Adjoint and Trivial Cohomology Tables for Indecomposable Nilpotent Lie Algebras of Dimension ≤ 7 over $\mathbb{C}$

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ABSTRACT

(back cover and inside back cover)

Due to the lack of a good invariant, a general classification of complex finite-dimensional Lie algebras seems to be, for the time being, hopeless. However several classifications have been computed "by hand" in different ways for Lie algebras of dimension ≤ 7. In particular, the weight system for the action of a maximal torus is an invariant of the Lie algebra under consideration and gives rise to a classification of Lie algebras of dimension ≤ 7 which separates all algebras except some algebras having a null weight or a continuous series and some of its "limit points".

Then an obvious question is how going to weight systems on the cohomology would refine this classification, i.e. separate algebras which are not separated by weights on the algebra itself.

But very few results are known on the cohomology of nilpotent Lie algebras, except for the very special case of the nilradical of a minimal parabolic subalgebra of a semisimple Lie algebra. Anyone interested in computing by hand cohomology , even in dimensions as low as 4,5,6, will be discouraged by the amount of calculation. This is naturally the domain of such a computer algebra system like REDUCE.

The aim of this book is to provide the reader with complete cohomological data for indecomposable complex Lie algebras of dimension ≤ 7, which can also be used for further calculations.

We consider only trivial and adjoint cohomology:

* Trivial cohomology because of the so-called "Riemann Hypothesis" on the spectrality of the maximal torus and the interest in the Poincaré polynomial.

* Adjoint cohomology because of its relevance to deformation theory and the amount of structural informations it contains.

To be precise , this book (which is self-contained) gives the following data which do not appear elsewhere in the litterature:

Trivial cohomology:

coboundaries, Betti numbers, characters of each cohomological space for the action of the maximal torus, and bases of eigenspaces, reduction over $\mathbb{Z}$ of the Poincaré polynomial, test of the Riemann Hypothesis.
These computations are performed in two distinct ways: the first way doesn't make use of Poincaré Duality , the second uses harmonic cohomology and Poincaré Duality since we explain in the Introduction why harmonic cohomology is needed in order to get explicitely the dual basis via Poincaré Duality.

Adjoint cohomology:

Structure of the derivation algebra and the 1-cohomology algebra (bases and commutation relations), adjoint Betti numbers, characters of all cohomological spaces, explicit basis for the 2-cohomological space consisting of eigenvectors for the action of the maximal torus.

Continuous series are carefully studied and "singular values" of the parameter (i.e. values which give rise to a cohomology different from the generic one) are computed.

From these data it is readily seen that trivial cohomology doesn't refine the classification but adjoint cohomology does except for the 2 couples $(\mathcal{G}_{7,1.2(i_\lambda)}, \mathcal{G}_{7,1.2(ii)})$, $(\mathcal{G}_{7,1.3(i_\lambda)}, \mathcal{G}_{7,1.3(iii)})$. In either of those 2 couples, the weight system on the cohomology is identical for the 2 components. It is also a fact that Riemann Hypothesis holds for Lie algebras of dimension ≤ 7 under restrictions that are precisely formulated in the Introduction.

This book may be of interest to people working in Lie algebra cohomology, deformation theory , classification or structure of Lie algebras, since the data it provides can be used to test new results.

Contents

Chapter 1

Introduction

1.1 Definitions and notations

1.1.1 Rank and weights for nilpotent Lie algebras

This work is based on the classification of complex nilpotent Lie Algebras of dimension ≤ 7 in [3]. This classification relies on the action of the maximal torus. Recall that a torus T of a finite dimensional Lie Algebra $\mathcal{G}$ is by definition a commutative Lie subalgebra of Der $\mathcal{G}$ consisting of semisimple derivations. By Mostow's theorem, over $\mathbb{C}$, any two maximal tori T_1 and T_2 are conjugate, i.e. $\exists \theta \in \mathrm{Aut}(\mathcal{G})\ \ \theta T_1 \theta^{-1} = T_2$. The dimension of any maximal torus is called the rank of $\mathcal{G}$. Note that for nilpotent nonabelian $\mathcal{G}$ this is not the dimension of a Cartan subalgebra, since the only Cartan subalgebra is then $\mathcal{G}$.

Let $\mathcal{G}$ be a nilpotent complex finite dimensional Lie algebra and T a maximal torus of $\mathcal{G}$. Let $\{x_1, ...x_\ell\}$ denote a minimal set of Lie-generators of $\mathcal{G}$ (i.e. generating $\mathcal{G}$ by commutators and linear combination) consisting of T-eigenvectors, i.e. $t(x_i) = \beta_i(t)x_i \forall t \in T$ where β_i belongs to the dual T^* of T and $\ell = \dim\ \ \mathcal{G}/[\mathcal{G},\mathcal{G}]$. Then $\dim T = \mathrm{rk}(\beta_1, ..., \beta_\ell)$ [11], hence $\mathrm{rk}(\mathcal{G}) \leq \ell$. If the equality holds then $\mathcal{G}$ is said to be of maximal rank. In this work, maximal rank nilpotent Lie algebras will be indicated by †.

1.1.2 Cohomology

Let E be a $\mathcal{G}$-module and $C^p(\mathcal{G}, E) = (\bigwedge^p \mathcal{G}^*) \otimes E$ the p-th cochain space ($p \geq 0$); the coboundary operator $d : C^{p-1}(\mathcal{G}, E) \longrightarrow C^p(\mathcal{G}, E)$ ($p \geq 1$) is defined by

$$\begin{aligned} df(X_1,...,X_p) &= \sum_{i=1}^{p}(-1)^{i-1}X_i \cdot f(X_1,...,\widehat{X_i},...,X_p) \\ &+ \sum_{1\le i<j\le p}(-1)^{i+j}f([X_i,X_j],...,\widehat{X_i},...,\widehat{X_j},...,X_p) \end{aligned}$$

for all $X_1,..,X_p \in \mathcal{G}$, and we denote:

$$\begin{aligned} Z_E^p &= \{f \in C^p(\mathcal{G},E);\ df = 0\} \\ B_E^p &= d(C^{p-1}(\mathcal{G},E)) \\ H_E^p &= Z_E^p/B_E^p \end{aligned}$$

In the present work, we are concerned only with the cases E=$\mathbb{C}$the trivial $\mathcal{G}$-module and $E = \mathcal{G}$ the adjoint $\mathcal{G}$-module. All the preceding spaces are semisimple T-modules (the dual $\mathcal{G}^*$ of $\mathcal{G}$ is considered as the contragredient T-module). For more details, see [8].

Recall that $H^0_{\mathcal{G}} = Z^0_{\mathcal{G}}$ is plainly the center of $\mathcal{G}$, $Z^1_{\mathcal{G}}$= Der $\mathcal{G}$, $H^1_{\mathcal{G}}$= Der $\mathcal{G}$ / ad $\mathcal{G}$.

In the trivial cohomology, $H^1_{\mathbb{C}} = (\mathcal{G}/[\mathcal{G},\mathcal{G}])^*$.

1.1.3 Computer Algebra System

All calculations have been performed by use of the computer algebra system REDUCE by A.C. HEARN, together with the EXCALC package by E.SCHRÜFER, on the IBM RISC 6000 platform of the CRI, CENTRE DE CALCUL DE L'UNIVERSITÉ DE BOURGOGNE (FRANCE). All results have been electronically converted into LaTeX.

Let us mention here the work [13], which gives lists of trivial and adjoint Betti numbers based on the classification of complex 7-dimensional Lie algebras in [1]. We did not use this latter classification since it is not related to the action of the maximal torus. The work [13] doesn't make use of any computer algebra system; calculations are there performed by reduction modulo p, with a finite set of typical primes p depending on the algebra into consideration. This implies difficulties with continuous series, and as a matter of fact singular values like $\lambda^2 - \lambda + 1 = 0$ do not appear.

1.2 Organization of results for adjoint cohomology

$(x_i)_{1\leq i\leq n}$, n=dim $\mathcal{G}$, denotes the basis of $\mathcal{G}$ in [3]. It consists of T-eigenvectors. The dual basis of $\mathcal{G}^*$ is denoted by $(\omega^i)_{1\leq i\leq n}$; $\omega^{i_1,\dots,i_p}$ denotes $\omega^{i_1}\wedge\dots\wedge\omega^{i_p}\in \bigwedge^p\mathcal{G}^*$ and $x_j\omega^{i_1,\dots,i_p}$ denotes $\omega^{i_1}\wedge\dots\wedge\omega^{i_p}\otimes x_j\in(\bigwedge^p\mathcal{G}^*)\otimes\mathcal{G}=C^p(\mathcal{G},\mathcal{G})$. We systematically make use of representatives to denote cohomology classes, e.g. $\omega^{i_1,\dots,i_p}$ stands for $[\omega^{i_1,\dots,i_p}]$.

The results are organized as follows. First, the computer recalls the commutations relations for $\mathcal{G}$ (Jacobi identities have been independently tested). Then it writes the matrix of the generic derivation δ of $\mathcal{G}$ in terms of a set of indeterminates $\{\xi_i^j,(i,j)\in\mathcal{P}_{\mathcal{G}}\}$ depending on $\mathcal{G}$. If δ_i^j $((i,j)\in\mathcal{P}_{\mathcal{G}})$ denotes the derivation obtained by substituting

$$\xi_{i'}^{j'}=\begin{cases}1 & \text{if } i'=i \text{ and } j'=j\\ 0 & \text{otherwise}\end{cases}$$

then $\{\delta_i^j,(i,j)\in\mathcal{P}_{\mathcal{G}}\}$ is a basis for Der $\mathcal{G}$ consisting of T-eigenvectors. The adjoint derivations ad x_i are given in terms of the preceding basis, writing t_i for δ_i^i.

The maximal torus is computed in the form $T=\langle t_i,i\in I\rangle$ ($\langle a,b,\dots\rangle$ will always denote the vector subspace with basis $\{a,b,\dots\}$). Then $(\beta_i)_{i\in I}$ will denote the basis of T^* dual to $(t_i)_{i\in I}$.

The character of a semisimple T-module M is

$$\mathrm{ch}(M)=\sum_{\alpha\in T^*}\mathrm{mult}(\alpha)\ e^\alpha\in\mathbb{Z}\,[T^*]$$

where mult(α) is the multiplicity of the weight α in M. In the present work, when we want to incorporate the eigenspace M^α we write

$$\mathrm{ch}(M)=\sum_{\alpha\in T^*}\mathrm{mult}(\alpha)\ e^\alpha\ M^\alpha$$

The computer gives the commutation relations for the elements of the basis of $Z^1_{\mathcal{G}}$ not in T , i.e. $\{\delta_i^j,(i,j)\in\mathcal{P}_{\mathcal{G}},\ i\neq j\}$ and the same for $H^1_{\mathcal{G}}$. When there are no nontrivial commutation relations (i.e. $H^1_{\mathcal{G}}=T\oplus\mathcal{A}$ with $\mathcal{A}$ commutative), the computer writes ":."

The computer gives ch$(\mathcal{G})$ and the characters and dimensions for the whole adjoint cohomology. Finally the sum S of all weights of $\mathcal{G}$ is computed.

1.3 Poincaré Duality for trivial cohomology

$\mathcal{G}$ denotes a Lie algebra with basis $(x_1, ..., x_n)$.

1.3.1 Trivial homology

Let $C_p = \bigwedge^p \mathcal{G}$ $(p \geq 0)$. C_p may be identified to the dual of $C^p = C^p(\mathcal{G}, \mathbb{C}) = \bigwedge^p \mathcal{G}^*$ by the canonical duality

$$\langle z, f \rangle = f(z) \quad (z \in C_p, f \in C^p).$$

Then the transpose of d is the boundary operator

$$\partial : C_{p+1} \longrightarrow C_p \ (p \geq 0)$$

defined for all $X_1, .., X_{p+1} \in \mathcal{G}$ by

$$\partial(X_1 \wedge ... \wedge X_{p+1}) = \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} [X_i, X_j] \wedge X_1 \wedge ... \wedge \widehat{X_i} \wedge ... \wedge \widehat{X_j} \wedge ... \wedge X_{p+1}$$

since

$$\langle \partial z, f \rangle = \langle z, df \rangle \quad \forall z \in C_p, \ \forall f \in C^p.$$

Hence

$$\text{Ker } \partial = (\text{Im } d)^{\perp}$$

$$\text{Ker } d = (\text{Im } \partial)^{\perp}$$

Introduce

$$\begin{aligned} (Z_p)_{\mathbb{C}} &= \{z \in C_p ; \partial z = 0\} \\ (B_p)_{\mathbb{C}} &= \partial(C_{p+1}) \\ (H_p)_{\mathbb{C}} &= (Z_p)_{\mathbb{C}} / (B_p)_{\mathbb{C}} \end{aligned}$$

As $(Z_p)_{\mathbb{C}} = (B^p_{\mathbb{C}})^{\perp}$, $(B_p)_{\mathbb{C}} = (Z^p_{\mathbb{C}})^{\perp}$ the duality restricts to a duality for $(H_p)_{\mathbb{C}} \times (H^p)_{\mathbb{C}}$ which permits identification of $(H_p)_{\mathbb{C}}$ to $(H^p)_{\mathbb{C}}{}^*$. The class $[z] \in (H_p)_{\mathbb{C}}$ $(z \in (Z_p)_{\mathbb{C}})$ is identified to the linear form $[f] \to \langle z, f \rangle$ on $(H^p)_{\mathbb{C}}$.

1.3.2 Poincaré Duality for unimodular Lie Algebras

Introduce for $f \in C^p$

$$\varrho(f) = \Omega \lfloor f \in C_{n-p}$$

where $\Omega = x_1 \wedge ... \wedge x_n$ and $\lfloor$ denotes interior product:

$$\langle \varrho(f), h \rangle = \langle \Omega \lfloor f, h \rangle = \langle \Omega, f \wedge h \rangle \ \ \forall f \in C^p \ \forall h \in C^{n-p}$$

Then $\varrho : C^p \to C_{n-p}$ is an isomorphism and we get a duality for $C^p \times C^{n-p}$ defined by

$$B(f,h) = \langle \varrho(f), h \rangle = \langle \Omega, f \wedge h \rangle \ \ \forall f \in C^p \ \forall h \in C^{n-p}$$

Lemma 1 ([6]) *If $\mathcal{G}$ is unimodular (i.e. $Tr(adX) = 0 \ \forall X \in \mathcal{G}$), then:*

$$\varrho(df) = (-1)^{p+1} \partial(\varrho(f)) \ \ \forall f \in C^p$$

Proof

For all $h \in C^{n-p-1}$,

$$\langle \Omega \lfloor df, h \rangle = \langle \Omega, df \wedge h \rangle = (-1)^{p+1} \langle \Omega, f \wedge dh \rangle$$

since by unimodularity $\partial \Omega = 0$ hence $0 = d(f \wedge h) = df \wedge h + (-1)^p f \wedge dh$. Hence

$$\langle \varrho(df), h \rangle = (-1)^{p+1} \langle \Omega \lfloor f, dh \rangle = (-1)^{p+1} \langle \partial(\varrho(f)), h \rangle$$

From the lemma, for unimodular $\mathcal{G}$ one has:

$$\varrho(Z^p_{\mathcal{C}}) = (Z_{n-p})_{\mathcal{C}} = (B^{n-p}_{\mathcal{C}})^{\perp}$$

$$\varrho(B^p_{\mathcal{C}}) = (B_{n-p})_{\mathcal{C}} = (Z^{n-p}_{\mathcal{C}})^{\perp}.$$

Hence ϱ defines an isomorphism $H^p_{\mathcal{C}} \to (H_{n-p})_{\mathcal{C}}$ and the duality restricts to a duality for $H^p_{\mathcal{C}} \times H^{n-p}_{\mathcal{C}}$ by means of which $H^p_{\mathcal{C}}$ may be identified to $(H^{n-p}_{\mathcal{C}})^*$: the class $[f]$ of $f \in Z^p_{\mathcal{C}}$ is identified to the linear form $[h] \to B(f,h)$ on $H^{n-p}_{\mathcal{C}}$.

Remark

Suppose $\mathcal{G}$ nilpotent with maximal torus T and basis $(x_1, ..., x_n)$ consisting of T-eigenvectors associated to weights α_i $1 \leq i \leq n$. The weight of Ω is the sum $S = \sum_{i=1}^n \alpha_i$ and one gets for all $f \in C^p$, $h \in C^{n-p}$ and $t \in T$:

$$\begin{aligned}\langle t(\varrho(f)), h\rangle &= -\langle \varrho(f), th\rangle \\ &= -\langle \Omega, f \wedge th\rangle \\ &= -\langle \Omega, t(f \wedge h)\rangle + \langle \Omega, tf \wedge h)\rangle \\ &= \langle t\Omega, f \wedge h\rangle + \langle \Omega, tf \wedge h)\rangle \\ &= \langle S(t)\varrho(f) + \varrho(tf), h\rangle\end{aligned}$$

i.e. $t(\varrho(f)) = S(t)\varrho(f) + \varrho(tf)$. Hence for any weight α one has $\varrho((C^p)^\alpha) \subset (C_{n-p})^{S+\alpha}$.

1.3.3 Dual bases for $H^p_{\mathbb{C}}$ and $H^{n-p}_{\mathbb{C}}$

1.3.3.1

Suppose $\mathcal{G}$ unimodular and we are given bases $(\varphi_i)_{1\leq i\leq s}$ for $H^p_{\mathbb{C}}$ and $(\psi_j)_{1\leq j\leq s}$ for $H^{n-p}_{\mathbb{C}}$ (we write representative cocycles, i.e. φ instead of $[\varphi]$). We look for the dual basis of $(\varphi_i)_{1\leq i\leq s}$ in $H^{n-p}_{\mathbb{C}}$, i.e. $(\psi'_j)_{1\leq j\leq s}$ such that

$$B(\varphi_i, \psi'_j) = \delta_{i,j} \quad \forall i, j. \tag{1.1}$$

This is well-kown: if we let $(\psi'_j) = \sum_{k=1}^s \eta^{j,k}\psi_k$, then 1.1 reads in matrix form $\eta\ {}^t g = I$ where $\eta = (\eta^{i,j}), g = (g_{i,j}), g_{i,j} = B(\varphi_i, \psi_j)$ and t denotes here transpose. If we denote g^{-1} by $(g^{i,j})$ then $\eta = {}^t g^{-1}$, i.e. $\psi'_j = \sum_{k=1}^s g^{k,j}\psi_k$.

Remark
For nilpotent $\mathcal{G}$ with maximal torus T, one has for $\varphi \in (H^p_{\mathbb{C}})^\alpha$, $\psi \in (H^{n-p}_{\mathbb{C}})^\beta$ and $t \in T$:

$$\begin{aligned}S(t)\langle \Omega, \varphi \wedge \psi\rangle &= \langle t\Omega, \varphi \wedge \psi\rangle \\ &= -\langle \Omega, t(\varphi \wedge \psi)\rangle \\ &= -(\alpha + \beta)(t)\ \langle \Omega, \varphi \wedge \psi\rangle\end{aligned}$$

Hence for $\beta \neq -S - \alpha$ one has $(H^{n-p}_{\mathbb{C}})^\beta \subset ((H^p_{\mathbb{C}})^\alpha)^\perp$. Computing the dual basis reduces then to doing it with $(H^p_{\mathbb{C}})^\alpha$ and $(H^{n-p}_{\mathbb{C}})^{-S-\alpha}$ for the various α's.

1.3.3.2

If we have got basis for $H^p_{\mathbb{C}}$ only for $2p \leq n$ and we want to make use of the Poincaré duality to get the dual basis for $H^{n-p}_{\mathbb{C}}$, then we face 2 steps:

1: use Poincaré duality to get a basis (ψ_j) for $H^{n-p}_{\mathbb{C}}$

2: compute the dual basis starting from (ψ_j) with the foregoing procedure

We now consider step 1.

1.3.3.3

For any subset H of cardinality p of $\{1, ..., n\}$

$$H = \{i_1, ..., i_p\}, \quad 1 \leq i_1 < ... < i_p \leq n$$

let

$$\omega^H = \omega^{i_1,...,i_p}, \; x_H = x_{i_1,...,i_p} = x_{i_1} \wedge ... \wedge x_{i_p}.$$

Then for all H, K of cardinality p

$$\langle x_H, \omega^K \rangle = \delta_{H,K} = \begin{cases} 1 & \text{if } H = K \\ 0 & \text{if } H \neq K \end{cases}$$

For all subsets I, K of respective cardinality p and $n-p$, one has

$$\begin{aligned} \langle \varrho(\omega^I), \omega^K \rangle &= \langle \Omega, \omega^I \wedge \omega^K \rangle \\ &= \sum_{\sigma \in \mathcal{S}'_n} \epsilon_\sigma \; \langle x_{\sigma(1),...,\sigma(p)}, \omega^I \rangle \; \langle x_{\sigma(p+1),...,\sigma(n)}, \omega^K \rangle \\ &= \delta_{K,I'} \; \rho_{I,I'} \end{aligned}$$

where $\mathcal{S}'_n = \{\sigma \in \mathcal{S}_n; \sigma(1) < ... < \sigma(p); \sigma(p+1) < ... < \sigma(n)\}$,
I' denotes the complementary subset to I and

$$\rho_{I,I'} = \text{card}\{(i,j) \in I \times I'; j < i\} = (-1)^{i_1+...+i_p}(-1)^{p(p+1)/2}.$$

Hence

$$\varrho(\omega^I) = \rho_{I,I'} \; x_{I'}. \tag{1.2}$$

Let $(.|.)$ denote either the canonical bilinear form or the canonical hermitian scalar product defined on C^p by $(\omega^H|\omega^K) = \delta_{H,K}$. For all $z \in C_p$ let $g_z \in C^p$ defined by

$$(f|g_z) = \langle z, f \rangle \; \forall f \in C^p$$

Then the map $z \to g_z$ from C_p to C^p is bijective; it is linear in the bilinear case and conjugate-linear in the hermitian case. Note that for any subset I of cardinality p $g_{x_I} = \omega^I$, hence:

$$g_{\varrho(\omega^I)} = \rho_{I,I'}\ \omega^{I'}. \tag{1.3}$$

Remark
If $\mathcal{G}$ is nilpotent with maximal torus T and $tx_i = \alpha_i(t)x_i\ \forall t \in T$, then $tg_{x_i} = t\ \omega^i = -\alpha_i(t)\omega^i = -\alpha_i(t)g_{x_i}$ hence $g_z \in (C^p)^{-\beta}\ \forall z \in (C_p)^\beta$ and

$$g_{\varrho(f)} \in (C^{n-p})^{-S-\alpha}\ \forall f \in (C^p)^\alpha. \tag{1.4}$$

1.3.3.4

Let $d^* : C^{p+1} \to C^p$ the adjoint of d for $(.|.)$. Then

$$\begin{aligned} (df|g_z) &= \langle z, df\rangle\ \forall f \in C^p\ \forall z \in C_{p+1} \\ &= \langle \partial z, f\rangle \text{ since } d = {}^t\partial \\ &= (f|g_{\partial z}) \end{aligned}$$

i.e.

$$d^* g_z = g_{\partial z}. \tag{1.5}$$

1.3.3.5

Lemma 2 *If $\mathcal{G}$ is unimodular , then*

$$d(g_{\varrho(f)}) = (-1)^p g_{\varrho(d^* f)}\ \ \forall f \in C^p$$

Proof
Denote by Φ the "Hodge operator" on the cochain space $\bigoplus_{p=0}^n C^p$, defined by

$$\Phi f = g_{\varrho(f)}\ \ \forall f \in C^p \tag{1.6}$$

(Φ is linear in the bilinear case and conjugate-linear in the hermitian case).

For I subset of cardinality p of $\{1, ..., n\}$

$$\Phi^2(\omega^I) = \rho_{I,I'}\ \Phi(\omega^{I'}) = \rho_{I,I'}\ \rho_{I',I}\ \omega^I = (-1)^{n(n+1)/2}\ \varepsilon_p\ \omega^I$$

where $\varepsilon_p = (-1)^{p(p+1)/2}\ (-1)^{(n-p)(n-p+1)/2}$. Hence

$$\Phi^2 = (-1)^{n(n+1)/2} \bigoplus_{p=0}^{n} \varepsilon_p\ Id_{C^p}. \tag{1.7}$$

From lemma 1, one has for $f \in C^p,\ 0 \leq p \leq n$.

$$d^*\Phi f = d^*(g_{\varrho(f)}) = g_{\partial\varrho(f)} = (-1)^{p+1} g_{\varrho(df)} = (-1)^{p+1}\Phi(df).$$

Hence

$$d^*\Phi^2 f = (-1)^{n-p+1}\Phi d\Phi f\ \ \forall f \in C^p$$

and

$$\Phi d^*\Phi^2 f = (-1)^{n-p+1}\Phi^2 d\Phi f\ \ \forall f \in C^p$$

i.e.

$$\varepsilon_p \Phi d^* f = (-1)^{n-p+1}\varepsilon_{n-p+1}\ d\Phi f\ \ \forall f \in C^p.$$

As $\varepsilon_{n-p+1} = (-1)^{(n-p+1)(n-p+2)/2}\ (-1)^{(p-1)p/2} = (-1)^{n-p+1}\ \varepsilon_p\ (-1)^p = (-1)^{n+1}\ \varepsilon_p$, one gets

$$\Phi d^* f = (-1)^p\ d\Phi f\ \ \forall f \in C^p$$

1.3.3.6

Definition 1 *A cocycle $f \in Z^p_{\mathbb{C}}$ is called harmonic if $df = d^*f = 0$.*

Corollary 3 *Let $f \in Z^p_{\mathbb{C}}$. Then $g_{\varrho(f)} \in Z^{n-p}_{\mathbb{C}}$ if and only if f is harmonic; in that case, $g_{\varrho(f)}$ is also harmonic.*

1.3.3.7

Lemma 4 ([5]) *let d, δ be endomorphisms of the finite-dimensional vector space E such that $d^2 = \delta^2 = 0$ and let $\Delta = d\delta + \delta d$. If d and δ are disjoint (i.e. $\forall x \in E$ $d\delta x = 0 \Rightarrow \delta x = 0$ and $\delta dx = 0 \Rightarrow dx = 0$) then*

$$\begin{aligned} Ker\ \Delta &= Ker\ d \cap Ker\ \delta \\ E &= Im\ d \oplus Im\ \delta \oplus Ker\ \Delta \\ Ker\ d/Im\ d &\cong Ker\ \Delta \end{aligned}$$

Proof. (We give the proof for the sake of completeness)

For $x \in$ Ker Δ, $d\delta x = -\delta dx$. If $y = d\delta x$, then $\delta y = \delta d\delta x = 0$ implies $d\delta x = 0$ i.e. $y = 0$, and then $\delta x = 0$ whence $\delta dx = 0$ and $dx = 0$. Hence Ker $\Delta \subset$ Ker $d \cap$ Ker δ , and the equality follows. Now Im $d \cap$ Im $\delta = \{0\}$ since $dx = \delta z$ implies $d\delta z = 0$ hence $\delta z = 0$. In the same way,(Im $d \oplus$ Im $\delta) \cap$ Ker $\Delta = 0$, hence one has a direct sum Im $d \oplus$ Im $\delta \oplus$ Ker Δ. As Im $\Delta \subset$ Im $d \oplus$ Im δ and dim Im $\Delta +$ dim Ker $\Delta =$ dim E one has Im $d \oplus$ Im $\delta =$ Im Δ and $E =$ Im $\Delta \oplus$ Ker Δ. The mapping $x \mapsto [x]$ from Ker Δ to Ker d/Im d is one-to-one; let us show that it is onto. Let $z \in$ Ker d. Then $z = \alpha + \beta$, $\alpha \in$ Im Δ, $\beta \in$ Ker Δ, $\alpha = d\delta\gamma + \delta d\gamma$, $\gamma \in E; dz = 0 = d\alpha = d\delta d\gamma$ hence $\delta d\gamma = 0$ and $\alpha \in$ Im d i.e. $[z] = [\beta]$.

The lemma is applied with $E = \bigoplus_{p=0}^{n} C^p$, Ker d/Im $d = \bigoplus_{p=0}^{n} H^p_{\mathbb{C}}$.

1.3.3.8

In the case where $(.|.)$ is the canonical bilinear form on C^p, d and d^* are not disjoint and the lemma doesn't apply: a cohomology class need not contain any harmonic cocycle.

Example

Consider the Lie algebra $\mathcal{G}_{7,1.3(i_\lambda)}$ for $\lambda = i\sqrt{3}$. Then

$$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-4\beta_1}\ \langle \omega^{1,5}, \omega^{1,4} \rangle + 3e^{-5\beta_1}\ \langle \omega^{2,4}, \omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4} \rangle.$$

$\omega^{2,4}$ is not cohomologous to any harmonic cocycle. In fact, the cocycles cohomologous to $\omega^{2,4}$ are

$$\pi = \omega^{2,4} + a\omega^{1,2} + b\omega^{1,3} + c(\omega^{1,4} + \omega^{2,3}) + s(\omega^{1,6} + \lambda\omega^{2,4} + \omega^{2,5} + \omega^{3,5})$$

where $a, b, c, s \in \mathbb{C}$. Now, in the bilinear case: $d^*\omega^{2,4} = -\lambda\omega^7, d^*\omega^{1,2} = -\omega^4, d^*\omega^{1,3} = -\omega^5, d^*\omega^{1,4} = -\omega^6, d^*\omega^{1,6} = -\omega^7, d^*\omega^{2,3} = -\omega^6, d^*\omega^{2,5} = -\omega^7, d^*\omega^{3,5} = -\omega^7$ hence $d^*\pi = -a\omega^4 - b\omega^5 - 2c\omega^6 - \omega^7\ (\lambda + s(\lambda^2+3)) \neq 0\ \ \forall a, b, c, s \in \mathbb{C}$i.e. π is never harmonic.

In the case where $(.|.)$ is the canonical hermitian scalar product on C^p , d and d^* are disjoint and the lemma applies.
In the case of the example, one then has $d^*\omega^{2,4} = -\bar{\lambda}\omega^7$ and π is cohomologous to the harmonic cocycle $\omega^{2,4} - \frac{\bar{\lambda}}{|\lambda|^2+3}(\omega^{1,6} + \lambda\omega^{2,4} + \omega^{2,5} + \omega^{3,5})$.

1.3.3.9

For the isomorphism $[f] \mapsto [\varrho(f)]$ from $H^p_{\mathbb{C}}$ onto $(H^{n-p}_{\mathbb{C}})^*$ to lift to a map $[f] \mapsto [g_{\varrho(f)}]$ from $H^p_{\mathbb{C}}$ onto $H^{n-p}_{\mathbb{C}}$ we must use harmonic cocycles and it is thus necessary to work with the case where $(.|.)$ is the canonical hermitian scalar product on C^p. Then $H^p_{\mathbb{C}}$ is isomorphic to the space of harmonic cocycles and the map $[f] \mapsto [g_{\varrho(f)}]$ which assigns to the class of the harmonic cocycle f the class of the harmonic cocycle $g_{\varrho(f)}$ is a conjugate-linear isomorphism from $H^p_{\mathbb{C}}$ onto $H^{n-p}_{\mathbb{C}}$

One has $[g_{\varrho(f)}] \in (H^{n-p})^{-S-\alpha}\ \ \forall\ [f] \in (H^p)^\alpha$.

