;0;1;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,2.18}}$ $(\varepsilon = 0)$ (V) $\underline{\mathfrak{g}_{7,1.9}}$ $(\varepsilon = 1)$ (V2)
				$\xi_2^6 = 0$, $\xi_2^5 = 0$	$\xi_1^6 = 1$	$(0;0;0;1,0)$	$\underline{\mathfrak{g}_{7,2.19}}$ (VI)

17 $\mathfrak{g}_{6,15}$

Commut. relations for $\mathfrak{g}_{6,15}$: $[x_1,x_2] = x_3$; $[x_1,x_3] = x_4$; $[x_1,x_5] = x_6$; $[x_2,x_3] = x_5$; $[x_2,x_4] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_2^5 & 0 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_2^4;\xi_1^5,\xi_2^5)$.

The automorphisms of $\mathfrak{g}_{6,15}$ fall into 2 kinds. The first kind is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_2^2 b_1^1 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_2^3 b_1^1 & b_2^2 {b_1^1}^2 & 0 & 0 \\ b_1^5 & b_2^5 & -b_1^3 b_2^2 & 0 & {b_2^2}^2 b_1^1 & 0 \\ b_1^6 & b_2^6 & -b_1^4 b_2^2 + b_2^5 b_1^1 & -b_1^3 b_2^2 b_1^1 & b_2^3 b_2^2 b_1^1 & {b_2^2}^2 b_1^1 \end{pmatrix} \tag{9}$$

where $b_1^1 b_2^2 \neq 0$. The second kind is comprised of the matrices

$$\Psi = \begin{pmatrix} 0 & b_2^1 & 0 & 0 & 0 & 0 \\ b_1^2 & 0 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & -b_1^2 b_2^1 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & -b_1^3 b_2^1 & 0 & -b_1^2 {b_2^1}^2 & 0 \\ b_1^5 & b_2^5 & b_2^3 b_1^2 & -{b_1^2}^2 b_2^1 & 0 & 0 \\ b_1^6 & b_2^6 & b_2^4 b_1^2 - b_1^5 b_2^1 & -b_1^3 b_1^2 b_2^1 & b_2^3 b_1^2 b_2^1 & -{b_1^2}^2 {b_2^1}^2 \end{pmatrix} \tag{10}$$

where $b_2^1 b_1^2 \neq 0$.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 4 nonisomorphic algebras :

$\mathfrak{g}_{6,15}$.

if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^4\xi_1^5 \neq 0$ (N1)	$\xi_2^4 = 1$, $\xi_1^5 = 1$	$(1;1,a)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.2(i_\lambda)}}$* $(a \neq 0)$ (I,I2) $\underline{\mathfrak{g}_{7,1.2(i_\lambda)}}$ $(\lambda = 1)$ $(a = 0)$ (I3)
$\xi_2^4 \neq 0$, $\xi_1^5 = 0$ or $\xi_2^4 = 0$, $\xi_1^5 \neq 0$ (N2)	$\xi_2^4 = 0$, $\xi_1^5 = 1$, $\xi_2^5 = 0,1$	$(0;1,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.41}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,1.2(ii)}}$ $(\varepsilon = 1)$ (II2)
$\xi_2^4 = 0$, $\xi_1^5 = 0$ (N3)	$\xi_2^5 = 1$	$(0;0,1)$	$\underline{\mathfrak{g}_{7,2.37}}$ (III)

18 $\mathfrak{g}_{6,16}$

Commut. relations for $\mathfrak{g}_{6,16}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & \xi_2^4 & 0 & 0 & 0 \\ 0 & \xi_2^6 & \xi_2^5 & \xi_2^4 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_2^4; \xi_2^5; \xi_2^6)$. Aut $\mathfrak{g}_{6,16}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_2^2 b_1^1 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_2^3 b_1^1 & b_2^2 {b_1^1}^2 & 0 & 0 \\ b_1^5 & b_2^5 & b_2^4 b_1^1 & b_2^3 {b_1^1}^2 & b_2^2 {b_1^1}^3 & 0 \\ b_1^6 & b_2^6 & b_2^5 b_1^1 & b_2^4 {b_1^1}^2 & b_2^3 {b_1^1}^3 & b_2^2 {b_1^1}^4 \end{pmatrix} \tag{11}$$

where $b_1^1 b_2^2 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 11 nonisomorphic algebras :

$\mathfrak{g}_{6,16}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$	$(1;1;a;1)$ † $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\underline{\mathfrak{g}_{7,0.2}}$ $(a^2 \neq \frac{4}{5})$ (I) $\underline{\mathfrak{g}_{7,1.1(i_\lambda)}}$ $(\lambda = 1)$ $\underline{(a^2 = \frac{4}{5})}$ (I2)
				$\xi_2^6 = 0$	$\xi_2^5 = 0,1$	$(1;1;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\mathfrak{g}_{7,1.1(i_\lambda)}$ $(\lambda = 1)$ $(\varepsilon = 0)$ (II) $\mathfrak{g}_{7,0.2}$ $(\varepsilon = 1)$ (II2)
		$\xi_2^4 = 0$ (N2)	$\longrightarrow\longrightarrow$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$ $\xi_2^6 = 0,1$	$(1;0;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,1.4}}$ $(\varepsilon = 0)$ (III) $\underline{\mathfrak{g}_{7,0.3}}$ $(\varepsilon = 1)$ (III2)
				$\xi_2^5 = 0$	$\xi_2^6 = 0,1$	$(1;0;0;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.3}}$ $(\varepsilon = 0)$ (IV) $\underline{\mathfrak{g}_{7,1.6}}$ $(\varepsilon = 1)$ (IV2)
$\xi_1^2 = 0$ (N3)	$\longrightarrow\longrightarrow$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$	$(0;1;a;1)$ ‡ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.01(i)}}$ $(a^2 \neq \frac{4}{5})$ (V) $\underline{\mathfrak{g}_{7,2.14}}$ $(a^2 = \frac{4}{5})$ (V2,V3)
				$\xi_2^6 = 0$	$\xi_2^5 = 0,1$	$(0;1;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\mathfrak{g}_{7,2.14}$ $(\varepsilon = 0)$ (VI) $\mathfrak{g}_{7,1.01(i)}$ $(\varepsilon = 1)$ (VI2)
		$\xi_2^4 = 0$ (N4)	$\longrightarrow\longrightarrow$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$ $\xi_2^6 = 0,1$	$(0;0;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.15}}$ $(\varepsilon = 0)$ (VII) $\underline{\mathfrak{g}_{7,1.01(ii)}}$ $(\varepsilon = 1)$ (VII2)
				$\xi_2^5 = 0$	$\xi_2^6 = 1$	$(0;0;0;1)$	$\underline{\mathfrak{g}_{7,2.16}}$ (VIII)

*$\lambda = \frac{2i+a}{2i-a}$ if $a \neq 2i$ and $\lambda = 0$ if $a = 2i$.

19 $\mathfrak{g}_{6,17}$

Commut. relations for $\mathfrak{g}_{6,17}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & \xi_2^4 & 0 & 0 & 0 \\ 0 & \xi_2^6 & \xi_2^5 & \xi_2^4 + \xi_1^2 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_1^2; \xi_2^4; \xi_2^5; \xi_2^6)$.

Aut $\mathfrak{g}_{6,17}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & {b_1^1}^3 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^4 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_2^3 b_1^1 & {b_1^1}^5 & 0 & 0 \\ b_1^5 & b_2^5 & b_2^4 b_1^1 & b_2^3 {b_1^1}^2 & {b_1^1}^6 & 0 \\ b_1^6 & b_2^6 & b_2^5 b_1^1 + b_2^3 b_1^2 - b_1^3 {b_1^1}^3 & {b_1^1}^2 (b_2^4 + b_1^2 {b_1^1}^2) & b_2^3 {b_1^1}^3 & {b_1^1}^7 \end{pmatrix}$$

We get for $\tilde{\mathfrak{g}}_\delta$ 7 nonisomorphic algebras :

$\mathfrak{g}_{6,17}$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_2^6 = 0$, $\xi_2^5 = 0,1$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$(1; a; \varepsilon; 0)$ $a \in \mathbb{C}$, $\varepsilon = 0,1$ $(a = \xi_2^4,\ \varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.1(i_\lambda)}}$ $(\lambda = 1 + \frac{1}{a})$ $(a \neq 0)$ (I,I3) $\underline{\mathfrak{g}_{7,1.1(ii)}}$ $(a = 0,\ \varepsilon = 0)$ (I2) $\underline{\mathfrak{g}_{7,0.1}}$ $(a = 0,\ \varepsilon = 1)$ (I4)
$\xi_1^2 = 0$, $\xi_2^4 \neq 0$ (N2)	$\xi_2^4 = 1$, $\xi_2^6 = 0$, $\xi_2^5 = 0,1$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$(0; 1; \varepsilon; 0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.1(v)}}$ (II,II2)
$\xi_1^2 = 0$, $\xi_2^4 = 0$ (N2)	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0,1$	$(0; 0; 1; \varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.15}}$ $(\varepsilon = 0)$ (III) $\underline{\mathfrak{g}_{7,1.01(ii)}}$ $(\varepsilon = 1)$ (III2)
		$\xi_2^5 = 0$	$\xi_2^6 = 1$	$(0; 0; 0; 1)$	$\underline{\mathfrak{g}_{7,1.15}}$ (IV)

20 $\mathfrak{g}_{6,18}$

Commut. rel. for $\mathfrak{g}_{6,18}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_5] = x_6$; $[x_3, x_4] = -x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & 0 & 0 & 0 & 0 \\ \xi_1^6 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_2^5; \xi_1^6)$.

† $(1; 1; a; 1)$ and $(1; 1; a'; 1)$ are proj. equiv. if and only if $a' = \pm a$.

‡ $(0; 1; a; 1)$ and $(0; 1; a'; 1)$ are proj. equiv. if and only if $a' = \pm a$.

Aut $\mathfrak{g}_{6,18}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_2^2 b_1^1 & 0 & 0 & 0 \\ b_1^4 & \frac{{b_2^3}^2}{2b_2^2} & b_2^3 b_1^1 & b_2^2 {b_1^1}^2 & 0 & 0 \\ b_1^5 & b_2^5 & \frac{{b_2^3}^2 b_1^1}{2b_2^2} & b_2^3 {b_1^1}^2 & b_2^2 {b_1^1}^3 & 0 \\ b_1^6 & b_2^6 & \frac{-{b_2^3}^2 b_1^3 + 2b_1^4 b_2^3 b_2^2 - 2b_1^5 {b_2^2}^2}{2b_2^2} & b_1^1(-b_2^3 b_1^3 + b_1^4 b_2^2) & -b_1^3 b_2^2 {b_1^1}^2 & {b_2^2}^2 {b_1^1}^3 \end{pmatrix} \tag{12}$$

where $b_1^1 b_2^2 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 3 nonisomorphic algebras :

$\mathfrak{g}_{6,18}$.

if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^5 \neq 0$ (N1)	$\xi_2^5 = 1$ $\xi_1^6 = 0,1$	$(1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,2.13}}$ $(\varepsilon = 0)$ (I) $\underline{\mathfrak{g}_{7,1.20}}$ $(\varepsilon = 1)$ (I2)
$\xi_2^5 = 0$	$\xi_2^6 = 1$	$(0;1)$	$\underline{\mathfrak{g}_{7,2.10}}$ (II)

21 $\mathfrak{g}_{6,19}$

Commut. relations for $\mathfrak{g}_{6,19}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & \xi_2^4 & 0 & 0 & 0 \\ 0 & \xi_2^6 & \xi_2^5 & \xi_2^4 & 0 & 0 \end{pmatrix}$$

We denote δ by $(\xi_2^4; \xi_2^5; \xi_2^6)$.

Aut $\mathfrak{g}_{6,19}$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & {b_1^1}^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_2^3 b_1^1 & {b_1^1}^4 & 0 & 0 \\ b_1^5 & b_2^5 & b_1^1(-b_1^3 b_1^1 + b_2^4) & b_2^3 {b_1^1}^2 & {b_1^1}^5 & 0 \\ b_1^6 & b_2^6 & b_1^1(-b_1^4 b_1^1 + b_2^5) & {b_1^1}^2(-b_1^3 b_1^1 + b_2^4) & b_2^3 {b_1^1}^3 & {b_1^1}^6 \end{pmatrix} \tag{13}$$

where $b_1^1 \neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 5 nonisomorphic algebras :

$\mathfrak{g}_{6,19}$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^4 \neq 0$ (N1)	$\xi_2^4 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$	$(1;a;1)$ $a \in \mathbb{C}$ $(a = \xi_2^5)$	$\underline{\mathfrak{g}_{7,1.1(v)}}$ $(a \neq 0)$ (I) $\underline{\mathfrak{g}_{7,1.01(i)}}$ $(a = 0)$ (I2)
		$\xi_2^6 = 0$	$\xi_2^5 = 0,1$	$(1;\varepsilon;0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.14}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,1.1(v)}}$ $(\varepsilon = 1)$ (II2)
$\xi_2^4 = 0$ (N2)	$\longrightarrow\!\!\longrightarrow$	$\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_2^6 = 0,1$	$(0;1;\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,1.11}}$ (III,III2)
		$\xi_2^5 = 0$	$\xi_2^6 = 1$	$(0;0;1)$	$\underline{\mathfrak{g}_{7,1.12}}$ (IV)

22 $\mathfrak{g}_{6,20}$

Commut. relations for $\mathfrak{g}_{6,20}$: $[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_2,x_3]=x_5$; $[x_2,x_5]=x_6$; $[x_3,x_4]=-x_6$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&\xi_2^5&0&0&0&0\\\xi_1^6&0&0&0&0&0\end{pmatrix}$$

We denote δ by $(\xi_2^5;\xi_1^6)$.

Aut $\mathfrak{g}_{6,20}$ is comprised of the matrices

$$\Phi=\begin{pmatrix}b_1^1&0&0&0&0&0\\0&{b_1^1}^2&0&0&0&0\\b_1^3&b_2^3&{b_1^1}^3&0&0&0\\b_1^4&\frac{{b_2^3}^2}{2{b_1^1}^2}&b_2^3b_1^1&{b_1^1}^4&0&0\\b_1^5&b_2^5&\frac{-2b_1^3{b_1^1}^3+{b_2^3}^2}{2b_1^1}&b_2^3{b_1^1}^2&{b_1^1}^5&0\\b_1^6&b_2^6&\frac{-{b_2^3}^2b_1^3+2b_1^4b_2^3{b_1^1}^2-2b_1^5{b_1^1}^4}{2{b_1^1}^2}&b_1^1(-b_2^3b_1^3+b_1^4{b_1^1}^2)&-b_1^3{b_1^1}^4&{b_1^1}^7\end{pmatrix}\tag{14}$$

where $b_1^1\neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 3 nonisomorphic algebras :

$\mathfrak{g}_{6,20}$.

if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_2^5\neq 0$ (N1)	$\xi_2^5=1$ $\xi_1^6=0,1$	$(1;\varepsilon)$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^6)$	$\underline{\mathfrak{g}_{7,1.21}}$ $(\varepsilon=0)$ (I) $\underline{\mathfrak{g}_{7,0.8}}$ $(\varepsilon=1)$ (I2)
$\xi_2^5=0$	$\xi_2^6=1$	$(0;1)$	$\underline{\mathfrak{g}_{7,1.1(vi)}}$ (II)

23 $\mathfrak{g}_{5,1}\times\mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,1}\times\mathbb{C}$: $[x_1,x_3]=x_5$; $[x_2,x_4]=x_5$. ($\mathfrak{g}_{5,1}$ is the 5-dim. complex Heisenberg Lie algebra.)