Solution to step 1 is thus to use harmonic cocycles and hermitian scalar product. If (ψ_j) is a basis for $H^p_{\mathbb{C}}$ consisting of harmonic cocycles (we drop [ ]), then $(g_{\varrho(\psi_j)})$ is a basis for $H^{n-p}_{\mathbb{C}}$ consisting of harmonic cocycles.

1.4 Riemann Hypothesis for trivial cohomology

1.4.1

Let T be a torus for the Lie algebra $\mathcal{G}$ and E a $(T \oplus \mathcal{G})$-module (semi-direct product). We suppose that E is semi-simple as T-module, and denote by $(\alpha_i)_{1\le i\le n}$ the T-roots of $\mathcal{G}$ associated to a basis $(x_i)_{1\le i\le n}$ of $\mathcal{G}$ consisting of T-eigenvectors.

Theorem 1 (Riemann-Roch [14]) *The following identity holds in $\mathbb{Z}[T^*]$:*

$$\sum_{p=0}^{n}(-1)^p ch(H^p(\mathcal{G}, E)) = \left[\prod_{i=1}^{n}(1 - e^{-\alpha_i})\right] ch(E)$$

where $ch(M)$ denotes the character of the semi-simple T-module M.

Proof

From

$$\mathrm{ch}(H^{q+1}(\mathcal{G},E)) = \mathrm{ch}(Z^{q+1}(\mathcal{G},E)) + \mathrm{ch}(Z^{q}(\mathcal{G},E)) - \mathrm{ch}(C^{q}(\mathcal{G},E))$$

one gets

$$\sum_{p=0}^{n}(-1)^p \mathrm{ch}(H^p(\mathcal{G},E)) = \sum_{p=0}^{n}(-1)^p \mathrm{ch}(C^p(\mathcal{G},E))$$

Since $C^p(\mathcal{G},E) = (\bigwedge^p \mathcal{G}^*) \otimes E$ one has

$$\mathrm{ch}(C^p(\mathcal{G},E)) = \mathrm{ch}(\bigwedge{}^p \mathcal{G}^*)\ \mathrm{ch}(E)$$

Finally:

$$\mathrm{ch}(\bigwedge{}^p \mathcal{G}^*) = \sum_{\nu_1+\ldots+\nu_n=p} e^{-\nu_1\alpha_1}\ldots e^{-\nu_n\alpha_n}$$

$$\sum_{p=0}^{n}(-1)^p \mathrm{ch}(\bigwedge{}^p \mathcal{G}^*) = \prod_{i=1}^{n}(1-e^{-\alpha_i})$$

Note that for $T=0$ one gets the formula

$$\sum_{p=0}^{n}(-1)^p \dim(H^p(\mathcal{G},E)) = 0$$

1.4.2

Now, take T maximal torus for $\mathcal{G}$ and $E=\mathbb{C}$ trivial $\mathcal{G}$-module. Then

$$P = \prod_{i=1}^{n}(1-e^{-\alpha_i}) \in \mathbb{Z}[T^*]$$

is called the "associated polynomial" [4]. From the Riemann-Roch Theorem, $\sum_{i=0}^{n}(\dim H^i_{\mathbb{C}})$ is at least equal to the sum of absolute values of all coefficients of P, i.e. the "length of P", and equality holds if and only if $H^i_{\mathbb{C}}$ and $H^j_{\mathbb{C}}$ have no common weight whenever $i \neq j$.

Definition 2 *A nilpotent Lie algebra $\mathcal{G}$ with maximal torus T is said to verify the Riemann Hypothesis if $H^i_{\mathbb{C}}$ and $H^j_{\mathbb{C}}$ have no common weight whenever $i \neq j$.*

Introduce

$$\tilde{\mathbb{C}} = \mathbb{C}(e^{\alpha_1}, ..., e^{\alpha_n})$$

$$\tilde{\mathcal{G}} = \mathcal{G} \otimes \tilde{\mathbb{C}}$$

$$f = \mathrm{diag}(e^{\alpha_1}, ..., e^{\alpha_n}) \in \mathrm{Aut}(\tilde{\mathcal{G}})$$

Then, denoting by $\check{f}$ the contragredient of f,

$$\mathrm{ch}(H^p(\mathcal{G}, \mathbb{C})) = \mathrm{Tr}(\check{f}|H^p(\tilde{\mathcal{G}}, \tilde{\mathbb{C}}))$$

Definition 3 ([4]) *The ζ-function of $\mathcal{G}$ is defined for $z \in \mathbb{C}$by*

$$\zeta(z) = \prod_{p=0}^{n} \left(det(1 - z\check{f}|H^p(\tilde{\mathcal{G}}, \tilde{\mathbb{C}}))\right)^{(-1)^{p+1}}$$

Then the Riemann Hypothesis (RH) is just that the various factors in $\zeta(z)$ are relatively prime.

1.4.3

If $\mathcal{G}_1$, $\mathcal{G}_2$ are nilpotent with maximal tori T_1, T_2, then $T_1 \otimes T_2$ is a maximal torus for the direct product $\mathcal{G}_1 \times \mathcal{G}_2$ and it follows from the Künneth formula

$$H^p(\mathcal{G}_1 \times \mathcal{G}_2, \mathbb{C}) = \sum_{p_1+p_2=p} H^{p_1}(\mathcal{G}_1, \mathbb{C}) \otimes H^{p_2}(\mathcal{G}_2, \mathbb{C})$$

that $\mathcal{G}_1 \times \mathcal{G}_2$ verifies RH if and only if both $\mathcal{G}_1$ and $\mathcal{G}_2$ verify RH . It is then enough to consider only indecomposable nilpotent Lie algebras.

1.4.4

It is known that RH is verified for all nilpotent Lie algebras of dimension $n \leq 6$. Consider $n = 7$. According to the rank, one has the following number of indecomposable (non equivalent) weight systems:

rank	number of weight systems
0	1
1	21 + (3 having a zero weight)
2	45
3	24
4	4
	total: 98

Clearly, Lie algebras having a zero weight don't verify RH. Hence we discard those systems having a zero weight. We then get two kinds of weight systems:

- Type 1: There exists only one nilpotent Lie algebra (isomorphic Lie algebras are identified) having that weight system.

- Type 2: There exist a continous 1-parameter family $\mathcal{G}_\lambda$, $\lambda \in \mathbb{C}$of nilpotent Lie algebras with $\mathcal{G}_\lambda$ isomorphic to $\mathcal{G}_{\lambda'}$ if and only if λ' is deduced from λ by a finite group of transformations (symmetry group of the Dynkin diagram), and a finite number of limit points of $\mathcal{G}_\lambda$'s having that weight system.

From our results, it is a fact that all Lie algebras belonging to a weight system of type 1 verify RH; for a type 2 system, the continuous series verifies RH, except for some singular values of the parameter, for which the cohomology is not the generic one, and the limit points may or may not verify RH.

For examples in higher dimensions, see [10].

1.5 Organization of results for trivial cohomology

The trivial cohomology is computed twice, in two distinct ways. The first way doesn't make use of Poincaré Duality. Hence, the basis exhibited for $H^{n-p}_{\mathbb{C}}$ is in general not the dual basis of the basis exhibited for $H^p_{\mathbb{C}}$. The computer gives characters and dimensions for the whole trivial cohomology, all nonzero coboundaries for the canonical basis of $\bigwedge^p \mathcal{G}^*$ and finally the Poincaré polynomial

$$P(Y) = \sum_{i=0}^{n} (\dim H^i_{\mathbb{C}})\, Y^i \qquad n = \dim \mathcal{G}$$

is factorized on $\mathbb{Z}[Y]$.

The second way is via harmonic cocycles and Poincaré Duality. The computer gives all nonzero d^*-coboundaries and all nonzero laplacians (nonzero coboundaries are not given to save place since they already appear in the first way); harmonic p-cocycles and characters of the corresponding $H^p_{\mathbb{C}}$'s are computed for $2p \leq n$; then the dual basis for $H^{n-p}_{\mathbb{C}}$ is computed via Poincaré Duality and characters of the corresponding $H^{n-p}_{\mathbb{C}}$'s are given with the dual basis. All dimensions are given; the length of the associated polynomial is computed and RH is tested. Finally the Poincaré polynomial is written again.

1.6 Some explicit formulae in arbitrary dimension

1.6.1 Heisenberg Lie Algebras

Let $\mathcal{G}$ be the $(2N+1)$-dimensional Heisenberg Lie algebra. Then [9] (see also [12]):

$$\dim(H^p_{\mathcal{G}}) = (2N+1)C^p_{2N+1} - C^{p+1}_{2N+1} - 2NC^{p-1}_{2N+1} \; (0 \leq p \leq N)$$
$$\dim(H^{N+1}_{\mathcal{G}}) = (2N+1)(C^N_{2N} - C^{N-2}_{2N}) - C^{N-1}_{2N} + C^{N-3}_{2N}$$
$$\dim(H^{2N+1-p}_{\mathcal{G}}) = 2N(C^p_{2N} - C^{p-2}_{2N}) - C^{p-1}_{2N} + C^{p-3}_{2N} \; (0 \leq p \leq N-1)$$

where C^q_p denotes the binomial coefficient extended by 0 for $q > p$ or $q < 0$.

One also has for $0 \leq p \leq N$

$$\dim(H^p_{\mathbb{C}}) = C^p_{2N} - C^{p-2}_{2N}$$

and the trivial cohomology has total dimension $2C^N_{2N+1}$.

It is easy to see that $\mathcal{G}$ verifies RH : if $(x_i)_{1\leq i\leq 2N+1}$, denotes a basis of $\mathcal{G}$ with the commutation relations $[x_i, x_{N+i}] = x_{2N+1}$ $(1 \leq i \leq N)$ then

$$T = \operatorname{diag}(\lambda_1, ..., \lambda_N, \lambda_{N+1}, \lambda_1 + \lambda_{N+1} - \lambda_2, ..., \lambda_1 + \lambda_{N+1} - \lambda_N, \lambda_1 + \lambda_{N+1})$$

is a maximal torus , and the weights $\beta_1, ..., \beta_{N+1}$ of $x_1, ..., x_{N+1}$ form a basis of T^*. If we define the height $h(\sum_{i=1}^{N+1} c_i\beta_i) = \sum_{i=1}^{N+1} c_i$, then all weights of $\mathcal{G}/[\mathcal{G},\mathcal{G}]$ have height 1. Since $\{\omega^{i_1,...,i_p}, 1 \leq i_1 < ... < i_p \leq 2N\}$ is a basis for $Z^p_{\mathbb{C}}$ $0 \leq p \leq N$, all weights of $Z^p_{\mathbb{C}}$ have height $-p$; since the weights of $H^{N+1}_{\mathbb{C}}$ have the form $-S - \alpha$, $S = (N+1)(\beta_1 + \beta_{N+1})$, α a weight of $H^N_{\mathbb{C}}$, they have height $-(N+2)$. By Poincaré Duality , RH is satisfied.

1.6.2 Generic filiform Lie algebra

Let $\mathcal{G}$ be the $(n+1)$-dimensional generic filiform Lie algebra with basis $(x_i)_{0\leq i\leq n}$ and commutation relations $[x_0, x_i] = x_{i+1}\ \forall i\ 1 \leq i < n$. $(\omega^i)_{0\leq i\leq n}$ denotes the dual basis of $\mathcal{G}^*$.

1.6.2.1

The following formula is proved in [2]:

$$\dim(H^k_{\mathbb{C}}) = \varphi(n,k,s(n,k)) + \varphi(n,k-1,s(n,k-1))\ (2 \leq k \leq n) \tag{1.8}$$

where $s(n,k) = \left[\frac{k(n+1)}{2}\right]$ (here [.] denotes entire part) and

$$\varphi(n,k,p) = \mathrm{card}(\{(i_1,...,i_k);\ 1 \leq i_1 < ... < i_k \leq n;\ i_1 + ... + i_k = p\}) \tag{1.9}$$

In particular,

$$\begin{aligned}\dim(H^2_{\mathbb{C}}) &= \left[\frac{n}{2}\right] + 1\\ \dim(H^3_{\mathbb{C}}) &= \begin{cases} C^2_{q+1} & \text{if } n = 2q\\ C^2_{q+1} + \left[\frac{q+1}{2}\right] & \text{if } n = 2q+1\end{cases}\end{aligned}$$

1.6.2.2

The derivation of 1.8 in [2] starts from the following facts. If π_{n-1} denotes the canonical irreducible representation of the Lie algebra $sl(2)$ on $V = \langle x_1,...,x_n\rangle$, then the adjoint action of x_0 on V coincides with the action of Y, where $\{H,X,Y\}$ is the Weyl basis for $sl(2)$ and one gets in the contragredient representation $(\bigwedge^k \pi_{n-1})^*$ of $\bigwedge^k \pi_{n-1}$ in $\bigwedge^k V^*$

$$d\omega = \omega^0 \wedge (Y\cdot\omega)\ \ \forall \omega \in \bigwedge^k V^* \tag{1.10}$$

$$Z^k_{\mathbb{C}} = \left(\mathbb{C}\omega^0 \wedge \bigwedge^{k-1} V^*\right) \oplus \left(\bigwedge^k V^*\right)^Y \tag{1.11}$$

$$H^k_{\mathbb{C}} = H^k_0 \oplus \left(\bigwedge^k V^*\right)^Y \tag{1.12}$$

where $(\bigwedge^k V^*)^Y$ denotes the annihilator of Y in $\bigwedge^k V^*$ and

$$H^k_0 = \mathbb{C}\omega^0 \wedge \left(\bigwedge^{k-1} V^* \Big/ Y\cdot(\bigwedge^{k-1} V^*)\right) \tag{1.13}$$

Then the decomposition into irreducible representations

$$\left(\bigwedge^k \pi_{n-1}\right)^* = \pi_{\ell_1} \oplus ... \oplus \pi_{\ell_s} \tag{1.14}$$

gives $\dim \left(\bigwedge^k V^*\right)^Y = s$; as the number of irreducible factors in an arbitrary representation of $sl(2)$ is the sum of the multiplicities of 0 and 1 (or -1) as eigenvalues of H , the formula (for $k \geq 1$)

$$\dim \left(\bigwedge^k V^*\right)^Y = \varphi(n, k, s(n,k))$$

results from the formula

$$H \cdot \omega^I = (2|I| - k(n+1))\, \omega^I \tag{1.15}$$

for all $I = \{i_1, ..., i_k\},\ 1 \leq i_1 < ... < i_k \leq n$ where we denote $|I| = i_1 + ... + i_k$.

1.6.2.3

We now prove that $\mathcal{G}$ satisfies RH.
$\mathcal{G}$ has rank 2;

$$T = \operatorname{diag}(\lambda_0, \lambda_1, \lambda_0 + \lambda_1, ..., (n-1)\lambda_0 + \lambda_1)$$

is a maximal torus , and the weights β_0, β_1 of x_0, x_1 form a basis of T^*.
π_{n-1} and T do not commute, however from 1.15 an element $\psi \in \bigwedge^k V^*$ is an eigenvector for H iff it is an eigenvector for T, since the T-weight of ω^I in 1.15 is $(k - |I|)\beta_0 - k\beta_1$.
Now ω^I $(I = \{i_1, ..., i_k\},\ 0 \leq i_1 < ... < i_k \leq n)$ and ω^J $(J = \{j_1, ..., j_\ell\},\ 0 \leq j_1 < ... < j_\ell \leq n)$ $\ell > k$ have identical T-weight iff

$$\ell = k + 1,\ i_1 \geq 1,\ j_1 = 0,\ |I| = |J| + 1.$$

Hence common weights can only occur between $\left(\bigwedge^k V^*\right)^Y$ and H_0^{k+1}.
For fixed k, let v_r (resp: w_r) a primitive vector (resp: minimal weight vector) for π_{ℓ_r} $(1 \leq r \leq s)$. By 1.14, $\left(\bigwedge^k V^*\right)^Y = \bigoplus_{r=1}^s \mathbb{C} w_{\ell_r}$, and $\bigoplus_{r=1}^s \mathbb{C} v_{\ell_r}$ is a T-invariant supplementary subspace to $Y \cdot (\bigwedge^k V^*)$. Hence $\mathcal{G}$ doesn't verify RH iff there exists an integer k and T-eigenvectors $\phi = \sum_I \phi_I \omega^I \in \left(\bigwedge^k V^*\right)^Y$, $\phi \neq 0$ and $\psi = \sum_J \psi_J \omega^J \in \bigoplus_{r=1}^s \mathbb{C} v_{\ell_r}$, $\psi \neq 0$ such that for all I, J $|I| = |J| + 1$. But then ψ and ϕ would have as H-weights $\alpha = 2|J| - k(n+1)$ and $\alpha + 2$ respectively which contradicts the fact that ψ is a highest weight vector for $\left(\bigwedge^k \pi_{n-1}\right)^*$.

1.6.2.4

$G_n(t,q) = \prod_{j=1}^{n}(1+tq^j)$ is a generating function for $\varphi(n,k,p)$, i.e.

$$\varphi(n,k,p) = \frac{1}{k!p!}\frac{\partial^{k+p}G_n}{\partial t^k \partial q^p}(0,0)$$

Chapter 2

3 and 4-dimensional Lie Algebras

2.1 Algebra $\mathcal{G}_3$ (3-dim Heisenberg Lie Algebra) †

Adjoint cohomology

$[x_1, x_2] = x_3$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 \\ \xi_1^2 & \xi_2^2 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_1+\beta_2}\ \langle x_3 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_3 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{\beta_2}\ \langle \delta_1^3 \rangle + e^{\beta_1}\ \langle \delta_2^3 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^2] = -t_2 + t_1$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_2}\ \langle x_1\omega^{2,3} \rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3} \rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + x_2\omega^{2,3} \rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2} \rangle + e^{-\beta_2}\ \langle x_1\omega^{1,2} \rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-2\beta_2} + e^{-2\beta_1-\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 3$: $(1, 6, 8, 3)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 3$: $(1, 4, 5, 2)$

S=$2(\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 3$: $(1, 2, 3, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 3$: $(1, 2, 2, 1)$

$d\omega^3 = -\omega^{1,2}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle \omega^1 \rangle + e^{-\beta_2}\ \langle \omega^2 \rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-2\beta_1-\beta_2}\ \langle \omega^{1,3} \rangle + e^{-\beta_1-2\beta_2}\ \langle \omega^{2,3} \rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-2\beta_1-2\beta_2}\ \langle \omega^{1,2,3} \rangle$.

$P(Y) = (1 + Y + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 3$: $(1,2,3,1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 3$: $(1,2,2,1)$
Total dimension of trivial cohomology: 6.
Length associated polynomial: 6.
Riemann Hyp.: yes
$d\omega^3 = -\omega^{1,2}$;
$d^\star\omega^{1,2} = -\omega^3$;
$\Delta\omega^3 = \omega^3$; $\Delta\omega^{1,2} = \omega^{1,2}$;
$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.
$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2} \langle-\omega^{1,3}\rangle$.
$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,3}\rangle$.
$P(Y) \;=\; (1+Y+Y^2)(1+Y)$.

2.2 Algebra $\mathcal{G}_4$ (4-dim generic filiform Lie algebra) †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 + \xi_2^2 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = 0.$

Maximal torus : $T{=}\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_4\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_2^3] = -\delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_2}\ \langle x_1\omega^{2,3}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,4}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + x_2\omega^{2,3}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2}\rangle + e^{-\beta_2}\ \langle -x_1\omega^{1,2} + x_4\omega^{2,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-4\beta_1-\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-3\beta_1-3\beta_2} + e^{-4\beta_1-2\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 4$: $(1, 7, 15, 14, 4)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 4$: $(1, 4, 6, 5, 2)$

S=$4\beta_1 + 3\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 4$: $(1, 2, 4, 4, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 4$: $(1, 2, 2, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{3,4} = -\omega^{1,2,4}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,4}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,4}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4}\rangle$.

$P(Y)\ =\ (1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 4$: $(1,2,4,4,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 4$: $(1,2,2,2,1)$

Total dimension of trivial cohomology: 8.

Length associated polynomial: 8.

Riemann Hyp.: yes

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{3,4}=-\omega^{1,2,4}$;

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,2,3}=-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{3,4}$;

$\Delta\omega^3=\omega^3$; $\Delta\omega^4=\omega^4$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{2,4}=\omega^{2,4}$; $\Delta\omega^{3,4}=\omega^{3,4}$; $\Delta\omega^{1,2,3}=\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^{1}\rangle + e^{-\beta_2}\ \langle\omega^{2}\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,4}\rangle + e^{-4\beta_1-2\beta_2}\ \langle-\omega^{1,3,4}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4}\rangle$.

$P(Y)\ =\ (1+Y^2)(1+Y)^2$.

Chapter 3

5-dimensional Lie Algebras

3.1 rank 1 Lie algebras

3.1.1 Algebra $\mathcal{G}_{5,6}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 4\xi_1^1 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 + \xi_2^4 & \xi_1^2 + \xi_2^3 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_1^5, ad\ x_5 = 0.$

Maximal torus : T=$\langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{2\beta_1}\ \langle x_2 \rangle + e^{3\beta_1}\ \langle x_3 \rangle + e^{4\beta_1}\ \langle x_4 \rangle + e^{5\beta_1}\ \langle x_5 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_5 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{3\beta_1}\ \langle \delta_1^4, \delta_2^5 \rangle + e^{4\beta_1}\ \langle \delta_1^5 \rangle + 2e^{2\beta_1}\ \langle \delta_1^3, \delta_2^4 \rangle + 2e^{\beta_1}\ \langle \delta_1^2, \delta_2^3 \rangle + e^0\ \langle t_1 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^2] = -\delta_1^5$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_1^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$; $[\delta_2^4, \delta_1^2] = -\delta_2^5 + \delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle \delta_1^4 \rangle + e^{2\beta_1}\ \langle \delta_2^4 \rangle + e^{\beta_1}\ \langle \delta_1^2 \rangle + e^0\ \langle t_1 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^2] = 2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $3e^{-2\beta_1}\ \langle -x_1\omega^{1,2} + 2x_3\omega^{1,4} + x_4\omega^{2,4}, x_1\omega^{1,2} - x_3\omega^{1,4} + x_4\omega^{1,5}, x_2\omega^{1,3} \rangle + 2e^{-3\beta_1}\ \langle x_3(\omega^{1,5} + \omega^{2,4}) + x_4\omega^{2,5}, -x_1\omega^{1,3} - x_2(\omega^{1,4} - \omega^{2,3}) \rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_4\omega^{2,3} \rangle + e^{-4\beta_1}\ \langle 3/2x_1(\omega^{1,4} - \omega^{2,3}) - x_2(\omega^{1,5} + \omega^{2,4}) + x_3(\omega^{2,5} - 1/2\omega^{3,4}) + x_4\omega^{3,5} \rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1} + 2e^{-6\beta_1} + 3e^{-7\beta_1} + 2e^{-8\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-8\beta_1} + e^{-9\beta_1} + e^{-10\beta_1} + e^{-11\beta_1} + e^{-12\beta_1} + e^{-13\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$e^{-13\beta_1} + e^{-14\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(1, 8, 24, 34, 22, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(1, 4, 7, 8, 6, 2)$

S=$15\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 5$: $(1, 2, 6, 7, 5, 1)$

Dimensions of $H_{\mathbb{C}}^{j}$'s $0 \le j \le 5$: $(1,2,3,3,2,1)$

$d\omega^{3} = -\omega^{1,2}$; $d\omega^{4} = -\omega^{1,3}$; $d\omega^{5} = -\omega^{1,4} - \omega^{2,3}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$.

$\mathrm{ch}(H_{\mathbb{C}}^{1}) = e^{-\beta_1}\ \langle\omega^{1}\rangle + e^{-2\beta_1}\ \langle\omega^{2}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{2}) = e^{-5\beta_1}\ \langle\omega^{1,4}\rangle + e^{-6\beta_1}\ \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-7\beta_1}\ \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{3}) = e^{-8\beta_1}\ \langle\omega^{1,2,5}\rangle + e^{-9\beta_1}\ \langle\omega^{1,3,5}\rangle + e^{-10\beta_1}\ \langle\omega^{1,4,5} - \omega^{2,3,5}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{4}) = e^{-13\beta_1}\ \langle\omega^{1,3,4,5}\rangle + e^{-14\beta_1}\ \langle\omega^{2,3,4,5}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{5}) = e^{-15\beta_1}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z_{\mathbb{C}}^{j}$'s $0 \le j \le 5$: $(1,2,6,7,5,1)$

Dimensions of $H_{\mathbb{C}}^{j}$'s $0 \le j \le 5$: $(1,2,3,3,2,1)$

Total dimension of trivial cohomology: 12.

Length associated polynomial: 12.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{3}$; $d^{\star}\omega^{1,3} = -\omega^{4}$; $d^{\star}\omega^{1,4} = -\omega^{5}$; $d^{\star}\omega^{2,3} = -\omega^{5}$; $d^{\star}\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^{\star}\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^{\star}\omega^{1,2,5} = -\omega^{3,5}$; $d^{\star}\omega^{1,3,4} = -\omega^{3,5}$; $d^{\star}\omega^{1,3,5} = -\omega^{4,5}$; $d^{\star}\omega^{2,3,4} = \omega^{4,5}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{1,4,5} - \omega^{2,3,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{2,4,5}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{3,4,5}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = 2\omega^{5}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$;

$\mathrm{ch}(H_{\mathbb{C}}^{1}) = e^{-\beta_1}\ \langle\omega^{1}\rangle + e^{-2\beta_1}\ \langle\omega^{2}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{2}) = e^{-5\beta_1}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-6\beta_1}\ \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-7\beta_1}\ \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{3}) = e^{-10\beta_1}\ \langle(-\omega^{1,4,5} + \omega^{2,3,5})/2\rangle + e^{-9\beta_1}\ \langle(-\omega^{1,3,5} - \omega^{2,3,4})/2\rangle + e^{-8\beta_1}\ \langle(-\omega^{1,2,5} + \omega^{1,3,4})/2\rangle$.

$\mathrm{ch}(H_{\mathbb{C}}^{4}) = e^{-14\beta_1}\ \langle\omega^{2,3,4,5}\rangle + e^{-13\beta_1}\ \langle-\omega^{1,3,4,5}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{A}})= e^{-15\beta_1}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)$.

3.2 rank 2 Lie algebras

3.2.1 Algebra $\mathcal{G}_{5,4}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_2, x_3] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1 + \xi_2^2 & \xi_2^1 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_1^2 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = 0, ad\ x_5 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+2\beta_2}\ \langle x_5\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+2\beta_2}\ \langle x_5\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_1+\beta_2}\ \langle \delta_1^4, \delta_2^5\rangle + e^{2\beta_1}\ \langle \delta_2^4\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1\rangle + e^{2\beta_2}\ \langle \delta_1^5\rangle + e^{\beta_2}\ \langle \delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^1] = \delta_2^4$; $[\delta_1^4, \delta_1^2] = -\delta_1^5$; $[\delta_2^4, \delta_1^2] = -\delta_2^5 + \delta_1^4$; $[\delta_2^3, \delta_1^3] = \delta_2^5 + \delta_1^4$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^1, \delta_1^5] = -\delta_2^5 + \delta_1^4$; $[\delta_2^1, \delta_2^5] = \delta_2^4$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_2^5, \delta_1^2] = \delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_1}\ \langle \delta_2^4\rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1\rangle + e^{2\beta_2}\ \langle \delta_1^5\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^2] = 2\delta_1^4$; $[\delta_2^1, \delta_1^5] = 2\delta_1^4$; $[\delta_2^1, \delta_1^4] = -\delta_2^4$; $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_1^4, \delta_1^2] = -\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $2e^{-\beta_2}\ \langle -x_1\omega^{1,2} + x_4\omega^{2,4}, x_1\omega^{1,2} + x_4\omega^{1,5}\rangle + e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,3}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_5\omega^{1,5}, x_4\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + x_2\omega^{2,3}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + e^{-4\beta_1-\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-2\beta_1-4\beta_2} + 2e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-5\beta_2} + e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-4\beta_2} + e^{-5\beta_1-3\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-4\beta_1-5\beta_2} + e^{-5\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(2, 10, 24, 35, 22, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(2, 7, 9, 9, 7, 2)$

S=5$(\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 5$: $(1,2,6,7,5,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 5$: $(1,2,3,3,2,1)$

$d\omega^{3}=-\omega^{1,2}$; $d\omega^{4}=-\omega^{1,3}$; $d\omega^{5}=-\omega^{2,3}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^{1}\rangle+e^{-\beta_2}\ \langle\omega^{2}\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,5}+\omega^{2,4}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle$.

ch$(H^3_{\mathbb{C}})$= $e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,4}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,5}\rangle+e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,3,4,5}\rangle+e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 5$: $(1,2,6,7,5,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 5$: $(1,2,3,3,2,1)$

Total dimension of trivial cohomology: 12.

Length associated polynomial: 12.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^{3}$; $d^\star\omega^{1,3}=-\omega^{4}$; $d^\star\omega^{2,3}=-\omega^{5}$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{2,3,4}=\omega^{4,5}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,4,5}$; $d^\star\omega^{1,2,4,5}=-\omega^{3,4,5}$;

$\Delta\omega^{3}=\omega^{3}$; $\Delta\omega^{4}=\omega^{4}$; $\Delta\omega^{5}=\omega^{5}$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,5}=\omega^{1,5}-\omega^{2,4}$; $\Delta\omega^{2,3}=\omega^{2,3}$; $\Delta\omega^{2,4}=-\omega^{1,5}+\omega^{2,4}$; $\Delta\omega^{3,4}=\omega^{3,4}$; $\Delta\omega^{3,5}=\omega^{3,5}$; $\Delta\omega^{4,5}=2\omega^{4,5}$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=\omega^{1,2,5}$; $\Delta\omega^{1,3,5}=\omega^{1,3,5}-\omega^{2,3,4}$; $\Delta\omega^{1,4,5}=\omega^{1,4,5}$; $\Delta\omega^{2,3,4}=-\omega^{1,3,5}+\omega^{2,3,4}$; $\Delta\omega^{2,4,5}=\omega^{2,4,5}$; $\Delta\omega^{3,4,5}=\omega^{3,4,5}$; $\Delta\omega^{1,2,3,4}=\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=\omega^{1,2,3,5}$; $\Delta\omega^{1,2,4,5}=\omega^{1,2,4,5}$;

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^{1}\rangle+e^{-\beta_2}\ \langle\omega^{2}\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,5}+\omega^{2,4}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle$.

ch$(H^3_{\mathbb{C}})$= $e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle+e^{-3\beta_1-3\beta_2}\ \langle(-\omega^{1,3,5}-\omega^{2,3,4})/2\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,4}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5}\rangle+e^{-5\beta_1-4\beta_2}\ \langle-\omega^{1,3,4,5}\rangle$.

$$\mathrm{ch}(H^5_{\mathcal{X}}) = e^{-5\beta_1-5\beta_2} \ \langle \omega^{1,2,3,4,5} \rangle.$$
$$P(Y) \ = \ (1+Y+Y^2)(1+Y^2)(1+Y).$$

3.2.2 Algebra $\mathcal{G}_{5,5}$ (5-dim generic filiform Lie Algebra) †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = -\delta_1^5, ad\ x_5 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1+\beta_2}\ \langle x_5\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_5\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^3, \delta_2^3] = -\delta_1^4$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3}\rangle + 2e^{-\beta_2}\ \langle x_1\omega^{1,2} + 2x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_5\omega^{2,5}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2}\rangle + e^{2\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + 2x_2\omega^{2,3} + x_3\omega^{2,4} + x_4\omega^{2,5} + x_5\omega^{3,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-2\beta_2} + 2e^{-4\beta_1-\beta_2} + e^{-5\beta_1-\beta_2} + e^{-6\beta_1-\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-3\beta_1-3\beta_2} + e^{-4\beta_1-2\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 5$: $(1, 9, 24, 34, 22, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 5$: $(1, 5, 8, 8, 6, 2)$

S=$7\beta_1 + 4\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1, 2, 6, 7, 5, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,2,3,3,2,1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,4}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,2,6,7,5,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,2,3,3,2,1)$

Total dimension of trivial cohomology: 12.

Length associated polynomial: 12.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,2,3} = -\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{3,5}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,4,5}$; $d^\star\omega^{1,2,4,5} = -\omega^{3,4,5}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-3\beta_1-3\beta_2}\ \langle-\omega^{2,3,4}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,5}\rangle + e^{-4\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5} + \omega^{1,3,4})/2\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5}\rangle + e^{-7\beta_1-3\beta_2}\ \langle-\omega^{1,3,4,5}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)$.

3.2.3 Algebra $\mathcal{G}_{5,3}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1 + \xi_2^2 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^3 + \xi_2^4 & 2\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^5, ad\ x_3 = -\delta_2^5, ad\ x_4 = -\delta_1^5, ad\ x_5 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_5\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle + 2e^{\beta_2}\ \langle\delta_1^4, \delta_3^5\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{2\beta_1}\ \langle\delta_2^5\rangle + 2e^{\beta_1}\ \langle\delta_1^3, \delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_2^4 + \delta_1^3$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_1^4, \delta_1^3] = \delta_1^5$; $[\delta_1^4, \delta_2^4] = -\delta_1^5$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_1^3] = -(\delta_3^5+\delta_1^4)$; $[\delta_1^2, \delta_2^4] = \delta_3^5 - \delta_1^4$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_1^3, \delta_2^4] = -\delta_2^5$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_1}\ \langle\delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_1^2, \delta_1^3] = -2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_2}\ \langle x_1\omega^{2,4}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,4} - \omega^{2,3}) + x_2\omega^{2,4}, x_1(\omega^{1,4} - \omega^{2,3}) + x_3(\omega^{1,5} - \omega^{3,4}) + x_4\omega^{2,5}\rangle + 2e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3\omega^{1,3}, x_2\omega^{1,2} + x_4\omega^{2,3}\rangle + e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle + 3e^{-2\beta_1}\ \langle -x_1\omega^{1,3} - 2x_2\omega^{1,4} + x_4\omega^{3,4}, -x_1\omega^{1,3} - x_2(\omega^{1,4} - \omega^{2,3}), -x_1\omega^{1,3} - x_2\omega^{1,4} + x_4\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle -x_2(\omega^{1,5} - \omega^{3,4}) + x_4\omega^{3,5}\rangle + 2e^{-\beta_2}\ \langle x_3\omega^{2,3}, -x_1\omega^{1,2} + x_3\omega^{1,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + e^{-5\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-3\beta_2} + 2e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-6\beta_1-\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-3\beta_2} + e^{-6\beta_1-2\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(1, 10, 28, 37, 23, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(1, 6, 13, 15, 10, 3)$

S=$3(2\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,3,6,8,5,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,3,4,4,3,1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-\beta_2}$ $\langle\omega^2\rangle + e^{-2\beta_1}$ $\langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-3\beta_1}$ $\langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}$ $\langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2}$ $\langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}$ $\langle\omega^{1,5} - \omega^{3,4}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-5\beta_1-\beta_2}$ $\langle\omega^{1,3,5}\rangle + e^{-3\beta_1-2\beta_2}$ $\langle\omega^{1,2,5}\rangle + e^{-4\beta_1-2\beta_2}$ $\langle\omega^{1,4,5} - \omega^{2,3,5}\rangle +$ $e^{-3\beta_1-3\beta_2}$ $\langle\omega^{2,4,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-4\beta_1-3\beta_2}$ $\langle\omega^{1,2,4,5}\rangle + e^{-6\beta_1-2\beta_2}$ $\langle\omega^{1,3,4,5}\rangle + e^{-5\beta_1-3\beta_2}$ $\langle\omega^{2,3,4,5}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-6\beta_1-3\beta_2}$ $\langle\omega^{1,2,3,4,5}\rangle$.

$P(Y) = (1+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,3,6,8,5,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1,3,4,4,3,1)$

Total dimension of trivial cohomology: 16.

Length associated polynomial: 16.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{3,4,5}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5}+\omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}+\omega^{2,3,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5}+\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$;

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-\beta_2}$ $\langle\omega^2\rangle + e^{-2\beta_1}$ $\langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-3\beta_1}$ $\langle\omega^{1,3}\rangle$ + $e^{-2\beta_1-\beta_2}$ $\langle\omega^{1,4} - \omega^{2,3}\rangle$ + $e^{-\beta_1-2\beta_2}$ $\langle\omega^{2,4}\rangle$ + $e^{-3\beta_1-\beta_2}$ $\langle\omega^{1,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= \quad e^{-3\beta_1-3\beta_2} \quad \langle -\omega^{2,4,5}\rangle \quad + \quad e^{-4\beta_1-2\beta_2} \quad \langle(-\omega^{1,4,5} \quad + \quad \omega^{2,3,5})/2\rangle \quad + e^{-5\beta_1-\beta_2} \ \langle -\omega^{1,3,5}\rangle + e^{-3\beta_1-2\beta_2} \ \langle(-\omega^{1,2,5} - \omega^{2,3,4})/2\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-5\beta_1-3\beta_2} \ \langle \omega^{2,3,4,5}\rangle + e^{-6\beta_1-2\beta_2} \ \langle -\omega^{1,3,4,5}\rangle + e^{-4\beta_1-3\beta_2} \ \langle \omega^{1,2,4,5}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-3\beta_2} \ \langle \omega^{1,2,3,4,5}\rangle.$

$P(Y) \ = \ (1+Y^2)(1+Y)^3.$

3.3 rank 3 Lie algebras

3.3.1 Algebra $\mathcal{G}_{5,2}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & \xi_3^2 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_1^1 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = 0, ad\ x_5 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{\beta_3} \langle x_3 \rangle + e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{\beta_1+\beta_3} \langle x_5 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{\beta_1+\beta_3} \langle x_5 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle + e^{\beta_2} \langle \delta_1^4 \rangle + 2e^{\beta_1} \langle \delta_2^4, \delta_3^5 \rangle + e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + 3e^0 \langle t_2, t_1, t_3 \rangle + e^{\beta_2-\beta_3} \langle \delta_3^2 \rangle + e^{\beta_3} \langle \delta_1^5 \rangle + e^{\beta_1-\beta_2+\beta_3} \langle \delta_2^5 \rangle + e^{-\beta_2+\beta_3} \langle \delta_2^3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_3^4] = -\delta_1^4$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_2^4, \delta_3^2] = \delta_3^4$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$; $[\delta_3^4, \delta_2^3] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^2, \delta_2^5] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_3^5] = \delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^5, \delta_2^3] = \delta_2^5$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle + e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + 3e^0 \langle t_2, t_1, t_3 \rangle + e^{\beta_2-\beta_3} \langle \delta_3^2 \rangle + e^{\beta_1-\beta_2+\beta_3} \langle \delta_2^5 \rangle + e^{\beta_1} \langle \delta_2^4 \rangle + e^{-\beta_2+\beta_3} \langle \delta_2^3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_3^4, \delta_2^3] = 2\delta_2^4$; $[\delta_3^2, \delta_2^5] = 2\delta_2^4$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{\beta_1-\beta_2-\beta_3} \langle x_1\omega^{2,3} \rangle + 2e^{-2\beta_1} \langle x_2\omega^{1,4}, x_3\omega^{1,5} \rangle + e^{-2\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,5} \rangle + 3e^{-\beta_3} \langle -x_1\omega^{1,3} + x_2\omega^{2,3}, -x_1\omega^{1,3} + x_4\omega^{3,4}, x_1\omega^{1,3} + x_4\omega^{2,5} \rangle + e^{-2\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,4} \rangle + 3e^{-\beta_2} \langle x_1\omega^{1,2} + x_3\omega^{2,3}, -x_1\omega^{1,2} + x_5\omega^{2,5}, x_4\omega^{2,4} \rangle + 2e^{-\beta_1} \langle x_2\omega^{1,2}, x_3\omega^{1,3} \rangle + e^{-\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,3} \rangle + e^{\beta_2-2\beta_3} \langle x_4\omega^{3,5} \rangle + e^{-\beta_1-\beta_3} \langle 2x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) - 2x_3\omega^{3,5} + x_4\omega^{4,5} \rangle + e^{-\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,2} \rangle + e^{-2\beta_2+\beta_3} \langle x_5\omega^{2,4} \rangle + e^{-\beta_1-\beta_2} \langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5} \rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 3e^{-\beta_1-\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + 3e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-\beta_3} + e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3}$.

ch($H^4_{\mathcal{G}}$)=$e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-2\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-2\beta_3}$.

ch($H^5_{\mathcal{G}}$)=$e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(2, 13, 31, 39, 23, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 5$: $(2, 10, 19, 20, 12, 3)$

S=$3\beta_1 + 2\beta_2 + 2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 5$: $(1, 3, 8, 8, 5, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 5$: $(1, 3, 6, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,5}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,5}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 5$: $(1, 3, 8, 8, 5, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 5$: $(1, 3, 6, 6, 3, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,2,3,4} = \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{3,4,5}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle-\omega^{2,3,4}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_3}\ \langle-\omega^{1,3,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle(\omega^{1,2,5}+\omega^{1,3,4})/2\rangle + e^{-2\beta_1-2\beta_2}\ \langle-\omega^{1,2,4}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,4,5}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle-\omega^{1,3,4,5}\rangle + e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,4,5}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,3,4,5}\rangle.$

$P(Y) = (1+2Y+4Y^2+2Y^3+Y^4)(1+Y).$

3.3.2 Algebra $\mathcal{G}_{5,1}$ (5-dim Heisenberg Lie Algebra)

Adjoint cohomology

$[x_1, x_3] = x_5$; $[x_2, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & \xi_4^1 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_4^1 & \xi_4^2 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & -\xi_1^2 & 0 \\ \xi_2^3 & \xi_2^4 & -\xi_2^1 & \xi_1^1 - \xi_2^2 + \xi_3^3 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_3^5, ad\ x_2 = \delta_4^5, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_2^5, ad\ x_5 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle x_4 \rangle + e^{\beta_1+\beta_3}\ \langle x_5 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_3}\ \langle x_5 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{\beta_1-\beta_3}\ \langle \delta_3^1 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_4^1 \rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^2 \rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4 \rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + 3e^0\ \langle t_2, t_1, t_3 \rangle + e^{\beta_3}\ \langle \delta_1^5 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^5 \rangle + e^{\beta_1}\ \langle \delta_3^5 \rangle + e^{\beta_2}\ \langle \delta_4^5 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^1, \delta_1^3] = -t_3 + t_1$; $[\delta_3^1, \delta_1^2] = -\delta_4^1$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_3^1, \delta_1^5] = -\delta_3^5$; $[\delta_4^1, \delta_1^3] = \delta_1^2$; $[\delta_4^1, \delta_1^2] = -2\delta_4^2$; $[\delta_4^1, \delta_2^3] = -t_3 + t_2 + t_1$; $[\delta_4^1, \delta_2^4] = \delta_2^1$; $[\delta_4^1, \delta_2^1] = -2\delta_3^1$; $[\delta_4^1, \delta_1^5] = -\delta_4^5$; $[\delta_4^1, \delta_2^5] = -\delta_3^5$; $[\delta_4^2, \delta_2^3] = \delta_1^2$; $[\delta_4^2, \delta_2^4] = t_2$; $[\delta_4^2, \delta_2^1] = -\delta_4^1$; $[\delta_4^2, \delta_2^5] = -\delta_4^5$; $[\delta_1^3, \delta_2^1] = \delta_2^3$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^3] = -2\delta_1^3$; $[\delta_1^2, \delta_2^4] = -\delta_2^3$; $[\delta_1^2, \delta_2^1] = t_3 + t_2 - t_1$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_3^5] = \delta_4^5$; $[\delta_2^3, \delta_2^1] = 2\delta_2^4$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_2^3, \delta_4^5] = -\delta_1^5$; $[\delta_2^4, \delta_4^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_4^5] = \delta_3^5$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1-\beta_3}\ \langle \delta_3^1 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_4^1 \rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^2 \rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4 \rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + 3e^0\ \langle t_2, t_1, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^1, \delta_1^3] = -t_3+t_1$; $[\delta_3^1, \delta_1^2] = -\delta_4^1$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_4^1, \delta_1^3] = \delta_1^2$; $[\delta_4^1, \delta_1^2] = -2\delta_4^2$; $[\delta_4^1, \delta_2^3] = -t_3 + t_2 + t_1$; $[\delta_4^1, \delta_2^4] = \delta_2^1$; $[\delta_4^1, \delta_2^1] = -2\delta_3^1$; $[\delta_4^2, \delta_2^3] = \delta_1^2$; $[\delta_4^2, \delta_2^4] = t_2$; $[\delta_4^2, \delta_2^1] = -\delta_4^1$; $[\delta_1^3, \delta_2^1] = \delta_2^3$; $[\delta_1^2, \delta_2^3] = -2\delta_1^3$; $[\delta_1^2, \delta_2^4] = -\delta_2^3$; $[\delta_1^2, \delta_2^1] = t_3+t_2-t_1$; $[\delta_2^3, \delta_2^1] = 2\delta_2^4$.

ch($H^2_{\mathcal{G}}$)= $e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3} \rangle + e^{\beta_2-2\beta_3}\ \langle x_1\omega^{3,4} \rangle + e^{-2\beta_1+2\beta_2-\beta_3}\ \langle x_2\omega^{1,4} \rangle + 3e^{-\beta_3}\ \langle x_1\omega^{2,4} + x_2\omega^{2,3}, -x_1\omega^{2,4} + x_4\omega^{3,4}, x_1\omega^{1,3} \rangle + e^{-\beta_1+2\beta_2-2\beta_3}\ \langle x_2\omega^{3,4} \rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2} \rangle + e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,4} \rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_3\omega^{2,3}, -x_1\omega^{1,2} + x_4\omega^{1,3}, x_4\omega^{2,4} \rangle + 3e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_1\omega^{1,4} + x_3\omega^{3,4}, x_1\omega^{1,4} + x_2\omega^{1,3}, x_2\omega^{2,4} \rangle +$

$e^{-2\beta_2+\beta_3}$ $\langle x_4\omega^{1,2}\rangle$ + $3e^{-\beta_1}$ $\langle -x_2\omega^{1,2}$ + $x_4\omega^{1,4}, x_3\omega^{1,3}, x_2\omega^{1,2}$ + $x_3\omega^{2,4}\rangle$ + $e^{\beta_1-2\beta_2}$ $\langle x_4\omega^{2,3}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{-\beta_1+\beta_2-2\beta_3} + e^{-\beta_1+\beta_2-3\beta_3} + 2e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2} + e^{-\beta_1-2\beta_2} + e^{-\beta_1-\beta_3} + 2e^{-\beta_1-2\beta_3} + e^{-2\beta_1+2\beta_2-3\beta_3} + e^{-2\beta_1+\beta_2-\beta_3} + 2e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-\beta_3} + e^{-3\beta_1+2\beta_2-2\beta_3} + e^{-3\beta_1+\beta_2-\beta_3} + e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3}$.

ch$(H^4_{\mathcal{G}})$=$2e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-3\beta_3} + 2e^{-2\beta_1+\beta_2-3\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-2\beta_3} + e^{-3\beta_1+2\beta_2-3\beta_3} + 2e^{-3\beta_1+\beta_2-2\beta_3} + e^{-3\beta_1-\beta_3}$.

ch$(H^5_{\mathcal{G}})$=$e^{-2\beta_1-\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_3} + e^{-3\beta_1+\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 5$: $(1, 15, 30, 41, 24, 5)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 5$: $(1, 11, 20, 21, 15, 4)$

S=$3(\beta_1 + \beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1, 4, 6, 9, 5, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1, 4, 5, 5, 4, 1)$

$d\omega^5 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^{1,5} = \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{1,3,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}$ $\langle\omega^1\rangle$ + $e^{-\beta_2}$ $\langle\omega^2\rangle$ + $e^{-\beta_3}$ $\langle\omega^3\rangle$ + $e^{-\beta_1+\beta_2-\beta_3}$ $\langle\omega^4\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-\beta_1-\beta_2}$ $\langle\omega^{1,2}\rangle$ + $e^{-2\beta_1+\beta_2-\beta_3}$ $\langle\omega^{1,4}\rangle$ + $e^{-\beta_2-\beta_3}$ $\langle\omega^{2,3}\rangle$ + $e^{-\beta_1-\beta_3}$ $\langle\omega^{1,3}\rangle$ + $e^{-\beta_1+\beta_2-2\beta_3}$ $\langle\omega^{3,4}\rangle$.

ch$(H^3_{\mathbb{C}})$= $e^{-2\beta_1-\beta_2-\beta_3}$ $\langle\omega^{1,2,5}\rangle$ + $e^{-3\beta_1+\beta_2-2\beta_3}$ $\langle\omega^{1,4,5}\rangle$ + $e^{-\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{2,3,5}\rangle$ + $e^{-2\beta_1-2\beta_3}$ $\langle\omega^{1,3,5} - \omega^{2,4,5}\rangle$ + $e^{-2\beta_1+\beta_2-3\beta_3}$ $\langle\omega^{3,4,5}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-2\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,2,3,5}\rangle$ + $e^{-3\beta_1-2\beta_3}$ $\langle\omega^{1,2,4,5}\rangle$ + $e^{-3\beta_1+\beta_2-3\beta_3}$ $\langle\omega^{1,3,4,5}\rangle$ + $e^{-2\beta_1-3\beta_3}$ $\langle\omega^{2,3,4,5}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-3\beta_1-3\beta_3}$ $\langle\omega^{1,2,3,4,5}\rangle$.

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1, 4, 6, 9, 5, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 5$: $(1, 4, 5, 5, 4, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^5$; $d^\star\omega^{1,2,3} = -\omega^{2,5}$; $d^\star\omega^{1,2,4} = \omega^{1,5}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,5} + \omega^{2,4,5}$;

$\Delta\omega^{5} = 2\omega^{5}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_2}\ \langle\omega^{1,2}\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3} - \omega^{2,4}\rangle + e^{-\beta_1+\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,4,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-3\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_3}\ \langle(\omega^{1,3,5} - \omega^{2,4,5})/2\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-3\beta_3}\ \langle\omega^{2,3,4,5}\rangle + e^{-3\beta_1+\beta_2-3\beta_3}\ \langle-\omega^{1,3,4,5}\rangle + e^{-3\beta_1-2\beta_3}\ \langle\omega^{1,2,4,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle-\omega^{1,2,3,5}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-3\beta_3}\ \langle\omega^{1,2,3,4,5}\rangle$.

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)(1 + Y)$.

Chapter 4

6-dimensional Lie Algebras

4.1 rank 1 Lie Algebras

4.1.1 Algebra $\mathcal{G}_{6,12}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & 4\xi_1^1 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3-\xi_1^4+\xi_2^5 & \xi_1^2+\xi_2^4 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-\delta_2^6, ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-\delta_1^6, ad\ x_6=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+2e^{3\beta_1}\ \langle x_3,x_4\rangle+e^{4\beta_1}\ \langle x_5\rangle+e^{5\beta_1}\ \langle x_6\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{5\beta_1}\ \langle x_6\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 3e^{\beta_1}\ \langle \delta_2^3,\delta_1^2,\delta_2^4\rangle+2e^{3\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+e^{4\beta_1}\ \langle\delta_1^6\rangle+4e^{2\beta_1}\ \langle\delta_3^6,\delta_1^3,\delta_1^4,\delta_2^5\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_1^5,\delta_2^4]=-\delta_1^6$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_1^4]=-\delta_1^6$; $[\delta_1^3,\delta_1^2]=\delta_1^5$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_2^5,\delta_1^2]=-\delta_2^6+\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^6-\delta_1^4$.

$\mathrm{ch}(H^1_{\mathcal{G}})= 2e^{\beta_1}\ \langle\delta_2^3,\delta_1^2\rangle+3e^{2\beta_1}\ \langle\delta_1^3,\delta_1^4,\delta_2^5\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 6e^{-2\beta_1}\ \langle x_2(\omega^{1,3}+\omega^{1,4}),-x_1\omega^{1,2}+2x_3\omega^{1,5}+x_5\omega^{2,5},x_3\omega^{2,3},x_3\omega^{2,4},-x_3\omega^{1,5}+x_4\omega^{1,5},x_1\omega^{1,2}-x_3\omega^{1,5}+x_4\omega^{2,4}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3(\omega^{1,3}+\omega^{1,4}),x_2\omega^{1,2}-x_3\omega^{1,4}+x_5\omega^{2,3},-x_2\omega^{1,2}+x_3\omega^{1,4}+x_4\omega^{1,3}\rangle+2e^{-4\beta_1}\ \langle x_1\omega^{2,4}-x_3(\omega^{2,6}-\omega^{4,5}),3/2x_1(\omega^{1,5}-\omega^{2,4})-x_2(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+x_4(\omega^{2,6}+\omega^{3,5}-1/2\omega^{4,5})+x_5(\omega^{3,6}+\omega^{4,6})\rangle+4e^{-3\beta_1}\ \langle x_1(\omega^{1,3}+\omega^{1,4})+x_2(\omega^{1,5}+\omega^{2,4})+2x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})-2x_4(\omega^{1,6}-1/2\omega^{3,4}),x_2\omega^{2,4}+x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+x_4\omega^{2,5}+x_5\omega^{2,6},-x_1(\omega^{1,3}+2\omega^{1,4})-2x_2(\omega^{1,5}-\omega^{2,3}-3/2\omega^{2,4})+x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})-x_4\omega^{1,6}+x_5\omega^{3,5},x_1\omega^{1,4}+x_2(\omega^{1,5}-2\omega^{2,3}-3\omega^{2,4})-2x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+2x_4\omega^{1,6}+x_6\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-4\beta_1}+6e^{-5\beta_1}+6e^{-6\beta_1}+6e^{-7\beta_1}+2e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=3e^{-8\beta_1}+4e^{-9\beta_1}+6e^{-10\beta_1}+4e^{-11\beta_1}+e^{-12\beta_1}+e^{-13\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-11\beta_1}+2e^{-12\beta_1}+3e^{-13\beta_1}+3e^{-14\beta_1}+e^{-15\beta_1}+e^{-16\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-15\beta_1}+e^{-16\beta_1}+e^{-17\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,11,40,71,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,6,15,21,19,11,3)$

S=$18\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}-\omega^{2,4}$; $d\omega^{1,6}=\omega^{1,2,3}+\omega^{1,2,4}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}+\omega^{2,4,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}-\omega^{1,2,4,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-4\beta_1}\ \langle\omega^{1,3}\rangle+2e^{-5\beta_1}\ \langle\omega^{1,5},\omega^{2,3}\rangle+e^{-6\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4}\rangle+e^{-7\beta_1}\ \langle\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=2e^{-8\beta_1}\ \langle\omega^{1,2,6},\omega^{1,3,5}\rangle+2e^{-9\beta_1}\ \langle\omega^{1,3,6}+\omega^{2,3,5},\omega^{1,4,6}-\omega^{2,3,5}\rangle+2e^{-10\beta_1}\ \langle\omega^{1,5,6}-\omega^{2,4,6}+\omega^{3,4,5},\omega^{2,3,6}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-11\beta_1}\ \langle\omega^{1,2,3,6}\rangle+2e^{-13\beta_1}\ \langle\omega^{1,3,5,6}-\omega^{2,3,4,6},\omega^{1,4,5,6}+\omega^{2,3,4,6}\rangle+e^{-12\beta_1}\ \langle\omega^{1,2,5,6}\rangle+e^{-14\beta_1}\ \langle\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-15\beta_1}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-16\beta_1}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-17\beta_1}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=e^{-18\beta_1}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 3\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}+\omega^{2,3}+\omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5}+\omega^{2,3}+\omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 3\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,5} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = -\omega^{3,5,6}+2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}+\omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,6} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,5,6} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,5} - \omega^{2,4}, \omega^{2,3} - \omega^{2,4}\rangle + e^{-6\beta_1}\ \langle\omega^{1,6} + \omega^{2,5} - \omega^{3,4}\rangle + e^{-7\beta_1}\ \langle\omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{-8\beta_1}\ \langle\omega^{1,2,6} - \omega^{1,4,5}, \omega^{1,3,5} - \omega^{1,4,5} - \omega^{2,3,4}\rangle + 2e^{-9\beta_1}\ \langle\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,3,5} + 2\omega^{2,4,5}\rangle + 2e^{-10\beta_1}\ \langle\omega^{1,5,6} - \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-14\beta_1}\ \langle-\omega^{2,4,5,6}\rangle + 2e^{-13\beta_1}\ \langle(\omega^{1,3,5,6} - \omega^{1,4,5,6} - 2\omega^{2,3,4,6})/3, (\omega^{1,3,5,6} + 2\omega^{1,4,5,6} + \omega^{2,3,4,6})/3\rangle + e^{-12\beta_1}\ \langle(-\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5})/3\rangle + e^{-11\beta_1}\ \langle(-\omega^{1,2,3,6} - \omega^{1,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= e^{-17\beta_1}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-16\beta_1}\ \langle-\omega^{1,3,4,5,6}\rangle + e^{-15\beta_1}\ \langle\omega^{1,2,4,5,6}\rangle.$
$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-18\beta_1}\ \langle\omega^{1,2,3,4,5,6}\rangle.$
$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2.$

4.1.2 Algebra $\mathcal{G}_{6,17}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 5\xi_1^1 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 6\xi_1^1 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^5 & \xi_1^2+\xi_2^4 & \xi_2^3 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^6), ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_1^6, ad\ x_6=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{3\beta_1}\ \langle x_2\rangle+e^{4\beta_1}\ \langle x_3\rangle+e^{5\beta_1}\ \langle x_4\rangle+e^{6\beta_1}\ \langle x_5\rangle+e^{7\beta_1}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{4\beta_1}\ \langle\delta_1^4,\delta_2^6\rangle+e^{5\beta_1}\ \langle\delta_1^5\rangle+e^{6\beta_1}\ \langle\delta_1^6\rangle+2e^{3\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+2e^{2\beta_1}\ \langle\delta_1^2,\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^2]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^3]=-\delta_1^5$; $[\delta_1^5,\delta_2^3]=-\delta_1^6$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^3,\delta_2^5]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_1^5$; $[\delta_1^3,\delta_2^3]=-(\delta_2^6+\delta_1^4)$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_2^6-\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle\delta_1^4\rangle+e^{3\beta_1}\ \langle\delta_2^5\rangle+2e^{2\beta_1}\ \langle\delta_1^2,\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^4]=-2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $3e^{-3\beta_1}\ \langle x_1\omega^{1,2}-x_3(\omega^{1,5}-2\omega^{2,3})+x_4\omega^{2,4},-x_1\omega^{1,2}+2x_3(\omega^{1,5}-1/2\omega^{2,3})+x_5\omega^{2,5},-x_3\omega^{2,3}+x_4\omega^{1,6}\rangle+3e^{-4\beta_1}\ \langle x_3(\omega^{1,6}+\omega^{2,4})+x_4\omega^{2,5}+x_5\omega^{2,6},-x_1\omega^{1,3}-2x_2(\omega^{1,5}-\omega^{2,3})+x_3\omega^{2,4}+x_4\omega^{2,5}+x_5\omega^{3,5},x_1\omega^{1,3}+x_2\omega^{1,5}+x_3\omega^{2,4}+x_4\omega^{3,4}\rangle+2e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_5\omega^{2,3},x_4\omega^{1,4}\rangle+2e^{-2\beta_1}\ \langle x_4\omega^{1,5},x_2\omega^{1,3}\rangle$.

ch($H^3_{\mathcal{G}}$)=$3e^{-7\beta_1}+3e^{-8\beta_1}+4e^{-9\beta_1}+e^{-10\beta_1}+2e^{-11\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$3e^{-12\beta_1}+e^{-13\beta_1}+3e^{-14\beta_1}+2e^{-15\beta_1}+e^{-16\beta_1}+e^{-17\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$e^{-17\beta_1}+e^{-18\beta_1}+e^{-19\beta_1}+e^{-20\beta_1}+e^{-22\beta_1}+e^{-24\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-23\beta_1}+e^{-25\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(1,10,36,67,64,32,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(1,5,10,13,11,6,2)$

S=$26\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,7,12,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,3,4,3,2,1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle \omega^1 \rangle + e^{-3\beta_1} \langle \omega^2 \rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-7\beta_1} \langle \omega^{1,5} \rangle + e^{-8\beta_1} \langle \omega^{1,6} + \omega^{2,4} \rangle + e^{-9\beta_1} \langle \omega^{2,5} - \omega^{3,4} \rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 2e^{-12\beta_1} \langle \omega^{1,3,6}, \omega^{1,4,5} \rangle + 2e^{-14\beta_1} \langle \omega^{1,5,6} - \omega^{2,4,5}, \omega^{2,3,6} - \omega^{2,4,5} \rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-17\beta_1} \langle \omega^{1,2,5,6} \rangle + e^{-18\beta_1} \langle \omega^{1,3,5,6} \rangle + e^{-19\beta_1} \langle \omega^{1,4,5,6} + \omega^{2,3,4,6} \rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-23\beta_1} \langle \omega^{1,3,4,5,6} \rangle + e^{-25\beta_1} \langle \omega^{2,3,4,5,6} \rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-26\beta_1} \langle \omega^{1,2,3,4,5,6} \rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,7,12,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,3,4,3,2,1)$

Total dimension of trivial cohomology: 16.