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta=\begin{pmatrix}\xi_1^1&\xi_2^1&\xi_3^1&\xi_4^1&0&0\\\xi_1^2&\xi_2^2&\xi_4^1&\xi_4^2&0&0\\\xi_1^3&\xi_2^3&-\xi_1^1&-\xi_1^2&0&0\\\xi_2^3&\xi_2^4&-\xi_2^1&-\xi_2^2&0&0\\0&0&0&0&0&\xi_6^5\\\xi_1^6&\xi_2^6&\xi_3^6&\xi_4^6&0&0\end{pmatrix}\text{ with }A=\begin{pmatrix}\xi_1^1&\xi_2^1&\xi_3^1&\xi_4^1\\\xi_1^2&\xi_2^2&\xi_4^1&\xi_4^2\\\xi_1^3&\xi_2^3&-\xi_1^1&-\xi_1^2\\\xi_2^3&\xi_2^4&-\xi_2^1&-\xi_2^2\end{pmatrix}\text{ nilpotent.}\tag{15}$$

The matrix A has to be a nilpotent element of the Lie algebra $\mathfrak{sp}(4,\mathbb{C})$. Now, from the usual root space decomposition of $\mathfrak{sp}(4,\mathbb{C})$, the set $\mathfrak{sp}(4,\mathbb{C})_+$ of positive elements consists of the matrices $M=\begin{pmatrix}0&u_2^1&u_3^1&u_4^1\\0&0&u_4^1&u_4^2\\0&0&0&0\\0&0&-u_2^1&0\end{pmatrix}$, and we know ([3] p.63, 6.2, [1], 14.26) that the nilpotent elements are simply the set $\mathrm{Ad}(G)(\mathfrak{sp}(4,\mathbb{C})_+)$ where G is the Lie group $SP(4,\mathbb{C})$. As $\mathrm{Ad}(P)(X)=PXP^{-1}$ and $\mathrm{diag}\,(P,\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right))\in\mathrm{Aut}(\mathfrak{g}_{5,1}\times\mathbb{C})$ $\forall\,P\in SP(4,\mathbb{C})$ we may suppose $A\in\mathfrak{sp}(4,\mathbb{C})_+$, that is $\xi_1^1=\xi_1^2=$

$\xi^2_2 = \xi^3_1 = \xi^3_2 = \xi^4_2 = 0:$

$$\delta = \begin{pmatrix} 0 & \xi^1_2 & \xi^1_3 & \xi^1_4 & 0 & 0 \\ 0 & 0 & \xi^1_4 & \xi^2_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi^1_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi^5_6 \\ \xi^6_1 & \xi^6_2 & \xi^6_3 & \xi^6_4 & 0 & 0 \end{pmatrix} \tag{16}$$

We will denote δ by $(\xi^1_2, \xi^1_3, \xi^1_4; \xi^2_4; \xi^5_6; \xi^6_1, \xi^6_2, \xi^6_3, \xi^6_4)$.

$\mathrm{Aut}(\mathfrak{g}_{5,1} \times \mathbb{C})$ clearly consists of the matrices

$$\begin{pmatrix} & & & & 0 & 0 \\ & & \boxed{s\,\Lambda} & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ b^5_1 & b^5_2 & b^5_3 & b^5_4 & s^2 & b^5_6 \\ b^6_1 & b^6_2 & b^6_3 & b^6_4 & 0 & b^6_6 \end{pmatrix} \tag{17}$$

where $s \in \mathbb{C}^*, \Lambda \in SP(4, \mathbb{C}), b^i_j \in \mathbb{C}, b^6_6 \neq 0$. However, as we need a more explicit expression, we record the following lemma.

Lemma 1. *The automorphisms of* $\mathfrak{g}_{5,1} \times \mathbb{C}$ *fall into 4 kinds.*
(i) First kind.

$$\Phi = \begin{pmatrix} b^1_1 & b^1_2 & b^1_3 & b^1_4 & 0 & 0 \\ * & b^2_2 & * & b^2_4 & 0 & 0 \\ b^3_1 & b^3_2 & b^3_3 & b^3_4 & 0 & 0 \\ * & b^4_2 & * & * & 0 & 0 \\ b^5_1 & b^5_2 & b^5_3 & b^5_4 & b^3_3 b^1_1 - b^3_1 b^1_3 + b^3_4 b^1_2 - b^3_2 b^1_4 & b^5_6 \\ b^6_1 & b^6_2 & b^6_3 & b^6_4 & 0 & b^6_6 \end{pmatrix} \tag{18}$$

where $b^2_1 = -\frac{(b^2_4 b^1_2 - b^2_2 b^1_4)b^3_1 - b^3_2 b^2_4 b^1_1 + b^3_4 b^2_2 b^1_1}{b^3_3 b^1_1 - b^3_1 b^1_3}$, $b^2_3 = -\frac{(b^2_4 b^1_2 - b^2_2 b^1_4)b^3_3 - b^3_2 b^2_4 b^1_3 + b^3_4 b^2_2 b^1_3}{b^3_3 b^1_1 - b^3_1 b^1_3}$, $b^4_1 = \frac{(b^3_3 b^1_1 - b^3_1 b^1_3)(b^3_2 b^1_1 - b^3_1 b^1_2) - b^4_2 b^3_4 b^2_2 b^1_1 - ((b^2_4 b^1_2 - b^2_2 b^1_4)b^3_1 - b^3_2 b^2_4 b^1_1)b^4_2}{(b^3_3 b^1_1 - b^3_1 b^1_3)b^2_2}$, $b^4_3 = -\frac{(b^3_3 b^1_1 - b^3_1 b^1_3)(b^3_3 b^1_2 - b^3_2 b^1_3) + b^4_2 b^3_4 b^2_2 b^1_3 + ((b^2_4 b^1_2 - b^2_2 b^1_4)b^3_3 - b^3_2 b^2_4 b^1_3)b^4_2}{(b^3_3 b^1_1 - b^3_1 b^1_3)b^2_2}$, $b^4_4 = \frac{b^3_3 b^1_1 - b^3_1 b^1_3 + b^4_2 b^2_4}{b^2_2}$, *and the parameters are subject to the 2 conditions* $\det\Phi \neq 0$ *(that is* $(b^3_4 b^1_2 - b^3_2 b^1_4 + b^3_3 b^1_1 - b^3_1 b^1_3)b^6_6 \neq 0$*) and* $(b^3_3 b^1_1 - b^3_1 b^1_3)b^2_2 \neq 0$.
(ii) Second kind.

$$\Psi = \begin{pmatrix} b^1_1 & b^1_2 & b^1_3 & b^1_4 & 0 & 0 \\ * & 0 & * & b^2_4 & 0 & 0 \\ b^3_1 & b^3_2 & b^3_3 & b^3_4 & 0 & 0 \\ * & -\frac{b^3_3 b^1_1 - b^3_1 b^1_3}{b^2_4} & * & b^4_4 & 0 & 0 \\ b^5_1 & b^5_2 & b^5_3 & b^5_4 & b^3_3 b^1_1 - b^3_1 b^1_3 + b^3_4 b^1_2 - b^3_2 b^1_4 & b^5_6 \\ b^6_1 & b^6_2 & b^6_3 & b^6_4 & 0 & b^6_6 \end{pmatrix} \tag{19}$$

where $b^2_1 = \frac{(b^3_2 b^1_1 + b^3_1 b^1_2)b^2_4}{b^3_3 b^1_1 - b^3_1 b^1_3}$, $b^2_3 = -\frac{(b^3_3 b^1_2 - b^3_2 b^1_3)b^2_4}{b^3_3 b^1_1 - b^3_1 b^1_3}$, $b^4_1 = \frac{(b^3_3 b^1_1 - b^3_1 b^1_3)(b^3_4 b^1_1 - b^3_1 b^1_4) + (b^3_2 b^1_1 - b^3_1 b^1_2)b^4_4 b^2_4}{(b^3_3 b^1_1 - b^3_1 b^1_3)b^2_4}$, $b^4_2 = -\frac{b^3_3 b^1_1 - b^3_1 b^1_3}{b^2_4}$, $b^4_3 = \frac{(b^3_3 b^1_1 - b^3_1 b^1_3)(b^3_4 b^1_3 - b^3_3 b^1_4) - (b^3_3 b^1_2 - b^3_2 b^1_3)b^4_4 b^2_4}{(b^3_3 b^1_1 - b^3_1 b^1_3)b^2_4}$, *and the parameters are subject to the 2 conditions* $\det\Psi \neq 0$ *(that is* $(b^3_4 b^1_2 - b^3_2 b^1_4 + b^3_3 b^1_1 - b^3_1 b^1_3)b^6_6 \neq 0$*) and* $b^3_3 b^1_1 - b^3_1 b^1_3 \neq 0$.
(iii) Third kind.

$$P = \begin{pmatrix} * & b^1_2 & * & \frac{b^4_1 b^2_3 - b^4_3 b^2_1 + b^3_4 b^1_2}{b^3_2} & 0 & 0 \\ b^2_1 & b^2_2 & b^2_3 & b^2_4 & 0 & 0 \\ * & b^3_2 & * & b^3_4 & 0 & 0 \\ b^4_1 & b^4_2 & b^4_3 & b^4_4 & 0 & 0 \\ b^5_1 & b^5_2 & b^5_3 & b^5_4 & b^4_4 b^2_2 - b^4_2 b^2_4 + b^4_3 b^2_1 - b^4_1 b^2_3 & b^5_6 \\ b^6_1 & b^6_2 & b^6_3 & b^6_4 & 0 & b^6_6 \end{pmatrix} \tag{20}$$

where $b_1^1 = -\frac{((b_2^4b_1^2-b_1^4b_2^2)b_3^4+b_4^4b_2^3b_2^1)b_1^2+(b_1^4b_3^2b_2^2+b_4^3b_2^1b_2^2-b_2^3b_4^2b_2^1)b_1^4-(b_1^4b_3^2+b_4^3b_2^1)b_2^4b_1^2}{(b_3^4b_1^2-b_1^4b_3^2)b_2^3}$, $b_3^1 = -\frac{((b_4^3b_2^2-b_2^3b_4^2)b_2^1+b_1^4b_3^2b_2^2)b_3^4+b_4^4b_2^3b_3^2b_2^1-(b_3^4b_2^2-b_2^4b_3^2)b_3^4b_1^2-(b_1^4b_3^2+b_4^3b_2^1)b_2^4b_3^2}{(b_3^4b_1^2-b_1^4b_3^2)b_2^3}$, $b_1^3 = -\frac{(b_4^3b_2^2-b_2^3b_4^2)b_1^4-b_2^4b_4^3b_1^2+b_4^4b_2^3b_1^2}{b_3^4b_1^2-b_1^4b_3^2}$, $b_3^3 = -\frac{(b_4^3b_2^2-b_2^3b_4^2)b_3^4-b_2^4b_4^3b_3^2+b_4^4b_2^3b_3^2}{b_3^4b_1^2-b_1^4b_3^2}$, *and the parameters are subject to the 2 conditions* $\det P \neq 0$ *(that is* $(b_4^4b_2^2 - b_2^4b_4^2 + b_3^4b_1^2 - b_1^4b_3^2)b_6^6 \neq 0$*) and* $(b_3^4b_1^2 - b_1^4b_3^2)b_2^3 \neq 0$.

(iv) Fourth kind.

$$X = \begin{pmatrix} * & b_2^1 & * & b_4^1 & 0 & 0 \\ b_1^2 & b_2^2 & b_3^2 & b_4^2 & 0 & 0 \\ \frac{b_2^4b_1^2-b_1^4b_2^2}{b_2^1} & 0 & \frac{b_2^4b_3^2-b_3^4b_2^2}{b_2^1} & \frac{b_3^4b_1^2-b_1^4b_3^2}{b_2^1} & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & b_4^4 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_4^5 & b_4^4b_2^2 - b_2^4b_4^2 + b_3^4b_1^2 - b_1^4b_3^2 & b_6^5 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & 0 & b_6^6 \end{pmatrix} \tag{21}$$

where $b_1^1 = \frac{(b_4^2b_2^1-b_2^2b_4^1)b_1^4+(b_2^4b_4^1-b_4^4b_2^1)b_1^2}{(b_3^4b_1^2-b_1^4b_3^2)}$, $b_3^1 = \frac{(b_4^2b_2^1-b_2^2b_4^1)b_3^4+(b_2^4b_4^1-b_4^4b_2^1)b_3^2}{(b_3^4b_1^2-b_1^4b_3^2)}$, *and the parameters are subject to the 2 conditions* $\det X \neq 0$ *(that is* $(b_4^4b_2^2 - b_2^4b_4^2 + b_3^4b_1^2 - b_1^4b_3^2)b_6^6 \neq 0$*) and* $(b_3^4b_1^2 - b_1^4b_3^2)b_2^1 \neq 0$.

Proof. Let $\varphi = \left(b_j^i\right)_{1 \leqslant i,j \leqslant 6} \in \mathrm{GL}(6, \mathbb{C})$. Denote by

$$ij|k \ (1 \leqslant i < j \leqslant 6, \ 1 \leqslant k \leqslant 6) \tag{22}$$

the equation obtained by projecting on x_k the equation $[\varphi(x_i), \varphi(x_j)] - \varphi([x_i, x_j]) = 0$. We have to solve equations (22). In a first step, we are left with

$$\varphi = \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 & b_4^1 & 0 & 0 \\ b_1^2 & b_2^2 & b_3^2 & b_4^2 & 0 & 0 \\ b_1^3 & b_2^3 & b_3^3 & b_4^3 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & b_4^4 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_4^5 & \Delta_{1,3}^{1,3} + \Delta_{1,3}^{2,4} & b_6^5 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & 0 & b_6^6 \end{pmatrix} \tag{23}$$

(where $\Delta_{k,\ell}^{i,j}$ is the minor formed with rows i, j and columns k, ℓ in the matrix) and the remaining equations $12|5, 14|5, 23|5, 24|5, 34|5$. The system 1 consisting of the 2 equations $12|5, 14|5$ reads $\Delta_{1,2}^{1,3} + \Delta_{1,2}^{2,4} = 0$, $\Delta_{1,4}^{1,3} + \Delta_{1,4}^{2,4} = 0$, and has principal determinant $\Delta_{2,4}^{2,4}$ with respect to the principal unknowns b_1^2, b_1^4. The system 2 consisting of the 2 equations $23|5, 34|5$ reads $\Delta_{2,3}^{1,3} + \Delta_{2,3}^{2,4} = 0$, $\Delta_{3,4}^{1,3} + \Delta_{3,4}^{2,4} = 0$, and has principal determinant $\Delta_{2,4}^{1,3}$ with respect to the principal unknowns b_3^1, b_3^3. The last equation $24|5$ reads $\Delta_{2,4}^{1,3} + \Delta_{2,4}^{2,4} - (\Delta_{1,3}^{1,3} + \Delta_{1,3}^{2,4}) = 0$. Hence, as $\Delta_{1,3}^{1,3} + \Delta_{1,3}^{2,4}$ has to be nonzero, $\Delta_{2,4}^{1,3} + \Delta_{2,4}^{2,4}$ has to be nonzero as well. Hence at least one of the 2 above systems is a Cramer system.

Case 1. $\Delta_{2,4}^{2,4} \neq 0$. From system 1: $b_1^2 = -\frac{(b_4^2b_2^1-b_2^2b_4^1)b_1^3-b_2^3b_4^2b_1^1+b_4^3b_2^2b_1^1}{\Delta_{2,4}^{2,4}}$, $b_1^4 = -\frac{(b_4^3b_1^1-b_1^3b_4^1)b_2^4-(b_2^3b_1^1+b_1^3b_2^1)b_4^4}{\Delta_{2,4}^{2,4}}$. Now, note that the system 2 has principal determinant $\Delta_{2,4}^{2,4}$ with respect to the principal unknowns b_3^2, b_3^4. Hence we get: $b_3^2 = -\frac{b_3^3b_4^2b_2^1-b_3^3b_2^2b_4^1-b_2^3b_4^2b_3^1+b_4^3b_2^2b_3^1}{\Delta_{2,4}^{2,4}}$, $b_3^4 = -\frac{(b_4^3b_3^1-b_3^3b_4^1)b_2^4+(b_3^3b_2^1-b_1^3b_3^1)b_4^4}{\Delta_{2,4}^{2,4}}$. The last equation $24|5$ now simply reads $\Delta_{1,3}^{1,3} = \Delta_{2,4}^{2,4}$. As b_2^2, b_4^2 cannot simultaneously vanish, one has 2 subcases. Subcase 1.1. $b_2^2 \neq 0$. From $24|5$, $b_4^4 = \frac{b_3^3b_1^1-b_1^3b_3^1+b_2^4b_4^2}{b_2^2}$. Then we get the first kind. Subcase 1.2. $b_2^2 = 0, b_4^2 \neq 0$. From $24|5$, $b_2^4 = -\frac{b_3^3b_1^1-b_1^3b_3^1}{b_4^2}$. Then we get the second kind.