Length associated polynomial: 16.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}+\omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6}-\omega^{2,4}$; $\Delta\omega^{2,3} = \omega^{1,5}+\omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,6}+\omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,4} = \omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6}+2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{4,5} = \omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}+\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}+\omega^{1,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5}+\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6}+2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}+\omega^{1,4,5}-\omega^{2,3,4}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6}+\omega^{1,4,5}-\omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}+\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6}+\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,5,6}+\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6}+2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}+\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6}+2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}-\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,3,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-3\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-7\beta_1}\ \langle\omega^{1,5}-\omega^{2,3}\rangle + e^{-8\beta_1}\ \langle\omega^{1,6}+\omega^{2,4}\rangle + e^{-9\beta_1}\ \langle\omega^{2,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{-12\beta_1}\ \langle\omega^{1,3,6}+\omega^{2,3,4},\omega^{1,4,5}+\omega^{2,3,4}\rangle + 2e^{-14\beta_1}\ \langle\omega^{1,5,6}-\omega^{2,4,5},\omega^{2,3,6}-\omega^{2,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-19\beta_1}\ \langle(-\omega^{1,4,5,6}-\omega^{2,3,4,6})/2\rangle + e^{-18\beta_1}\ \langle(-\omega^{1,3,5,6}+\omega^{2,3,4,5})/2\rangle + e^{-17\beta_1}\ \langle(-\omega^{1,2,5,6}+\omega^{1,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-25\beta_1}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-23\beta_1}\ \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-26\beta_1}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^2$.

4.1.3 Algebra $\mathcal{G}_{6,19}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 4\xi_1^1 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 5\xi_1^1 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 6\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -\delta_1^6, ad\ x_6 = 0.$

Maximal torus : $T = \langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{3\beta_1}\ \langle x_3\rangle + e^{4\beta_1}\ \langle x_4\rangle + e^{5\beta_1}\ \langle x_5\rangle + e^{6\beta_1}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{4\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + e^{5\beta_1}\ \langle\delta_1^6\rangle + 2e^{3\beta_1}\ \langle\delta_1^4, \delta_2^5\rangle + 2e^{2\beta_1}\ \langle\delta_1^3, \delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^4, \delta_2^3] = -(\delta_2^6 + \delta_1^5)$; $[\delta_2^5, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_2^4] = -(\delta_2^6 + \delta_1^5)$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle\delta_1^5\rangle + e^{3\beta_1}\ \langle\delta_1^4\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $2e^{-3\beta_1}\ \langle x_3(\omega^{1,5} + \omega^{2,4}) + x_4\omega^{2,5}, -x_2(\omega^{1,4} - \omega^{2,3}) + x_3\omega^{2,4} + x_4\omega^{3,4}\rangle + e^{-4\beta_1}\ \langle x_3(\omega^{1,6} + \omega^{2,5}) + x_4\omega^{2,6}\rangle + 2e^{-\beta_1}\ \langle x_3\omega^{1,3} + x_4\omega^{2,3}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4}\rangle + 3e^{-2\beta_1}\ \langle x_3(\omega^{1,4} + \omega^{2,3}) + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_3(\omega^{1,4} - 2\omega^{2,3}) + x_4\omega^{1,5}, x_3\omega^{1,4} + x_5\omega^{1,6}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1} + 2e^{-6\beta_1} + 4e^{-7\beta_1} + 2e^{-8\beta_1} + 2e^{-9\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-9\beta_1} + e^{-10\beta_1} + 2e^{-11\beta_1} + 3e^{-12\beta_1} + 2e^{-13\beta_1} + e^{-14\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$2e^{-14\beta_1} + e^{-16\beta_1} + e^{-17\beta_1} + e^{-18\beta_1} + e^{-19\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-19\beta_1} + e^{-20\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 9, 35, 66, 64, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 4, 8, 11, 10, 6, 2)$

S=$21\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 7, 12, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 3, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-2\beta_1}$ $\langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-5\beta_1}$ $\langle\omega^{1,4}\rangle + 2e^{-7\beta_1}$ $\langle\omega^{1,6} + \omega^{3,4}, \omega^{2,5} - \omega^{3,4}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-9\beta_1}$ $\langle\omega^{1,2,6}\rangle$ + $e^{-10\beta_1}$ $\langle\omega^{1,3,6} + \omega^{2,3,5}\rangle$ + $e^{-11\beta_1}$ $\langle\omega^{1,4,6}\rangle$ + $e^{-12\beta_1}$ $\langle\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $2e^{-14\beta_1}$ $\langle\omega^{1,2,5,6}, \omega^{1,3,4,6}\rangle + e^{-16\beta_1}$ $\langle\omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-19\beta_1}$ $\langle\omega^{1,3,4,5,6}\rangle + e^{-20\beta_1}$ $\langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-21\beta_1}$ $\langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y^2)^2(1 + Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 7, 12, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 3, 4, 3, 2, 1)$

Total dimension of trivial cohomology: 16.

Length associated polynomial: 16.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} =$

$-\omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^{\star}\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,5} = \omega^{3,5,6}$; $d^{\star}\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 3\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-5\beta_1}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,6} + \omega^{3,4}, \omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-9\beta_1}\ \langle 2\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}\rangle + e^{-10\beta_1}\ \langle\omega^{1,3,6} - \omega^{1,4,5} + 2\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle 2\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-12\beta_1}\ \langle\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-16\beta_1}\ \langle(-\omega^{1,4,5,6} + \omega^{2,3,5,6})/2\rangle + 2e^{-14\beta_1}\ \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5})/3, (-\omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-20\beta_1}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-19\beta_1}\ \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-21\beta_1}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y^2)^2(1 + Y)^2$.

4.1.4 Algebra $\mathcal{G}_{6,20}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_5$; $[x_2, x_5] = x_6$; $[x_3, x_4] = -x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_2^3 & 4\xi_1^1 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_2^3 & 5\xi_1^1 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^5 & \xi_1^4 & -\xi_1^3 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_2^6, ad\ x_6 = 0$.

Maximal torus : T=$\langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{3\beta_1}\ \langle x_3\rangle + e^{4\beta_1}\ \langle x_4\rangle + e^{5\beta_1}\ \langle x_5\rangle + e^{7\beta_1}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{3\beta_1}\ \langle \delta_2^5, \delta_1^4\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^{6\beta_1}\ \langle \delta_1^6\rangle + e^{5\beta_1}\ \langle \delta_2^6\rangle + e^{4\beta_1}\ \langle \delta_1^5\rangle + e^{2\beta_1}\ \langle \delta_1^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^3] = \delta_2^6$; $[\delta_2^3, \delta_1^5] = \delta_2^6$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_1^3] = \delta_2^5 + \delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle \delta_1^6\rangle + e^{3\beta_1}\ \langle \delta_1^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $2e^{-2\beta_1}\ \langle -2x_1\omega^{1,2} + 3x_3(\omega^{1,4} - 1/3\omega^{2,3}) + x_4\omega^{2,4}, x_1\omega^{1,2} + x_2\omega^{1,3} - x_3\omega^{1,4} + x_4\omega^{1,5}\rangle + 2e^{-4\beta_1}\ \langle 3/2x_1(\omega^{1,4} - \omega^{2,3}) - x_2(\omega^{1,5} + \omega^{2,4}) + x_3(\omega^{2,5} - 1/2\omega^{3,4}) + x_4\omega^{3,5}, -x_1\omega^{2,3} + x_5\omega^{2,6}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_6\omega^{1,6}\rangle + e^{-3\beta_1}\ \langle -x_1\omega^{1,3} - x_2(\omega^{1,4} - \omega^{2,3}) + x_3(\omega^{1,5} + \omega^{2,4}) + x_4\omega^{2,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1} + e^{-6\beta_1} + 2e^{-7\beta_1} + 2e^{-8\beta_1} + e^{-9\beta_1} + e^{-10\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-10\beta_1} + 2e^{-12\beta_1} + e^{-13\beta_1} + e^{-14\beta_1} + 2e^{-15\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$e^{-16\beta_1} + e^{-17\beta_1} + e^{-18\beta_1} + e^{-19\beta_1} + e^{-20\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-20\beta_1} + e^{-21\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 6$: $(1, 8, 34, 64, 63, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 6$: $(1, 3, 6, 8, 7, 5, 2)$

S=$22\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,6,11,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,2,2,2,2,1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{2,5} + \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} - \omega^{2,3,6} - \omega^{3,4,5}$; $d\omega^{1,2,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,6} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{1,2,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,4,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1} \langle\omega^{1,4}\rangle + e^{-6\beta_1} \langle\omega^{1,5} + \omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-10\beta_1} \langle\omega^{1,4,5} - \omega^{2,3,5}\rangle + e^{-12\beta_1} \langle\omega^{1,4,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-16\beta_1} \langle\omega^{1,3,5,6}\rangle + e^{-17\beta_1} \langle\omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-20\beta_1} \langle\omega^{1,3,4,5,6}\rangle + e^{-21\beta_1} \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-22\beta_1} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y+Y^2)(1-Y+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,6,11,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,2,2,2,2,2,1)$

Total dimension of trivial cohomology: 12.

Length associated polynomial: 12.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{3,4} = \omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,4} = -\omega^{1,6} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{2,6} + \omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{3,6}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,4,5} = -\omega^{4,6}$; $d^\star\omega^{3,4,5} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} =$

$\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{4,5}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{4,5} = -\omega^{2,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{3,4,5} = \omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-5\beta_1} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-6\beta_1} \langle\omega^{1,5} + \omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-10\beta_1} \langle\omega^{1,2,6} + 2\omega^{1,4,5} - \omega^{2,3,5}\rangle + e^{-12\beta_1} \langle\omega^{1,4,6} - 2\omega^{2,3,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-17\beta_1} \langle(-\omega^{1,4,5,6} + \omega^{2,3,5,6})/2\rangle + e^{-16\beta_1} \langle(-\omega^{1,3,5,6} - \omega^{2,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-21\beta_1} \langle\omega^{2,3,4,5,6}\rangle + e^{-20\beta_1} \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-22\beta_1} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^2$.

4.2 rank 2 Lie Algebras

4.2.1 Algebra $\mathcal{G}_{6,5}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -(\xi_1^3+\xi_1^4) & 0 & \xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-\delta_2^6, ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=0, ad\ x_6=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+2e^{\beta_1+\beta_2}\ \langle x_3,x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle\delta_2^3,\delta_3^5,\delta_2^4\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+3e^{\beta_2}\ \langle\delta_3^6,\delta_1^3,\delta_1^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^5]=-\delta_2^5$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_2^4,\delta_1^3]=\delta_2^6$; $[\delta_2^4,\delta_1^4]=\delta_2^6+\delta_1^5$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_1^4]=-\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $2e^{\beta_1}\ \langle\delta_2^3,\delta_3^5\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+2e^{\beta_2}\ \langle\delta_1^3,\delta_1^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^5]=-\delta_2^5$; $[\delta_1^3,\delta_1^4]=-\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $3e^{-2\beta_1}\ \langle x_2(\omega^{1,3}+\omega^{1,4}),-x_3\omega^{1,5}+x_4\omega^{1,5},x_2\omega^{1,4}+x_6\omega^{3,5}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3(\omega^{1,3}+\omega^{1,4}),x_2\omega^{1,2}+x_6\omega^{1,6},-x_2\omega^{1,2}+x_3\omega^{1,4}+x_4\omega^{1,3}\rangle+3e^{-2\beta_2}\ \langle x_3\omega^{2,6},-x_1\omega^{2,3}+x_5\omega^{3,6},x_1\omega^{2,4}\rangle+4e^{-\beta_1-\beta_2}\ \langle x_1(\omega^{1,3}+3\omega^{1,4})-2x_2(\omega^{2,3}+3/2\omega^{2,4})+x_4\omega^{3,4},-x_1(\omega^{1,3}+\omega^{1,4})+x_2(\omega^{2,3}+\omega^{2,4})+x_5\omega^{3,5},x_1\omega^{1,4}-x_2(\omega^{2,3}+2\omega^{2,4})-x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+x_4\omega^{1,6},-x_1\omega^{1,4}+x_2\omega^{2,4}+x_6\omega^{3,6}\rangle+3e^{-\beta_2}\ \langle -x_1\omega^{1,2}+x_5\omega^{2,5},x_3\omega^{2,3},x_3\omega^{2,4}\rangle+e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-\beta_1-2\beta_2}+4e^{-\beta_1-3\beta_2}+e^{-\beta_1-4\beta_2}+2e^{-2\beta_1-\beta_2}+4e^{-2\beta_1-2\beta_2}+2e^{-2\beta_1-3\beta_2}+4e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+e^{-4\beta_1-\beta_2}+e^{-3\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$3e^{-2\beta_1-4\beta_2}+e^{-2\beta_1-5\beta_2}+4e^{-3\beta_1-3\beta_2}+3e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2}+3e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}$.

ch$(H^5_{\mathcal{G}})$=$e^{-3\beta_1-5\beta_2}+2e^{-4\beta_1-4\beta_2}+2e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2}+e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+e^{-6\beta_1-4\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-5\beta_1-5\beta_2}+e^{-5\beta_1-6\beta_2}+e^{-6\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,12,42,72,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,8,18,24,20,11,3)$

S=$6(\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{2,3}-\omega^{2,4}$; $d\omega^{1,6}=\omega^{1,2,3}+\omega^{1,2,4}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}+\omega^{2,4,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}-\omega^{1,2,4,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_1-\beta_2}\ \langle\omega^3\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4}\rangle$.

ch$(H^3_{\mathbb{C}})$= $2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5},\omega^{1,4,5}\rangle+2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}+\omega^{2,3,5},\omega^{1,4,6}-\omega^{2,3,5}\rangle+2e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6},\omega^{2,4,6}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5}\rangle+e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6}+\omega^{1,4,5,6}\rangle+e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,2,5,6}\rangle+e^{-3\beta_1-5\beta_2}\ \langle\omega^{2,3,4,6}\rangle+e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,4,5,6}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-5\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

ch$(H^6_{\mathbb{C}})$= $e^{-6\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} + \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 2\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = -\omega^{3,5,6} + 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,6} + \omega^{1,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_1-\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3} - \omega^{2,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,5}, \omega^{1,4,5}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6}, \omega^{2,4,6}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,3,5} + 2\omega^{2,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-4\beta_1-5\beta_2} \langle-\omega^{2,4,5,6}\rangle + e^{-3\beta_1-5\beta_2} \langle-\omega^{2,3,4,6}\rangle + e^{-5\beta_1-4\beta_2} \langle(\omega^{1,3,5,6} + \omega^{1,4,5,6})/2\rangle + e^{-5\beta_1-3\beta_2} \langle-\omega^{1,3,4,5}\rangle + e^{-4\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-5\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-5\beta_2} \langle-\omega^{1,3,4,5,6}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-6\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 + Y^2)(1 + Y)^2$.

4.2.2 Algebra $\mathcal{G}_{6,7}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_2, x_3] = -x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 3\xi_1^1 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_1^3+\xi_2^4 & -\xi_1^2+\xi_3^4 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4+\delta_3^5, ad\ x_2 = -(\delta_1^4+\delta_3^6), ad\ x_3 = -\delta_1^5+\delta_2^6, ad\ x_4 = -\delta_1^6, ad\ x_5 = 0, ad\ x_6 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{3\beta_1}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{3\beta_1}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_2}\ \langle \delta_1^4, \delta_3^6\rangle + 2e^{2\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_2^5\rangle + 3e^{\beta_1}\ \langle \delta_3^5, \delta_1^3, \delta_2^4\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^3\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^6\rangle + 2e^{-\beta_1+\beta_2}\ \langle \delta_1^2, \delta_3^4\rangle + 2e^0\ \langle t_1, t_2\rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^4, \delta_1^3] = -\delta_1^6$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^5, \delta_1^2] = \delta_1^6$; $[\delta_1^5, \delta_3^4] = -\delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_2^6 + \delta_1^5$; $[\delta_2^5, \delta_3^4] = -\delta_2^6$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_3^5, \delta_1^2] = \delta_3^6$; $[\delta_3^5, \delta_3^4] = -\delta_3^6$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_2^4] = \delta_2^5$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^3, \delta_3^4] = \delta_3^5 - \delta_2^4$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_2^4] = \delta_2^6$; $[\delta_1^3, \delta_3^4] = \delta_3^6 - \delta_1^4$; $[\delta_2^4, \delta_1^2] = \delta_1^4$; $[\delta_2^4, \delta_3^4] = \delta_3^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{3\beta_1-\beta_2}\ \langle \delta_2^5\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^3\rangle + e^{2\beta_1}\ \langle \delta_1^5\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + 2e^{\beta_1}\ \langle \delta_1^3, \delta_2^4\rangle + 2e^{-\beta_1+\beta_2}\ \langle \delta_1^2, \delta_3^4\rangle + 2e^0\ \langle t_1, t_2\rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -2\delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_3^4] = -2\delta_1^4$; $[\delta_1^3, \delta_2^4] = \delta_1^5$; $[\delta_3^4, \delta_2^5] = \delta_1^5$; $[\delta_3^4, \delta_2^4] = \delta_1^4$; $[\delta_3^4, \delta_2^3] = 2\delta_2^4$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$; $[\delta_2^3, \delta_1^4] = \delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 5e^{-2\beta_1}\ \langle -x_1\omega^{1,3} + 2x_2\omega^{1,4} + x_4\omega^{3,4}, -x_1\omega^{1,3} + x_2(\omega^{1,4} + \omega^{2,3}), x_1\omega^{1,3} - x_2\omega^{1,4} + x_4\omega^{1,6}, x_1\omega^{1,3} - x_2\omega^{1,4} - x_3\omega^{1,5} + x_4\omega^{2,5}, x_5\omega^{3,5}\rangle + e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,5} + x_4\omega^{3,5}\rangle + e^{-3\beta_1}\ \langle x_2(\omega^{1,6} + \omega^{3,4}) + x_4\omega^{3,6}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle + 4e^{-\beta_2}\ \langle -x_1\omega^{1,2} - x_3\omega^{1,4} + x_5\omega^{2,5}, -x_3\omega^{1,4} + x_4\omega^{2,4}, x_1\omega^{1,2} + x_3(\omega^{1,4} + \omega^{2,3}), -x_1\omega^{1,2} - x_3\omega^{1,4} + x_5\omega^{1,6}\rangle + e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4} + x_5\omega^{2,6}\rangle + 3e^{-\beta_1-\beta_2}\ \langle -2x_1(\omega^{1,4} + 3/2\omega^{2,3}) + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5}, x_1(\omega^{1,4} + \omega^{2,3}) - x_3(\omega^{1,6} + \omega^{3,4}) + x_4\omega^{2,6}, x_1(\omega^{1,4} + \omega^{2,3}) - x_2\omega^{2,4} + x_5\omega^{3,6}\rangle + e^{\beta_1-\beta_2}\ \langle x_3\omega^{1,2}\rangle +$

$3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3}, x_5\omega^{1,5}, -x_2\omega^{1,2}+x_4\omega^{2,3}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}, x_4\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-\beta_1-2\beta_2}+3e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+6e^{-3\beta_1-\beta_2}+3e^{-4\beta_1-\beta_2}+3e^{-4\beta_1}+4e^{-5\beta_1}+e^{-6\beta_1+\beta_2}+e^{-6\beta_1}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-2\beta_2}+2e^{-3\beta_1-3\beta_2}+5e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+3e^{-5\beta_1-\beta_2}+3e^{-5\beta_1-2\beta_2}+4e^{-6\beta_1-\beta_2}+e^{-6\beta_1}+e^{-7\beta_1-\beta_2}+e^{-7\beta_1}+e^{-8\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_1-3\beta_2}+2e^{-5\beta_1-3\beta_2}+2e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-7\beta_1-2\beta_2}+e^{-7\beta_1-3\beta_2}+e^{-8\beta_1-\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-9\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-3\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-9\beta_1-2\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,14,44,75,69,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,10,22,29,24,12,3)$

S=$3(3\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,6,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}+\omega^{2,3}$; $d\omega^{1,6}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}-\omega^{2,3,5}$; $d\omega^{1,4,6}=\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,5}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-4\beta_1}\ \langle\omega^{1,5}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}+\omega^{3,4},\omega^{2,5}+\omega^{3,4}\rangle+e^{-5\beta_1}\ \langle\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle+2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6},\omega^{1,4,5}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6}-\omega^{2,4,5},\omega^{2,3,6}+\omega^{2,4,5}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle+e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,6}-\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6}\rangle+2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,2,5,6},\omega^{1,3,4,6}\rangle+e^{-8\beta_1-\beta_2}\ \langle\omega^{1,3,5,6}\rangle+e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,3,4,6}\rangle+e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,4,5,6}+\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-8\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,6,8,6,3,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{2,3} = \omega^6$; $d^\star\omega^{1,2,3} = -\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} - \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,3} = -\omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 3\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4} + \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + 2e^{-3\beta_1-\beta_2} \langle\omega^{1,6} + \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-5\beta_1} \langle\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6} - \omega^{2,3,4}\rangle + 2e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6} - \omega^{2,3,5}, \omega^{1,4,5} + \omega^{2,3,5}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6} - \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle +$

$e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,6}-\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-3\beta_2}\ \langle-\omega^{2,3,4,6}\rangle + e^{-7\beta_1-2\beta_2}\ \langle(\omega^{1,4,5,6}+\omega^{2,3,5,6})/2\rangle + e^{-8\beta_1-\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle(\omega^{1,2,5,6}-\omega^{1,3,4,6}+2\omega^{2,3,4,5})/3, (\omega^{1,2,5,6}+2\omega^{1,3,4,6}-\omega^{2,3,4,5})/3\rangle + e^{-4\beta_1-3\beta_2}\ \langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-8\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-9\beta_1-2\beta_2}\ \langle-\omega^{1,3,4,5,6}\rangle + e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2$.

4.2.3 Algebra $\mathcal{G}_{6,8}$

Adjoint cohomology

$[x_1,x_2]=x_4;\ [x_1,x_4]=x_5;\ [x_2,x_3]=x_5;\ [x_2,x_4]=x_6.$

$$\delta=\begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^3+\xi_2^4 & 2\xi_1^1+\xi_2^2 & \xi_2^1\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=0, ad\ x_6=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+2e^{\beta_2}\ \langle\delta_3^5,\delta_1^4\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle+e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^5]=-\delta_2^5$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_1^5,\delta_2^1]=\delta_2^5$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_1^3,\delta_2^4]=-\delta_2^5$; $[\delta_1^3,\delta_2^1]=\delta_2^3$; $[\delta_1^3,\delta_3^6]=-\delta_1^6$; $[\delta_1^3,\delta_1^4]=-\delta_1^5$; $[\delta_2^4,\delta_1^4]=\delta_2^6+\delta_1^5$; $[\delta_2^1,\delta_1^6]=-\delta_2^6+\delta_1^5$; $[\delta_2^1,\delta_2^6]=\delta_2^5$; $[\delta_2^1,\delta_3^6]=\delta_3^5$; $[\delta_2^1,\delta_1^4]=-\delta_2^4$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle+e^{\beta_1}\ \langle\delta_1^3\rangle+e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^1]=\delta_2^3$; $[\delta_1^3,\delta_3^6]=-\delta_1^6$; $[\delta_1^3,\delta_1^4]=-\delta_1^5$; $[\delta_2^3,\delta_3^6]=\delta_1^5$; $[\delta_2^1,\delta_1^6]=2\delta_1^5$; $[\delta_2^1,\delta_3^6]=\delta_1^4$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{\beta_1-3\beta_2}\ \langle x_3\omega^{2,6}\rangle+3e^{-2\beta_1}\ \langle -2x_1\omega^{1,3}-3x_2(\omega^{1,4}-1/3\omega^{2,3})+x_4\omega^{3,4}, x_1\omega^{1,3}+x_2(\omega^{1,4}-\omega^{2,3})+x_6\omega^{3,6}, x_2\omega^{1,4}+x_5\omega^{3,5}\rangle+3e^{-\beta_2}\ \langle x_3\omega^{2,3}, -x_1\omega^{1,2}+x_3\omega^{1,4}, x_1\omega^{1,2}+x_5\omega^{1,6}\rangle+e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+3e^{-\beta_1-\beta_2}\ \langle x_1(\omega^{1,4}-\omega^{2,3})+x_3(\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,5}, -x_1(\omega^{1,4}-\omega^{2,3})+x_2\omega^{2,4}, -x_1\omega^{2,3}+x_5\omega^{3,6}\rangle+2e^{-2\beta_2}\ \langle x_1\omega^{2,4}, x_3(\omega^{1,6}+\omega^{2,5})+x_4\omega^{2,6}\rangle+e^{-2\beta_1+2\beta_2}\ \langle x_6\omega^{1,3}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}+x_6\omega^{1,5}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}-x_3\omega^{1,3}+x_6\omega^{1,6}, x_3\omega^{1,3}+x_4\omega^{2,3}, x_3\omega^{1,3}+x_5\omega^{1,5}\rangle+e^{-3\beta_1+\beta_2}\ \langle -x_2\omega^{1,3}+x_6\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1-2\beta_2}+3e^{-\beta_1-3\beta_2}+e^{-\beta_1-4\beta_2}+2e^{-2\beta_1-\beta_2}+4e^{-2\beta_1-2\beta_2}+2e^{-2\beta_1-3\beta_2}+3e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+2e^{-3\beta_1}+e^{-4\beta_1+\beta_2}+e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-2\beta_1-4\beta_2}+e^{-2\beta_1-5\beta_2}+4e^{-3\beta_1-3\beta_2}+2e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2}+3e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2}+e^{-6\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-4\beta_2}+2e^{-4\beta_1-5\beta_2}+e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-5\beta_1-5\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,13,42,72,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,9,19,24,20,11,3)$

S=$7\beta_1+5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{2,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}+\omega^{2,4,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{-3\beta_1}\ \langle\omega^{1,3}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})$= $e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,5}\rangle+2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,6},\omega^{2,3,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,6}+\omega^{2,3,5},\omega^{1,4,5}-\omega^{2,3,5}\rangle+e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})$= $e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,3,4,5}\rangle+e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,3,4,6}\rangle+e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,2,5,6}\rangle+e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,4,5,6}-\omega^{2,3,5,6}\rangle+e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})$= $e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})$= $e^{-7\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathfrak{a}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}}) = e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}}) = e^{-5\beta_1-\beta_2} \langle\omega^{1,3,5}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,5}, \omega^{1,4,5} - \omega^{2,3,5}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}}) = e^{-4\beta_1-5\beta_2} \langle-\omega^{2,4,5,6}\rangle + e^{-5\beta_1-4\beta_2} \langle(-\omega^{1,4,5,6} + \omega^{2,3,5,6})/2\rangle + e^{-5\beta_1-3\beta_2} \langle(\omega^{1,3,4,6} + \omega^{2,3,4,5})/2\rangle + e^{-6\beta_1-2\beta_2} \langle-\omega^{1,3,4,5}\rangle + e^{-4\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6} - \omega^{2,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}}) = e^{-6\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-7\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}}) = e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6}\rangle$.

$$P(Y) \;=\; (1+Y+Y^2)(1+Y^2)(1+Y)^2.$$

4.2.4 Algebra $\mathcal{G}_{6,10}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_3, x_5] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 \\ -\xi_3^2 & 0 & (\xi_1^1 + \xi_2^2)/2 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & \xi_3^2 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & (3\xi_1^1 + \xi_2^2)/2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & \xi_3^4 - \xi_1^5 & 2\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_1^6, ad\ x_5 = -\delta_3^6, ad\ x_6 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{(\beta_1+\beta_2)/2}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{(3\beta_1+\beta_2)/2}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + e^{(-\beta_1+\beta_2)/2}\ \langle\delta_3^2\rangle + 2e^{\beta_1}\ \langle\delta_3^5, \delta_2^4\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{2\beta_1}\ \langle\delta_2^6\rangle + e^{(3\beta_1+\beta_2)/2}\ \langle\delta_3^6\rangle + 2e^{(\beta_1+\beta_2)/2}\ \langle\delta_3^4, \delta_1^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_3^2, \delta_3^5] = \delta_1^5 + \delta_3^4$; $[\delta_3^2, \delta_2^6] = -\delta_3^6$; $[\delta_3^2, \delta_3^6] = \delta_1^6$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_3^2, \delta_3^4] = \delta_1^4$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^5, \delta_3^4] = -\delta_3^6$; $[\delta_3^5, \delta_1^5] = \delta_3^6$; $[\delta_2^4, \delta_3^4] = \delta_3^6$; $[\delta_3^4, \delta_1^5] = \delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{(-\beta_1+\beta_2)/2}\ \langle\delta_3^2\rangle + e^{2\beta_1}\ \langle\delta_2^6\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle + e^{(\beta_1+\beta_2)/2}\ \langle\delta_3^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^2, \delta_2^4] = -\delta_3^4$.

ch($H^2_{\mathcal{G}}$)= $e^{-5/2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,5}\rangle + 2e^{-2\beta_1}\ \langle x_3\omega^{1,5}, x_2\omega^{1,4}\rangle + e^{-3/2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,3}\rangle + 3e^{-1/2\beta_1-1/2\beta_2}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_4\omega^{3,4}, x_2\omega^{2,3} + x_3\omega^{1,2} + x_4\omega^{2,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_5\omega^{1,4}\rangle + 2e^{-3/2\beta_1-1/2\beta_2}\ \langle 2x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) + 3x_3(\omega^{1,4} - 2/3\omega^{3,5}) + x_4\omega^{4,5}, -x_1\omega^{1,5} + x_2(\omega^{2,5} + \omega^{3,4}) - x_3\omega^{1,4} + x_4\omega^{3,6} + x_5\omega^{1,6}\rangle + e^{1/2\beta_1-3/2\beta_2}\ \langle -x_1\omega^{2,3} + x_5\omega^{2,4}\rangle + 2e^{-\beta_2}\ \langle -2x_1\omega^{1,2} - x_3\omega^{2,3} - 2x_4\omega^{2,4} + x_5\omega^{2,5}, x_1\omega^{1,2} + x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_6\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5}, x_1\omega^{1,4} - x_2\omega^{2,4} + x_5\omega^{3,6}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2}, x_3\omega^{1,3} + x_4\omega^{3,5}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-\beta_1-3\beta_2/2}+e^{-\beta_1-5\beta_2/2}+3e^{-3\beta_1-3\beta_2/2}+e^{-3\beta_1-5\beta_2/2}+2e^{-5\beta_1-\beta_2/2}+2e^{-5\beta_1-3\beta_2/2}+2e^{-7\beta_1-\beta_2/2}+e^{-9\beta_1-\beta_2/2}+3e^{-\beta_1-2\beta_2}+3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+2e^{-3\beta_1-\beta_2}+e^{-4\beta_1-\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-3\beta_1-5\beta_2/2}+2e^{-5\beta_1-5\beta_2/2}+e^{-5\beta_1-7\beta_2/2}+e^{-7\beta_1-3\beta_2/2}+2e^{-7\beta_1-5\beta_2/2}+3e^{-9\beta_1-3\beta_2/2}+e^{-11\beta_1-3\beta_2/2}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2}+3e^{-4\beta_1-2\beta_2}+e^{-5\beta_1-\beta_2}+e^{-5\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-7\beta_1-7\beta_2/2}+e^{-9\beta_1-5\beta_2/2}+e^{-9\beta_1-7\beta_2/2}+2e^{-11\beta_1-5\beta_2/2}+e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-2\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-6\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-11\beta_1-7\beta_2/2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,12,41,72,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,7,17,23,20,11,3)$

S=$2(3\beta_1+2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}-\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}+\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}$; $d\omega^{1,2,6}=-\omega^{1,2,3,5}$; $d\omega^{1,4,6}=\omega^{1,3,4,5}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,6}=\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{1,2,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{(-\beta_1-\beta_2)/2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{(-5\beta_1-\beta_2)/2}\ \langle\omega^{1,5}\rangle+e^{(-\beta_1-3\beta_2)/2}\ \langle\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{(-3\beta_1-3\beta_2)/2}\ \langle\omega^{2,5}+\omega^{3,4}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})$= $e^{(-7\beta_1-3\beta_2)/2}\ \langle\omega^{1,3,6}\rangle+e^{(-9\beta_1-3\beta_2)/2}\ \langle\omega^{1,5,6}\rangle+e^{(-3\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle+e^{(-5\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,6}+\omega^{2,4,5}\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6}-\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})$= $e^{(-9\beta_1-5\beta_2)/2}\ \langle\omega^{1,2,5,6}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6}\rangle+e^{(-11\beta_1-5\beta_2)/2}\ \langle\omega^{1,4,5,6}\rangle+e^{(-7\beta_1-7\beta_2)/2}\ \langle\omega^{2,3,4,6}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6}+\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})$= $e^{(-11\beta_1-7\beta_2)/2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-5\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})$= $e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 6$: $(1, 3, 8, 13, 12, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 6$: $(1, 3, 5, 6, 5, 3, 1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,6}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{2,3,5} = \omega^{2,6}$; $d^\star\omega^{3,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,6}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,6}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}+\omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{2,5} = \omega^{2,5}-\omega^{3,4}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{3,4} = -\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}-\omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,6}+2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}-\omega^{2,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}-\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5}+2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}-\omega^{1,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6}+\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}+\omega^{3,5,6}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4}+\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}-\omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{1,4,6}+\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}-\omega^{2,3,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6}+\omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{(-\beta_1-\beta_2)/2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{(-5\beta_1-\beta_2)/2}\ \langle\omega^{1,5}\rangle + e^{(-\beta_1-3\beta_2)/2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{(-3\beta_1-3\beta_2)/2}\ \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{(-7\beta_1-3\beta_2)/2}\ \langle\omega^{1,3,6}+\omega^{1,4,5}\rangle + e^{(-9\beta_1-3\beta_2)/2}\ \langle\omega^{1,5,6}\rangle + e^{(-3\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,4}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}+\omega^{2,3,5}\rangle + e^{(-5\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6}-\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{(-7\beta_1-7\beta_2)/2}\ \langle-\omega^{2,3,4,6}\rangle + e^{(-11\beta_1-5\beta_2)/2}\ \langle\omega^{1,4,5,6}\rangle + e^{-5\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + e^{(-9\beta_1-5\beta_2)/2}\ \langle(\omega^{1,2,5,6}+\omega^{1,3,4,6})/2\rangle + e^{-4\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,6}+$

$\omega^{2,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= \quad e^{-5\beta_1-4\beta_2} \quad \langle\omega^{2,3,4,5,6}\rangle \quad + \quad e^{-6\beta_1-3\beta_2} \quad \langle-\omega^{1,3,4,5,6}\rangle \quad +$
$e^{(-11\beta_1-7\beta_2)/2}\ \langle\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) \ = \ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

4.2.5 Algebra $\mathcal{G}_{6,11}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^5 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-\delta_2^6, ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_1^6, ad\ x_6=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+2e^{\beta_2}\ \langle\delta_1^4,\delta_3^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_1^4,\delta_1^3]=\delta_1^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^5$; $[\delta_1^5,\delta_2^4]=-\delta_1^6$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_1^3]=-\delta_1^5$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^6-\delta_1^4$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$.

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3}, x_2\omega^{1,2}+x_5\omega^{2,3}, x_4\omega^{1,4}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+e^{-2\beta_2}\ \langle x_1\omega^{2,4}-x_3(\omega^{2,6}-\omega^{4,5})\rangle+3e^{-3\beta_1}\ \langle x_1\omega^{1,3}+x_2(\omega^{1,5}-2\omega^{2,3})+x_4\omega^{3,4}, -x_1\omega^{1,3}-2x_2(\omega^{1,5}-1/2\omega^{2,3})+x_5\omega^{3,5}, -x_2\omega^{2,3}+x_4\omega^{1,6}\rangle+3e^{-\beta_2}\ \langle -x_1\omega^{1,2}+2x_3\omega^{1,5}+x_5\omega^{2,5}, x_3\omega^{2,3}, x_1\omega^{1,2}-x_3\omega^{1,5}+x_4\omega^{2,4}\rangle+e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,4}+x_3(\omega^{1,6}-\omega^{3,4})+x_4\omega^{2,5}+x_5\omega^{2,6}\rangle+e^{-4\beta_1}\ \langle -x_2(\omega^{1,6}-\omega^{3,4})+x_4\omega^{3,5}+x_5\omega^{3,6}\rangle+e^{2\beta_1-\beta_2}\ \langle x_3\omega^{1,2}\rangle+e^{-2\beta_1}\ \langle x_4\omega^{1,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$3e^{-\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+2e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2}+4e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+e^{-5\beta_1-\beta_2}+e^{-5\beta_1}+e^{-6\beta_1-\beta_2}+e^{-6\beta_1}+e^{-7\beta_1}+e^{-3\beta_2}$.

ch$(H^4_{\mathcal{G}})$=$2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+4e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-8\beta_1-\beta_2}+e^{-9\beta_1-\beta_2}$.

ch$(H^5_{\mathcal{G}})$=$e^{-4\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-2\beta_2}+2e^{-7\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-2\beta_2}+e^{-9\beta_1-3\beta_2}+e^{-10\beta_1-2\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-7\beta_1-4\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-10\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,12,40,71,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,7,16,21,19,11,3)$

S=$2(5\beta_1+2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-4\beta_1}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{3,4}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,6}-\omega^{4,5}\rangle$.

ch$(H^3_{\mathbb{C}})$= $2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6},\omega^{1,4,5}\rangle+e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,5}\rangle+2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,6}+\omega^{3,4,5},\omega^{2,3,6}+\omega^{3,4,5}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,3,6}\rangle+e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6}\rangle+e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,2,5,6}\rangle+e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}+\omega^{2,3,4,6}\rangle+e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,4,5,6}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-9\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

ch$(H^6_{\mathbb{C}})$= $e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{4}$; $d^{\star}\omega^{1,4} = -\omega^{5}$; $d^{\star}\omega^{1,5} = -\omega^{6}$; $d^{\star}\omega^{2,3} = -\omega^{6}$; $d^{\star}\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^{\star}\omega^{1,2,4} = -\omega^{2,5}$; $d^{\star}\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^{\star}\omega^{1,2,6} = -\omega^{4,6}$; $d^{\star}\omega^{1,3,4} = -\omega^{3,5}$; $d^{\star}\omega^{1,3,5} = -\omega^{3,6}$; $d^{\star}\omega^{1,4,5} = -\omega^{4,6}$; $d^{\star}\omega^{1,4,6} = -\omega^{5,6}$; $d^{\star}\omega^{2,3,4} = \omega^{4,6}$; $d^{\star}\omega^{2,3,5} = \omega^{5,6}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^{\star}\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^{\star}\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^{1}\rangle + e^{-\beta_2}\ \langle\omega^{2}\rangle + e^{-3\beta_1}\ \langle\omega^{3}\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6} + \omega^{2,3,4}, \omega^{1,4,5} + \omega^{2,3,4}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,5}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,6} + \omega^{3,4,5}, \omega^{2,3,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2}\ \langle-\omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6} - \omega^{2,3,4,6})/2\rangle + e^{-9\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + e^{-6\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6} + \omega^{2,3,4,5})/2\rangle + e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,2,3,6} - \omega^{1,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-10\beta_1-3\beta_2}\ \langle-\omega^{1,3,4,5,6}\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle$.

$$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-10\beta_1 - 4\beta_2}\ \langle \omega^{1,2,3,4,5,6} \rangle.$$
$$P(Y) \ = \ (1 + Y + Y^2)(1 + Y^2)(1 + Y)^2.$$

4.2.6 Algebra $\mathcal{G}_{6,13}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_5$; $[x_3,x_4]=-x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0\\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^4 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^5 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_2^5, ad\ x_4=-\delta_1^5+\delta_3^6, ad\ x_5=-\delta_1^6, ad\ x_6=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{3\beta_1+\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^{0}\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^5]=\delta_3^6-\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^5,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=\delta_3^6-\delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+2e^{0}\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+e^{-3\beta_1}\ \langle -x_2(\omega^{1,5}-\omega^{3,4})+x_4\omega^{3,5}\rangle+e^{-4\beta_1}\ \langle -x_2(\omega^{1,6}-\omega^{3,5})+x_4\omega^{3,6}\rangle+e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})+2x_2\omega^{2,4}+x_3(\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,5}\rangle+2e^{-\beta_2}\ \langle -x_1\omega^{1,2}+x_3(\omega^{1,4}+\omega^{2,3})+x_6\omega^{2,6}, x_3(\omega^{1,4}-\omega^{2,3})+x_4\omega^{2,4}\rangle+2e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_4\omega^{2,3}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}\rangle+3e^{-2\beta_1}\ \langle -x_2(\omega^{1,4}+\omega^{2,3})+x_4\omega^{3,4}, x_1\omega^{1,3}+x_2(\omega^{1,4}-2\omega^{2,3})+x_4\omega^{1,5}, x_2\omega^{1,4}+x_5\omega^{1,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+3e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+e^{-5\beta_1-\beta_2}+e^{-5\beta_1}+e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+2e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-\beta_2}+e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-8\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-4\beta_2}+2e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-9\beta_1-2\beta_2}$.

ch($H^6_{\mathcal{G}}$)=$e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,10,38,68,67,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,5,12,16,15,10,3)$

S=$9\beta_1+4\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,7,12,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}+\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=-\omega^{1,3,4}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}-\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{1,2,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{1,2,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-3\beta_1}\ \langle\omega^{1,3}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{3,5}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6}+\omega^{2,3,5}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,5}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6}\rangle+e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6}\rangle+e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}-\omega^{2,3,5,6}\rangle+e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,4,5,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-8\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,7,12,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,4,4,4,3,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{3,4} = \omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = -\omega^{1,6} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{2,6} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{3,4,5} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})= e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6} - \omega^{1,4,5} + 2\omega^{2,3,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle 2\omega^{1,4,6} - \omega^{2,3,6} - \omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{G}})= e^{-6\beta_1-4\beta_2}\ \langle -\omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6} + \omega^{2,3,5,6})/2\rangle + e^{-8\beta_1-2\beta_2}\ \langle -\omega^{1,3,5,6}\rangle + e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,6} + \omega^{2,3,4,5})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{G}})= e^{-8\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-9\beta_1-3\beta_2}\ \langle -\omega^{1,3,4,5,6}\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{G}})= e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle.$

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y)^2.$

4.2.7 Algebra $\mathcal{G}_{6,14}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_3] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 3\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^3 & \xi_1^2 & 0 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^6), ad\ x_4 = -\delta_1^5, ad\ x_5 = 0, ad\ x_6 = 0$.

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{\beta_1+\beta_2} \langle x_3 \rangle + e^{2\beta_1+\beta_2} \langle x_4 \rangle + e^{3\beta_1+\beta_2} \langle x_5 \rangle + e^{\beta_1+2\beta_2} \langle x_6 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{3\beta_1+\beta_2} \langle x_5 \rangle + e^{\beta_1+2\beta_2} \langle x_6 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = 2e^{\beta_1+\beta_2} \langle \delta_1^4, \delta_2^6 \rangle + e^{2\beta_1+\beta_2} \langle \delta_1^5 \rangle + e^{3\beta_1} \langle \delta_2^5 \rangle + e^{2\beta_1} \langle \delta_2^4 \rangle + e^{\beta_1} \langle \delta_2^3 \rangle + e^{2\beta_2} \langle \delta_1^6 \rangle + e^{\beta_2} \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^4, \delta_1^2] = -\delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^4, \delta_1^3] = \delta_1^5$; $[\delta_2^4, \delta_1^2] = -\delta_2^6 + \delta_1^4$; $[\delta_2^3, \delta_1^3] = \delta_2^6 + \delta_1^4$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^6, \delta_1^2] = \delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{3\beta_1} \langle \delta_2^5 \rangle + e^{2\beta_1} \langle \delta_2^4 \rangle + e^{2\beta_2} \langle \delta_1^6 \rangle + e^{\beta_1+\beta_2} \langle \delta_1^4 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^2] = 2\delta_1^4$; $[\delta_1^4, \delta_1^2] = -\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 3e^{-\beta_2} \langle x_1\omega^{1,2} + 2x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_5\omega^{2,5}, -x_3\omega^{2,3} + x_4\omega^{1,6} \rangle + e^{2\beta_1-2\beta_2} \langle x_5\omega^{2,6} \rangle + e^{-\beta_1-\beta_2} \langle -x_1\omega^{1,3} + 2x_2\omega^{2,3} + x_3\omega^{2,4} + x_4\omega^{2,5} + x_5\omega^{3,5} \rangle + e^{\beta_1-2\beta_2} \langle x_4\omega^{2,6} + x_5\omega^{3,6} \rangle + e^{-3\beta_1+\beta_2} \langle x_6\omega^{1,5} \rangle + 2e^{-\beta_1} \langle x_2\omega^{1,2} + x_6\omega^{1,6}, x_4\omega^{1,4} \rangle + 2e^{-2\beta_1} \langle x_4\omega^{1,5}, x_2\omega^{1,3} \rangle + e^{-4\beta_1} \langle x_2\omega^{1,5} + x_6\omega^{4,5} \rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{\beta_1-3\beta_2} + 2e^{-\beta_1-3\beta_2} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 4e^{-3\beta_1-2\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-2\beta_1-4\beta_2} + 2e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-5\beta_2} + e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_2}$.

ch$(H^5_{\mathcal{G}})$=$e^{-4\beta_1-5\beta_2}$ + $e^{-5\beta_1-4\beta_2}$ + $2e^{-6\beta_1-4\beta_2}$ + $e^{-6\beta_1-6\beta_2}$ + $e^{-7\beta_1-3\beta_2}$ + $e^{-7\beta_1-5\beta_2}$ + $e^{-8\beta_1-4\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-7\beta_1-6\beta_2}$ + $e^{-8\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(2, 11, 37, 68, 66, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(2, 7, 12, 15, 14, 8, 2)$

S=$2(4\beta_1 + 3\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 8, 13, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 4, 6, 4, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-\beta_2}$ $\langle\omega^2\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-4\beta_1-\beta_2}$ $\langle\omega^{1,5}\rangle$ + $e^{-2\beta_1-2\beta_2}$ $\langle\omega^{1,6} + \omega^{2,4}\rangle$ + $e^{-\beta_1-3\beta_2}$ $\langle\omega^{2,6}\rangle$ + $e^{-3\beta_1-2\beta_2}$ $\langle\omega^{2,5} - \omega^{3,4}\rangle$.

ch$(H^3_{\mathbb{C}})$= $e^{-4\beta_1-2\beta_2}$ $\langle\omega^{1,2,5}\rangle$ + $e^{-6\beta_1-2\beta_2}$ $\langle\omega^{1,4,5}\rangle$ + $e^{-3\beta_1-3\beta_2}$ $\langle\omega^{1,3,6}\rangle$ + $e^{-2\beta_1-4\beta_2}$ $\langle\omega^{2,3,6}\rangle + e^{-5\beta_1-3\beta_2}$ $\langle\omega^{1,5,6} - \omega^{2,4,5}\rangle + e^{-4\beta_1-4\beta_2}$ $\langle\omega^{2,5,6} - \omega^{3,4,6}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-7\beta_1-3\beta_2}$ $\langle\omega^{1,3,4,5}\rangle + e^{-5\beta_1-4\beta_2}$ $\langle\omega^{1,2,5,6}\rangle + e^{-6\beta_1-4\beta_2}$ $\langle\omega^{1,3,5,6}\rangle$ + $e^{-4\beta_1-5\beta_2}$ $\langle\omega^{2,3,4,6}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-8\beta_1-5\beta_2}$ $\langle\omega^{1,3,4,5,6}\rangle + e^{-7\beta_1-6\beta_2}$ $\langle\omega^{2,3,4,5,6}\rangle$.

ch$(H^6_{\mathbb{C}})$= $e^{-8\beta_1-6\beta_2}$ $\langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 8, 13, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 4, 6, 4, 2, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,6} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,4,5} = \omega^{1,5,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-4\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,5} - \omega^{1,3,4}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,4}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,5,6} - \omega^{2,4,5}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,5,6} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-4\beta_1-5\beta_2} \langle-\omega^{2,3,4,6}\rangle + e^{-6\beta_1-4\beta_2} \langle(-\omega^{1,3,5,6} + \omega^{2,3,4,5})/2\rangle + e^{-7\beta_1-3\beta_2} \langle-\omega^{1,3,4,5}\rangle + e^{-5\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6} + \omega^{1,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-7\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-8\beta_1-5\beta_2} \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^2$.

4.2.8 Algebra $\mathcal{G}_{6,15}$ †

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 2(\xi_1^1+\xi_2^2) \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_2^6, ad\ x_5=-\delta_1^6, ad\ x_6=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{2\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+2\beta_2}\ \langle x_5\rangle+e^{2\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{2\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_1+2\beta_2}\ \langle\delta_1^6\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_2^6\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^4,\delta_2^5\rangle+e^{\beta_2}\ \langle\delta_1^3\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4,\delta_1^3]=\delta_2^6$; $[\delta_1^5,\delta_2^3]=-\delta_1^6$; $[\delta_1^4,\delta_2^3]=-\delta_2^6$; $[\delta_2^5,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{2\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}\rangle+e^{-2\beta_2}\ \langle x_3\omega^{2,5}+x_4\omega^{2,6}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle+e^{-2\beta_1}\ \langle x_3\omega^{1,4}+x_5\omega^{1,6}\rangle+e^{-2\beta_1-2\beta_2}\ \langle -x_1(\omega^{1,6}+\omega^{3,4})+x_2(\omega^{2,6}-\omega^{3,5})-x_3\omega^{4,5}-x_4\omega^{4,6}+x_5\omega^{5,6}\rangle+2e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, x_3\omega^{1,3}+x_5\omega^{1,5}\rangle+2e^{-\beta_2}\ \langle x_3\omega^{2,3}+x_4\omega^{2,4}, -x_1\omega^{1,2}-2x_3\omega^{2,3}+x_4\omega^{1,5}\rangle+e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,3}+x_3\omega^{2,4}+x_4\omega^{3,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-3\beta_2}+2e^{-\beta_1-4\beta_2}+e^{-2\beta_1-2\beta_2}+3e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-\beta_2}+3e^{-3\beta_1-2\beta_2}+2e^{-4\beta_1-\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-2\beta_1-4\beta_2}+e^{-3\beta_1-3\beta_2}+2e^{-3\beta_1-5\beta_2}+e^{-3\beta_1-6\beta_2}+e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+e^{-5\beta_1-7\beta_2}+e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-6\beta_2}+e^{-7\beta_1-5\beta_2}$.

ch($H^6_{\mathcal{G}}$)=$e^{-6\beta_1-7\beta_2}+e^{-7\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 10, 36, 67, 66, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 5, 10, 13, 13, 8, 2)$

S=$7(\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 8, 13, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 4, 6, 4, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-3\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,6} - \omega^{3,5}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-3\beta_1-3\beta_2} \langle\omega^{1,2,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,4}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,5}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,4,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,2,5,6}\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{2,3,5,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-7\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6}\rangle + e^{-6\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-7\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 8, 13, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 4, 6, 4, 2, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$;

$d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathfrak{a}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}}) = e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,6} + \omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,6} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}}) = e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,4}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle 2\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-3\beta_1-5\beta_2}\ \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}}) = e^{-4\beta_1-6\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,3,4,6}\rangle + e^{-4\beta_1-5\beta_2}\ \langle(\omega^{1,2,5,6} + \omega^{2,3,4,5})/2\rangle + e^{-5\beta_1-4\beta_2}\ \langle(\omega^{1,2,4,6} - \omega^{1,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}}) = e^{-6\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-7\beta_1-6\beta_2}\ \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}}) = e^{-7\beta_1-7\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^2$.

4.2.9 Algebra $\mathcal{G}_{6,16}$ (6-dim generic filiform Lie Algebra) †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 4\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6, ad\ x_6 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1+\beta_2}\ \langle x_5\rangle + e^{4\beta_1+\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{4\beta_1+\beta_2}\ \langle x_6\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_2^5] = -\delta_1^6$; $[\delta_1^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^3, \delta_2^3] = -\delta_1^4$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-5\beta_1}\ \langle x_2\omega^{1,6}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3}\rangle + 2e^{-\beta_2}\ \langle x_1\omega^{1,2} + 3x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_5\omega^{2,5}, -x_1\omega^{1,2} - 2x_3\omega^{2,3} - x_4\omega^{2,4} + x_6\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} + 2x_2\omega^{2,3} + 3x_3\omega^{2,4} + x_4(\omega^{2,5} + 2\omega^{3,4}) + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4(\omega^{2,5} - \omega^{3,4}) + x_5\omega^{2,6} + x_6\omega^{3,6}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2}\rangle + e^{3\beta_1-\beta_2}\ \langle x_6\omega^{2,3}\rangle + e^{2\beta_1-\beta_2}\ \langle x_5\omega^{2,3} + x_6\omega^{2,4}\rangle + e^{\beta_1-\beta_2}\ \langle x_4\omega^{2,3} + x_5\omega^{2,4} + x_6\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2} + 2e^{-\beta_1-2\beta_2} + e^{-2\beta_1-2\beta_2} + 2e^{-3\beta_1-2\beta_2} + e^{-4\beta_1-\beta_2} + 2e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + 2e^{-6\beta_1-\beta_2} + e^{-7\beta_1-\beta_2} + e^{-8\beta_1-\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-3\beta_1-3\beta_2} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_2} + 2e^{-8\beta_1-2\beta_2} + e^{-9\beta_1-2\beta_2} + e^{-10\beta_1-2\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-6\beta_1-4\beta_2}$ + $e^{-8\beta_1-3\beta_2}$ + $e^{-9\beta_1-5\beta_2}$ + $e^{-10\beta_1-3\beta_2}$ + $e^{-10\beta_1-4\beta_2}$ + $e^{-11\beta_1-3\beta_2}$.

ch($H^6_{\mathcal{G}}$)=$e^{-10\beta_1-5\beta_2} + e^{-11\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 11, 37, 67, 64, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(1, 6, 12, 14, 11, 6, 2)$

S=$11\beta_1 + 5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 7, 12, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 3, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-5\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,5} - \omega^{3,4}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,3,6} - \omega^{2,4,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-8\beta_1-3\beta_2}\ \langle\omega^{1,2,5,6}\rangle + e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-11\beta_1-4\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle + e^{-10\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-11\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 7, 12, 11, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 2, 3, 4, 3, 2, 1)$

Total dimension of trivial cohomology: 16.

Length associated polynomial: 16.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,4,5} =$

$-\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,4} = \omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6}+2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{4,5} = \omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}+\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}+\omega^{1,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5}+\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6}+2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}+\omega^{1,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6}+\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6}+2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}+\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6}+2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-5\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,3,6}-\omega^{1,4,5}\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,3,6}-\omega^{2,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5}\rangle + e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle + e^{-8\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6}+\omega^{1,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-10\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-11\beta_1-4\beta_2}\ \langle-\omega^{1,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-11\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^2$.

4.2.10 Algebra $\mathcal{G}_{6,18}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_2, x_5] = x_6$; $[x_3, x_4] = -x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^3 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^5 & \xi_1^4 & -\xi_1^3 & 3\xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_2^6, ad\ x_6 = 0$.

Maximal torus : T=$\langle t_1, t_2 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1+\beta_2}\ \langle x_5\rangle + e^{3\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+2\beta_2}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^6\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_2^6\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^3] = \delta_2^6$; $[\delta_2^3, \delta_1^5] = \delta_2^6$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_1^3] = \delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^6\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{2\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,3} + x_5\omega^{2,6}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + 2x_2\omega^{2,3} + x_3\omega^{2,4} + x_4\omega^{2,5} + x_5\omega^{3,5}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_3\omega^{1,3} + x_6\omega^{1,6}\rangle + e^{-\beta_2}\ \langle -2/3x_1\omega^{1,2} - 1/3x_3\omega^{2,3} + 1/3x_4\omega^{2,4} + x_5\omega^{2,5}\rangle + e^{-2\beta_1}\ \langle x_3\omega^{1,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-6\beta_1-\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-4\beta_1-4\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-3\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-6\beta_1-5\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-9\beta_1-4\beta_2} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2}$.

ch($H^6_{\mathcal{G}}$)=$e^{-9\beta_1-6\beta_2} + e^{-10\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 6$: $(1, 9, 34, 64, 63, 32, 6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 6$: $(1, 4, 7, 8, 7, 5, 2)$

S=$2(5\beta_1+3\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,2,6,11,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,2,2,2,2,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{3,4,5}$; $d\omega^{1,2,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,5}$; $d\omega^{1,4,6}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,3,4,5}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}$; $d\omega^{1,2,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,4,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-4\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,5}\rangle+e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,3,6}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,4,5,6}\rangle+e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,3,4,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-9\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-10\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,2,6,11,11,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,2,2,2,2,2,1)$

Total dimension of trivial cohomology: 12.

Length associated polynomial: 12.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{2,5}=-\omega^6$; $d^\star\omega^{3,4}=\omega^6$; $d^\star\omega^{1,2,3}=-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{3,6}$; $d^\star\omega^{1,3,4}=-\omega^{1,6}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,4}=-\omega^{2,6}$; $d^\star\omega^{2,3,5}=-\omega^{3,6}$; $d^\star\omega^{2,4,5}=-\omega^{4,6}$; $d^\star\omega^{3,4,5}=-\omega^{5,6}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,5,6}=-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,5,6}$; $d^\star\omega^{1,3,5,6}=$

$-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,3,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1-5\beta_2} \langle-\omega^{2,3,4,6}\rangle + e^{-9\beta_1-4\beta_2} \langle\omega^{1,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-9\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-10\beta_1-5\beta_2} \langle-\omega^{1,3,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-10\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6}\rangle.$

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^2.$

4.3 rank 3 Lie Algebras

4.3.1 Algebra $\mathcal{G}_{6,1}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_4] = x_6$; $[x_2, x_3] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & -\xi_2^1 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & -\xi_1^1+\xi_2^2+\xi_3^3 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1+\xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & -\xi_1^3+\xi_2^4 & \xi_2^2+\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_4^6, ad\ x_2 = -\delta_1^5 + \delta_3^6, ad\ x_3 = -\delta_2^6, ad\ x_4 = -\delta_1^6, ad\ x_5 = 0, ad\ x_6 = 0.$

Maximal torus : T=$\langle t_1, t_2, t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_2+\beta_3}\ \langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_2+\beta_3}\ \langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle + e^{-2\beta_1+\beta_2+\beta_3}\ \langle\delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + 2e^{\beta_2}\ \langle\delta_1^5, \delta_3^6\rangle + 2e^{\beta_1}\ \langle\delta_2^5, \delta_4^6\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{2\beta_1-\beta_3}\ \langle\delta_4^5\rangle + 3e^0\ \langle t_1, t_2, t_3\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle\delta_1^6\rangle + e^{\beta_3}\ \langle\delta_2^6\rangle + 2e^{-\beta_1+\beta_3}\ \langle\delta_1^3, \delta_2^4\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_2^4 + \delta_1^3$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^1, \delta_1^4] = -(\delta_2^4 + \delta_1^3)$; $[\delta_2^1, \delta_1^2] = t_3 - t_2 + t_1$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_3^5] = \delta_4^5$; $[\delta_2^1, \delta_1^6] = -\delta_2^6$; $[\delta_2^1, \delta_3^6] = \delta_4^6$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^1, \delta_2^4] = -\delta_2^3$; $[\delta_1^4, \delta_4^5] = -\delta_1^5$; $[\delta_1^4, \delta_4^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_4^5] = \delta_3^5$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_4^6] = \delta_3^6$; $[\delta_1^2, \delta_1^3] = -\delta_1^4$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^5, \delta_1^3] = \delta_1^6$; $[\delta_1^5, \delta_2^4] = -\delta_1^6$; $[\delta_2^5, \delta_1^3] = \delta_2^6$; $[\delta_2^5, \delta_2^4] = -\delta_2^6$; $[\delta_3^5, \delta_1^3] = \delta_3^6 + \delta_1^5$; $[\delta_3^5, \delta_2^4] = -\delta_3^6$; $[\delta_4^5, \delta_1^3] = \delta_4^6$; $[\delta_4^5, \delta_2^4] = -\delta_4^6 + \delta_2^5$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_4^6, \delta_2^4] = \delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle + e^{-2\beta_1+\beta_2+\beta_3}\ \langle\delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{2\beta_1-\beta_3}\ \langle\delta_4^5\rangle + 3e^0\ \langle t_1, t_2, t_3\rangle + e^{\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1}\ \langle\delta_2^5\rangle + 2e^{-\beta_1+\beta_3}\ \langle\delta_1^3, \delta_2^4\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_2^1] = \delta_2^3$; $[\delta_1^3, \delta_1^2] = \delta_1^4$; $[\delta_1^3, \delta_3^5] = -2\delta_1^5$; $[\delta_1^3, \delta_4^5] = \delta_2^5$; $[\delta_2^3, \delta_1^2] = \delta_2^4 + \delta_1^3$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^4] = -\delta_2^4 - \delta_1^3$; $[\delta_2^1, \delta_2^4] = -\delta_2^3$; $[\delta_2^1, \delta_1^2] = t_3 - t_2 + t_1$; $[\delta_2^1, \delta_3^5] = \delta_4^5$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$;