Case 2. $\Delta^{1,3}_{2,4} \neq 0$. From system 2: $b^1_3 = \frac{(b^2_4 b^1_2 - b^2_2 b^1_4)b^4_3 + (b^4_2 b^1_4 - b^4_4 b^1_2)b^2_3}{\Delta^{1,3}_{2,4}}$, $b^3_3 = \frac{(b^3_2 b^2_4 - b^3_4 b^2_2)b^4_3 - (b^4_4 b^3_2 - b^4_2 b^3_4)b^2_3}{\Delta^{1,3}_{2,4}}$. Now, note that the system 1 has principal determinant $\Delta^{1,3}_{2,4}$ with respect to the principal unknowns b^1_1, b^3_1. Hence we get: $b^1_1 = \frac{(b^2_4 b^1_2 - b^2_2 b^1_4)b^4_1 - (b^4_4 b^1_2 - b^4_2 b^1_4)b^2_1}{\Delta^{1,3}_{2,4}}$, $b^1_1 = -\frac{(b^3_4 b^2_2 - b^3_2 b^2_4)b^4_1 + (b^4_4 b^3_2 - b^4_2 b^3_4)b^2_1}{\Delta^{1,3}_{2,4}}$. Now $\det\varphi = \frac{(\Delta^{2,3}_{2,4} + \Delta^{1,3}_{2,4})^6 (\Delta^{2,4}_{1,3})^2 b^6_6}{(\Delta^{1,3}_{2,4})^2}$ and hence the last equation $24|5$ simply amounts to $\Delta^{1,3}_{2,4} = \Delta^{2,4}_{1,3}$. As b^3_2, b^1_2 cannot simultaneously vanish, one has 2 subcases. Subcase 2.1. $b^3_2 \neq 0$. From $24|5$, $b^1_4 = \frac{b^4_1 b^2_3 - b^4_3 b^2_1 + b^3_4 b^1_2}{b^3_2}$. Then we get the third kind. Subcase 2.2. $b^3_2 = 0, b^1_2 \neq 0$. From $24|5$, $b^3_4 = \frac{b^4_3 b^2_1 - b^4_1 b^2_3 +}{b^1_2}$. Then we get the fourth kind. $\square$

We get for $\tilde{\mathfrak{g}}_\delta$ 15 nonisomorphic algebras:

$\mathfrak{g}_{5,1} \times \mathbb{C}$. **Table 1.** $\xi^1_2 \neq 0$.
Then one may suppose $\xi^1_2 = 1,\ \xi^1_3 = 0,\ \xi^1_4 = 0,\ \xi^6_2 = 0,\ \xi^6_3 = 0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi^2_4 \neq 0$ (N1)	$\xi^2_4 = 1$, $\xi^6_4 = 0$	$\xi^5_6 \neq 0$	$\xi^5_6 = 1$, $\xi^6_1 = 0, 1$	$(1,0,0;1;1;\varepsilon,0,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_1)$	$\underline{\mathfrak{g}_{7,2.10}}$ $(\varepsilon = 0)$ (I) $\underline{\mathfrak{g}_{7,1.1(ii)}}$ $(\varepsilon = 1)$ (I2)
		$\xi^5_6 = 0$	$\xi^6_1 = 1$	$(1,0,0;1;0;1,0,0,0)$	$\underline{\mathfrak{g}_{7,2.4}}$ (II)
$\xi^2_4 = 0$ (N2)		$\xi^5_6 \neq 0$	$\xi^5_6 = 1$, $\xi^6_1 = 0,1$ $\xi^6_4 = 0,1$	$(1,0,0;0;1;\varepsilon,0,0,\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi^6_1, \eta = \xi^6_4)$	$\underline{\mathfrak{g}_{7,3.1(iii)}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (III) $\underline{\mathfrak{g}_{7,2.1(ii)}}$ $(\varepsilon = 1\ ,\ \eta = 0)$ $(\varepsilon = 0\ ,\ \eta = 1)$ (III2,3) * $\underline{\mathfrak{g}_{7,1.3(iii)}}$ $(\varepsilon = 1\ ,\ \eta = 1)$ (III4)
		$\xi^5_6 = 0$, $\xi^6_1 \neq 0$	$\xi^6_1 = 1$, $\xi^6_4 = 0,1$	$(1,0,0;0;0;1,0,0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_4)$	$\underline{\mathfrak{g}_{7,3.22}}$ $(\varepsilon = 0)$ (IV)† $\underline{\mathfrak{g}_{7,2.12}}$ $(\varepsilon = 1)$ (IV2)
		$\xi^5_6 = 0$, $\xi^6_1 = 0$	$\xi^6_4 = 1$	$(1,0,0;0;0;0,0,0,1)$	$\mathfrak{g}_{7,3.22}$† (V)

$\mathfrak{g}_{5,1} \times \mathbb{C}$. **Table 2.** $\xi^1_2 = 0,\ \xi^1_3 \neq 0$ **(N31).**
Then one may suppose $\xi^1_3 = 1,\ \xi^1_4 = 0,\ \xi^6_3 = 0,\ \xi^2_4 = 0$ ‡. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi^5_6 \neq 0$	$\xi^5_6 = 1$	$\xi^6_2 \neq 0$	$\xi^6_2 = 1$, $\xi^6_4 = 0$, $\xi^6_1 = 0,1$	$(0,1,0;0;1;\varepsilon,1,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_1)$	$\underline{\mathfrak{g}_{7,2.28}}$ $(\varepsilon = 0)$ (X) § $\underline{\mathfrak{g}_{7,2.30}}$ $(\varepsilon = 1)$ (X2) ¶
		$\xi^6_2 = 0$	$\xi^6_1 = 0,1$ $\xi^6_4 = 0,1$	$(0,1,0;0;1;\varepsilon,0,0,\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi^6_1, \eta = \xi^6_4)$	$\underline{\mathfrak{g}_{7,3.18}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (XI) ‖ $\mathfrak{g}_{7,2.30}$ $(\varepsilon = 1\ ,\ \eta = 0)$ $(\varepsilon = 1\ ,\ \eta = 1)$ (XI2,4) ¶ $\mathfrak{g}_{7,2.28}$ $(\varepsilon = 0\ ,\ \eta = 1)$ (XI3) §
$\xi^5_6 = 0$	$\longrightarrow\longrightarrow$	$\xi^6_2 \neq 0$	$\xi^6_2 = 1$, $\xi^6_4 = 0$, $\xi^6_1 = 0,1$	$(0,1,0;0;0;\varepsilon,1,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_1)$	$\underline{\mathfrak{g}_{7,3.15}}$ $(\varepsilon = 0)$ (XII) ** $\underline{\mathfrak{g}_{7,3.13}}$ $(\varepsilon = 1)$ (XII2) ††
		$\xi^6_2 = 0$, $\xi^6_1 \neq 0$	$\xi^6_1 = 1$, $\xi^6_4 = 0,1$	$(0,1,0;0;0;1,0,0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi^6_4)$	$\mathfrak{g}_{7,3.13}$ (XIII,XIII2) ††
		$\xi^6_2 = 0$, $\xi^6_1 = 0$	$\xi^6_4 = 1$	$(0,1,0;0;0;0,0,0,1)$	$\mathfrak{g}_{7,3.15}$ (XIV) **

* These two cases are proj. equiv.
† These two cases are proj. equiv.
‡ The case $\xi^2_4 \neq 0$ is proj. equiv. to some case $\xi^1_2 \neq 0, \xi^2_4 = 0$ (N31).
§ These 2 cases are proj. equiv.
¶ These 3 cases are proj. equiv.
‖ These 2 cases are proj. equiv.
** These 2 cases are proj. equiv.
†† These 2 cases are proj. equiv.

$\mathfrak{g}_{5,1} \times \mathbb{C}$. **Table 3.** $\xi_2^1 = 0,\ \xi_3^1 = 0$.
Then one may suppose [‡‡]**:** $\xi_4^2 = 0$, **(N4)** $\xi_4^1 = 0$ **(N51). Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^6 \neq 0$ (N6)	$\xi_2^6 = 1$, $\xi_1^6 = 0$, $\xi_3^6 = 0$	$\longrightarrow\longrightarrow$	$\xi_4^6 = 0$ $\xi_6^5 = 0, 1$	$(0,0,0;0;\varepsilon;0,1,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_6^5)$	$\underline{\mathfrak{g}_{7,4.3}}$ $(\varepsilon = 0)$ (XIX) $\mathfrak{g}_{7,3.18}$ $(\varepsilon = 1)$ (XIX2) [‖]
$\xi_2^6 = 0$	$\xi_1^6 = 0$, $\xi_3^6 = 0$	$\longrightarrow\longrightarrow$	$\xi_4^6 = 0$, $\xi_6^5 = 1$	$(0,0,0;0;1;0,0,0,0)$	$\underline{\mathfrak{g}_{7,4.4}}$ (XXII)

24 $\mathfrak{g}_{5,2} \times \mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,2} \times \mathbb{C}$: $[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & \xi_2^2 & \xi_3^2 & \xi_6^4 \\ 0 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_3^3 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & 0 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_2^2 & \xi_3^2 \\ \xi_2^3 & \xi_3^3 \end{pmatrix} \text{ nilpotent.} \tag{24}$$

Aut $(\mathfrak{g}_{5,2} \times \mathbb{C})$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & b_3^2 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_3^3 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_3^4 & b_1^1 b_2^2 & b_1^1 b_3^2 & b_6^4 \\ b_1^5 & b_2^5 & b_3^5 & b_1^1 b_2^3 & b_1^1 b_3^3 & b_6^5 \\ b_1^6 & b_2^6 & b_3^6 & 0 & 0 & b_6^6 \end{pmatrix} \tag{25}$$

where $(b_1^1 b_2^2 - b_2^1 b_1^2) b_1^1 b_6^6 \neq 0$. By Jordan reduction we get 2 cases : case 1. $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$, case 2. $A = 0$. We will denote δ by $(\xi_1^2, \xi_1^3, \xi_1^6; \xi_2^5, \xi_2^6; \xi_3^4, \xi_3^5, \xi_3^6; \xi_6^4, \xi_6^5)$ (resp. $[\xi_1^2, \xi_1^3, \xi_1^6; \xi_2^5, \xi_2^6; \xi_3^4, \xi_3^5, \xi_3^6; \xi_6^4, \xi_6^5]$ in case 1 (resp. 2). The cases where $\xi_6^4 = \xi_6^5 = \xi_1^6 = \xi_2^6 = \xi_3^6 = 0$ yield for $\tilde{\mathfrak{g}}_\delta$ a direct product by $\mathbb{C}x_7$, hence are dismissed. We get finally for $\tilde{\mathfrak{g}}_\delta$ 40 nonisomorphic algebras :

[‡‡] The case $\xi_4^2 \neq 0$ is proj. equiv. to some above case $\xi_3^1 \neq 0, \xi_4^2 = 0$ (N4), and the case $\xi_4^1 \neq 0$ is proj. equiv. to some above case $\xi_2^1 \neq 0, \xi_3^1 = \xi_4^1 = \xi_4^2 = 0$ (N51).

$\mathfrak{g}_{5,2} \times \mathbb{C}$. **Table 1. Case 1 :** $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. **Then one may suppose** $\xi_1^2 = \xi_3^4 = \xi_3^5 = \xi_3^6 = \xi_6^4 = 0$. **Then:**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_2^6 \neq 0$ (case1N1)	$\xi_2^6 = 1$, $\xi_1^6 = 0$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_2^5 = 0,1$, $\xi_6^5 = 0,1$	$(0,1,0;\varepsilon,1;0,0,0;0,\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_2^5,\ \eta = \xi_6^5)$	$\underline{\mathfrak{g}_{7,2.8}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (case1I) $\underline{\mathfrak{g}_{7,1.13}}$ $(\varepsilon = 1\ ,\ \eta = 0)$ (case1I2) $\underline{\mathfrak{g}_{7,1.4}}$ $(\varepsilon = 0\ ,\ \eta = 1)$ (case1I3) $\underline{\mathfrak{g}_{7,0.3}}$ $(\varepsilon = 1\ ,\ \eta = 1)$ (case1I4)
		$\xi_1^3 = 0$ (case1N2)	$\xi_2^5 = 0,1$, $\xi_6^5 = 0,1$	$(0,0,0;\varepsilon,1;0,0,0;0,\eta)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_2^5,\ \eta = \xi_6^5)$	$\underline{\mathfrak{g}_{7,3.5}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (case1II) $\underline{\mathfrak{g}_{7,2.24}}$ $(\varepsilon = 1\ ,\ \eta = 0)$ (case1II2) $\underline{\mathfrak{g}_{7,2.15}}$ $(\varepsilon = 0\ ,\ \eta = 1)$ (case1II3) $\underline{\mathfrak{g}_{7,1.01(ii)}}$ $(\varepsilon = 1\ ,\ \eta = 1)$ (case1II4)
$\xi_2^6 = 0$ (case1N3)	$\longrightarrow\longrightarrow$	$\xi_1^3 \neq 0$	$\xi_1^3 = 1$, $\xi_1^6 = 0$, $\xi_2^5 = 0,1$, $\xi_6^5 = 0,1$	$(0,1,0;\varepsilon,0;0,0,0;0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,2.41}}$ $(\varepsilon = 0)$ (case1III) $\underline{\mathfrak{g}_{7,1.12}}$ $(\varepsilon = 1)$ (case1III2)
		$\xi_1^3 = 0$, $\xi_6^5 \neq 0$	$\xi_6^5 = 1$, $\xi_2^5 = 0,1$, $\xi_1^6 = 0,1$	$(0,0,\varepsilon;\eta,0;0,0,0;0,1)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^6,\ \eta = \xi_2^5)$	$\underline{\mathfrak{g}_{7,3.10}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (case1IV) $\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = 1)$ $(\varepsilon = 1\ ,\ \eta = 0)$ (case1IV2) $\underline{\mathfrak{g}_{7,2.25}}$ $(\varepsilon = 0\ ,\ \eta = 1)$ (case1IV3) $\underline{\mathfrak{g}_{7,1.3(i_\lambda)}}$ $(\lambda = 0)$ $(\varepsilon = 1\ ,\ \eta = 1)$ (case1IV4)
		$\xi_1^3 = 0$, $\xi_6^5 = 0$	$\xi_2^5 = 0,1$, $\xi_1^6 = 1$	$(0,0,1;\varepsilon,0;0,0,0;0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,3.1(i_\lambda)}}$ $(\lambda = 0)$ $(\varepsilon = 0)$ (case1V) $\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = 0)$ $(\varepsilon = 1)$ (case1V2)

$\mathfrak{g}_{5,2}\times\mathbb{C}$. **Table 2. Case 2 :** $A=0$. **Subcase 2.1 :** $\begin{pmatrix}\xi_1^2\\ \xi_1^3\end{pmatrix}\neq 0$ **(case2N1).**

Then one may suppose $\xi_1^2=1$, $\xi_1^3=0$, $\xi_1^6=0$. **Then**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_2^6\neq 0$	$\xi_2^6=1$, $\xi_2^5=0$, $\xi_3^5=0$, $\xi_3^6=0$	$\xi_6^4\neq 0$	$\xi_6^4=1$, $\xi_3^4=0$, $\xi_6^5=0,1$	$[1,0,0;0,1;0,0,0;1,\varepsilon]$ $\varepsilon=0,1$ $(\varepsilon=\xi_6^5)$	$\underline{\mathfrak{g}_{7,2.22}}$ $(\varepsilon=0)$ (case2I) $\underline{\mathfrak{g}_{7,1.18}}$ $(\varepsilon=1)$ (case2I2)
		$\xi_6^4=0$	$\xi_3^4=0,1$ $\xi_6^5=0,1$	$[1,0,0;0,1;\varepsilon,0,0;0,\eta]$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_3^4,\eta=\xi_6^5)$	$\underline{\mathfrak{g}_{7,3.23}}$ $(\varepsilon=0\ ,\ \eta=0)$ (case2II) $\underline{\mathfrak{g}_{7,2.40}}$ $(\varepsilon=1\ ,\ \eta=0)$ (case2II2) $\underline{\mathfrak{g}_{7,2.18}}$ $(\varepsilon=0\ ,\ \eta=1)$ (case2II3) $\underline{\mathfrak{g}_{7,1.9}}$ $(\varepsilon=1\ ,\ \eta=1)$ (case2II4)
$\xi_2^6=0$, $\xi_6^5\neq 0$ (case2N2)	$\xi_6^5=1$, $\xi_3^5=0$, $\xi_6^4=0$	$\xi_3^6\neq 0$	$\xi_3^6=1$ $\xi_2^5=0$ $\xi_3^4=0$	$[1,0,0;0,0;0,0,1;0,1]$	$\underline{\mathfrak{g}_{7,2.42}}$ (case2III)
		$\xi_3^6=0$	$\xi_2^5=0,1$ $\xi_3^4=0,1$	$[1,0,0;\varepsilon,0;\eta,0,0;0,1]$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^5,\eta=\xi_3^4)$	$\underline{\mathfrak{g}_{7,3.11}}$ $(\varepsilon=0\ ,\ \eta=0)$ (case2IV) $\underline{\mathfrak{g}_{7,2.27}}$ $(\varepsilon=1\ ,\ \eta=0)$ (case2IV2) $\underline{\mathfrak{g}_{7,2.45}}$ $(\varepsilon=0\ ,\ \eta=1)$ (case2IV3) $\underline{\mathfrak{g}_{7,2.38}}$ $(\varepsilon=1\ ,\ \eta=1)$ (case2IV4)
$\xi_2^6=0$, $\xi_6^5=0$, $\xi_3^6\neq 0$ (case2N22)	$\xi_3^6=1$, $\xi_3^4=0$, $\xi_3^5=0$, $\xi_2^5=0,1$ $\xi_6^4=0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$[1,0,0;\varepsilon,0;0,0,1;\eta,0]$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^5,\eta=\xi_6^4)$	$\underline{\mathfrak{g}_{7,3.6}}$ $(\varepsilon=0\ ,\ \eta=0)$ (case2IVbis) $\mathfrak{g}_{7,2.40}$ $(\varepsilon=1\ ,\ \eta=0)$ (case2IVbis2) $\underline{\mathfrak{g}_{7,2.44}}$ $(\varepsilon=0\ ,\ \eta=1)$ (case2IVbis3) $\underline{\mathfrak{g}_{7,1.7}}$ $(\varepsilon=1\ ,\ \eta=1)$ (case2IVbis4)
$\xi_2^6=0$, $\xi_6^5=0$, $\xi_3^6=0$ (case2N22)	$\xi_6^4=1$, $\xi_3^4=0$	$\xi_2^5\neq 0$	$\xi_2^5=1$, $\xi_3^5=0$	$[1,0,0;1,0;0,0,0;1,0]$	$\mathfrak{g}_{7,2.38}$ (case2IVter)
		$\xi_2^5=0$	$\xi_3^5=0,1$	$[1,0,0;0,0;0,\varepsilon,0;1,0]$ $\varepsilon=0,1$ $(\varepsilon=\xi_3^5)$	$\underline{\mathfrak{g}_{7,3.8}}$ $(\varepsilon=0)$ (case2IVquater) $\underline{\mathfrak{g}_{7,3.9}}$ $(\varepsilon=1)$ (case2IVquater2)

$\mathfrak{g}_{5,2} \times \mathbb{C}$. **Table 3. Case 2 :** $A=0$. **Subcase 2.2 :** $\begin{pmatrix} \xi_1^2 \\ \xi_1^3 \end{pmatrix} = 0$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\begin{pmatrix} \xi_6^4 \\ \xi_6^5 \end{pmatrix} \neq 0$ (case2N3)	$\xi_6^4 = 1$, $\xi_6^5 = 0$, $\xi_3^4 = 0$, $\xi_3^5 = 0$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_1^6 = 0$, $\xi_2^5 = 0$, $\xi_3^6 = 0$	$[0,0,0;0,1;0,0,0;1,0]$	$\underline{\mathfrak{g}_{7,3.7}}$ (case2V)
		$\xi_2^6 = 0$ $\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_1^6 = 0$, $\xi_2^5 = 0,1$	$[0,0,0;\varepsilon,0;0,0,1;1,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^5)$	$\underline{\mathfrak{g}_{7,3.15}}$ $(\varepsilon = 0)$ (case2VI) $\underline{\mathfrak{g}_{7,2.36}}$ $(\varepsilon = 1)$ (case2VI2)
		$\xi_2^6 = 0$ $\xi_3^6 = 0$	$\xi_1^6 = 0,1$ $\xi_2^5 = 0,1$	$[0,0,\varepsilon;\eta,0;0,0,0;1,0]$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^6, \eta = \xi_2^5)$	$\underline{\mathfrak{g}_{7,4.3}}$ $(\varepsilon = 0, \eta = 0)$ (case2VII) $\mathfrak{g}_{7,3.9}$ $(\varepsilon = 1, \eta = 0)$ (case2VII2) $\underline{\mathfrak{g}_{7,3.19}}$ $(\varepsilon = 0, \eta = 1)$ (case2VII3) $\mathfrak{g}_{7,2.45}$ $(\varepsilon = 1, \eta = 1)$ (case2VII4)
$\begin{pmatrix} \xi_6^4 \\ \xi_6^5 \end{pmatrix} = 0$ (case2N4)	$\longrightarrow\longrightarrow$	$\begin{pmatrix} \xi_2^6 \\ \xi_3^6 \end{pmatrix} \neq 0$	$\xi_2^6 = 1$, $\xi_3^6 = 0$, $\xi_1^6 = 0$, $\xi_2^5 = 0$, $\xi_3^5 = 0$, $\xi_3^4 = 0,1$	$[0,0,0;0,1;\varepsilon,0,0;0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^4)$	$\underline{\mathfrak{g}_{7,4.1}}$ $(\varepsilon = 0)$ (case2VIII) $\underline{\mathfrak{g}_{7,3.12}}$ $(\varepsilon = 1)$ (case2VIII2)
		$\begin{pmatrix} \xi_2^6 \\ \xi_3^6 \end{pmatrix} = 0$ ${\xi_3^5}^2 \neq -4\xi_3^4\xi_2^5$	$\xi_1^6 = 1$, $\xi_3^4 = 0$, $\xi_2^5 = 0$, $\xi_3^5 = 0,1$	$[0,0,1;0,0;0,\varepsilon,0;0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,4.2}}$ $(\varepsilon = 0)$ (case2IX) $\mathfrak{g}_{7,4.1}$ $(\varepsilon = 1)$ (case2IX2)
		$\begin{pmatrix} \xi_2^6 \\ \xi_3^6 \end{pmatrix} = 0$ ${\xi_3^5}^2 = -4\xi_3^4\xi_2^5$	$\xi_1^6 = 1$, $\xi_3^4 = 0$, $\xi_2^5 = 1$, $\xi_3^5 = 0,1$	$[0,0,1;1,0;0,\varepsilon,0;0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.24}}$ $(\varepsilon = 0)$ (case2X) $\mathfrak{g}_{7,4.1}$ $(\varepsilon = 1)$ (case2X2)

25 $\mathfrak{g}_{5,3} \times \mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,3} \times \mathbb{C}$: $[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_5$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi_1^2 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^5 & -\xi_1^3 & 0 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & 0 \end{pmatrix} \tag{26}$$

Aut $(\mathfrak{g}_{5,3} \times \mathbb{C})$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & {b_1^1}^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & -b_1^1 b_1^2 & b_1^1 b_2^2 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_1^1 b_2^4 + b_1^2 b_2^3 - b_2^2 b_1^3 & {b_1^1}^2 b_2^2 & b_6^5 \\ b_1^6 & b_2^6 & b_3^6 & 0 & 0 & b_6^6 \end{pmatrix} \tag{27}$$

where $b_1^1 b_2^2 b_6^6 \neq 0$. We will denote δ by $(\xi_1^2; \xi_1^3, \xi_2^3; \xi_3^5, \xi_6^5; \xi_1^6, \xi_2^6, \xi_3^6)$. The cases where $\xi_6^5 = \xi_1^6 = \xi_2^6 = \xi_3^6 = 0$ yield for $\tilde{\mathfrak{g}}_\delta$ a direct product by $\mathbb{C}x_7$, hence are dismissed.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 23 nonisomorphic algebras :

$\mathfrak{g}_{5,3} \times \mathbb{C}$. (Continues on next page)

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_1^3 = 0$, $\xi_1^6 = 0$, $\xi_3^5 = 0$	$\xi_2^3 \neq 0$	$\xi_2^3 = 1$, $\xi_2^6 = 0$	$\xi_6^5 \neq 0$	$\xi_6^5 = 1$, $\xi_3^6 = 0,1$	$(1;0,1;0,1;0,0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^6)$	$\mathfrak{g}_{7,1.1(vi)}$ $(\varepsilon = 0)$ (I) $\mathfrak{g}_{7,0.7}$ $(\varepsilon = 1)$ (I2)
				$\xi_6^5 = 0$	$\xi_3^6 = 0,1$	$(1;0,1;0,0;0,0,1)$	$\mathfrak{g}_{7,1.14}$ (II)
		$\xi_2^3 = 0$, $\xi_6^5 \neq 0$	$\xi_6^5 = 1$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$	$(1;0,0;0,1;0,a,1)$ $a \in \mathbb{C}$ $(a = \xi_2^6)$	$\mathfrak{g}_{7,1.2(i_\lambda)}$ * $(a \neq -1,0)$ (III) $\mathfrak{g}_{7,1.2(ii)}$ $(a = -1)$ (III2) $\mathfrak{g}_{7,1.2(iv)}$ $(a = 0)$ (III3)
				$\xi_3^6 = 0$	$\xi_2^6 = 0,1$	$(1;0,0;0,1;0,\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\mathfrak{g}_{7,2.1(iii)}$ $(\varepsilon = 0)$ (IV) $\mathfrak{g}_{7,1.2(i_\lambda)}$ $(\lambda = 0)$ $(\varepsilon = 1)$ (IV2)
		$\xi_2^3 = 0$, $\xi_6^5 = 0$	$\longrightarrow\longrightarrow$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$ $\xi_2^6 = 0,1$	$(1;0,0;0,0;0,\varepsilon,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\mathfrak{g}_{7,2.24}$ $(\varepsilon = 0)$ (V) $\mathfrak{g}_{7,1.2(iii)}$ $(\varepsilon = 1)$ (V2)
				$\xi_3^6 = 0$	$\xi_2^6 = 1$	$(1;0,0;0,0;0,1,0)$	$\mathfrak{g}_{7,2.17}$ (VI)
$\xi_1^2 = 0$ $\xi_2^3 \neq 0$ (N2)	$\xi_2^3 = 1$, $\xi_1^3 = 0$, $\xi_2^6 = 0$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_3^5 = 0$, $\xi_1^6 = 0$, $\xi_6^5 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$(0;0,1;0,\varepsilon;0,0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_6^5)$	direct product $(\varepsilon = 0)$ (VII) $\mathfrak{g}_{7,3.1(iii)}$ $(\varepsilon = 1)$ (VII2)
		$\xi_3^6 = 0$	$\longrightarrow\longrightarrow$	$\xi_6^5 \neq 0$, $\xi_1^6 \neq 0$	$\xi_6^5 = 1$, $\xi_1^6 = 1$	$(0;0,1;a,1;1,0,0)$ $a \in \mathbb{C}$ $(a = \xi_3^5)$	$\mathfrak{g}_{7,3.1(i_\lambda)}$ † $(4a^3 + 27 \neq 0)$ (VIII) $\mathfrak{g}_{7,2.2}$ $(4a^3 + 27 = 0)$ (VIII2)
				$\xi_6^5 \neq 0$, $\xi_1^6 = 0$	$\xi_3^5 = 0,1$	$(0;0,1;\varepsilon,1;0,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\mathfrak{g}_{7,2.28}$ $(\varepsilon = 0)$ (IX) $\mathfrak{g}_{7,3.1(iii)}$ $(\varepsilon = 1)$ (IX2)
				$\xi_6^5 = 0$	$\xi_1^6 = 1$, $\xi_3^5 = 0,1$	$(0;0,1;a,0;1,0,0)$ $a \in \mathbb{C}$ $(a = \xi_3^5)$	$\mathfrak{g}_{7,3.1(i_\lambda)}$ $(\lambda = 1)$ $(a \neq 0)$ (X) $\mathfrak{g}_{7,2.44}$ $(a = 0)$ (X2)

* $\lambda = 1 + 2a + 2\sqrt{a(a+1)}$. Note that $\lambda \neq -1, 1$ from $a \neq -1, 0$. Changing the sign of the square root changes λ to $\frac{1}{\lambda}$ which is in line with the fact that $\mathfrak{g}_{7,1.2(i_{\lambda'})} \cong \mathfrak{g}_{7,1.2(i_\lambda)} \Leftrightarrow \lambda' = \lambda, \frac{1}{\lambda}$.

† $\lambda = -\lambda_1 \lambda_2{}^2 \frac{(9\lambda_2{}^2 + 2a^2\lambda_2 + 6a)(2a\lambda_2 + 3)}{(\lambda_2 - \lambda_3)^2(2a\lambda_1 + 3)^2(a\lambda_2 + 1)}$ where $\lambda_1, \lambda_2, \lambda_3$ are the 3 roots of $X^3 + aX + 1 = 0$.

$\mathfrak{g}_{5,3} \times \mathbb{C}$. (Continued)

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 = 0$ $\xi_2^3 = 0$ (N3)	$\longrightarrow\!\!\longrightarrow$	$\xi_3^6 \neq 0$	$\xi_3^6 = 1$, $\xi_3^5 = 0$, $\xi_1^6 = 0$, $\xi_2^6 = 0$, $\xi_6^5 = 0,1$, $\xi_1^3 = 0,1$	$\longrightarrow\!\!\longrightarrow$	$\longrightarrow\!\!\longrightarrow$	$(0;\varepsilon,0;0,\eta;0,0,1)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_1^3, \eta = \xi_6^5)$	$\underline{\mathfrak{g}_{7,3.7}}$ $(\varepsilon = 0, \eta = 0)$ (XI) $\underline{\mathfrak{g}_{7,2.26}}$ $(\varepsilon = 1, \eta = 0)$ (XI2) $\underline{\mathfrak{g}_{7,2.23}}$ $(\varepsilon = 0, \eta = 1)$ (XI3) $\mathfrak{g}_{7,1.2(iv)}$ $(\varepsilon = 1, \eta = 1)$ (XI4)
		$\xi_3^6 = 0$, $\xi_1^3 \neq 0$ (N4)	$\xi_1^3 = 1$, $\xi_3^5 = 0$	$\xi_2^6 \neq 0$ $\xi_6^5 \neq -2$	$\xi_2^6 = 1$, $\xi_1^6 = 0$	$(0;1,0;0,a;0,1,0)$ $a \in \mathbb{C}, a \neq -2$ $(a = \xi_6^5)$	$\mathfrak{g}_{7,3.1(i_\lambda)}$ ‡ $(a \neq \frac{1}{4})$ (XII,XII2) $\mathfrak{g}_{7,2.2}$ $(a = \frac{1}{4})$ (XII3)
				$\xi_2^6 \neq 0$ $\xi_6^5 = -2$	$\xi_1^6 = 0,1$	$(0;1,0;0,-2;\varepsilon,1,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\mathfrak{g}_{7,3.1(i_\lambda)}$ $(\lambda = 2)$ $(\varepsilon = 0)$ (XIII) $\mathfrak{g}_{7,2.2}$ $(\varepsilon = 1)$ (XIII2)
				$\xi_2^6 = 0$	$\xi_1^6 = 0$, $\xi_6^5 = 1$	$(0;1,0;0,1;0,0,0)$	$\mathfrak{g}_{7,3.1(iii)}$ (XIV)
		$\xi_3^6 = 0$, $\xi_1^3 = 0$ (N5)	$\longrightarrow\!\!\longrightarrow$	$\xi_2^6 \neq 0$ $\xi_6^5 \neq 0$	$\xi_2^6 = 1$, $\xi_1^6 = 0$, $\xi_6^5 = 1$, $\xi_3^5 = 0$	$(0;0,0;0,1;0,1,0)$	$\mathfrak{g}_{7,3.1(iii)}$ (XV)
				$\xi_2^6 \neq 0$ $\xi_6^5 = 0$	$\xi_2^6 = 1$, $\xi_1^6 = 0$, $\xi_3^5 = 0,1$	$(0;0,0;\varepsilon,0;0,1,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.9}}$ $(\varepsilon = 0)$ (XVI) $\underline{\mathfrak{g}_{7,3.15}}$ $(\varepsilon = 1)$ (XVI2)
				$\xi_2^6 = 0$ $\xi_6^5 \neq 0$	$\xi_6^5 = 1$, $\xi_3^5 = 0$, $\xi_1^6 = 0,1$	$(0;0,0;0,1;\varepsilon,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,3.18}}$ $(\varepsilon = 0)$ (XVII) $\mathfrak{g}_{7,2.28}$ $(\varepsilon = 1)$ (XVII2)
				$\xi_2^6 = 0$ $\xi_6^5 = 0$	$\xi_1^6 = 1$, $\xi_3^5 = 0,1$	$(0;0,0;\varepsilon,0;1,0,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.8}}$ $(\varepsilon = 0)$ (XVIII) $\underline{\mathfrak{g}_{7,3.14}}$ $(\varepsilon = 1)$ (XVIII2)

26 $\mathfrak{g}_{5,4} \times \mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,4} \times \mathbb{C}$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_2, x_3] = x_5$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 2\xi_1^1 + \xi_2^2 & \xi_2^1 & \xi_6^4 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_1^2 & \xi_1^1 + 2\xi_2^2 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_1^1 & \xi_2^1 \\ \xi_1^2 & \xi_2^2 \end{pmatrix} \text{ nilpotent.} \tag{28}$$

Aut $(\mathfrak{g}_{5,4} \times \mathbb{C})$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_1^1 b_2^2 - b_2^1 b_1^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_1^1 b_2^3 + b_2^1 b_1^3 & b_1^1(b_1^1 b_2^2 - b_2^1 b_1^2) & b_2^1(b_1^1 b_2^2 - b_2^1 b_1^2) & b_6^4 \\ b_1^5 & b_2^5 & b_1^2 b_2^3 + b_2^2 b_1^3 & b_1^2(b_1^1 b_2^2 - b_2^1 b_1^2) & b_2^2(b_1^1 b_2^2 - b_2^1 b_1^2) & b_6^5 \\ b_1^6 & b_2^6 & 0 & 0 & 0 & b_6^6 \end{pmatrix} \tag{29}$$

where $(b_1^1 b_2^2 - b_2^1 b_1^2) b_6^6 \neq 0$. By Jordan reduction we get 2 cases : case 1. $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$, case 2. $A = 0$. We will denote δ by $(\xi_2^4, \xi_6^4; \xi_1^5, \xi_2^5, \xi_6^5; \xi_1^6, \xi_2^6)$ (resp. $[\xi_2^4, \xi_6^4; \xi_1^5, \xi_2^5, \xi_6^5; \xi_1^6, \xi_2^6]$) in case 1 (resp. 2). As

‡ $\lambda = 1 - \frac{1+\sqrt{1-4a}}{2a}$ if $a \neq 0$; $\lambda = 0$ if $a = 0$.