$[\delta_1^4,\delta_4^5] = -\delta_1^5$; $[\delta_2^4,\delta_1^2] = \delta_1^4$; $[\delta_2^4,\delta_3^5] = \delta_1^5$; $[\delta_2^4,\delta_4^5] = -2\delta_2^5$; $[\delta_1^2,\delta_4^5] = \delta_3^5$; $[\delta_1^2,\delta_2^5] = -\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{2\beta_1-2\beta_2-\beta_3}\ \langle x_1\omega^{2,4}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + 4e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1(\omega^{1,4}-\omega^{2,3}) + x_2\omega^{2,4}, x_1\omega^{2,3} + x_3\omega^{3,4}, -x_1\omega^{1,4} + x_5\omega^{4,5}, x_1\omega^{1,4} + x_5\omega^{2,6}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3\omega^{1,3}, x_2\omega^{1,2} + x_4\omega^{2,3}, x_4\omega^{1,4}, x_5\omega^{1,5}\rangle + e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,5}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,5} + x_3(\omega^{2,6}+\omega^{4,5})\rangle + e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}\rangle + e^{-3\beta_1+\beta_3}\ \langle x_4\omega^{1,5}\rangle + 4e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_4\omega^{2,4}, x_3\omega^{2,3}, -x_1\omega^{1,2} + x_3\omega^{1,4}, x_5\omega^{2,5}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5} + x_4\omega^{2,5}\rangle + 4e^{-\beta_3}\ \langle -x_2\omega^{1,4} + x_4\omega^{3,4}, -x_1\omega^{1,3} - x_2\omega^{1,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} - x_2(\omega^{1,4}-\omega^{2,3}), -x_1\omega^{1,3} - x_2\omega^{1,4} + x_5\omega^{1,6}\rangle + e^{-2\beta_1}\ \langle -x_2\omega^{1,5} - x_4(\omega^{1,6}-\omega^{3,5})\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,5} + x_2\omega^{2,5} + x_3(\omega^{1,6}-\omega^{3,5}) + x_4(\omega^{2,6}+\omega^{4,5})\rangle + e^{2\beta_1-2\beta_3}\ \langle x_5\omega^{3,4}\rangle + e^{\beta_1-2\beta_3}\ \langle x_2\omega^{3,4} + x_5\omega^{3,6}\rangle + e^{2\beta_1-\beta_2-2\beta_3}\ \langle x_1\omega^{3,4} + x_5\omega^{4,6}\rangle + e^{-2\beta_1+\beta_3}\ \langle x_4\omega^{1,2}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{2\beta_1-\beta_2-3\beta_3} + e^{2\beta_1-2\beta_2-2\beta_3} + e^{2\beta_1-3\beta_2-2\beta_3} + 3e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + 2e^{\beta_1-2\beta_2-2\beta_3} + 2e^{\beta_1-3\beta_2-\beta_3} + 5e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + 4e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + 2e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2+\beta_3} + e^{-3\beta_1} + 2e^{-\beta_2-\beta_3} + 2e^{-\beta_2-2\beta_3} + 5e^{-2\beta_2-\beta_3} + e^{-3\beta_2} + e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{2\beta_1-3\beta_2-3\beta_3} + 2e^{\beta_1-2\beta_2-3\beta_3} + 3e^{\beta_1-3\beta_2-2\beta_3} + e^{\beta_1-4\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-2\beta_3} + 4e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + 2e^{-\beta_1-3\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2} + e^{-\beta_2-3\beta_3} + 5e^{-2\beta_2-2\beta_3} + 2e^{-3\beta_2-\beta_3} + e^{-3\beta_2-2\beta_3} + e^{-4\beta_2-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{\beta_1-3\beta_2-3\beta_3} + e^{\beta_1-4\beta_2-3\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-3\beta_3} + 3e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + 2e^{-2\beta_1-2\beta_2-2\beta_3} + 2e^{-2\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + e^{-2\beta_2-3\beta_3} + e^{-3\beta_2-2\beta_3} + e^{-3\beta_2-3\beta_3} + 2e^{-4\beta_2-2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-\beta_1-3\beta_2-3\beta_3} + e^{-\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(2,17,50,82,74,34,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 6$: $(2,13,31,42,36,18,4)$

S$=\beta_1 + 4\beta_2 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,4,10,15,13,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,4,8,10,8,4,1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,5,6} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{\beta_1-\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6} + \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + 2e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,6},\omega^{1,4,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-3\beta_2-\beta_3}\ \langle\omega^{2,4,5}\rangle + e^{\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,4,6}\rangle + 2e^{-2\beta_2-2\beta_3}\ \langle\omega^{1,4,6} - \omega^{3,4,5}, \omega^{2,3,6} - \omega^{3,4,5}\rangle + e^{\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,5,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,3,6}\rangle + e^{-2\beta_2-3\beta_3}\ \langle\omega^{1,3,4,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,3,5,6}\rangle + e^{-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,6}\rangle + e^{\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,4,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle + e^{-4\beta_2-2\beta_3}\ \langle\omega^{2,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,3,5,6}\rangle + e^{-\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle + e^{-\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,3,4,5,6}\rangle + e^{-4\beta_2-3\beta_3}\ \langle\omega^{2,3,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{1,2,3,4,5,6}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y)^2.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,4,10,15,13,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,4,8,10,8,4,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,6} + \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6}-\omega^{2,3,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{3,4,5,6}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{2,6}-\omega^{4,5}$; $\Delta\omega^{3,5} = \omega^{1,6}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = -\omega^{2,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}-\omega^{1,4,5}-\omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,6}+\omega^{1,4,5}+\omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}+\omega^{2,3,6}+\omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6}+\omega^{1,4,5}+\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6}+\omega^{2,3,6}+\omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6}+\omega^{2,3,6}+\omega^{3,4,5}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$;

$\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{\beta_1-\beta_2-\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{\beta_1-2\beta_2-\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{\beta_1-\beta_2-2\beta_3} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6}\rangle + 2e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6} + \omega^{2,3,5}, \omega^{1,4,5} - \omega^{2,3,5}\rangle + e^{-3\beta_2-\beta_3} \langle\omega^{2,4,5}\rangle + e^{\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,4,6}\rangle + 2e^{-2\beta_2-2\beta_3} \langle\omega^{1,4,6} - \omega^{3,4,5}, \omega^{2,3,6} - \omega^{3,4,5}\rangle + e^{\beta_1-2\beta_2-3\beta_3} \langle\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_2-2\beta_3} \langle-\omega^{2,4,5,6}\rangle + e^{\beta_1-3\beta_2-3\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle(-\omega^{1,4,5,6} + \omega^{2,3,5,6})/2\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle-\omega^{1,3,5,6}\rangle + e^{-2\beta_2-3\beta_3} \langle\omega^{1,3,4,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_2-2\beta_3} \langle(\omega^{1,2,4,6} + \omega^{2,3,4,5})/2\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,2,3,6} - \omega^{1,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-4\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-\beta_1-3\beta_2-3\beta_3} \langle-\omega^{1,3,4,5,6}\rangle + e^{-\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle-\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-\beta_1-4\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y)^2$.

4.3.2 Algebra $\mathcal{G}_{6,2}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_3, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 \\ \xi_1^4 & 0 & \xi_3^4 & 2\xi_1^1+\xi_2^2-\xi_3^3 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_1^4 & -\xi_1^3 & \xi_1^1+\xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^5 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5, ad\ x_3 = \delta_4^6, ad\ x_4 = -\delta_3^6, ad\ x_5 = -\delta_1^6, ad\ x_6 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

ch($\mathcal{G}$)= e^{β_1} $\langle x_1\rangle$ + e^{β_2} $\langle x_2\rangle$ + e^{β_3} $\langle x_3\rangle$ + $e^{2\beta_1+\beta_2-\beta_3}$ $\langle x_4\rangle$ + $e^{\beta_1+\beta_2}$ $\langle x_5\rangle$ + $e^{2\beta_1+\beta_2}$ $\langle x_6\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}$ $\langle x_6\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}$ $\langle\delta_1^2\rangle$+$e^{-2\beta_1-\beta_2+2\beta_3}$ $\langle\delta_4^3\rangle$+$e^{2\beta_1+\beta_2-2\beta_3}$ $\langle\delta_3^4\rangle$+$3e^0$ $\langle t_3, t_1, t_2\rangle$+ e^{β_2} $\langle\delta_1^5\rangle$+$e^{\beta_1+\beta_2-\beta_3}$ $\langle\delta_1^4\rangle$+$e^{-\beta_1+\beta_3}$ $\langle\delta_1^3\rangle$+$e^{\beta_1+\beta_2}$ $\langle\delta_1^6\rangle$+$e^{2\beta_1}$ $\langle\delta_2^6\rangle$+$e^{2\beta_1+\beta_2-\beta_3}$ $\langle\delta_3^6\rangle$+ e^{β_3} $\langle\delta_4^6\rangle$ + e^{β_1} $\langle\delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_4^3, \delta_3^6] = -\delta_4^6$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_1^5, \delta_2^5] = -\delta_1^6$; $[\delta_1^4, \delta_1^3] = 2\delta_1^5$; $[\delta_1^4, \delta_4^6] = -\delta_1^6$; $[\delta_1^4, \delta_2^5] = -\delta_3^6$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_2^5] = \delta_4^6$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}$ $\langle\delta_1^2\rangle$+$e^{-2\beta_1-\beta_2+2\beta_3}$ $\langle\delta_4^3\rangle$+$e^{2\beta_1+\beta_2-2\beta_3}$ $\langle\delta_3^4\rangle$+$3e^0$ $\langle t_3, t_1, t_2\rangle$+ $e^{\beta_1+\beta_2-\beta_3}$ $\langle\delta_1^4\rangle$ + $e^{-\beta_1+\beta_3}$ $\langle\delta_1^3\rangle$ + $e^{2\beta_1}$ $\langle\delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_3^4, \delta_1^3] = \delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_2}$ $\langle x_1\omega^{2,5}\rangle$ + $e^{-\beta_1+\beta_2-\beta_3}$ $\langle x_2\omega^{1,3}\rangle$ + $e^{-3\beta_1+\beta_3}$ $\langle x_2\omega^{1,4}\rangle$ + $e^{-\beta_1-\beta_2}$ $\langle -x_1\omega^{1,5} + x_2\omega^{2,5}\rangle$ + $e^{-3\beta_1-\beta_2+2\beta_3}$ $\langle x_3\omega^{1,4}\rangle$ + $e^{-2\beta_1-2\beta_2+2\beta_3}$ $\langle x_3\omega^{2,4}\rangle$ + $e^{-\beta_1-2\beta_2+\beta_3}$ $\langle x_1\omega^{2,4} + x_3\omega^{2,5}\rangle$ + $e^{\beta_1+\beta_2-2\beta_3}$ $\langle x_4\omega^{1,3}\rangle$ + $3e^{-\beta_1}$ $\langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2}, x_3\omega^{1,3} + x_5\omega^{3,4}\rangle$ + $e^{2\beta_1-2\beta_3}$ $\langle x_4\omega^{2,3}\rangle$ + $2e^{-\beta_2}$ $\langle -x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_3\omega^{2,3} + x_6\omega^{2,6}\rangle$ + $e^{\beta_1-\beta_2-\beta_3}$ $\langle -x_1\omega^{2,3} + x_4\omega^{2,5}\rangle$ + $e^{-2\beta_1}$ $\langle x_2\omega^{1,5}\rangle$ + $3e^{-\beta_3}$ $\langle x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_5\omega^{3,5}, x_4\omega^{3,4}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{1,5}\rangle$+$3e^{-2\beta_1-\beta_2+\beta_3}$ $\langle x_1\omega^{1,4}-2x_2\omega^{2,4}+x_5\omega^{4,5}, x_3\omega^{3,4}, x_1\omega^{1,4}-x_2\omega^{2,4}+x_3\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-2\beta_3}+e^{-\beta_1-\beta_2-\beta_3}+3e^{-\beta_1-2\beta_2}+e^{-\beta_1-\beta_3}+e^{-\beta_1-2\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+2e^{-2\beta_1-\beta_2}+2e^{-2\beta_1-2\beta_2+\beta_3}+e^{-2\beta_1-3\beta_2+\beta_3}+e^{-2\beta_1-\beta_3}+e^{-3\beta_1-\beta_2+\beta_3}+2e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2+\beta_3}+e^{-3\beta_1-3\beta_2+2\beta_3}+e^{-3\beta_1-\beta_3}+e^{-4\beta_1-\beta_2+\beta_3}+e^{-4\beta_1-2\beta_2+\beta_3}+e^{-5\beta_1-\beta_2+\beta_3}+e^{-5\beta_1-2\beta_2+2\beta_3}+2e^{-\beta_2-\beta_3}+e^{-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-2\beta_2-\beta_3}+e^{-\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-\beta_2-2\beta_3}+e^{-2\beta_1-2\beta_2-\beta_3}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2+\beta_3}+2e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-\beta_2-\beta_3}+3e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2+\beta_3}+e^{-4\beta_1-4\beta_2+\beta_3}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2+\beta_3}+e^{-5\beta_1-3\beta_2+\beta_3}+e^{-5\beta_1-4\beta_2+2\beta_3}+e^{-6\beta_1-2\beta_2+\beta_3}+e^{-6\beta_1-3\beta_2+2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-2\beta_2-2\beta_3}+2e^{-3\beta_1-3\beta_2-\beta_3}+2e^{-4\beta_1-2\beta_2-\beta_3}+2e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2+\beta_3}+e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2+\beta_3}+e^{-6\beta_1-4\beta_2+2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-4\beta_1-3\beta_2-\beta_3}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-4\beta_2+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(1,14,44,73,72,34,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(1,9,22,27,25,16,4)$

S=$2(3\beta_1+2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 6$: $(1,4,8,13,13,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 6$: $(1,4,6,6,6,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,5}-\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=-\omega^{1,4,5}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{3,4,5}$; $d\omega^{1,2,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,3,4,5}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{2,3,4,5}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,4,6}=-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}$; $d\omega^{1,2,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{3,4,5,6}=-\omega^{1,2,3,4,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle+e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6}\rangle+e^{-5\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,6}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle+e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle+e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4,5}\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,5,6}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,4,6}\rangle+e^{-4\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,3,5,6}\rangle+e^{-6\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,4,5,6}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,5,6}+\omega^{2,3,4,6}\rangle+e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,5,6}\rangle+e^{-5\beta_1-4\beta_2+\beta_3}\ \langle\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-4\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,3,5,6}\rangle+e^{-6\beta_1-4\beta_2+\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle+e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle+e^{-5\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) \ = \ (1+2Y+Y^2+2Y^3+Y^4)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,4,8,13,13,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,4,6,6,6,4,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{3,4}=-\omega^6$; $d^\star\omega^{1,2,3}=\omega^{3,5}$; $d^\star\omega^{1,2,4}=\omega^{4,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}$; $d^\star\omega^{1,2,6}=-\omega^{5,6}$; $d^\star\omega^{1,3,4}=\omega^{1,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{2,3,4}=\omega^{2,6}$; $d^\star\omega^{3,4,5}=\omega^{5,6}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,6}=\omega^{3,5,6}$; $d^\star\omega^{1,2,4,5}=-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6}=\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,6}-\omega^{3,4,6}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,6}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{3,4,5,6}$;

$\Delta\omega^5=\omega^5$; $\Delta\omega^6=2\omega^6$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,5}=\omega^{1,5}+\omega^{3,4}$; $\Delta\omega^{1,6}=\omega^{1,6}$; $\Delta\omega^{2,6}=2\omega^{2,6}$; $\Delta\omega^{3,4}=\omega^{1,5}+\omega^{3,4}$; $\Delta\omega^{3,5}=\omega^{3,5}$; $\Delta\omega^{3,6}=\omega^{3,6}$; $\Delta\omega^{4,5}=\omega^{4,5}$; $\Delta\omega^{4,6}=\omega^{4,6}$; $\Delta\omega^{5,6}=2\omega^{5,6}$; $\Delta\omega^{1,2,3}=\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=\omega^{1,2,5}-\omega^{2,3,4}$; $\Delta\omega^{1,2,6}=2\omega^{1,2,6}$; $\Delta\omega^{1,3,4}=\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=\omega^{1,3,5}$; $\Delta\omega^{1,4,5}=\omega^{1,4,5}$; $\Delta\omega^{1,5,6}=\omega^{1,5,6}+\omega^{3,4,6}$; $\Delta\omega^{2,3,4}=-\omega^{1,2,5}+\omega^{2,3,4}$; $\Delta\omega^{2,3,6}=\omega^{2,3,6}$; $\Delta\omega^{2,4,6}=\omega^{2,4,6}$; $\Delta\omega^{2,5,6}=\omega^{2,5,6}$; $\Delta\omega^{3,4,5}=2\omega^{3,4,5}$; $\Delta\omega^{3,4,6}=\omega^{1,5,6}+\omega^{3,4,6}$; $\Delta\omega^{3,5,6}=\omega^{3,5,6}$; $\Delta\omega^{4,5,6}=\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4}=2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6}=\omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5}=\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,5,6}-\omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5}=2\omega^{1,3,4,5}$; $\Delta\omega^{2,3,4,5}=\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{3,4,5,6}=\omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5}=2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle+e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6}\rangle+e^{-5\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,6}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}+\omega^{2,3,4}\rangle+e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle+e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4,5}\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,5,6}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-4\beta_2+\beta_3}\ \langle-\omega^{2,4,5,6}\rangle+e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,5,6}\rangle+e^{-6\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,4,5,6}\rangle+e^{-4\beta_1-2\beta_2-\beta_3}\ \langle-\omega^{1,3,5,6}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,4,6}\rangle+e^{-4\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6}-\omega^{2,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})=\quad e^{-5\beta_1-4\beta_2}\quad \langle\omega^{2,3,4,5,6}\rangle \quad+\quad e^{-6\beta_1-3\beta_2}\quad \langle-\omega^{1,3,4,5,6}\rangle \quad+$ $e^{-6\beta_1-4\beta_2+\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle + e^{-4\beta_1-3\beta_2-\beta_3}\ \langle-\omega^{1,2,3,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6}\rangle.$

$P(Y) \ = \ (1+2Y+Y^2+2Y^3+Y^4)(1+Y)^2.$

4.3.3 Algebra $\mathcal{G}_{6,3}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_2, x_3] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & \xi_3^2 & -\xi_3^1 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_1^1 + \xi_3^3 & \xi_2^1 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & \xi_1^2 & \xi_2^2 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4 + \delta_3^6, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = 0, ad\ x_5 = 0, ad\ x_6 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

ch$(\mathcal{G})$= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{\beta_2+\beta_3}\ \langle x_6\rangle$.

ch$(H^0_{\mathcal{G}})$= $e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{\beta_2+\beta_3}\ \langle x_6\rangle$.

ch$(Z^1_{\mathcal{G}})$= $2e^{\beta_2}\ \langle \delta_1^4, \delta_3^6\rangle + 2e^{\beta_1}\ \langle \delta_2^4, \delta_3^5\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^4\rangle + e^{\beta_2-\beta_3}\ \langle \delta_3^2\rangle + e^{\beta_1-\beta_3}\ \langle \delta_3^1\rangle + 2e^{\beta_3}\ \langle \delta_1^5, \delta_2^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^5\rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + 3e^0\ \langle t_1, t_2, t_3\rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_3^1] = \delta_3^4$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^4, \delta_2^1] = \delta_2^4$; $[\delta_1^4, \delta_1^3] = \delta_1^6$; $[\delta_2^4, \delta_3^2] = \delta_3^4$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$; $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_2^4, \delta_1^2] = \delta_1^4$; $[\delta_3^4, \delta_2^3] = -\delta_3^5 + \delta_2^4$; $[\delta_3^4, \delta_1^3] = \delta_3^6 + \delta_1^4$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^2, \delta_2^5] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_3^5] = \delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^2, \delta_2^1] = -\delta_3^1$; $[\delta_3^2, \delta_2^6] = -\delta_3^6$; $[\delta_3^2, \delta_1^3] = \delta_1^2$; $[\delta_3^1, \delta_1^5] = -\delta_3^5$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_3^1, \delta_1^6] = -(\delta_3^6 + \delta_1^4)$; $[\delta_3^1, \delta_2^6] = -\delta_2^4$; $[\delta_3^1, \delta_3^6] = -\delta_3^4$; $[\delta_3^1, \delta_1^3] = -t_3 + t_1$; $[\delta_3^1, \delta_1^2] = -\delta_3^2$; $[\delta_1^5, \delta_2^1] = \delta_2^5$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_2^5, \delta_1^2] = -\delta_2^6 + \delta_1^5$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_3^5, \delta_1^2] = -\delta_3^6$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^1, \delta_1^6] = -\delta_2^6 + \delta_1^5$; $[\delta_2^1, \delta_2^6] = \delta_2^5$; $[\delta_2^1, \delta_3^6] = \delta_3^5$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_3^6, \delta_1^3] = \delta_1^6$.

ch$(H^1_{\mathcal{G}})$= $e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^4\rangle + e^{\beta_2-\beta_3}\ \langle \delta_3^2\rangle + e^{\beta_1-\beta_3}\ \langle \delta_3^1\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^5\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + 3e^0\ \langle t_1, t_2, t_3\rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6\rangle + e^{\beta_3}\ \langle \delta_1^5\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4, \delta_2^3] = 2\delta_2^4$; $[\delta_3^4, \delta_1^3] = 2\delta_1^4$; $[\delta_3^2, \delta_2^5] = 2\delta_2^4$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^2, \delta_2^1] = -\delta_3^1$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^2, \delta_1^3] = \delta_1^2$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_3^1, \delta_1^6] = -2\delta_1^4$; $[\delta_3^1, \delta_1^5] = \delta_2^4$; $[\delta_3^1, \delta_1^4] = -\delta_3^4$; $[\delta_3^1, \delta_1^3] = -t_3 + t_1$; $[\delta_3^1, \delta_1^2] = -\delta_3^2$; $[\delta_2^5, \delta_1^2] = 2\delta_1^5$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$; $[\delta_2^4, \delta_1^3] = -\delta_1^5$;

$[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_2^3,\delta_1^4]=\delta_1^5$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_2^1,\delta_1^6]=2\delta_1^5$; $[\delta_2^1,\delta_1^5]=-\delta_2^5$; $[\delta_2^1,\delta_1^4]=-\delta_2^4$; $[\delta_2^1,\delta_1^3]=-\delta_2^3$; $[\delta_2^1,\delta_1^2]=-t_2+t_1$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_1^4,\delta_1^3]=\delta_1^6$.

ch$(H^2_{\mathcal{G}})$= $4e^{-\beta_3}\ \langle -x_1\omega^{1,3}+x_4\omega^{3,4}, x_1\omega^{1,3}+x_4\omega^{2,5}, -x_1\omega^{1,3}+x_2\omega^{2,3}, -x_1\omega^{1,3}+x_4\omega^{1,6}\rangle + e^{\beta_2-2\beta_3}\ \langle x_4\omega^{3,5}\rangle + e^{\beta_1-2\beta_3}\ \langle x_4\omega^{3,6}\rangle + e^{-2\beta_3}\ \langle x_1\omega^{3,5}+x_2\omega^{3,6}+x_4\omega^{5,6}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + 4e^{-\beta_2}\ \langle -x_1\omega^{1,2}+x_5\omega^{2,5}, x_4\omega^{2,4}, x_1\omega^{1,2}+x_3\omega^{2,3}, x_1\omega^{1,2}+x_5\omega^{1,6}\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle + 2e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}, x_4\omega^{2,6}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,4}+x_3\omega^{2,6}+x_5\omega^{4,6}\rangle + e^{-\beta_2-\beta_3}\ \langle -x_1(\omega^{2,5}+\omega^{3,4})-x_2\omega^{2,6}+x_3\omega^{3,6}-x_4\omega^{4,6}+x_5\omega^{5,6}\rangle + e^{-2\beta_1+\beta_3}\ \langle x_6\omega^{1,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle + 4e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_6\omega^{1,6}, -x_2\omega^{1,2}+x_3\omega^{1,3}, x_5\omega^{1,5}, x_4\omega^{1,4}\rangle + 2e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}, x_5\omega^{1,4}\rangle + 2e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}, x_4\omega^{1,5}\rangle + e^{-2\beta_1}\ \langle -x_2\omega^{1,4}-x_3\omega^{1,5}+x_6\omega^{4,5}\rangle + e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4}-x_2\omega^{2,4}-x_3(\omega^{1,6}+\omega^{2,5})-x_5\omega^{4,5}+x_6\omega^{4,6}\rangle + e^{-\beta_1-\beta_3}\ \langle x_1\omega^{1,5}+x_2(\omega^{1,6}-\omega^{3,4})-x_3\omega^{3,5}+x_4\omega^{4,5}+x_6\omega^{5,6}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2-2\beta_3}+6e^{-\beta_1-\beta_2-\beta_3}+4e^{-\beta_1-2\beta_2}+4e^{-\beta_1-2\beta_3}+e^{-2\beta_1+\beta_2-2\beta_3}+4e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+\beta_3}+4e^{-2\beta_1-\beta_3}+e^{-3\beta_1}+4e^{-\beta_2-2\beta_3}+4e^{-2\beta_2-\beta_3}+e^{-3\beta_2}+e^{-3\beta_3}$.

ch$(H^4_{\mathcal{G}})$=$e^{-\beta_1-\beta_2-2\beta_3}+2e^{-\beta_1-\beta_2-3\beta_3}+e^{-\beta_1-2\beta_2-\beta_3}+4e^{-\beta_1-2\beta_2-2\beta_3}+2e^{-\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+4e^{-2\beta_1-\beta_2-2\beta_3}+4e^{-2\beta_1-2\beta_2-\beta_3}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-2\beta_3}+e^{-2\beta_1-3\beta_3}+2e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-2\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-2\beta_2-3\beta_3}+e^{-3\beta_2-2\beta_3}$.

ch$(H^5_{\mathcal{G}})$=$e^{-\beta_1-2\beta_2-3\beta_3}+e^{-\beta_1-3\beta_2-2\beta_3}+e^{-\beta_1-3\beta_2-3\beta_3}+e^{-2\beta_1-\beta_2-3\beta_3}+3e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-2\beta_2-3\beta_3}+e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-3\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2-2\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}$.

ch$(H^6_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2-3\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_2-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(3,18,48,78,72,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(3,15,30,36,30,15,3)$

S=$3(\beta_1+\beta_2+\beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,11,16,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,12,8,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{2,3}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,5,6}+\omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,4,5}\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,4,6}\rangle + 2e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,5,6}, \omega^{1,3,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,4,5}\rangle + e^{-2\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,3,5,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,4,6}\rangle + e^{-\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,3,4,5,6}\rangle + e^{-2\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y + 5Y^2 + Y^3 + Y^4)(1 + Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 3, 11, 16, 12, 6, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1, 3, 8, 12, 8, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} - \omega^{3,4,5}$;

$\Delta\omega^{2,3,4} = -\omega^{1,2,6}+\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6}+\omega^{2,3,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6}+\omega^{2,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6}+\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{4,5,6} = 3\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6}-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathfrak{a}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle+e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,5}+\omega^{1,3,4}\rangle+e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,5}\rangle+ e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,6} + \omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6} + \omega^{2,3,5}\rangle + e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,4,6}+\omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,5,6}+\omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= e^{-\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3}\ \langle-\omega^{2,3,4,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3}\ \langle-\omega^{1,3,5,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle-\omega^{1,3,4,5}\rangle + 2e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle(-\omega^{1,2,5,6}+ \omega^{1,3,4,6}+2\omega^{2,3,4,5})/3, (\omega^{1,2,5,6}+2\omega^{1,3,4,6}+\omega^{2,3,4,5})/3\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\ \langle-\omega^{1,2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,4,5}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= e^{-2\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3}\ \langle-\omega^{1,3,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1 + Y + 5Y^2 + Y^3 + Y^4)(1 + Y)^2$.