$\mathbb{C} = \mathbb{C}x_7$, the cases where $\xi_6^4 = \xi_6^5 = \xi_1^6 = \xi_2^6 = 0$ yield for $\tilde{\mathfrak{g}}_\delta$ a direct product by $\mathbb{C}x_7$. Hence we discard those cases.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 13 nonisomorphic algebras :

$\mathfrak{g}_{5,4} \times \mathbb{C}$. **Table 1. Case 1 :** $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. **Then one may suppose** $\xi_2^4 = \xi_6^4 = \xi_2^5 = \xi_2^6 = 0$. **Then:**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^6 \neq 0$ (case1N1)	$\xi_1^6 = 1$	$\xi_1^5 \neq 0$	$\xi_1^5 = 1$	$(0,0;1,0,a;1,0)$ $a \in \mathbb{C}$ $(a = \xi_6^5)$	$\underline{\mathfrak{g}_{7,0.5}}$ $(a \neq 0)$ (I) $\underline{\mathfrak{g}_{7,1.14}}$ $(a = 0)$ (I2)
		$\xi_1^5 = 0$	$\xi_6^5 = 0,1,$	$(0,0;0,0,\varepsilon;1,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_6^5)$	$\underline{\mathfrak{g}_{7,2.6}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,1.17}}$ $(\varepsilon = 1)$ (II2)
$\xi_1^6 = 0$ (case1N2)	$\longrightarrow\longrightarrow$	$\xi_1^5 \neq 0$	$\xi_1^5 = 1,$ $\xi_6^5 = 1$	$(0,0;1,0,1;0,0)$	$\underline{\mathfrak{g}_{7,1.21}}$ (III)
		$\xi_1^5 = 0$	$\xi_6^5 = 1$	$(0,0;0,0,1;0,0)$	$\underline{\mathfrak{g}_{7,2.13}}$ (IV)

$\mathfrak{g}_{5,4} \times \mathbb{C}$. **Table 2. Case 2 :** $A = 0$. **Then:**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\begin{pmatrix} \xi_1^6 \\ \xi_2^6 \end{pmatrix} \neq 0$ (case2N1)	$\begin{pmatrix} \xi_1^6 \\ \xi_2^6 \end{pmatrix} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, $\xi_1^5 = 0$, $\xi_2^5 = 0$	$\xi_6^4 \neq 0$	$\xi_6^4 = 1, \xi_2^4 = 0$, $\xi_6^5 = 0$	$[0,1;0,0,0;1,0]$	$\underline{\mathfrak{g}_{7,2.34}}$ (V)
		$\xi_6^4 = 0$	$\xi_6^4 = 1,$ $\xi_2^4 = 0,1,$ $\xi_6^5 = 0,1,$	$[\varepsilon,0;0,0,\eta;1,0]$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_2^4, \eta = \xi_6^5)$	$\underline{\mathfrak{g}_{7,3.23}}$ $(\varepsilon = 0\ ,\ \eta = 0)$ (VI) $\underline{\mathfrak{g}_{7,2.40}}$ $(\varepsilon = 1\ ,\ \eta = 0)$ (VI2) $\underline{\mathfrak{g}_{7,2.12}}$ $(\varepsilon = 0\ ,\ \eta = 1)$ (VI3) $\underline{\mathfrak{g}_{7,1.3(iv)}}$ $(\varepsilon = 1\ ,\ \eta = 1)$ (VI4)
$\begin{pmatrix} \xi_1^6 \\ \xi_2^6 \end{pmatrix} = 0$ (case2N2)	$\begin{pmatrix} \xi_6^4 \\ \xi_6^5 \end{pmatrix} = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, $\xi_2^4 = 0$, $\xi_2^5 = 0$, $\xi_1^5 = 0,1$	$\longrightarrow\longrightarrow$	$\longrightarrow\longrightarrow$	$[0,1;\varepsilon,0,0;0,0]$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^5)$	$\underline{\mathfrak{g}_{7,3.13}}$ $(\varepsilon = 0)$ (VII) $\underline{\mathfrak{g}_{7,2.29}}$ $(\varepsilon = 1)$ (VII2)

27 $\mathfrak{g}_{5,5} \times \mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,5} \times \mathbb{C} : [x_1, x_2] = x_3;\ [x_1, x_3] = x_4;\ [x_1, x_4] = x_5.$

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^5 & \xi_2^4 & 0 & 0 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{30}$$

Aut $(\mathfrak{g}_{5,5} \times \mathbb{C})$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_1^1 b_2^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_1^1 b_2^3 & {b_1^1}^2 b_2^2 & 0 & 0 \\ b_1^5 & b_2^5 & b_1^1 b_2^4 & {b_1^1}^2 b_2^3 & {b_1^1}^3 b_2^2 & b_6^5 \\ b_1^6 & b_2^6 & 0 & 0 & 0 & b_6^6 \end{pmatrix} \tag{31}$$

where $b_1^1 b_2^2 b_6^6 \neq 0$. We will denote δ by $(\xi_1^2; \xi_2^4; \xi_2^5, \xi_6^5; \xi_1^6, \xi_2^6)$. As $\mathbb{C} = \mathbb{C}x_7$, the cases where $\xi_6^5 = \xi_1^6 = \xi_2^6 = 0$ yield for $\tilde{\mathfrak{g}}_\delta$ a direct product by $\mathbb{C}x_7$. Hence we discard those cases.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 16 nonisomorphic algebras :

$\mathfrak{g}_{5,5}\times\mathbb{C}$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_2^6\neq 0$ (N1)	$\xi_2^6=1$, $\xi_1^6=0$, $\xi_2^5=0$,	$\xi_1^2\neq 0$	$\xi_1^2=1$, $\xi_2^4=0,1$, $\xi_6^5=0,1$,	$(1;\varepsilon;0,\eta;0,1)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^4,\eta=\xi_6^5)$	$\underline{\mathfrak{g}_{7,2.7}}$ $(\varepsilon=0\ ,\ \eta=0)$ (I) $\underline{\mathfrak{g}_{7,1.13}}$ $(\varepsilon=1\ ,\ \eta=0)$ (I2) $\underline{\mathfrak{g}_{7,1.10}}$ $(\eta=1)$ (I3,I4)
		$\xi_1^2=0$	$\xi_2^4=0,1$, $\xi_6^5=0,1$,	$(0;\varepsilon;0,\eta;0,1)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^4,\eta=\xi_6^5)$	$\underline{\mathfrak{g}_{7,3.3}}$ $(\varepsilon=0\ ,\ \eta=0)$ (II) $\underline{\mathfrak{g}_{7,2.24}}$ $(\varepsilon=1\ ,\ \eta=0)$ (II2) $\underline{\mathfrak{g}_{7,2.33}}$ $(\eta=1)$ (II3,II4)
$\xi_2^6=0$ (N2)	$\longrightarrow\longrightarrow$	$\xi_1^2\neq 0$	$\xi_1^2=1$, $\xi_1^6=0$, $\xi_6^5=1$, $\xi_2^4=0,1$, $\xi_2^5=0,1$,	$(1;\varepsilon;\eta,1;0,0)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^4,\eta=\xi_2^5)$	$\underline{\mathfrak{g}_{7,2.16}}$ $(\varepsilon=0\ ,\ \eta=0)$ (III) $\underline{\mathfrak{g}_{7,1.12}}$ $(\varepsilon=1)$ (III2,III4) $\underline{\mathfrak{g}_{7,1.15}}$ $(\varepsilon=0\ ,\ \eta=1)$ (III3)
		$\xi_1^2=0$, $\xi_6^5\neq 0$	$\xi_6^5=1$, $\xi_2^5=0$, $\xi_2^4=0,1$, $\xi_1^6=0,1$	$(0;\varepsilon;0,1;\eta,0)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^4,\eta=\xi_1^6)$	$\underline{\mathfrak{g}_{7,3.17}}$ $(\varepsilon=0\ ,\ \eta=0)$ (IV) $\underline{\mathfrak{g}_{7,2.25}}$ $(\varepsilon=1\ ,\ \eta=0)$ (IV2) $\underline{\mathfrak{g}_{7,2.32}}$ $(\varepsilon=0\ ,\ \eta=1)$ (IV3) $\underline{\mathfrak{g}_{7,1.3(i_\lambda)}}$ $(\lambda=0)$ $(\varepsilon=1\ ,\ \eta=1)$ (IV4)
		$\xi_1^2=0$, $\xi_6^5=0$	$\xi_1^6=1$, $\xi_2^4=0,1$, $\xi_2^5=0,1$	$(0;\varepsilon;\eta,0;1,0)$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^4,\eta=\xi_2^5)$	$\underline{\mathfrak{g}_{7,3.2}}$ $(\varepsilon=0\ ,\ \eta=0)$ (V) $\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda=0)$ $(\varepsilon=1)$ (V2,V4) $\underline{\mathfrak{g}_{7,2.20}}$ $(\varepsilon=0\ ,\ \eta=1)$ (V3)

28 $\mathfrak{g}_{5,6}\times\mathbb{C}$

Commut. relations for $\mathfrak{g}_{5,6}\times\mathbb{C}$: $[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_2,x_3]=x_5$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta=\begin{pmatrix}0&0&0&0&0&0\\ \xi_1^2&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&\xi_2^4&0&0&0&0\\ 0&\xi_2^5&\xi_2^4&\xi_1^2&0&\xi_6^5\\ \xi_1^6&\xi_2^6&0&0&0&0\end{pmatrix}$$

We denote δ by $(\xi_1^2;\xi_2^4;\xi_2^5,\xi_6^5;\xi_1^6,\xi_2^6)$. Recall that the direct products are dismissed.

Aut $(\mathfrak{g}_{5,6}\times\mathbb{C})$ is comprised of the matrices

$$\Phi=\begin{pmatrix}b_1^1&0&0&0&0&0\\ b_1^2&{b_1^1}^2&0&0&0&0\\ b_1^3&b_2^3&{b_1^1}^3&0&0&0\\ b_1^4&b_2^4&b_2^3b_1^1&{b_1^1}^4&0&0\\ b_1^5&b_2^5&-b_1^3{b_1^1}^2+b_2^3b_1^2+b_2^4b_1^1&{b_1^1}^2(b_1^2b_1^1+b_2^3)&{b_1^1}^5&b_6^5\\ b_1^6&b_2^6&0&0&0&b_6^6\end{pmatrix}\tag{32}$$

where $b_1^1b_6^6\neq 0$. We get for $\tilde{\mathfrak{g}}_\delta$ 16 nonisomorphic algebras :

$\mathfrak{g}_{5,6} \times \mathbb{C}$.

if	toms	if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (N1)	$\xi_1^2 = 1$, $\xi_2^5 = 0$, $\xi_1^6 = 0$	$\xi_6^5 \neq 0$	$\xi_6^5 = 1$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$	$(1;1;0,1;0,a)$ $a \in \mathbb{C}$ $(a = \xi_2^6)$	$\underline{\mathfrak{g}_{7,0.7}}$ $(a \neq 0,-1)$ (I) $\underline{\mathfrak{g}_{7,1.1(vi)}}$ $(a = 0)$ (I2) $\underline{\mathfrak{g}_{7,1.03}}$ $(a = -1)$ (I3)
				$\xi_2^4 = 0$	$\xi_2^6 = 0,1$	$(1;0;0,1;0,\varepsilon)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^6)$	$\underline{\mathfrak{g}_{7,2.10}}$ $(\varepsilon = 0)$ (II) $\underline{\mathfrak{g}_{7,0.7}}$ $(\varepsilon = 1)$ (II2)
		$\xi_6^5 = 0$ (N2)	$\xi_2^4 = 0,1$, $\xi_2^6 = 1$	$\longrightarrow$	$\longrightarrow$	$(1;\varepsilon;0,0;0,1)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_2^4)$	$\underline{\mathfrak{g}_{7,2.6}}$ $(\varepsilon = 0)$ (III) $\underline{\mathfrak{g}_{7,1.14}}$ $(\varepsilon = 1)$ (III2)
$\xi_1^2 = 0$, $\xi_2^6 \neq 0$ (N3)	$\xi_2^6 = 1$, $\xi_2^5 = 0$, $\xi_1^6 = 0$, $\xi_2^4 = 0,1$, $\xi_6^5 = 0,1$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$(0;\varepsilon;0,\eta;0,1)$ $\varepsilon,\eta = 0,1$ $(\varepsilon = \xi_2^4,\ \eta = \xi_6^5)$	$\underline{\mathfrak{g}_{7,2.22}}$ $(\varepsilon = \eta = 0)$ (IV) $\underline{\mathfrak{g}_{7,2.24}}$ $(\varepsilon = 1, \eta = 0)$ (IV2) $\underline{\mathfrak{g}_{7,1.8}}$ $(\eta = 1)$ (IV3,IV4)
$\xi_1^2 = 0$, $\xi_2^6 = 0$ (N3)	$\longrightarrow$	$\xi_2^4 \neq 0$	$\xi_2^4 = 1$, $\xi_2^5 = 0$	$\xi_1^6 \neq 0$	$\xi_1^6 = 1$	$(0;1;0,a;1,0)$ $a \in \mathbb{C}$ $(a = \xi_6^5)$	$\underline{\mathfrak{g}_{7,1.3(i_\lambda)}}$ $(\lambda = -a)$ $(a \neq 0)$ (V) $\underline{\mathfrak{g}_{7,2.1(i_\lambda)}}$ $(\lambda = 0)$ $(a = 0)$ (V2)
				$\xi_1^6 = 0$	$\xi_6^5 = 1$	$(0;1;0,1;0,0)$	$\underline{\mathfrak{g}_{7,1.3(v)}}$ (VI)
		$\xi_2^4 = 0$	$\longrightarrow$	$\xi_6^5 \neq 0$	$\xi_6^5 = 1$, $\xi_2^5 = 0$	$(0;0;0,1;\varepsilon,0)$ $\varepsilon = 0,1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,2.30}}$ $(\varepsilon = 0)$ (VII) $\underline{\mathfrak{g}_{7,1.3(iii)}}$ $(\varepsilon = 1)$ (VII2)
				$\xi_6^5 = 0$, $\xi_2^5 \neq 0$	$\xi_2^5 = 1$, $\xi_1^6 = 1$	$(0;0;1,0;1,0)$	$\underline{\mathfrak{g}_{7,1.16}}$ (VIII)
				$\xi_6^5 = 0$, $\xi_2^5 = 0$	$\xi_1^6 = 1$	$(0;0;0,0;1,0)$	$\underline{\mathfrak{g}_{7,2.20}}$ (IX)

29 $\mathfrak{g}_4 \times \mathbb{C}^2$

Commut. relations for $\mathfrak{g}_4 \times \mathbb{C}^2$: $[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^4 & 0 & 0 & \xi_5^4 & \xi_6^4 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_5^5 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & 0 & 0 & \xi_5^6 & \xi_6^6 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_5^5 & \xi_6^5 \\ \xi_5^6 & \xi_6^6 \end{pmatrix} \text{ nilpotent.} \tag{33}$$

Aut $(\mathfrak{g}_4 \times \mathbb{C}^2)$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & 0 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_1^1 b_2^2 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & b_1^1 b_2^3 & {b_1^1}^2 b_2^2 & b_5^4 & b_6^4 \\ b_1^5 & b_2^5 & 0 & 0 & b_5^5 & b_6^5 \\ b_1^6 & b_2^6 & 0 & 0 & b_5^6 & b_6^6 \end{pmatrix} \tag{34}$$

where $(b_5^5 b_6^6 - b_5^6 b_6^5) b_1^1 b_2^2 \neq 0$. By Jordan reduction we get 2 cases :
case 1. $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$, case 2. $A = 0$. We will denote δ by $(\xi_1^2; \xi_2^4, \xi_5^4, \xi_6^4; \xi_1^5, \xi_2^5; \xi_1^6, \xi_2^6)$ (resp. $[\xi_1^2; \xi_2^4, \xi_5^4, \xi_6^4; \xi_1^5, \xi_2^5; \xi_1^6, \xi_2^6]$) in case 1 (resp. 2). We get finally for $\tilde{\mathfrak{g}}_\delta$ 17 nonisomorphic algebras :

$\mathfrak{g}_4 \times \mathbb{C}^2$. **Table 1. Case 1 :** $A = \left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$. **Then one may suppose** $\xi_6^4 = \xi_1^5 = \xi_2^5 = 0$. **Then:**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\xi_1^2 \neq 0$ (case1N1)	$\xi_1^2 = 1$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_2^4 = 0$, $\xi_1^6 = 0$, $\xi_5^4 = 0, 1$	$(1;0,\varepsilon,0;0,0;0,1)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_5^4$)	$\underline{\mathfrak{g}_{7,2.9}}$ ($\varepsilon = 0$) (case1I) $\underline{\mathfrak{g}_{7,1.10}}$ ($\varepsilon = 1$) (case1I2)
		$\xi_2^6 = 0$, $\xi_5^4 \neq 0$	$\xi_5^4 = 1$, $\xi_2^4 = 0$, $\xi_1^6 = 0, 1$	$(1;0,1,0;0,0;\varepsilon,0)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_1^6$)	$\underline{\mathfrak{g}_{7,2.31}}$ ($\varepsilon = 0$) (case1II) $\underline{\mathfrak{g}_{7,1.2(i_\lambda)}}$ ($\lambda = 1$) ($\varepsilon = 1$) (case1II2)
		$\xi_2^6 = 0$, $\xi_5^4 = 0$	$\xi_1^6 = 1$, $\xi_2^4 = 0, 1$	$(1;\varepsilon,0,0;0,0;1,0)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_2^4$)	$\underline{\mathfrak{g}_{7,2.11}}$ ($\varepsilon = 0$) (case1III) $\underline{\mathfrak{g}_{7,1.18}}$ ($\varepsilon = 1$) (case1III2)
$\xi_1^2 = 0$ (case1N2)	$\longrightarrow\!\!\!\longrightarrow$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_2^4 = 0$, $\xi_1^6 = 0$, $\xi_5^4 = 0, 1$	$(0;0,\varepsilon,0;0,0;0,1)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_5^4$)	$\underline{\mathfrak{g}_{7,3.4}}$ ($\varepsilon = 0$) (case1IV) $\underline{\mathfrak{g}_{7,2.33}}$ ($\varepsilon = 1$) (case1IV2)
		$\xi_2^6 = 0$, $\xi_5^4 \neq 0$	$\xi_5^4 = 1$, $\xi_2^4 = 0$, $\xi_1^6 = 0, 1$	$(0;0,1,0;0,0;\varepsilon,0)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_1^6$)	$\underline{\mathfrak{g}_{7,3.16}}$ ($\varepsilon = 0$) (case1V) $\underline{\mathfrak{g}_{7,2.31}}$ ($\varepsilon = 1$) (case1V2)
		$\xi_2^6 = 0$, $\xi_5^4 = 0$	$\xi_1^6 = 1$, $\xi_2^4 = 0, 1$	$(0;\varepsilon,0,0;0,0;1,0)$ $\varepsilon = 0,1$ ($\varepsilon = \xi_2^4$)	$\underline{\mathfrak{g}_{7,3.21}}$ ($\varepsilon = 0$) (case1VI) $\underline{\mathfrak{g}_{7,2.42}}$ ($\varepsilon = 1$) (case1VI2)

$\mathfrak{g}_4 \times \mathbb{C}^2$. **Table 2. Case 2 :** $A = 0$

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\begin{pmatrix}\xi_5^4\\ \xi_6^4\end{pmatrix} \neq 0$ (case2N1)	$\xi_5^4 = 0$, $\xi_6^4 = 1$, $\xi_2^4 = 0$	$\xi_2^6 \neq 0$	$\xi_2^6 = 1$, $\xi_2^5 = 0$, $\xi_1^6 = 0$, $\xi_1^2 = 0$, $\xi_1^5 = 1$	$[0;0,0,1;1,0;0,1]$	$\underline{\mathfrak{g}_{7,3.1(i_\lambda)}}$ ($\lambda = 0$) (case2I)
		$\xi_2^6 = 0$, $\xi_1^2 \neq 0$ (case2N2)	$\xi_1^2 = 1$, $\xi_1^5 = 0$, $\xi_2^5 = 1$, $\xi_1^6 = 0$	$[1;0,0,1;0,1;0,0]$	$\underline{\mathfrak{g}_{7,2.18}}$ (case2II)
		$\xi_2^6 = 0$, $\xi_1^2 = 0$, $\xi_2^5 \neq 0$ (case2N3)	$\xi_2^5 = 1$, $\xi_1^5 = 0$	$[0;0,0,1;0,1;\varepsilon,0]$ $\varepsilon = 0,1$ ($\varepsilon = \xi_1^6$)	$\underline{\mathfrak{g}_{7,3.15}}$ ($\varepsilon = 0$) (case2III) $\underline{\mathfrak{g}_{7,2.44}}$ ($\varepsilon = 1$) (case2III2)
		$\xi_2^6 = 0$, $\xi_1^2 = 0$, $\xi_2^5 = 0$	$\xi_1^6 = 0$, $\xi_1^5 = 1$	$[0;0,0,1;1,0;0,0]$	$\underline{\mathfrak{g}_{7,3.14}}$ (case2IV)
$\begin{pmatrix}\xi_5^4\\ \xi_6^4\end{pmatrix} = 0$ $\begin{pmatrix}\xi_2^5\\ \xi_2^6\end{pmatrix} \neq 0$ (case2N4)*	$\xi_2^5 = 0$, $\xi_2^6 = 1$, $\xi_2^4 = 0$, $\xi_1^6 = 0$	$\longrightarrow\!\!\!\longrightarrow$	$\xi_1^2 = 0$, $\xi_1^5 = 1$	$[0;0,0,0;1,0;0,1]$	$\underline{\mathfrak{g}_{7,3.6}}$ (case2V)

30 $\mathfrak{n} \times \mathfrak{n}$ ($\mathfrak{n}$ 3-dim. complex Heisenberg Lie algebra)

Commutation relations for $\mathfrak{g} = \mathfrak{n} \times \mathfrak{n}$: $[x_1, x_2] = x_5$; $[x_3, x_4] = x_6$.

*The case $\begin{pmatrix}\xi_5^4\\ \xi_6^4\end{pmatrix} = \begin{pmatrix}\xi_2^5\\ \xi_2^6\end{pmatrix} = 0$ (case2N5) is dismissed as it yields only direct products for $\tilde{\mathfrak{g}}_\delta$.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 \\ 0 & 0 & \xi_3^4 & \xi_4^4 & 0 & 0 \\ 0 & 0 & \xi_3^5 & \xi_4^5 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ with } A = \begin{pmatrix} \xi_1^1 & \xi_2^1 \\ \xi_1^2 & \xi_2^2 \end{pmatrix} \text{ and } B = \begin{pmatrix} \xi_3^3 & \xi_4^3 \\ \xi_3^4 & \xi_4^4 \end{pmatrix} \text{ nilpotent.} \tag{35}$$

The automorphisms of $\mathfrak{g} = \mathfrak{n} \times \mathfrak{n}$: fall into 2 kinds. The first kind is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3^3 & b_4^3 & 0 & 0 \\ 0 & 0 & b_3^4 & b_4^4 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_4^5 & b_1^1 b_2^2 - b_2^1 b_1^2 & 0 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & 0 & b_3^3 b_4^4 - b_4^3 b_3^4 \end{pmatrix} \tag{36}$$

where $(b_1^1 b_2^2 - b_2^1 b_1^2)(b_3^3 b_4^4 - b_4^3 b_3^4) \neq 0$. The second kind is comprised of the matrices

$$\Psi = \begin{pmatrix} 0 & 0 & b_3^1 & b_4^1 & 0 & 0 \\ 0 & 0 & b_3^2 & b_4^2 & 0 & 0 \\ b_1^3 & b_2^3 & 0 & 0 & 0 & 0 \\ b_1^4 & b_2^4 & 0 & 0 & 0 & 0 \\ b_1^5 & b_2^5 & b_3^5 & b_4^5 & 0 & b_3^1 b_4^2 - b_4^1 b_3^2 \\ b_1^6 & b_2^6 & b_3^6 & b_4^6 & b_1^3 b_2^4 - b_2^3 b_1^4 & 0 \end{pmatrix} \tag{37}$$

where $(b_3^1 b_4^2 - b_4^1 b_3^2)(b_1^3 b_2^4 - b_2^3 b_1^4) \neq 0$. By Jordan reduction using a first kind automorphism, we get 4 cases : case 1. $A = B = 0$; case 2. $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right), B = 0$; case 2'. $A = 0, B = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$; case 3. $A = B = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. Since cases 2 and 2' are equivalent by some second kind automorphism, we are left only with the 3 cases 1-3. We will denote δ by $(\xi_3^5, \xi_4^5; \xi_1^6, \xi_2^6)$ (resp. $[\xi_3^5, \xi_4^5; \xi_1^6, \xi_2^6]$, $\langle \xi_3^5, \xi_4^5; \xi_1^6, \xi_2^6 \rangle$) in case 1 (resp. 2, 3)

We get finally for $\tilde{\mathfrak{g}}_\delta$ 8 nonisomorphic algebras :

$\mathfrak{g} = \mathfrak{n} \times \mathfrak{n}$.

if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
case 1 (case1)	$\xi_3^5 = 1$, $\xi_4^5 = 0$ $\xi_1^6 = 0, 1$, $\xi_2^6 = 0$	$(1, 0; \varepsilon, 0)$ $\varepsilon = 0, 1$ $(\varepsilon = \xi_1^6)$	$\underline{\mathfrak{g}_{7,4.3}}$ $(\varepsilon = 0)$ (I) $\underline{\mathfrak{g}_{7,3.19}}$ $(\varepsilon = 1)$ (I2)
case 2 (case2)	$\xi_3^5 = 0, 1$, $\xi_4^5 = 0$ $\xi_1^6 = 0, 1$, $\xi_2^6 = 0$	$[\eta, 0; \varepsilon, 0]$ $\varepsilon = 0, 1$, $\eta = 0, 1$ $(\varepsilon = \xi_1^6$, $\eta = \xi_3^5)$	direct product $\mathfrak{g}_4 \times \mathfrak{n}$ $(\varepsilon = 0,\ \eta = 0)$ (II) $\underline{\mathfrak{g}_{7,3.7}}$ $(\varepsilon = 0, \eta = 1)$ (II2) $\underline{\mathfrak{g}_{7,3.13}}$ $(\varepsilon = 1, \eta = 0)$ (II3) $\underline{\mathfrak{g}_{7,2.29}}$ $(\varepsilon = 1, \eta = 1)$ (II4)
case 3 (case3)	$\xi_3^5 = 0, 1$, $\xi_4^5 = 0$ $\xi_1^6 = 0, 1$, $\xi_2^6 = 0$	$\langle \eta, 0; \varepsilon, 0 \rangle$ $\varepsilon = 0, 1$, $\eta = 0, 1$ $(\varepsilon = \xi_1^6$, $\eta = \xi_3^5)$	$\underline{\mathfrak{g}_{7,3.4}}$ $(\varepsilon = 0, \eta = 0)$ (III) $\underline{\mathfrak{g}_{7,2.34}}$ $(\varepsilon = 0, \eta = 1)$ (III2) or $(\varepsilon = 1, \eta = 0)$ (III3) $\underline{\mathfrak{g}_{7,1.19}}$ $(\varepsilon = 1,\ \eta = 1)$ (III4) *

31 $\mathfrak{n} \times \mathbb{C}^3$

Commut. relations for $\mathfrak{n} \times \mathbb{C}3$: $[x_1, x_2] = x_3$.

*Those 2 cases are proj. equiv. under some 2d kind automorphism.

Generic nilpotent derivation mod $ad(\mathfrak{g})$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_4^3 & \xi_5^3 & \xi_6^3 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_4^4 & \xi_5^4 & \xi_6^4 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_4^5 & \xi_5^5 & \xi_6^5 \\ \xi_1^6 & \xi_2^6 & 0 & \xi_4^6 & \xi_5^6 & \xi_6^6 \end{pmatrix} \tag{38}$$

with $A = \begin{pmatrix} \xi_1^1 & \xi_2^1 \\ \xi_1^2 & \xi_2^2 \end{pmatrix}$, $B = \begin{pmatrix} \xi_4^4 & \xi_5^4 & \xi_6^4 \\ \xi_4^5 & \xi_5^5 & \xi_6^5 \\ \xi_4^6 & \xi_5^6 & \xi_6^6 \end{pmatrix}$ nilpotent matrices.

Aut $(\mathfrak{n} \times \mathbb{C}^3)$ is comprised of the matrices

$$\Phi = \begin{pmatrix} b_1^1 & b_2^1 & 0 & 0 & 0 & 0 \\ b_1^2 & b_2^2 & 0 & 0 & 0 & 0 \\ b_1^3 & b_2^3 & b_1^1 b_2^2 - b_2^1 b_1^2 & b_4^3 & b_5^3 & b_6^3 \\ b_1^4 & b_2^4 & 0 & b_4^4 & b_5^4 & b_6^4 \\ b_1^5 & b_2^5 & 0 & b_4^5 & b_5^5 & b_6^5 \\ b_1^6 & b_2^6 & 0 & b_4^6 & b_5^6 & b_6^6 \end{pmatrix} \tag{39}$$

with $U = \begin{pmatrix} b_1^1 & b_2^1 \\ b_1^2 & b_2^2 \end{pmatrix}$, $V = \begin{pmatrix} b_4^4 & b_5^4 & b_6^4 \\ b_4^5 & b_5^5 & b_6^5 \\ b_4^6 & b_5^6 & b_6^6 \end{pmatrix}$ inversible matrices. By Jordan reduction we get 6 cases : case 1. $A = 0, B = 0$; case 2. $A = 0, B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; case 3. $A = 0, B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; and cases 4-6 analogous with $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. In case $\#i$, we will denote δ by $(\xi_1^4, \xi_2^4; \xi_1^5, \xi_2^5; \xi_1^6, \xi_2^6; \xi_4^3, \xi_5^3, \xi_6^3)_i$. As usual, we discard direct products.

We get finally for $\tilde{\mathfrak{g}}_\delta$ 25 nonisomorphic algebras :

$\mathfrak{n} \times \mathbb{C}^3$. **Table 1. Case 1 : $A = B = 0$. Then we get :**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta \cong$
$\longrightarrow\!-\!\!\!\to$	$\xi_1^4 = 1,$ $\xi_2^4 = 0,$ $\xi_1^5 = 0,$ $\xi_2^5 = 1,$ $\xi_1^6 = 0,$ $\xi_2^6 = 0$ (case1N1)	$\longrightarrow\!-\!\!\!\to$	$\xi_4^3 = 0,$ $\xi_5^3 = 0,$ $\xi_6^3 = 1.$	$(1,0;0,1;0,0;0,0,1)_1$	$\underline{\mathfrak{g}_{7,3.24}}$ (case1I)

$\mathfrak{n}\times\mathbb{C}^3$. **Table 2. Case 2 :** $A=0,\ B=\left(\begin{smallmatrix}0&1&0\\0&0&0\\0&0&0\end{smallmatrix}\right)$. **Then one may suppose** $\xi_1^4=\xi_2^4=0$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_4^3=0$, $\begin{pmatrix}\xi_1^5\\\xi_2^5\end{pmatrix}=0$	$\xi_5^3=0$ (case2N00)	$\begin{pmatrix}\xi_1^6\\\xi_2^6\end{pmatrix}=0$	$\xi_6^3=1$	$(0,0;0,0;0,0;0,0,1)_2$	$\underline{\mathfrak{g}_{7,4.3}}$ (case2-00I)
		$\begin{pmatrix}\xi_1^6\\\xi_2^6\end{pmatrix}\neq 0$	$\xi_1^6=0$, $\xi_2^6=1$, $\xi_6^3=0,1$	$(0,0;0,0;0,1;0,0,\varepsilon)_2$ $\varepsilon=0,1$ $(\varepsilon=\xi_6^3)$	$\underline{\mathfrak{g}_{7,4.1}}$ $(\varepsilon=0)$ (case2-00II) $\underline{\mathfrak{g}_{7,3.8}}$ $(\varepsilon=1)$ (case2-00II2)
$\xi_4^3=0$ $\begin{pmatrix}\xi_1^5\\\xi_2^5\end{pmatrix}\neq 0$	$\xi_1^5=0$, $\xi_2^5=1$, $\xi_2^6=0$, $\xi_5^3=0$ (case2N0)	$\xi_1^6\neq 0$	$\xi_1^6=1$, $\xi_6^3=0,1$	$(0,0;0,1;1,0;0,0,\varepsilon)_2$ $\varepsilon=0,1$ $(\varepsilon=\xi_6^3)$	$\underline{\mathfrak{g}_{7,3.6}}$ $(\varepsilon=0)$ (case2-0I) $\underline{\mathfrak{g}_{7,2.39}}$ $(\varepsilon=1)$ (case2-0I2)
		$\xi_1^6=0$	$\xi_6^3=1$	$(0,0;0,1;0,0;0,0,1)_2$	$\underline{\mathfrak{g}_{7,3.11}}$ (case2-0II)
$\xi_4^3\neq 0$	$\xi_4^3=1$, $\xi_5^3=0$, $\xi_6^3=0$ (case2N1)	$\begin{pmatrix}\xi_1^5\\\xi_2^5\end{pmatrix}=0$	$\xi_1^6=0$, $\xi_2^6=1$	$(0,0;0,0;0,1;1,0,0)_2$	$\underline{\mathfrak{g}_{7,3.14}}$ (case2I)
		$\begin{pmatrix}\xi_1^5\\\xi_2^5\end{pmatrix}\neq 0$	$\xi_1^5=1$, $\xi_2^5=0$, $\xi_1^6=0$, $\xi_2^6=1$	$(0,0;1,0;0,1;1,0,0)_2$	$\underline{\mathfrak{g}_{7,2.21}}$ (case2II)

$\mathfrak{n}\times\mathbb{C}^3$. **Table 3. Case 3 :** $A=0,\ B=\left(\begin{smallmatrix}0&1&0\\0&0&1\\0&0&0\end{smallmatrix}\right)$. **Then one may suppose** $\xi_1^4=\xi_2^4=0$.