4.3.4 Algebra $\mathcal{G}_{6,4}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_6$; $[x_2, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^4 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^3 & 0 & \xi_1^1+\xi_3^3 \end{pmatrix}$$

$ad\, x_1 = \delta_2^4 + \delta_3^6, ad\, x_2 = -\delta_1^4, ad\, x_3 = -\delta_1^6, ad\, x_4 = -\delta_2^5, ad\, x_5 = 0, ad\, x_6 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1+\beta_2}\ \langle x_4 \rangle + e^{\beta_1+2\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+2\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + 2e^{\beta_1}\ \langle \delta_2^4, \delta_3^6 \rangle + e^{2\beta_2}\ \langle \delta_1^5 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_2^5 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{\beta_2}\ \langle \delta_1^4 \rangle + 3e^0\ \langle t_2, t_1, t_3 \rangle + e^{\beta_3}\ \langle \delta_1^6 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_2^4, \delta_1^4] = \delta_2^5$; $[\delta_2^4, \delta_2^3] = -\delta_2^6$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_1^4, \delta_2^3] = -\delta_1^6$; $[\delta_3^6, \delta_2^3] = \delta_2^6$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + e^{2\beta_2}\ \langle \delta_1^5 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + 3e^0\ \langle t_2, t_1, t_3 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{\beta_1}\ \langle \delta_2^4 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_2^4, \delta_2^3] = -\delta_2^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3} \rangle + 2e^{-2\beta_1}\ \langle x_2\omega^{1,4}, x_3\omega^{1,6} \rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,4} \rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_3\omega^{2,3}, -x_1\omega^{1,2} + x_6\omega^{2,6}, x_4\omega^{2,4} \rangle + 3e^{-\beta_3}\ \langle -2x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} + x_5\omega^{3,5}, x_1\omega^{1,3} + x_4\omega^{2,6} \rangle + e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,6} - x_2(\omega^{2,6} + \omega^{3,4}) - 2x_3\omega^{3,6} + x_4\omega^{4,6} \rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_5\omega^{1,5}, x_3\omega^{1,3} \rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle x_5\omega^{1,6} \rangle + e^{2\beta_2-2\beta_3}\ \langle x_5\omega^{3,6} \rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2} \rangle + e^{-3\beta_2+\beta_3}\ \langle x_6\omega^{2,5} \rangle + e^{-2\beta_2}\ \langle x_4\omega^{2,5} \rangle + e^{-\beta_1-3\beta_2+\beta_3}\ \langle x_3\omega^{2,5} + x_6\omega^{4,5} \rangle + e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,6} + \omega^{3,4}) + x_6\omega^{4,6} \rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{-\beta_1+2\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2+\beta_3} + 2e^{-\beta_1-2\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-3\beta_2} + 3e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2} +$

$e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2+\beta_3}+e^{-3\beta_1-\beta_3}+e^{-\beta_2-\beta_3}+e^{-\beta_2-2\beta_3}+e^{-2\beta_2-\beta_3}+e^{-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-2\beta_3}+e^{-\beta_1-2\beta_2-2\beta_3}+e^{-\beta_1-3\beta_2-\beta_3}+e^{-\beta_1-3\beta_2-2\beta_3}+e^{-\beta_1-4\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-2\beta_3}+3e^{-2\beta_1-3\beta_2-\beta_3}+2e^{-2\beta_1-4\beta_2}+e^{-2\beta_1-2\beta_3}+2e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+3e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2+\beta_3}+e^{-3\beta_1-2\beta_3}+e^{-4\beta_1-2\beta_2-\beta_3}+e^{-4\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-2\beta_1-4\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-\beta_3}+e^{-3\beta_1-4\beta_2}+e^{-4\beta_1-2\beta_2-2\beta_3}+2e^{-4\beta_1-3\beta_2-\beta_3}+e^{-4\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-4\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-2\beta_3}+e^{-4\beta_1-4\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,13,43,74,69,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(2,9,20,27,23,12,3)$

S=$2(2\beta_1+2\beta_2+\beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,6,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,4}$; $d\omega^6=-\omega^{1,3}$; $d\omega^{1,5}=\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,3}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{1,3,4}$; $d\omega^{5,6}=\omega^{1,3,5}-\omega^{2,4,6}$; $d\omega^{1,3,5}=\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,4,6}$; $d\omega^{2,4,6}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=\omega^{1,2,3,5}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=-\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,5}$; $d\omega^{1,3,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\langle\omega^1\rangle+e^{-\beta_2}\langle\omega^2\rangle+e^{-\beta_3}\langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-2\beta_1-\beta_2}\langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_3}\langle\omega^{1,6}\rangle+e^{-\beta_2-\beta_3}\langle\omega^{2,3}\rangle+e^{-\beta_1-3\beta_2}\langle\omega^{2,5}\rangle+e^{-\beta_1-\beta_2-\beta_3}\langle\omega^{2,6}+\omega^{3,4}\rangle+e^{-\beta_1-2\beta_3}\langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-2\beta_1-\beta_2-\beta_3}\langle\omega^{1,2,6}\rangle+e^{-2\beta_1-2\beta_3}\langle\omega^{1,3,6}\rangle+e^{-3\beta_1-3\beta_2}\langle\omega^{1,4,5}\rangle+e^{-3\beta_1-\beta_2-\beta_3}\langle\omega^{1,4,6}\rangle+e^{-\beta_1-3\beta_2-\beta_3}\langle\omega^{2,3,5}\rangle+e^{-\beta_1-\beta_2-2\beta_3}\langle\omega^{2,3,6}\rangle+e^{-2\beta_1-4\beta_2}\langle\omega^{2,4,5}\rangle+e^{-2\beta_1-3\beta_2-\beta_3}\langle\omega^{2,5,6}-\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-3\beta_1-4\beta_2}\langle\omega^{1,2,4,5}\rangle+e^{-3\beta_1-3\beta_2-\beta_3}\langle\omega^{1,2,5,6}\rangle+e^{-3\beta_1-\beta_2-2\beta_3}\langle\omega^{1,3,4,6}\rangle+e^{-4\beta_1-3\beta_2-\beta_3}\langle\omega^{1,4,5,6}\rangle+e^{-2\beta_1-4\beta_2-\beta_3}\langle\omega^{2,3,4,5}\rangle+e^{-2\beta_1-3\beta_2-2\beta_3}\langle\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-4\beta_1-4\beta_2-\beta_3}\langle\omega^{1,2,4,5,6}\rangle+e^{-4\beta_1-3\beta_2-2\beta_3}\langle\omega^{1,3,4,5,6}\rangle+e^{-3\beta_1-4\beta_2-2\beta_3}\langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=e^{-4\beta_1-4\beta_2-2\beta_3}\langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,6,8,6,3,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^5$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,5}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,5}$; $d^\star\omega^{2,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,5}+\omega^{2,4,6}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{1,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{3,4} = -\omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,5} = \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,5,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\,\langle\omega^1\rangle + e^{-\beta_2}\,\langle\omega^2\rangle + e^{-\beta_3}\,\langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2}\,\langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3}\,\langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3}\,\langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2}\,\langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3}\,\langle\omega^{2,6}+\omega^{3,4}\rangle + e^{-\beta_1-2\beta_3}\,\langle\omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-\beta_2-\beta_3}\,\langle\omega^{1,2,6}+\omega^{1,3,4}\rangle + e^{-2\beta_1-2\beta_3}\,\langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2}\,\langle\omega^{1,4,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\,\langle\omega^{1,4,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\,\langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\,\langle\omega^{2,3,6}\rangle + e^{-2\beta_1-4\beta_2}\,\langle\omega^{2,4,5}\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\,\langle\omega^{2,5,6}-\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-2\beta_1-3\beta_2-2\beta_3}\,\langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-4\beta_2-\beta_3}\,\langle\omega^{2,3,4,5}\rangle + e^{-4\beta_1-3\beta_2-\beta_3}\,\langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\,\langle\omega^{1,3,4,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\,\langle(\omega^{1,2,5,6}-\omega^{1,3,4,5})/2\rangle + e^{-3\beta_1-4\beta_2}\,\langle\omega^{1,2,4,5}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-3\beta_1-4\beta_2-2\beta_3}\,\langle\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\,\langle-\omega^{1,3,4,5,6}\rangle + e^{-4\beta_1-4\beta_2-\beta_3}\,\langle\omega^{1,2,4,5,6}\rangle.$

$$\mathrm{ch}(H^6_{\mathfrak{a}}) = e^{-4\beta_1-4\beta_2-2\beta_3}\ \langle \omega^{1,2,3,4,5,6} \rangle.$$
$$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2.$$

4.3.5 Algebra $\mathcal{G}_{6,6}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^4 & \xi_1^1 + 2\xi_2^2 & \xi_3^4 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & 0 & \xi_2^2 + \xi_3^3 \end{pmatrix}$$

$ad\, x_1 = \delta_2^4, ad\, x_2 = -\delta_1^4 + \delta_3^6, ad\, x_3 = -\delta_2^6, ad\, x_4 = -\delta_2^5, ad\, x_5 = 0, ad\, x_6 = 0$.

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{\beta_3} \langle x_3 \rangle + e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{\beta_1+2\beta_2} \langle x_5 \rangle + e^{\beta_2+\beta_3} \langle x_6 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+2\beta_2} \langle x_5 \rangle + e^{\beta_2+\beta_3} \langle x_6 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{\beta_1-\beta_2} \langle \delta_2^1 \rangle + e^{-\beta_2+\beta_3} \langle \delta_2^3 \rangle + e^{\beta_1} \langle \delta_2^4 \rangle + e^{2\beta_2} \langle \delta_1^5 \rangle + e^{\beta_1+\beta_2} \langle \delta_2^5 \rangle + e^{\beta_1+2\beta_2-\beta_3} \langle \delta_3^5 \rangle + 2e^{\beta_2} \langle \delta_1^4, \delta_3^6 \rangle + 3e^0 \langle t_1, t_2, t_3 \rangle + e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + e^{-\beta_1+\beta_2+\beta_3} \langle \delta_1^6 \rangle + e^{\beta_3} \langle \delta_2^6 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^4] = -\delta_2^4$; $[\delta_2^1, \delta_1^6] = -\delta_2^6$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_2^3, \delta_3^4] = -\delta_2^4$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^4, \delta_1^4] = \delta_2^5$; $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_1^4, \delta_3^4] = -\delta_3^5$; $[\delta_1^4, \delta_1^3] = \delta_1^6$; $[\delta_3^4, \delta_1^6] = \delta_1^5$; $[\delta_3^4, \delta_2^6] = \delta_2^5$; $[\delta_3^4, \delta_3^6] = \delta_3^5$; $[\delta_3^4, \delta_1^3] = \delta_3^6 + \delta_1^4$; $[\delta_3^6, \delta_1^3] = \delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{\beta_1-\beta_2} \langle \delta_2^1 \rangle + e^{-\beta_2+\beta_3} \langle \delta_2^3 \rangle + e^{2\beta_2} \langle \delta_1^5 \rangle + e^{\beta_1+2\beta_2-\beta_3} \langle \delta_3^5 \rangle + 3e^0 \langle t_1, t_2, t_3 \rangle + e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + e^{-\beta_1+\beta_2+\beta_3} \langle \delta_1^6 \rangle + e^{\beta_2} \langle \delta_1^4 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_3^4, \delta_1^6] = \delta_1^5$; $[\delta_3^4, \delta_1^4] = \delta_3^5$; $[\delta_3^4, \delta_1^3] = 2\delta_1^4$; $[\delta_1^4, \delta_1^3] = \delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-3\beta_2} \langle x_1\omega^{2,5} \rangle + e^{\beta_1-2\beta_2-\beta_3} \langle x_1\omega^{2,6} \rangle + e^{-\beta_1-3\beta_2+\beta_3} \langle x_3\omega^{2,5} \rangle + 2e^{-2\beta_2} \langle x_3\omega^{2,6}, x_1\omega^{2,4} \rangle + e^{\beta_1-\beta_2-\beta_3} \langle x_1\omega^{2,3} \rangle + 3e^{-\beta_3} \langle -2x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} + x_5\omega^{3,5}, x_2\omega^{2,3} - x_4\omega^{1,6} + x_5\omega^{4,6} \rangle + e^{2\beta_2-\beta_3} \langle x_5\omega^{1,3} \rangle + 3e^{-\beta_1} \langle x_2\omega^{1,2} - x_3\omega^{1,3} + x_5\omega^{1,5}, x_3\omega^{1,3} + x_6\omega^{1,6}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4} \rangle + e^{\beta_2-\beta_3} \langle x_4\omega^{1,3} + x_5\omega^{1,6} \rangle + 2e^{-\beta_2} \langle x_1\omega^{1,2}, x_3\omega^{2,3} \rangle + e^{\beta_1+\beta_2-2\beta_3} \langle x_5\omega^{3,6} \rangle + e^{-\beta_2-\beta_3} \langle -x_1(\omega^{1,6} - \omega^{3,4}) + x_2\omega^{2,6} - x_3\omega^{3,6} - x_4\omega^{3,5} + x_5\omega^{5,6} \rangle + e^{-\beta_1+\beta_2} \langle x_5\omega^{1,4} \rangle + e^{-2\beta_1+\beta_3} \langle x_6\omega^{1,4} \rangle + e^{-2\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,4} + x_6\omega^{1,5} \rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle x_3\omega^{2,4} \rangle +$

$2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4}-x_4\omega^{1,5}+x_5\omega^{4,5}, 2x_1\omega^{1,4}-2x_2\omega^{2,4}-x_3(\omega^{1,6}-\omega^{3,4})+x_6\omega^{4,6}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2-2\beta_3}+3e^{-\beta_1-\beta_2-\beta_3}+e^{-\beta_1-2\beta_2-\beta_3}+e^{-\beta_1-2\beta_2}+3e^{-\beta_1-3\beta_2}+2e^{-\beta_1-4\beta_2}+2e^{-\beta_1-\beta_3}+3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2+\beta_3}+e^{-2\beta_1-4\beta_2+\beta_3}+e^{-2\beta_1-\beta_3}+e^{-3\beta_1-2\beta_2+\beta_3}+e^{\beta_2-2\beta_3}+2e^{-\beta_2-2\beta_3}+3e^{-2\beta_2-\beta_3}+2e^{-3\beta_2-\beta_3}+e^{-4\beta_2-\beta_3}$.

ch$(H^4_{\mathcal{G}})$=$e^{-\beta_1-\beta_2-2\beta_3}+e^{-\beta_1-2\beta_2-2\beta_3}+2e^{-\beta_1-3\beta_2-\beta_3}+3e^{-\beta_1-4\beta_2-\beta_3}+e^{-\beta_1-5\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-2\beta_3}+2e^{-2\beta_1-2\beta_2-\beta_3}+e^{-2\beta_1-3\beta_2-\beta_3}+3e^{-2\beta_1-4\beta_2}+e^{-2\beta_1-5\beta_2}+e^{-3\beta_1-2\beta_2-\beta_3}+2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2+\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-3\beta_2-2\beta_3}+e^{-4\beta_2-2\beta_3}$.

ch$(H^5_{\mathcal{G}})$=$e^{-\beta_1-4\beta_2-2\beta_3}+e^{-\beta_1-5\beta_2-2\beta_3}+e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-4\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2-2\beta_3}+2e^{-2\beta_1-5\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-\beta_3}+e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-2\beta_1-5\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-2\beta_3}+e^{-3\beta_1-5\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(2,15,45,75,69,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 6$: $(2,11,24,30,24,12,3)$

S=$3\beta_1+5\beta_2+2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 6$: $(1,3,6,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,4}$; $d\omega^6=-\omega^{2,3}$; $d\omega^{1,5}=\omega^{1,2,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{5,6}=\omega^{2,3,5}-\omega^{2,4,6}$; $d\omega^{1,3,5}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}+\omega^{1,2,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,6}=-\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{1,3,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle+e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle+e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6}-\omega^{3,4}\rangle+e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle$.

ch$(H^3_{\mathbb{C}})$= $e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,4}\rangle+e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,5}\rangle+e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,3,5}+\omega^{1,4,6}\rangle+e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle+e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,4,5}\rangle+e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle+e^{-\beta_1-4\beta_2-\beta_3}\ \langle\omega^{2,5,6}\rangle$.

ch$(H^4_{\mathbb{C}})$= $e^{-3\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5}\rangle+e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,3,4,5}\rangle+e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,3,4,6}\rangle+e^{-2\beta_1-4\beta_2-\beta_3}\ \langle\omega^{1,2,5,6}\rangle+e^{-\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6}\rangle+e^{-2\beta_1-5\beta_2-\beta_3}\ \langle\omega^{2,4,5,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-3\beta_1-5\beta_2-\beta_3}$ $\langle\omega^{1,2,4,5,6}\rangle$ + $e^{-3\beta_1-4\beta_2-2\beta_3}$ $\langle\omega^{1,3,4,5,6}\rangle$ + $e^{-2\beta_1-5\beta_2-2\beta_3}$ $\langle\omega^{2,3,4,5,6}\rangle$.

ch($H^6_{\mathbb{C}}$)= $e^{-3\beta_1-5\beta_2-2\beta_3}$ $\langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,9,14,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 6$: $(1,3,6,8,6,3,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^5$; $d^\star\omega^{1,2,3} = \omega^{1,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,5}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{2,3,4} = -\omega^{3,5}+\omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,5}-\omega^{1,4,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{1,5,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,3,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{3,4} = \omega^{1,6}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}-\omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,5}+2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}-\omega^{2,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}-\omega^{1,4,6}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,5}+\omega^{1,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6}-\omega^{3,4,5}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6}+2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}-\omega^{2,4,6}$; $\Delta\omega^{2,4,6} = -\omega^{2,3,5}+\omega^{2,4,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6}+\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}-\omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,5}+\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-\beta_2}$ $\langle\omega^2\rangle + e^{-\beta_3}$ $\langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-\beta_1-\beta_3}$ $\langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}$ $\langle\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2}$ $\langle\omega^{2,5}\rangle + e^{-2\beta_2-\beta_3}$ $\langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_3}$ $\langle\omega^{1,6}-\omega^{3,4}\rangle + e^{-\beta_2-2\beta_3}$ $\langle\omega^{3,6}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-2\beta_1-\beta_2-\beta_3}$ $\langle\omega^{1,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2}$ $\langle\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,3,5}+\omega^{1,4,6}\rangle + e^{-2\beta_2-2\beta_3}$ $\langle\omega^{2,3,6}\rangle + e^{-2\beta_1-4\beta_2}$ $\langle\omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,3,5}+\omega^{2,4,6}\rangle + e^{-\beta_1-4\beta_2-\beta_3}$ $\langle\omega^{2,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=$ $e^{-2\beta_1-5\beta_2-\beta_3}$ $\langle -\omega^{2,4,5,6}\rangle$ + $e^{-\beta_1-4\beta_2-2\beta_3}$ $\langle \omega^{2,3,5,6}\rangle$ + $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle \omega^{1,3,4,6}\rangle$ + $e^{-3\beta_1-3\beta_2-\beta_3}$ $\langle -\omega^{1,3,4,5}\rangle$ + $e^{-2\beta_1-4\beta_2-\beta_3}$ $\langle (-\omega^{1,2,5,6} + \omega^{2,3,4,5})/2\rangle$ + $e^{-3\beta_1-4\beta_2}$ $\langle \omega^{1,2,4,5}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=$ $e^{-2\beta_1-5\beta_2-2\beta_3}$ $\langle \omega^{2,3,4,5,6}\rangle$ + $e^{-3\beta_1-4\beta_2-2\beta_3}$ $\langle -\omega^{1,3,4,5,6}\rangle$ + $e^{-3\beta_1-5\beta_2-\beta_3}$ $\langle \omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=$ $e^{-3\beta_1-5\beta_2-2\beta_3}$ $\langle \omega^{1,2,3,4,5,6}\rangle$.

$P(Y) \;=\; (1+Y+3Y^2+Y^3+Y^4)(1+Y)^2.$

4.3.6 Algebra $\mathcal{G}_{6,9}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_2, x_5] = x_6$; $[x_3, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & 0 & \xi_1^1 + \xi_3^3 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^5 & -\xi_1^4 & \xi_1^1 + \xi_2^2 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_3^6, ad\ x_5 = -\delta_2^6, ad\ x_6 = 0.$

Maximal torus : T=$\langle t_1, t_2, t_3 \rangle$.

ch($\mathcal{G}$)= e^{β_1} $\langle x_1 \rangle$ + e^{β_2} $\langle x_2 \rangle$ + e^{β_3} $\langle x_3 \rangle$ + $e^{\beta_1+\beta_2}$ $\langle x_4 \rangle$ + $e^{\beta_1+\beta_3}$ $\langle x_5 \rangle$ + $e^{\beta_1+\beta_2+\beta_3}$ $\langle x_6 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2+\beta_3}$ $\langle x_6 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{\beta_1+\beta_2-\beta_3}$ $\langle \delta_3^4 \rangle$+$e^{\beta_1-\beta_2+\beta_3}$ $\langle \delta_2^5 \rangle$+$e^{\beta_1}$ $\langle \delta_2^4 \rangle$+$e^{\beta_2+\beta_3}$ $\langle \delta_1^6 \rangle$+$e^{\beta_1+\beta_3}$ $\langle \delta_2^6 \rangle$+ $e^{\beta_1+\beta_2}$ $\langle \delta_3^6 \rangle$ + e^{β_3} $\langle \delta_1^5 \rangle$ + e^{β_2} $\langle \delta_1^4 \rangle$ + $3e^0$ $\langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^5] = \delta_3^6$; $[\delta_2^5, \delta_1^4] = \delta_2^6$; $[\delta_2^4, \delta_1^5] = \delta_2^6$; $[\delta_2^4, \delta_1^4] = \delta_3^6$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1+\beta_2-\beta_3}$ $\langle \delta_3^4 \rangle$ + $e^{\beta_1-\beta_2+\beta_3}$ $\langle \delta_2^5 \rangle$ + $e^{\beta_2+\beta_3}$ $\langle \delta_1^6 \rangle$ + $3e^0$ $\langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{\beta_1-\beta_2-\beta_3}$ $\langle x_1\omega^{2,3} \rangle$ + $e^{-2\beta_1+\beta_2-\beta_3}$ $\langle x_2\omega^{1,5} \rangle$ + $e^{-2\beta_1-\beta_2+\beta_3}$ $\langle x_3\omega^{1,4} \rangle$ + $e^{-2\beta_1}$ $\langle x_2\omega^{1,4} + x_3\omega^{1,5} \rangle$ + $e^{\beta_2-2\beta_3}$ $\langle x_4\omega^{3,5} \rangle$ + $e^{-2\beta_3}$ $\langle -x_1\omega^{3,5} + x_4\omega^{3,6} \rangle$ + $e^{-\beta_1-\beta_3}$ $\langle 2x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) - 2x_3\omega^{3,5} + x_4\omega^{4,5} \rangle$ + $e^{-2\beta_2+\beta_3}$ $\langle x_5\omega^{2,4} \rangle$ + $e^{-2\beta_2}$ $\langle -x_1\omega^{2,4} + x_5\omega^{2,6} \rangle$ + $e^{-\beta_1-\beta_2}$ $\langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5} \rangle$ + $2e^{-\beta_1}$ $\langle x_2\omega^{1,2} + x_3\omega^{1,3} + x_6\omega^{1,6}, -x_3\omega^{1,3} + x_4\omega^{1,4} \rangle$ + $2e^{-\beta_2}$ $\langle -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_5\omega^{2,5} \rangle$ + $2e^{-\beta_3}$ $\langle -x_1\omega^{1,3} + x_4\omega^{3,4}, 2x_1\omega^{1,3} - x_2\omega^{2,3} + x_4\omega^{2,5} \rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-\beta_2-2\beta_3}$ + $e^{-\beta_1-2\beta_2-\beta_3}$ + $2e^{-\beta_1-2\beta_2}$ + $e^{-\beta_1-3\beta_2}$ + $2e^{-\beta_1-2\beta_3}$ + $e^{-\beta_1-3\beta_3}$ + $e^{-2\beta_1+\beta_2-2\beta_3}$ + $e^{-2\beta_1-\beta_2-\beta_3}$ + $2e^{-2\beta_1-\beta_2}$ + $e^{-2\beta_1-2\beta_2+\beta_3}$ + $2e^{-2\beta_1-\beta_3}$ + $e^{-3\beta_1-\beta_2}$ + $e^{-3\beta_1-\beta_3}$ + $e^{-\beta_2-\beta_3}$ + $e^{-\beta_2-2\beta_3}$ + $e^{-2\beta_2-\beta_3}$ + $e^{-2\beta_2-2\beta_3}$.

ch($H^4_{\mathcal{G}}$)=$2e^{-\beta_1-2\beta_2-2\beta_3}$ + $e^{-\beta_1-2\beta_2-3\beta_3}$ + $e^{-\beta_1-3\beta_2-2\beta_3}$ + $2e^{-2\beta_1-\beta_2-3\beta_3}$ + $e^{-2\beta_1-2\beta_2-2\beta_3}$ + $e^{-2\beta_1-2\beta_2}$ + $2e^{-2\beta_1-3\beta_2-\beta_3}$ + $e^{-2\beta_1-2\beta_3}$ + $e^{-3\beta_1-\beta_2-\beta_3}$ +

$e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-3\beta_3}+e^{-4\beta_1-\beta_2-2\beta_3}+e^{-4\beta_1-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-2\beta_2-3\beta_3}+e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-2\beta_1-3\beta_2-3\beta_3}+e^{-3\beta_1-\beta_2-3\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-4\beta_1-\beta_2-3\beta_3}+2e^{-4\beta_1-2\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-3\beta_2-3\beta_3}+e^{-4\beta_1-2\beta_2-3\beta_3}+e^{-4\beta_1-3\beta_2-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,11,41,70,68,33,6)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 6$: $(1,6,16,21,18,11,3)$

S$=4\beta_1+3\beta_2+3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,5,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{3,4,5}$; $d\omega^{1,2,6}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,5}$; $d\omega^{1,4,6}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,3,4,5}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}-\omega^{2,3,4,5}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{1,2,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,4,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\langle\omega^1\rangle+e^{-\beta_2}\langle\omega^2\rangle+e^{-\beta_3}\langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-2\beta_1-\beta_2}\langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_3}\langle\omega^{1,5}\rangle+e^{-\beta_2-\beta_3}\langle\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_3}\langle\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-2\beta_1-2\beta_2}\langle\omega^{1,2,4}\rangle+e^{-2\beta_1-2\beta_3}\langle\omega^{1,3,5}\rangle+e^{-3\beta_1-\beta_2-\beta_3}\langle\omega^{1,4,5}\rangle+e^{-\beta_1-2\beta_2-2\beta_3}\langle\omega^{2,3,6}\rangle+e^{-2\beta_1-3\beta_2-\beta_3}\langle\omega^{2,4,6}\rangle+e^{-2\beta_1-\beta_2-3\beta_3}\langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-3\beta_1-3\beta_2-\beta_3}\langle\omega^{1,2,4,6}\rangle+e^{-3\beta_1-\beta_2-3\beta_3}\langle\omega^{1,3,5,6}\rangle+e^{-4\beta_1-2\beta_2-2\beta_3}\langle\omega^{1,4,5,6}\rangle+e^{-2\beta_1-3\beta_2-2\beta_3}\langle\omega^{2,3,4,6}\rangle+e^{-2\beta_1-2\beta_2-3\beta_3}\langle\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-4\beta_1-3\beta_2-2\beta_3}\langle\omega^{1,2,4,5,6}\rangle+e^{-4\beta_1-2\beta_2-3\beta_3}\langle\omega^{1,3,4,5,6}\rangle+e^{-3\beta_1-3\beta_2-3\beta_3}\langle\omega^{2,3,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=e^{-4\beta_1-3\beta_2-3\beta_3}\langle\omega^{1,2,3,4,5,6}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^2$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 6$: $(1,3,8,13,12,6,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 6$: $(1,3,5,6,5,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{3,4} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,4} = \omega^{1,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,4} = \omega^{2,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,6}$; $d^\star\omega^{2,4,5} = -\omega^{4,6}$; $d^\star\omega^{3,4,5} = \omega^{5,6}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle-\omega^{1,3,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,4,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-3\beta_1-3\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle-\omega^{1,3,4,5,6}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6}\rangle.$

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^2.$

4.4 Appendix: correspondance with Morozov types

The Morozov classification of 6-dimensional indecomposable nilpotent Lie algebras over a 0-characteristic field K consists of 22 types (see [7]). For the sake of completeness, we here list the types.

The Morozov Types

m=5

1) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_5] = e_6$.
2) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_1, e_5] = e_6$.

m=4

3-dimensional center

3) $[e_1, e_2] = e_6$, $[e_1, e_3] = e_4$, $[e_2, e_3] = e_5$.

2-dimensional center

4) $[e_1, e_2] = e_5$, $[e_1, e_3] = e_6$, $[e_2, e_4] = e_6$.
5) $[e_1, e_3] = e_5$, $[e_1, e_4] = e_6$, $[e_2, e_4] = e_5$, $[e_2, e_3] = \gamma e_6$ ($\gamma \in K$, $\gamma \neq 0, \gamma$ not a square).
6) $[e_1, e_2] = e_6$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_2, e_3] = e_5$.
7) $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_2, e_3] = e_6$.
8) $[e_1, e_2] = e_3 + e_5$, $[e_1, e_3] = e_4$, $[e_2, e_5] = e_6$.
9) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_6$.
10) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_5$, $[e_1, e_4] = e_6$, $[e_2, e_4] = e_5$, $[e_2, e_3] = \gamma e_6$ ($\gamma \in K, \gamma \neq 0, \gamma$ not a square).
11) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_2, e_3] = e_6$.

1-dimensional center

12) $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_5] = e_6$.
13) $[e_1, e_2] = e_5$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_5] = e_6$.
14) $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_3] = e_5$, $[e_2, e_5] = \gamma e_6$ ($\gamma \neq 0$).
15) $[e_1, e_2] = e_3 + e_5$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_5] = e_6$.
16) $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_5$, $[e_2, e_4] = e_6$.
17) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_5] = e_6$.
18) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_6$, $[e_2, e_3] = e_5$, $[e_2, e_5] = \gamma e_6$ ($\gamma \neq 0$).

19) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_6$.

20) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_4$, $[e_1, e_4] = e_5$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_5$, $[e_2, e_4] = e_6$.

m=3

21) $[e_1, e_2] = e_3$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_4$, $[e_2, e_4] = e_5$, $[e_3, e_4] = e_6$.

22) $[e_1, e_2] = e_3$, $[e_1, e_3] = e_5$, $[e_1, e_5] = e_6$, $[e_2, e_3] = e_4$, $[e_2, e_4] = e_5$, $[e_3, e_4] = e_6$.

Here m denotes the largest dimension of an abelian ideal, and $\gamma \in K$; algebras corresponding to values of γ which differ by a square factor in K are isomorphic. Over $I\!R$, there are only one type 5 and one type 10 $(\gamma = -1)$, and type 14 (resp: 18) is splitted into two nonisomorphic 14_{-1} and 14_1 (resp: 18_{-1} and 18_1). Over $\mathbb{C}$, type 5 and 10 do not occur , and types 14_{-1} and 14_1 (resp: 18_{-1} and 18_1) are isomorphic.

The following table gives the correspondance between Morozov types over $\mathbb{C}$and the used types, and also the base change from Morozov $\{e_i, 1 \leq i \leq 6\}$ to the used $\{x_i, 1 \leq i \leq 6\}$.

correspondance table over $\mathbb{C}$

Morozov	Actual	Base change
1	$\mathcal{G}_{6,6}$	$x_1 = e_2, x_2 = e_1, x_3 = e_5, x_4 = -e_3, x_5 = -e_4, x_6 = e_6.$
2	$\mathcal{G}_{6,16}$	no change
3	$\mathcal{G}_{6,3}$	$x_1 = e_1, x_2 = e_2, x_3 = e_3, x_4 = e_6, x_5 = e_4, x_6 = e_5.$
4	$\mathcal{G}_{6,1}$	$x_1 = e_2, x_2 = e_1, x_3 = e_3, x_4 = e_4, x_5 = -e_5, x_6 = e_6.$
6	$\mathcal{G}_{6,7}$	$x_1 = e_1, x_2 = e_3, x_3 = e_2, x_4 = e_4, x_5 = e_6, x_6 = e_5.$
7	$\mathcal{G}_{6,4}$	$x_1 = e_3, x_2 = e_1, x_3 = e_2, x_4 = -e_4, x_5 = -e_5, x_6 = -e_6.$
8	$\mathcal{G}_{6,5}$	$x_1 = e_1, x_2 = e_2, x_3 = e_5, x_4 = e_3 + e_5, x_5 = e_4, x_6 = e_6.$
9	$\mathcal{G}_{6,8}$	$x_1 = e_2, x_2 = e_1, x_3 = -e_5, x_4 = -e_3, x_5 = -e_6, x_6 = -e_4.$
11	$\mathcal{G}_{6,14}$	no change
12	$\mathcal{G}_{6,2}$	$x_1 = e_1, x_2 = e_3, x_3 = e_2, x_4 = e_5, x_5 = e_4, x_6 = e_6.$
13	$\mathcal{G}_{6,10}$	$x_1 = e_1, x_2 = e_3, x_3 = e_2, x_4 = e_4, x_5 = e_5, x_6 = e_6.$
14_{-1}	$\mathcal{G}_{6,9}$	$x_1 = e_3, x_2 = e_1 + e_2, x_3 = e_1 - e_2, x_4 = -(e_4 + e_5),$ $x_5 = -(e_4 - e_5), x_6 = -2e_6.$
15	$\mathcal{G}_{6,12}$	$x_1 = e_1, x_2 = e_2, x_3 = e_5, x_4 = e_3 + e_5, x_5 = e_4, x_6 = e_6.$
16	$\mathcal{G}_{6,13}$	$x_1 = e_1, x_2 = e_3, x_3 = -e_2, x_i = e_i, 4 \leq i \leq 6.$
17	$\mathcal{G}_{6,11}$	$x_1 = e_1, x_2 = e_2, x_3 = e_5, x_4 = e_3, x_5 = e_4, x_6 = e_6.$
18_{-1}	$\mathcal{G}_{6,15}$	$x_1 = e_1 + e_2, x_2 = e_1 - e_2, x_3 = -2e_3, x_4 = -2(e_4 + e_5),$ $x_5 = -2(e_4 - e_5), x_6 = -4e_6.$
19	$\mathcal{G}_{6,17}$	no change
20	$\mathcal{G}_{6,19}$	no change
21	$\mathcal{G}_{6,18}$	$x_1 = e_2, x_2 = e_1, x_i = -e_i, 3 \leq i \leq 6.$
22	$\mathcal{G}_{6,20}$	$x_1 = e_2, x_2 = e_1, x_i = -e_i, 3 \leq i \leq 6.$

The basis in the Morozov classification diagonalizes the action of the maximal torus except for types 14 and 18.