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_4^3=0$	$\xi_5^3=0$, $\xi_6^3=0$ (case3N0)	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\xi_1^5=0$, $\xi_2^5=0$, $\xi_1^6=1$, $\xi_2^6=0$	$(0,0;0,0;1,0;0,0,0)_3$	$\underline{\mathfrak{g}_{7,3.3}}$ (case3-0I)
$\xi_4^3\neq 0$	$\xi_4^3=1$, $\xi_5^3=0$, $\xi_6^3=0$ (case3N1)	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\xi_1^5=0$, $\xi_2^5=0$, $\xi_1^6=0,1$, $\xi_2^6=0$	$(0,0;0,0;\varepsilon,0;1,0,0)_3$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^6)$	$\underline{\mathfrak{g}_{7,3.17}}$ $(\varepsilon=0)$ (case3I) $\underline{\mathfrak{g}_{7,2.16}}$ $(\varepsilon=1)$ (case3I2)

$\mathfrak{n}\times\mathbb{C}^3$. **Table 4. Case 4 :** $A=\left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, $B=0$. **Dismissed as yielding only direct products (case4N1).**

$\mathfrak{n}\times\mathbb{C}^3$. **Table 5. Case 5 :** $A=\left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, $B=\left(\begin{smallmatrix}0&1&0\\0&0&0\\0&0&0\end{smallmatrix}\right)$. **Then we may suppose** $\xi_2^4=\xi_2^5=\xi_2^6=\xi_5^3=0$. **Then :**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_4^3\neq 0$ (case5N1)	$\xi_4^3=1$, $\xi_6^3=0$, $\xi_1^5=0$, $\xi_1^6=1$, $\xi_1^4=0,1$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$(\varepsilon,0;0,0;1,0;1,0,0)_5$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^4)$	$\underline{\mathfrak{g}_{7,2.43}}$ $(\varepsilon=0)$ (case5I) $\underline{\mathfrak{g}_{7,1.16}}$ $(\varepsilon=1)$ (case5I2)
$\xi_4^3=0$ (case5N2)	$\longrightarrow\!\!\!-\!\!\!\!\longrightarrow$	$\xi_1^5\neq 0$	$\xi_1^5=1$, $\xi_1^4=0$, $\xi_1^6=0$, $\xi_6^3=1$	$(0,0;1,0;0,0;0,0,1)_5$	$\underline{\mathfrak{g}_{7,2.19}}$ $(\varepsilon=1)$ (case5II)
		$\xi_1^5=0$, $\xi_1^6\neq 0$	$\xi_1^4=0$, $\xi_1^6=1$, $\xi_6^3=0,1$	$(0,0;0,0;1,0;0,0,\varepsilon)_5$ $\varepsilon=0,1$ $(\varepsilon=\xi_6^3)$	$\underline{\mathfrak{g}_{7,3.23}}$ $(\varepsilon=0)$ (case5III) $\underline{\mathfrak{g}_{7,2.20}}$ $(\varepsilon=1)$ (case5III2)
		$\xi_1^5=0$, $\xi_1^6=0$	$\xi_1^4=0,1$ $\xi_6^3=1$	$(\varepsilon,0;0,0;0,0;0,0,1)_5$ $\varepsilon=0,1$ $(\varepsilon=\xi_1^4)$	$\underline{\mathfrak{g}_{7,3.9}}$ $(\varepsilon=0)$ (case5IV) $\underline{\mathfrak{g}_{7,2.27}}$ $(\varepsilon=1)$ (case5IV2)

$\mathfrak{n}\times\mathbb{C}^3$. **Table 6. Case 6 :** $A=\left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, $B=\left(\begin{smallmatrix}0&1&0\\0&0&1\\0&0&0\end{smallmatrix}\right)$. **We may suppose** $\xi_1^4=\xi_2^4=\xi_1^5=\xi_2^5=\xi_5^3=\xi_6^3=0$. **Then :**

if	toms	if	toms	δ	$\tilde{\mathfrak{g}}_\delta\cong$
$\xi_1^6\neq 0$ (case6N1)	$\xi_1^6=1$, $\xi_2^6=0$, $\xi_4^3=0,1$	$\longrightarrow\!-\!\!\rightarrow$	$-\!\!\rightarrow\!\longrightarrow$	$(0,0;0,0;1,0;\varepsilon,0,0)_6$ $\varepsilon=0,1$ $(\varepsilon=\xi_4^3)$	$\underline{\mathfrak{g}_{7,2.7}}$ $(\varepsilon=0)$ (case6I) $\underline{\mathfrak{g}_{7,1.6}}$ $(\varepsilon=1)$ (case6I2)
$\xi_1^6=0$	$\xi_2^6=0,1$ $\xi_4^3=0,1$	$\longrightarrow\!-\!\!\rightarrow$	$-\!\!\rightarrow\!\longrightarrow$	$(0,0;0,0;0,\varepsilon;\eta,0,0)_6$ $\varepsilon,\eta=0,1$ $(\varepsilon=\xi_2^6,\eta=\xi_4^3)$	$\underline{\mathfrak{g}_{7,3.21}}$ $(\varepsilon=0\ ,\ \eta=0)$ (case6II) $\underline{\mathfrak{g}_{7,2.11}}$ $(\varepsilon=1\ ,\ \eta=0)$ (case6II2) $\underline{\mathfrak{g}_{7,2.32}}$ $(\varepsilon=0\ ,\ \eta=1)$ (case6II3) $\underline{\mathfrak{g}_{7,1.15}}$ $(\varepsilon=1\ ,\ \eta=1)$ (case6II4)

32 $\mathbb{C}^6$

Discarding as usual direct products, we get only 4 cases: 1) $\delta=J_6$, 2) $\delta=\mathrm{diag}(J_4,J_2)$, 3) $\delta=\mathrm{diag}(J_3,J_3)$, 4) $\delta=\mathrm{diag}(J_2,J_2,J_2)$, where J_k denotes the standard $k\times k$ Jordan block. Then $\tilde{\mathfrak{g}}_\delta$ is simply $\underline{\mathfrak{g}_{7,2.3}}$ in case 1, $\underline{\mathfrak{g}_{7,3.2}}$ in case 2, $\underline{\mathfrak{g}_{7,3.20}}$ in case 3, $\underline{\mathfrak{g}_{7,4.2}}$ in case 4.

Cross Table: cross table for strains of complex 6-dimensional nilpotent Lie algebras

	$\mathfrak{g}_{6,1}$	$\mathfrak{g}_{6,2}$	$\mathfrak{g}_{6,3}$	$\mathfrak{g}_{6,4}$	$\mathfrak{g}_{6,5}$	$\mathfrak{g}_{6,6}$	$\mathfrak{g}_{6,7}$	$\mathfrak{g}_{6,8}$	$\mathfrak{g}_{6,9}$	$\mathfrak{g}_{6,10}$	$\mathfrak{g}_{6,11}$	$\mathfrak{g}_{6,12}$	$\mathfrak{g}_{6,13}$	$\mathfrak{g}_{6,14}$	$\mathfrak{g}_{6,15}$	$\mathfrak{g}_{6,16}$	$\mathfrak{g}_{6,17}$	$\mathfrak{g}_{6,18}$	$\mathfrak{g}_{6,19}$	$\mathfrak{g}_{6,20}$	$\mathfrak{g}_{5,1}\times\mathbb{C}$	$\mathfrak{g}_{5,2}\times\mathbb{C}$	$\mathfrak{g}_{5,3}\times\mathbb{C}$	$\mathfrak{g}_{5,4}\times\mathbb{C}$	$\mathfrak{g}_{5,5}\times\mathbb{C}$	$\mathfrak{g}_{5,6}\times\mathbb{C}$	$\mathfrak{g}_4\times\mathbb{C}^2$	$\mathfrak{n}\times\mathfrak{n}$	$\mathfrak{n}\times\mathbb{C}^3$	$\mathbb{C}^6$
#(strain)	27	15	11	17	15	35	28	19	5	17	17	21	6	10	4	11	7	3	5	3	15	40	23	13	16	16	17	8	25	4
$\mathfrak{g}_{7,0.1}$	•																•													
$\mathfrak{g}_{7,0.2}$						•										•														
$\mathfrak{g}_{7,0.3}$																•						•								
$\mathfrak{g}_{7,0.4}(\lambda)$								$\bullet^{\P}$		$\bullet^{\P}$	$\bullet^{\P}$	$\bullet^{\P}$																		
$\mathfrak{g}_{7,0.5}$										•	•													•						
$\mathfrak{g}_{7,0.6}$							•			•		•																		
$\mathfrak{g}_{7,0.7}$								•															•			•				
$\mathfrak{g}_{7,0.8}$		•						•				•								•										
$\mathfrak{g}_{7,1.01}(i)$						•							•			•			•											
$\mathfrak{g}_{7,1.01}(ii)$											•					•	•					•								
$\mathfrak{g}_{7,1.02}$			•							•		•																		
$\mathfrak{g}_{7,1.03}$	•							•																		•				
$\mathfrak{g}_{7,1.1}(i_\lambda)$			$\bullet^{*}$			$\bullet^{\dagger}$	$\bullet^{\ddagger\S}$									$\bullet^{\dagger}$	$\bullet^{\ddagger}$													
$\mathfrak{g}_{7,1.1}(ii)$																	•				•									
$\mathfrak{g}_{7,1.1}(iii)$							•							•																
$\mathfrak{g}_{7,1.1}(iv)$							•			•		•																		
$\mathfrak{g}_{7,1.1}(v)$							•						•				•		•											
$\mathfrak{g}_{7,1.1}(vi)$								•												•			•			•				
$\mathfrak{g}_{7,1.2}(i_\lambda)^{\|}$						$\bullet^{\dagger}$	$\bullet^{\ddagger\|}$								$\bullet^{\|}$								$\bullet^{\ddagger\|}$				$\bullet^{\dagger}$			
$\mathfrak{g}_{7,1.2}(ii)$	•						•								•								•							
$\mathfrak{g}_{7,1.2}(iii)$			•				•							•									•							
$\mathfrak{g}_{7,1.2}(iv)$							•				•	•											•							
$\mathfrak{g}_{7,1.3}(i_\lambda)$	$\bullet^{\S}$									$\bullet^{\S}$	$\bullet^{\P}$	$\bullet^{\P}$										$\bullet^{*}$			$\bullet^{*}$	$\bullet^{\S}$				
$\mathfrak{g}_{7,1.3}(ii)$	•						•																							
$\mathfrak{g}_{7,1.3}(iii)$										•	•	•									•					•				
$\mathfrak{g}_{7,1.3}(iv)$	•			•	•			•																•						
$\mathfrak{g}_{7,1.3}(v)$	•	•									•	•	•													•				
$\mathfrak{g}_{7,1.4}$																•						•								
$\mathfrak{g}_{7,1.5}$	•													•																
$\mathfrak{g}_{7,1.6}$																•													•	
$\mathfrak{g}_{7,1.7}$						•		•														•								
$\mathfrak{g}_{7,1.8}$		•								•	•	•														•				
$\mathfrak{g}_{7,1.9}$						•								•								•								
$\mathfrak{g}_{7,1.10}$																									•		•			
$\mathfrak{g}_{7,1.11}$	•										•								•											
$\mathfrak{g}_{7,1.12}$																			•			•			•					
$\mathfrak{g}_{7,1.13}$																						•			•					
$\mathfrak{g}_{7,1.14}$																							•	•		•				
$\mathfrak{g}_{7,1.15}$																	•								•				•	
$\mathfrak{g}_{7,1.16}$						•																				•			•	
$\mathfrak{g}_{7,1.17}$										•	•	•												•						
$\mathfrak{g}_{7,1.18}$						•								•								•					•			
$\mathfrak{g}_{7,1.19}$				•	•	•	•	•																				•		
$\mathfrak{g}_{7,1.20}$		•						•				•						•												
$\mathfrak{g}_{7,1.21}$		•										•								•				•						

$*$: $\lambda = 0$. $\dagger$: $\lambda = 1$. $\ddagger$: $\lambda \neq 1$. $\S$: $\lambda \neq 0$. $\P$: any $\lambda \in \mathbb{C}$. $\|$: $\lambda \neq -1$.

Cross Table continued

	$\mathfrak{g}_{6,1}$	$\mathfrak{g}_{6,2}$	$\mathfrak{g}_{6,3}$	$\mathfrak{g}_{6,4}$	$\mathfrak{g}_{6,5}$	$\mathfrak{g}_{6,6}$	$\mathfrak{g}_{6,7}$	$\mathfrak{g}_{6,8}$	$\mathfrak{g}_{6,9}$	$\mathfrak{g}_{6,10}$	$\mathfrak{g}_{6,11}$	$\mathfrak{g}_{6,12}$	$\mathfrak{g}_{6,13}$	$\mathfrak{g}_{6,14}$	$\mathfrak{g}_{6,15}$	$\mathfrak{g}_{6,16}$	$\mathfrak{g}_{6,17}$	$\mathfrak{g}_{6,18}$	$\mathfrak{g}_{6,19}$	$\mathfrak{g}_{6,20}$	$\mathfrak{g}_{5,1}\times\mathbb{C}$	$\mathfrak{g}_{5,2}\times\mathbb{C}$	$\mathfrak{g}_{5,3}\times\mathbb{C}$	$\mathfrak{g}_{5,4}\times\mathbb{C}$	$\mathfrak{g}_{5,5}\times\mathbb{C}$	$\mathfrak{g}_{5,6}\times\mathbb{C}$	$\mathfrak{g}_4\times\mathbb{C}^2$	$\mathfrak{n}\times\mathfrak{n}$	$\mathfrak{n}\times\mathbb{C}^3$	$\mathbb{C}^6$
$\mathfrak{g}_{7,2.1}(i_\lambda)$	$\bullet^{\ddagger\S}$					$\bullet^{\dagger}$	$\bullet^{\ddagger}$				$\bullet^{\S}$											$\bullet^{*\ \dagger}$			$\bullet^{*}$	$\bullet^{*}$				
$\mathfrak{g}_{7,2.1}(ii)$							•				•	•									•									
$\mathfrak{g}_{7,2.1}(iii)$	•										•	•	•										•							
$\mathfrak{g}_{7,2.1}(iv)$	•	•					•		•	•																				
$\mathfrak{g}_{7,2.1}(v)$	•		•	•	•		•																							
$\mathfrak{g}_{7,2.2}$	•																						•							
$\mathfrak{g}_{7,2.3}$																•														•
$\mathfrak{g}_{7,2.4}$														•							•									
$\mathfrak{g}_{7,2.5}$			•							•																				
$\mathfrak{g}_{7,2.6}$	•																							•		•				
$\mathfrak{g}_{7,2.7}$																									•				•	
$\mathfrak{g}_{7,2.8}$						•																•								
$\mathfrak{g}_{7,2.9}$						•																					•			
$\mathfrak{g}_{7,2.10}$								•										•			•					•				
$\mathfrak{g}_{7,2.11}$						•								•													•		•	
$\mathfrak{g}_{7,2.12}$				•	•			•													•			•						
$\mathfrak{g}_{7,2.13}$		•										•						•						•						
$\mathfrak{g}_{7,2.14}$						•							•			•			•											
$\mathfrak{g}_{7,2.15}$											•					•	•					•								
$\mathfrak{g}_{7,2.16}$																•									•				•	
$\mathfrak{g}_{7,2.17}$			•				•							•									•							
$\mathfrak{g}_{7,2.18}$														•								•					•			
$\mathfrak{g}_{7,2.19}$														•															•	
$\mathfrak{g}_{7,2.20}$						•																			•	•			•	
$\mathfrak{g}_{7,2.21}$						•																							•	
$\mathfrak{g}_{7,2.22}$						•																•				•				
$\mathfrak{g}_{7,2.23}$		•					•		•	•													•							
$\mathfrak{g}_{7,2.24}$							•															•	•		•	•				
$\mathfrak{g}_{7,2.25}$		•										•	•									•			•					
$\mathfrak{g}_{7,2.26}$	•		•	•	•		•																•							
$\mathfrak{g}_{7,2.27}$					•			•														•							•	
$\mathfrak{g}_{7,2.28}$										•											•		•							
$\mathfrak{g}_{7,2.29}$	•				•			•																•				•		
$\mathfrak{g}_{7,2.30}$		•										•									•					•				
$\mathfrak{g}_{7,2.31}$						•					•	•															•			
$\mathfrak{g}_{7,2.32}$										•		•													•				•	
$\mathfrak{g}_{7,2.33}$		•								•	•	•													•		•			
$\mathfrak{g}_{7,2.34}$				•	•			•																•				•		
$\mathfrak{g}_{7,2.35}$	•			•	•		•	•																						
$\mathfrak{g}_{7,2.36}$	•			•		•	•															•								
$\mathfrak{g}_{7,2.37}$	•						•								•															
$\mathfrak{g}_{7,2.38}$					•			•														•								
$\mathfrak{g}_{7,2.39}$						•																							•	
$\mathfrak{g}_{7,2.40}$																						•		•						
$\mathfrak{g}_{7,2.41}$						•									•							•								
$\mathfrak{g}_{7,2.42}$				•	•	•																•					•			
$\mathfrak{g}_{7,2.43}$						•		•																					•	
$\mathfrak{g}_{7,2.44}$																						•	•				•			
$\mathfrak{g}_{7,2.45}$				•			•															•								

*: $\lambda = 0$. †: $\lambda = 1$. ‡: $\lambda \neq 1$. §: $\lambda \neq 0$.

Cross Table: cross table for strains of complex 6-dimensional nilpotent Lie algebras

	$\mathfrak{g}_{6,1}$	$\mathfrak{g}_{6,2}$	$\mathfrak{g}_{6,3}$	$\mathfrak{g}_{6,4}$	$\mathfrak{g}_{6,5}$	$\mathfrak{g}_{6,6}$	$\mathfrak{g}_{6,7}$	$\mathfrak{g}_{6,8}$	$\mathfrak{g}_{6,9}$	$\mathfrak{g}_{6,10}$	$\mathfrak{g}_{6,11}$	$\mathfrak{g}_{6,12}$	$\mathfrak{g}_{6,13}$	$\mathfrak{g}_{6,14}$	$\mathfrak{g}_{6,15}$	$\mathfrak{g}_{6,16}$	$\mathfrak{g}_{6,17}$	$\mathfrak{g}_{6,18}$	$\mathfrak{g}_{6,19}$	$\mathfrak{g}_{6,20}$	$\mathfrak{g}_{5,1}\times\mathbb{C}$	$\mathfrak{g}_{5,2}\times\mathbb{C}$	$\mathfrak{g}_{5,3}\times\mathbb{C}$	$\mathfrak{g}_{5,4}\times\mathbb{C}$	$\mathfrak{g}_{5,5}\times\mathbb{C}$	$\mathfrak{g}_{5,6}\times\mathbb{C}$	$\mathfrak{g}_4\times\mathbb{C}^2$	$\mathfrak{n}\times\mathfrak{n}$	$\mathfrak{n}\times\mathbb{C}^3$	$\mathbb{C}^6$
#(strain)	27	15	11	17	15	35	28	19	5	17	17	21	6	10	4	11	7	3	5	3	15	40	23	13	16	16	17	8	25	4
$\mathfrak{g}_{7,0.1}$	•																•													
$\mathfrak{g}_{7,0.2}$						•										•														
$\mathfrak{g}_{7,0.3}$																•						•								
$\mathfrak{g}_{7,0.4}(\lambda)$								$\bullet^{\P}$		$\bullet^{\P}$	$\bullet^{\P}$	$\bullet^{\P}$																		
$\mathfrak{g}_{7,0.5}$										•	•													•						
$\mathfrak{g}_{7,0.6}$							•			•		•																		
$\mathfrak{g}_{7,0.7}$								•															•			•				
$\mathfrak{g}_{7,0.8}$		•						•				•								•										
$\mathfrak{g}_{7,1.01}(i)$						•							•			•			•											
$\mathfrak{g}_{7,1.01}(ii)$											•					•	•					•								
$\mathfrak{g}_{7,1.02}$			•							•		•																		
$\mathfrak{g}_{7,1.03}$	•							•																		•				
$\mathfrak{g}_{7,1.1}(i_\lambda)$			$\bullet^{*}$			$\bullet^{\dagger}$	$\bullet^{\ddagger\S}$									$\bullet^{\dagger}$	$\bullet^{\ddagger}$													
$\mathfrak{g}_{7,1.1}(ii)$																	•				•									
$\mathfrak{g}_{7,1.1}(iii)$							•							•																
$\mathfrak{g}_{7,1.1}(iv)$							•			•		•																		
$\mathfrak{g}_{7,1.1}(v)$							•						•				•		•											
$\mathfrak{g}_{7,1.1}(vi)$								•												•			•			•				
$\mathfrak{g}_{7,1.2}(i_\lambda)^{\|}$						$\bullet^{\dagger}$	$\bullet^{\ddagger\|}$								$\bullet^{\|}$								$\bullet^{\ddagger\|}$				$\bullet^{\dagger}$			
$\mathfrak{g}_{7,1.2}(ii)$	•						•								•								•							
$\mathfrak{g}_{7,1.2}(iii)$			•				•							•									•							
$\mathfrak{g}_{7,1.2}(iv)$							•				•	•											•							
$\mathfrak{g}_{7,1.3}(i_\lambda)$	$\bullet^{\S}$									$\bullet^{\P}$	$\bullet^{\P}$	$\bullet^{\P}$										$\bullet^{*}$			$\bullet^{*}$	$\bullet^{\S}$				
$\mathfrak{g}_{7,1.3}(ii)$	•						•																							
$\mathfrak{g}_{7,1.3}(iii)$										•	•	•									•					•				
$\mathfrak{g}_{7,1.3}(iv)$	•			•	•			•																•						
$\mathfrak{g}_{7,1.3}(v)$	•	•									•	•	•													•				
$\mathfrak{g}_{7,1.4}$																•						•								
$\mathfrak{g}_{7,1.5}$	•													•																
$\mathfrak{g}_{7,1.6}$																•													•	
$\mathfrak{g}_{7,1.7}$						•		•														•								
$\mathfrak{g}_{7,1.8}$		•								•	•	•														•				
$\mathfrak{g}_{7,1.9}$						•								•								•								
$\mathfrak{g}_{7,1.10}$																									•		•			
$\mathfrak{g}_{7,1.11}$	•										•								•											
$\mathfrak{g}_{7,1.12}$																			•			•			•					
$\mathfrak{g}_{7,1.13}$																						•			•					
$\mathfrak{g}_{7,1.14}$																							•	•		•				
$\mathfrak{g}_{7,1.15}$																	•								•				•	
$\mathfrak{g}_{7,1.16}$						•																				•			•	
$\mathfrak{g}_{7,1.17}$										•	•	•												•						
$\mathfrak{g}_{7,1.18}$						•								•								•					•			
$\mathfrak{g}_{7,1.19}$				•	•	•	•	•																				•		
$\mathfrak{g}_{7,1.20}$		•						•				•						•												
$\mathfrak{g}_{7,1.21}$		•										•								•				•						

*: $\lambda = 0$. †: $\lambda = 1$. ‡: $\lambda \neq 1$. §: $\lambda \neq 0$. ¶: any $\lambda \in \mathbb{C}$. $^{\|}$: $\lambda \neq -1$.

Cross Table continued

	$\mathfrak{g}_{6,1}$	$\mathfrak{g}_{6,2}$	$\mathfrak{g}_{6,3}$	$\mathfrak{g}_{6,4}$	$\mathfrak{g}_{6,5}$	$\mathfrak{g}_{6,6}$	$\mathfrak{g}_{6,7}$	$\mathfrak{g}_{6,8}$	$\mathfrak{g}_{6,9}$	$\mathfrak{g}_{6,10}$	$\mathfrak{g}_{6,11}$	$\mathfrak{g}_{6,12}$	$\mathfrak{g}_{6,13}$	$\mathfrak{g}_{6,14}$	$\mathfrak{g}_{6,15}$	$\mathfrak{g}_{6,16}$	$\mathfrak{g}_{6,17}$	$\mathfrak{g}_{6,18}$	$\mathfrak{g}_{6,19}$	$\mathfrak{g}_{6,20}$	$\mathfrak{g}_{5,1}\times\mathbb{C}$	$\mathfrak{g}_{5,2}\times\mathbb{C}$	$\mathfrak{g}_{5,3}\times\mathbb{C}$	$\mathfrak{g}_{5,4}\times\mathbb{C}$	$\mathfrak{g}_{5,5}\times\mathbb{C}$	$\mathfrak{g}_{5,6}\times\mathbb{C}$	$\mathfrak{g}_4\times\mathbb{C}^2$	$\mathfrak{n}\times\mathfrak{n}$	$\mathfrak{n}\times\mathbb{C}^3$	$\mathbb{C}^6$
$\mathfrak{g}_{7,2.1}(i_\lambda)$	•‡§					•†	•‡				•§											•* †			•*	•*				
$\mathfrak{g}_{7,2.1}(ii)$							•				•	•									•									
$\mathfrak{g}_{7,2.1}(iii)$	•										•	•	•										•							
$\mathfrak{g}_{7,2.1}(iv)$	•	•					•		•	•																				
$\mathfrak{g}_{7,2.1}(v)$	•		•	•	•		•																							
$\mathfrak{g}_{7,2.2}$	•																						•							
$\mathfrak{g}_{7,2.3}$																•														•
$\mathfrak{g}_{7,2.4}$														•							•									
$\mathfrak{g}_{7,2.5}$			•							•																				
$\mathfrak{g}_{7,2.6}$	•																							•		•				
$\mathfrak{g}_{7,2.7}$																									•				•	
$\mathfrak{g}_{7,2.8}$						•																•								
$\mathfrak{g}_{7,2.9}$						•																					•			
$\mathfrak{g}_{7,2.10}$								•										•			•					•				
$\mathfrak{g}_{7,2.11}$						•								•													•		•	
$\mathfrak{g}_{7,2.12}$				•	•			•													•			•						
$\mathfrak{g}_{7,2.13}$		•										•						•						•						
$\mathfrak{g}_{7,2.14}$						•							•			•			•											
$\mathfrak{g}_{7,2.15}$											•					•	•					•								
$\mathfrak{g}_{7,2.16}$																•									•				•	
$\mathfrak{g}_{7,2.17}$			•				•							•									•							
$\mathfrak{g}_{7,2.18}$														•								•					•			
$\mathfrak{g}_{7,2.19}$														•															•	
$\mathfrak{g}_{7,2.20}$						•																			•	•			•	
$\mathfrak{g}_{7,2.21}$						•																							•	
$\mathfrak{g}_{7,2.22}$						•																•				•				
$\mathfrak{g}_{7,2.23}$		•					•		•	•													•							
$\mathfrak{g}_{7,2.24}$							•															•	•		•	•				
$\mathfrak{g}_{7,2.25}$		•										•	•									•			•					
$\mathfrak{g}_{7,2.26}$	•		•	•	•		•																•							
$\mathfrak{g}_{7,2.27}$					•			•														•							•	
$\mathfrak{g}_{7,2.28}$										•											•		•							
$\mathfrak{g}_{7,2.29}$	•				•			•																•				•		
$\mathfrak{g}_{7,2.30}$		•										•									•					•				
$\mathfrak{g}_{7,2.31}$						•					•	•															•			
$\mathfrak{g}_{7,2.32}$										•		•													•				•	
$\mathfrak{g}_{7,2.33}$		•								•	•	•													•		•			
$\mathfrak{g}_{7,2.34}$				•	•			•																•				•		
$\mathfrak{g}_{7,2.35}$	•			•	•		•	•																						
$\mathfrak{g}_{7,2.36}$	•			•		•	•															•								
$\mathfrak{g}_{7,2.37}$	•						•								•															
$\mathfrak{g}_{7,2.38}$					•			•														•								
$\mathfrak{g}_{7,2.39}$						•																							•	
$\mathfrak{g}_{7,2.40}$																						•		•						
$\mathfrak{g}_{7,2.41}$						•									•							•								
$\mathfrak{g}_{7,2.42}$				•	•	•																•					•			
$\mathfrak{g}_{7,2.43}$						•		•																					•	
$\mathfrak{g}_{7,2.44}$																						•	•				•			
$\mathfrak{g}_{7,2.45}$				•			•															•								

*: $\lambda = 0$. †: $\lambda = 1$. ‡: $\lambda \neq 1$. §: $\lambda \neq 0$.

Cross Table continued

	$\mathfrak{g}_{6,1}$	$\mathfrak{g}_{6,2}$	$\mathfrak{g}_{6,3}$	$\mathfrak{g}_{6,4}$	$\mathfrak{g}_{6,5}$	$\mathfrak{g}_{6,6}$	$\mathfrak{g}_{6,7}$	$\mathfrak{g}_{6,8}$	$\mathfrak{g}_{6,9}$	$\mathfrak{g}_{6,10}$	$\mathfrak{g}_{6,11}$	$\mathfrak{g}_{6,12}$	$\mathfrak{g}_{6,13}$	$\mathfrak{g}_{6,14}$	$\mathfrak{g}_{6,15}$	$\mathfrak{g}_{6,16}$	$\mathfrak{g}_{6,17}$	$\mathfrak{g}_{6,18}$	$\mathfrak{g}_{6,19}$	$\mathfrak{g}_{6,20}$	$\mathfrak{g}_{5,1} \times \mathbb{C}$	$\mathfrak{g}_{5,2} \times \mathbb{C}$	$\mathfrak{g}_{5,3} \times \mathbb{C}$	$\mathfrak{g}_{5,4} \times \mathbb{C}$	$\mathfrak{g}_{5,5} \times \mathbb{C}$	$\mathfrak{g}_{5,6} \times \mathbb{C}$	$\mathfrak{g}_4 \times \mathbb{C}^2$	$\mathfrak{n} \times \mathfrak{n}$	$\mathfrak{n} \times \mathbb{C}^3$	$\mathbb{C}^6$
$\mathfrak{g}_{7,3,1}(i_\lambda)$	•¶																					•*	•¶				•*			
$\mathfrak{g}_{7,3,1}(iii)$	•								•												•		•							
$\mathfrak{g}_{7,3,2}$						•																			•					•
$\mathfrak{g}_{7,3,3}$						•																			•				•	
$\mathfrak{g}_{7,3,4}$				•	•	•																					•	•		
$\mathfrak{g}_{7,3,5}$	•					•	•	•														•								
$\mathfrak{g}_{7,3,6}$																						•					•		•	
$\mathfrak{g}_{7,3,7}$				•		•	•															•	•					•		
$\mathfrak{g}_{7,3,8}$						•	•															•	•						•	
$\mathfrak{g}_{7,3,9}$				•																		•	•						•	
$\mathfrak{g}_{7,3,10}$		•				•			•	•												•								
$\mathfrak{g}_{7,3,11}$				•		•																•							•	
$\mathfrak{g}_{7,3,12}$			•																			•								
$\mathfrak{g}_{7,3,13}$					•			•													•			•				•		
$\mathfrak{g}_{7,3,14}$	•					•	•																•				•		•	
$\mathfrak{g}_{7,3,15}$	•			•																	•	•	•				•			
$\mathfrak{g}_{7,3,16}$		•				•			•	•																	•			
$\mathfrak{g}_{7,3,17}$		•									•														•				•	
$\mathfrak{g}_{7,3,18}$		•																			•		•							
$\mathfrak{g}_{7,3,19}$	•																					•						•		
$\mathfrak{g}_{7,3,20}$						•																								•
$\mathfrak{g}_{7,3,21}$				•	•	•																					•		•	
$\mathfrak{g}_{7,3,22}$			•	•	•		•														•									
$\mathfrak{g}_{7,3,23}$																						•		•					•	
$\mathfrak{g}_{7,3,24}$			•																			•							•	
$\mathfrak{g}_{7,4,1}$			•																			•							•	
$\mathfrak{g}_{7,4,2}$																						•								•
$\mathfrak{g}_{7,4,3}$	•																				•	•						•	•	
$\mathfrak{g}_{7,4,4}$																					•									

*: $\lambda = 0$. ¶: **any** $\lambda \in \mathbb{C}$.

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