Chapter 5

7-dimensional Lie Algebras

5.1 rank 0 Lie Algebras

5.1.1 Algebra $\mathcal{G}_{7,0.1}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$; $[x_2, x_5] = x_7$; $[x_3, x_4] = -x_7$.

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_1^2/2 & \xi_2^3 & 0 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_1^2/2 & \xi_2^3 & 0 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^5 & (3\xi_1^2)/2 & \xi_2^3 & 0 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^4-\xi_1^5+\xi_2^6 & -\xi_1^3+\xi_1^4+\xi_2^5 & (-2\xi_1^3+5\xi_1^2)/2 & \xi_2^3+\xi_1^2 & 0 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^6), ad\ x_4 = -(\delta_1^5 + \delta_2^7), ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle\rangle$.

ch($\mathcal{G}$)= $7e^0\ \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^0\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0\ \langle \delta_1^6, \delta_1^7, \delta_2^7, \delta_1^5, \delta_2^6, \delta_1^4, \delta_2^5, \delta_1^3, \delta_1^2, \delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_1^5, \delta_1^2] = -5/2\delta_1^7$; $[\delta_1^5, \delta_2^3] = -(\delta_2^7 + \delta_1^6)$; $[\delta_2^6, \delta_1^3] = \delta_1^7$; $[\delta_2^6, \delta_1^2] = -\delta_2^7 + \delta_1^6$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_1^2] = 1/2\delta_2^7 - 3/2\delta_1^6$; $[\delta_1^4, \delta_2^3] = -(\delta_2^7 + \delta_1^5)$; $[\delta_2^5, \delta_1^3] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^2] = -2\delta_2^7 + \delta_1^5$; $[\delta_1^3, \delta_1^2] = -1/2(\delta_2^7 + \delta_1^5)$; $[\delta_1^3, \delta_2^3] = -(\delta_2^6 + \delta_1^4)$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $4e^0\ \langle \delta_1^5, \delta_1^4, \delta_2^5, \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^2] = 3\delta_1^5$; $[\delta_1^2, \delta_1^4] = 2\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $9e^0\ \langle 1/3x_1(\omega^{1,2} + 2\omega^{1,3}) - 1/3x_2(\omega^{1,5} - 6\omega^{2,3}) - 1/3x_3(\omega^{1,5} - 8\omega^{2,4}) + 2/3x_4(\omega^{1,6} + 3/2\omega^{2,3} + 1/2\omega^{2,4} + 3/2\omega^{2,5} + 5/2\omega^{3,4}) - x_5(\omega^{1,6} - \omega^{3,5}), -x_3(\omega^{1,2} - \omega^{1,3}) - x_4\omega^{1,4} + x_5\omega^{2,3}, -x_2(\omega^{1,2} - \omega^{1,3}) - x_3(\omega^{1,2} - \omega^{1,3}) + x_4\omega^{1,5}, x_3\omega^{1,4} + x_4\omega^{2,3} + x_5\omega^{2,4}, -2/3x_1(\omega^{1,2} - 5/2\omega^{1,3}) + 5/3x_2\omega^{1,5} + 5/3x_3(\omega^{1,5} - 6/5\omega^{2,3} + \omega^{2,4}) + 5/3x_4(\omega^{1,6} + 3/5\omega^{2,3} + 1/5\omega^{2,4} + \omega^{3,4}) - x_5(\omega^{1,6} - \omega^{2,5}), 2x_1\omega^{1,3} + x_2(\omega^{1,4} + 2\omega^{1,5}) + x_3(\omega^{1,4} + \omega^{1,5} - 2\omega^{2,3} + 2\omega^{2,4}) + 2x_4(\omega^{1,6} + \omega^{2,3} + \omega^{3,4}) - 2x_5\omega^{1,6} + x_6\omega^{1,7}, -x_3(\omega^{1,2} + \omega^{1,4}) - x_4\omega^{1,4} + x_6\omega^{3,4}, -2x_1\omega^{1,3} -$

$x_2\omega^{1,5} + 2x_3(\omega^{2,3} - \omega^{2,4}) - 2x_4(\omega^{1,6} + \omega^{2,3} + \omega^{3,4}) + 2x_5\omega^{1,6} + x_6\omega^{4,5}, x_1\omega^{1,4} - x_3(\omega^{1,6} + \omega^{2,4} - \omega^{2,5}) - x_4(\omega^{2,5} - \omega^{3,5}) - x_5(\omega^{2,6} - \omega^{4,5})\rangle$.

ch($H^3_{\mathcal{G}}$)=$15e^0$.

ch($H^4_{\mathcal{G}}$)=$16e^0$.

ch($H^5_{\mathcal{G}}$)=$11e^0$.

ch($H^6_{\mathcal{G}}$)=$6e^0$.

ch($H^7_{\mathcal{G}}$)=$2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 10, 48, 114, 147, 109, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 4, 9, 15, 16, 11, 6, 2)$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,4} - \omega^{2,5} + \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4} + \omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} - \omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,4} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,4,5} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,4,6} + \omega^{2,5,6} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,4} + \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6} - \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} =$

$-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathfrak{g}}$)= $2e^0$ $\langle\omega^1, \omega^2\rangle$.

ch($H^2_{\mathfrak{g}}$)= $3e^0$ $\langle\omega^{1,5}, \omega^{1,6} + \omega^{2,4}, \omega^{1,7} + 2\omega^{2,6} + \omega^{3,4} - \omega^{3,5}\rangle$.

ch($H^3_{\mathfrak{g}}$)= $4e^0$ $\langle\omega^{1,2,7} + \omega^{2,3,5}, \omega^{2,3,6} - \omega^{2,4,5}, \omega^{1,4,7} + \omega^{3,4,5}, \omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,4,5} + \omega^{3,4,6}\rangle$.

ch($H^4_{\mathfrak{g}}$)= $4e^0$ $\langle\omega^{1,3,6,7} - \omega^{2,3,5,6} - \omega^{2,4,5,6}, \omega^{1,2,6,7} - \omega^{2,3,5,6}, \omega^{1,4,5,7}, \omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}\rangle$.

ch($H^5_{\mathfrak{g}}$)= $3e^0$ $\langle\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}, \omega^{1,2,4,6,7}, \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle$.

ch($H^6_{\mathfrak{g}}$)= $2e^0$ $\langle\omega^{1,3,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle$.

ch($H^7_{\mathfrak{g}}$)= e^0 $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{g}}$'s $0 \leq j \leq 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathfrak{g}}$'s $0 \leq j \leq 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7} - \omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} - \omega^{3,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} + \omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,7} - \omega^{2,4,6} - \omega^{3,4,5}$;

$d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} + \omega^{3,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7} - \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,3,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} + \omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 4\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{1,7} = 3\omega^{1,7} - \omega^{2,5} - \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,5} = \omega^{1,6} - \omega^{1,7} + \omega^{2,4} + 2\omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6} + \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} - \omega^{1,7} - \omega^{2,4} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{2,7} + 4\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{2,7} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6} - \omega^{5,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,7} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4} + \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,4} + 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} + \omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,4} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,2,7} + 3\omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,5} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,2,7} + 3\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{1,5,6} - \omega^{1,5,7} - \omega^{2,4,5} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,7} + 3\omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 4\omega^{1,6,7} + \omega^{2,5,6} + \omega^{2,5,7} + \omega^{3,4,6} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + \omega^{1,2,7} - \omega^{1,4,5} + 3\omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} - \omega^{1,3,7} + \omega^{2,3,4} + 3\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} - \omega^{1,4,7} + \omega^{2,3,6} + 3\omega^{2,4,5} - \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + 3\omega^{2,4,6} +$

$\omega^{2,5,6} + \omega^{3,4,5} - \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{1,6,7} - \omega^{2,3,7} + \omega^{2,4,6} + \omega^{2,4,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{3,5,6} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7} + \omega^{3,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{1,5,7} - \omega^{2,4,5} + \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{2,4,6} + \omega^{2,4,7} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 3\omega^{3,4,7} + \omega^{3,5,6} - \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,4,7} + 4\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{2,6,7} + 4\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 4\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7} + 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4} + \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 4\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 4\omega^{1,2,4,5} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,6} = 4\omega^{1,2,4,6} + \omega^{1,2,5,6} + \omega^{1,3,4,5} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{1,3,6,7} - \omega^{2,3,4,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,5} + \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,2,4,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,5,7} + \omega^{1,2,6,7} - \omega^{1,3,4,7} + 3\omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,6,7} + 3\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 3\omega^{1,4,6,7} - \omega^{1,5,6,7} + \omega^{2,4,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,4,6,7} + 3\omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,6} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} - \omega^{1,2,5,7} + \omega^{1,3,4,6} - \omega^{1,3,4,7} + 4\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,4,5,6} + 3\omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{1,3,6,7} + \omega^{2,3,4,6} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + 3\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{1,4,6,7} + \omega^{2,3,4,7} + 3\omega^{2,4,5,6} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,5,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7} - \omega^{3,5,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7} + 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,6} + \omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 4\omega^{1,2,4,5,6} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{1,3,5,6,7} + \omega^{2,3,4,5,6} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,5,7} - \omega^{1,2,4,5,6} + \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} - \omega^{1,3,5,6,7} + \omega^{2,3,4,5,6} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{1,3,4,6,7} + 3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} =$

$-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})=2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})=3e^0\ \langle\omega^{1,5}-\omega^{2,3},\omega^{1,6}+\omega^{2,4}-\omega^{2,5}+\omega^{3,4},2\omega^{1,7}+\omega^{2,5}+4\omega^{2,6}+\omega^{3,4}-2\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})=4e^0\ \langle 5\omega^{1,2,7}-15\omega^{1,3,7}-5\omega^{1,4,6}-8\omega^{1,5,7}-5\omega^{2,3,5}-35\omega^{2,3,6}+12\omega^{2,3,7}+20\omega^{2,4,5}-12\omega^{2,4,6}+8\omega^{2,5,6}+20\omega^{3,4,5}-8\omega^{3,4,6},5\omega^{1,3,6}-10\omega^{1,3,7}-5\omega^{1,4,5}-5\omega^{1,4,6}-6\omega^{1,5,7}-5\omega^{2,3,5}-25\omega^{2,3,6}+9\omega^{2,3,7}+15\omega^{2,4,5}-9\omega^{2,4,6}+6\omega^{2,5,6}+15\omega^{3,4,5}-6\omega^{3,4,6},5\omega^{1,4,7}+4\omega^{1,5,7}-6\omega^{2,3,7}+6\omega^{2,4,6}-4\omega^{2,5,6}-5\omega^{3,4,5}+4\omega^{3,4,6},5\omega^{1,5,6}+2\omega^{1,5,7}-5\omega^{2,3,6}-3\omega^{2,3,7}+3\omega^{2,4,6}-2\omega^{2,5,6}-5\omega^{3,4,5}+2\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})=4e^0\ \langle(15\omega^{1,2,6,7}-11\omega^{1,3,6,7}+34\omega^{1,4,5,7}+31\omega^{1,4,6,7}-19\omega^{2,3,4,7}+15\omega^{2,3,5,6}-31\omega^{2,3,5,7}+135\omega^{2,3,6,7}+42\omega^{2,4,5,6}-135\omega^{2,4,5,7}+104\omega^{3,4,5,6})/2550,(\omega^{1,2,6,7}-3\omega^{1,3,6,7}-7\omega^{1,4,6,7}+\omega^{2,3,4,7}+\omega^{2,3,5,6}+7\omega^{2,3,5,7}-25\omega^{2,3,6,7}-4\omega^{2,4,5,6}+25\omega^{2,4,5,7}-18\omega^{3,4,5,6})/340,(-8\omega^{1,2,5,7}+5\omega^{1,2,6,7}+8\omega^{1,3,4,7}+12\omega^{1,3,5,7}-11\omega^{1,3,6,7}-12\omega^{1,4,5,6}-3\omega^{1,4,6,7}-8\omega^{2,3,4,6}-19\omega^{2,3,4,7}+25\omega^{2,3,5,6}+3\omega^{2,3,5,7}-\omega^{2,3,6,7}+8\omega^{2,4,5,6}+\omega^{2,4,5,7}+2\omega^{3,4,5,6})/340,(-285\omega^{1,2,6,7}+107\omega^{1,3,6,7}+272\omega^{1,4,5,7}+23\omega^{1,4,6,7}+463\omega^{2,3,4,7}-285\omega^{2,3,5,6}-23\omega^{2,3,5,7}-15\omega^{2,3,6,7}-84\omega^{2,4,5,6}+15\omega^{2,4,5,7}-38\omega^{3,4,5,6})/5100\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})=3e^0\ \langle(-\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7})/2,(\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7})/4,(2\omega^{1,2,4,6,7}+\omega^{1,2,5,6,7}-4\omega^{1,3,4,5,7}+\omega^{1,3,4,6,7}-2\omega^{2,3,4,5,6})/26\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})=2e^0\ \langle\omega^{2,3,4,5,6,7},-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})=e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

5.1.2 Algebra $\mathcal{G}_{7,0.2}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5+x_7$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 0 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 0 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^3-\xi_1^5+\xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 0 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5+\delta_2^7), ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0\ \langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^0\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0\ \langle\delta_1^6,\delta_1^7,\delta_2^7,\delta_1^5,\delta_2^6,\delta_1^4,\delta_2^5,\delta_1^3,\delta_2^4,\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-(\delta_2^7+\delta_1^6)$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^4,\delta_2^3]=-(\delta_2^6+\delta_1^5)$; $[\delta_2^5,\delta_1^3]=\delta_2^7+\delta_1^6$; $[\delta_1^3,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^7+\delta_2^5+\delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $4e^0\ \langle\delta_1^5,\delta_1^4,\delta_2^5,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $10e^0\ \langle -x_2\omega^{1,2}+x_3(\omega^{1,3}+\omega^{1,5}+\omega^{2,4})+x_4(\omega^{1,4}+\omega^{2,5}), x_1\omega^{1,2}-x_3(\omega^{1,6}-3\omega^{2,3}+\omega^{2,5})-x_4(\omega^{1,5}-\omega^{2,4}+\omega^{2,6}), x_1\omega^{1,2}-2x_3(\omega^{1,6}-\omega^{2,3}+\omega^{2,5})-x_4(\omega^{1,5}+2\omega^{2,6})+x_5\omega^{1,6}, -x_3(\omega^{1,7}+\omega^{2,4}+\omega^{2,6})-x_4(\omega^{2,3}+\omega^{2,5}+\omega^{2,7})+x_5\omega^{1,7}, x_1\omega^{1,2}-x_3(\omega^{1,4}+\omega^{1,6}-2\omega^{2,3}+\omega^{2,5})-x_4(\omega^{1,5}+\omega^{2,6})+x_5\omega^{1,4}, -x_3\omega^{1,3}-x_4\omega^{2,3}+x_5\omega^{1,3}, -x_3(\omega^{1,5}+\omega^{2,4})-x_4(\omega^{2,3}+\omega^{2,5})+x_5\omega^{1,5}, -x_3\omega^{2,3}+x_4(\omega^{1,7}+\omega^{2,6})+x_5\omega^{2,7}, x_1\omega^{1,2}-x_3(\omega^{1,4}+\omega^{1,6}-2\omega^{2,3}+\omega^{2,5})-x_4(\omega^{1,5}+\omega^{2,6})+x_7\omega^{3,4}, -4/3x_1\omega^{1,3}-8/3x_2\omega^{2,3}-x_3(\omega^{1,7}+5\omega^{2,4}+\omega^{2,6})+1/3x_4(\omega^{1,6}-5\omega^{2,5}-3\omega^{2,7}-9\omega^{3,4})-5/3x_5\omega^{3,5}-1/3x_6\omega^{3,6}+x_7\omega^{3,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$15e^0$.

ch($H^4_{\mathcal{G}}$)=$15e^0$.

$\mathrm{ch}(H^5_{\mathcal{G}})=11e^0$.

$\mathrm{ch}(H^6_{\mathcal{G}})=6e^0$.

$\mathrm{ch}(H^7_{\mathcal{G}})=2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1,10,49,113,147,109,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1,4,10,15,15,11,6,2)$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3} - \omega^{2,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3} + \omega^{1,2,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,3,4} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5} - \omega^{2,3,7}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6} - \omega^{2,4,7} + \omega^{2,5,6}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4} + \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5} + \omega^{1,2,3,7}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5} - \omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} + \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,5,6} + \omega^{2,3,4,6,7}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6} -$

$\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^0\ \langle\omega^{1,4},\omega^{1,7}+\omega^{2,4}+\omega^{2,6},\omega^{1,6}+\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 4e^0\ \langle\omega^{1,2,7},\omega^{1,3,7}+\omega^{2,4,5},\omega^{1,4,7}-\omega^{2,3,5}+\omega^{3,4,5},\omega^{1,5,6}+\omega^{2,4,6}-2\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 4e^0\ \langle\omega^{1,2,6,7},\omega^{1,3,5,7},\omega^{1,3,6,7}+\omega^{2,3,5,7}-2\omega^{2,4,5,6},\omega^{1,4,6,7}+\omega^{2,3,5,6}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^0\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,5,7},\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^0\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5-\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}+\omega^{1,7}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}+\omega^{4,5}+\omega^{4,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}+\omega^{5,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}+\omega^{6,7}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6}-\omega^{1,4,5}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{1,5,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{1,6,7}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7}+\omega^{3,5,6}-\omega^{4,5,7}$;

$d^\star\omega^{2,3,4,6} = \omega^{4,5,6} - \omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7} - \omega^{5,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} + \omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} + \omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{3,4,6,7} + \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} - \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5 + \omega^7$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^5 + 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5} + \omega^{1,7}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,3} - \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,5} + 2\omega^{1,7} - \omega^{2,4} - \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{1,6} + 2\omega^{2,3} + \omega^{2,5}$; $\Delta\omega^{2,4} = -\omega^{1,7} + 2\omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,3} + 2\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 3\omega^{3,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5} + \omega^{4,7}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,6} + \omega^{4,5} + 4\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6} - \omega^{6,7}$; $\Delta\omega^{5,7} = -\omega^{3,7} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = -\omega^{5,6} + 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3} + \omega^{1,2,5}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,3} + 3\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 3\omega^{1,3,6} - \omega^{1,4,7}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,3,4} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} + \omega^{1,4,7} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} - \omega^{2,3,4} + \omega^{2,3,6}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + \omega^{1,4,5} + 3\omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6} - \omega^{1,6,7} - \omega^{2,3,5} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,3,7} + 3\omega^{1,5,7} + \omega^{2,4,5} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = -\omega^{1,5,6} + 3\omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} - \omega^{1,3,7} - \omega^{1,4,6} + 3\omega^{2,3,4} - \omega^{2,4,5}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{1,5,6} + 3\omega^{2,3,5} - \omega^{2,3,7}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,4,6} + 3\omega^{2,3,6} - \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + \omega^{1,6,7} - \omega^{2,3,5} + 2\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,5,7} - \omega^{2,3,4} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{1,6,7} + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{2,3,6} + 3\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,3,6} + 2\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + \omega^{2,5,7} + 3\omega^{3,4,7} + \omega^{4,5,7}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + 3\omega^{3,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$;

$\Delta\omega^{3,6,7} = 4\omega^{3,6,7} - \omega^{5,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + 2\omega^{4,5,6} - \omega^{4,6,7}$; $\Delta\omega^{4,5,7} = \omega^{3,4,7} - \omega^{3,5,6} + 4\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,6} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{3,6,7} + 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} - \omega^{1,2,3,7}$; $\Delta\omega^{1,2,3,6} = 4\omega^{1,2,3,6} - \omega^{1,2,4,7} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,3,7} = -\omega^{1,2,3,5} + 2\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,4} + 4\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + 3\omega^{1,2,4,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,6} + 3\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 3\omega^{1,3,5,6} - \omega^{1,4,5,7}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7} - \omega^{1,5,6,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 2\omega^{1,4,5,6} - \omega^{1,4,6,7} + \omega^{2,3,4,5} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,4,7} - \omega^{1,3,5,6} + 3\omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,4,5,6} + 3\omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,3,6,7} + 3\omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,5,7} + \omega^{1,4,5,6} + 3\omega^{2,3,4,5} - \omega^{2,3,4,7}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{1,3,6,7} + 3\omega^{2,3,4,6} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,4,6,7} - \omega^{2,3,4,5} + 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} + 3\omega^{2,3,5,6} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7} = \omega^{1,4,5,7} - \omega^{1,5,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,5,6} + 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{1,5,6,7} - \omega^{2,3,4,6} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,5,6} + 3\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 3\omega^{3,4,6,7} + \omega^{4,5,6,7}$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{3,4,6,7} + 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5} - \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,4,7} = -\omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 4\omega^{1,2,3,5,6} + \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,5,6} + 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,6} + \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,5,6} + 3\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} + \omega^{1,4,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,3,4,6,7} + 2\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} - \omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,6} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,4,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,5,6} + 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6} + \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,5,6} + 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}}) = 2e^0\ \langle\omega^1, \omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 3e^0\ \langle\omega^{1,5} - 2\omega^{1,7} - \omega^{2,4} - 2\omega^{2,6}, \omega^{1,4} - \omega^{2,3} + \omega^{2,5} - \omega^{3,4}, \omega^{1,6} -$

$\omega^{2,5} + 2\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 4e^0\ \langle 11\omega^{1,3,7} - 10\omega^{1,4,6} + 3\omega^{1,5,7} + 9\omega^{2,3,6} - \omega^{2,4,5} + 3\omega^{2,4,7} - 3\omega^{2,5,6}, \omega^{1,2,7} + \omega^{1,4,7} - \omega^{2,3,5} - 2\omega^{2,3,7} + 2\omega^{2,4,6} - \omega^{3,4,5}, \omega^{1,3,6} - \omega^{1,4,5} + 3\omega^{1,4,7} - \omega^{2,3,5} - 4\omega^{2,3,7} + 4\omega^{2,4,6} - \omega^{3,4,5}, \omega^{1,5,6} - \omega^{2,3,7} + 2\omega^{2,4,6} - 3\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 4e^0\ \langle (3\omega^{1,3,4,7} - 3\omega^{1,3,5,6} + \omega^{1,3,6,7} - 9\omega^{1,4,5,7} - 3\omega^{2,3,4,6} + 10\omega^{2,3,5,7} - 11\omega^{2,4,5,6})/330, (-\omega^{1,4,5,6} - 3\omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6} + 2\omega^{2,3,6,7} - 2\omega^{2,4,5,7} + 5\omega^{3,4,5,6})/9, (6\omega^{1,2,6,7} + 6\omega^{1,3,5,7} - 5\omega^{1,4,5,6} + 9\omega^{1,4,6,7} + \omega^{2,3,4,7} + 13\omega^{2,3,5,6} - 11\omega^{2,3,6,7} + 11\omega^{2,4,5,7} - 20\omega^{3,4,5,6})/90, (-24\omega^{1,2,6,7} + 6\omega^{1,3,5,7} + 5\omega^{1,4,5,6} + 9\omega^{1,4,6,7} + 11\omega^{2,3,4,7} - 7\omega^{2,3,5,6} - \omega^{2,3,6,7} + \omega^{2,4,5,7} - 10\omega^{3,4,5,6})/90\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 3e^0\ \langle (2\omega^{1,3,4,5,7} + \omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,6} - \omega^{2,3,4,6,7})/10, (\omega^{1,3,4,6,7} - 2\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7} + 2\omega^{2,3,5,6,7})/5, (5\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - 3\omega^{1,4,5,6,7} + 4\omega^{2,3,4,5,7} + 3\omega^{2,3,5,6,7})/15\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= 2e^0\ \langle \omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^0\ \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.1.3 Algebra $\mathcal{G}_{7,0.3}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6+x_7$; $[x_2,x_4]=x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^5 & \xi_1^2+\xi_2^4 & \xi_2^3 & 0 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^3-\xi_1^4+\xi_2^6 & \xi_1^2-\xi_1^3+\xi_2^5 & 2\xi_1^2+\xi_2^4 & \xi_2^3 & 0 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^6+\delta_2^7), ad\ x_4=-(\delta_1^5+\delta_2^7), ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0$ $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= e^0 $\langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $11e^0$ $\langle \delta_1^5,\delta_1^6,\delta_1^7,\delta_2^7,\delta_1^4,\delta_2^6,\delta_1^3,\delta_2^5,\delta_1^2,\delta_2^4,\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^2]=-2\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_1^6$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_1^2]=-(\delta_1^7+\delta_1^6)$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^5)$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^3,\delta_2^5]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_1^5)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^7+\delta_2^6+\delta_1^4)$; $[\delta_2^5,\delta_1^2]=-2\delta_2^7+\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_2^7+\delta_2^6-\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $5e^0$ $\langle \delta_1^4,\delta_1^5,\delta_2^5,\delta_1^2,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=3\delta_1^5$; $[\delta_1^2,\delta_2^4]=-2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $12e^0$ $\langle -x_1\omega^{1,2}+2x_3(\omega^{1,5}-\omega^{2,3})-2x_4(\omega^{1,5}-1/2\omega^{1,6})+x_5\omega^{2,5}, x_3(\omega^{1,6}+\omega^{2,3}+\omega^{2,4})-x_4(\omega^{1,6}-\omega^{2,5})+x_5\omega^{2,6}, x_3(\omega^{1,7}+\omega^{2,3}+\omega^{2,4}+\omega^{2,5})-x_4(\omega^{1,7}-\omega^{2,6})+x_5\omega^{2,7}, x_3\omega^{1,3}-x_4\omega^{1,3}+x_5\omega^{2,3}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, -x_2\omega^{1,2}+x_3(\omega^{1,3}+\omega^{1,4})+x_4\omega^{2,3}+x_5\omega^{2,4}, x_5\omega^{1,5}, x_1\omega^{1,2}-x_3(\omega^{1,5}-3\omega^{2,3})+x_4(\omega^{1,5}-\omega^{1,6}+\omega^{2,4}), x_1\omega^{1,2}-x_3(\omega^{1,5}-2\omega^{2,3})+x_4(\omega^{1,5}-\omega^{1,6}-\omega^{2,3})+x_5\omega^{1,7}, -x_1\omega^{1,2}-x_2(\omega^{1,3}+\omega^{1,4})+x_3(\omega^{1,5}-2\omega^{2,3})-x_4(\omega^{1,5}-\omega^{1,6})+x_5\omega^{3,4}, -x_2\omega^{1,3}-x_4\omega^{1,5}+x_7\omega^{3,5}, -2x_1\omega^{1,3}-6x_2\omega^{2,3}+x_3(\omega^{1,6}+2\omega^{2,3}-7\omega^{2,4})-x_4(\omega^{1,6}+\omega^{1,7}+3\omega^{2,5}+5\omega^{3,4})-3x_5\omega^{3,5}-x_6\omega^{3,6}+x_7\omega^{3,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$19e^0$.

$\mathrm{ch}(H^4_{\mathcal{G}})=20e^0$.

$\mathrm{ch}(H^5_{\mathcal{G}})=14e^0$.

$\mathrm{ch}(H^6_{\mathcal{G}})=7e^0$.

$\mathrm{ch}(H^7_{\mathcal{G}})=2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 11, 50, 116, 149, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 5, 12, 19, 20, 14, 7, 2)$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 9, 18, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 4, 6, 6, 4, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3} - \omega^{2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6} + \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} =$

$-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 4e^0\ \langle\omega^{1,5},\omega^{1,7}+\omega^{2,4}+\omega^{3,4},\omega^{2,5}-\omega^{3,4},\omega^{2,7}-\omega^{3,6}+\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 6e^0\ \langle\omega^{1,2,7},\omega^{1,3,6},\omega^{1,4,7}-\omega^{2,3,5},\omega^{2,3,6}-\omega^{2,4,5},\omega^{1,5,7}-\omega^{2,4,5}-\omega^{3,4,5},\omega^{1,6,7}-\omega^{2,4,6}+\omega^{2,4,7}-2\omega^{2,5,6}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 6e^0\ \langle\omega^{1,3,4,7},\omega^{1,2,6,7},\omega^{1,3,6,7}-\omega^{2,3,5,6},\omega^{1,4,5,7}+\omega^{2,3,4,6},\omega^{1,5,6,7}+\omega^{2,4,5,6}-\omega^{2,4,5,7}+2\omega^{3,4,5,6},\omega^{2,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 4e^0\ \langle\omega^{1,2,5,6,7},\omega^{1,2,3,6,7},\omega^{1,3,4,6,7},\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}+\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^0\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+3Y^2+3Y^3+3Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,18,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,6,6,4,2,1)$

Total dimension of trivial cohomology: 26.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6-\omega^7$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6}+\omega^{1,7}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}+\omega^{4,6}+\omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,6}+\omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7}-\omega^{1,4,6}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6}-\omega^{1,5,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}-\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7}-\omega^{4,5,6}-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}-\omega^{4,6,7}$;

$d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} + \omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7} + \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6 + \omega^7$; $\Delta\omega^7 = \omega^6 + 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{1,7} + \omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,6} + 2\omega^{1,7} - \omega^{2,4} - \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{1,6} + 2\omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = -\omega^{1,7} + \omega^{2,3} + 2\omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,7} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 3\omega^{3,7} - \omega^{4,7}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6} + \omega^{4,7}$; $\Delta\omega^{4,7} = -\omega^{3,7} + \omega^{4,6} + 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6} + \omega^{5,7}$; $\Delta\omega^{5,7} = \omega^{5,6} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,4,7} - \omega^{2,3,4} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 3\omega^{1,4,6} + \omega^{1,4,7} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,7} + \omega^{1,4,6} + 2\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{1,5,7} - \omega^{2,3,5} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,6} + 3\omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,4,5} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,4,6} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} - \omega^{1,3,7} - \omega^{1,4,5} - \omega^{1,4,6} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,7} - \omega^{1,5,6} + 3\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6} - \omega^{2,3,7} + \omega^{2,4,5} + \omega^{2,4,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{1,6,7} - \omega^{2,3,6} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,7} + \omega^{1,5,7} + \omega^{2,3,5} + \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,6,7} + \omega^{2,3,6} + 3\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,7} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 4\omega^{3,5,7} - \omega^{4,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6} + \omega^{4,5,7}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,7} + \omega^{3,6,7} + \omega^{4,5,6} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} =$

$3\omega^{1,2,3,6}-\omega^{1,2,3,7}+\omega^{1,2,4,5}+\omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,7}=-\omega^{1,2,3,6}+2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5}=\omega^{1,2,3,5}+\omega^{1,2,3,6}+3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=\omega^{1,2,3,6}+4\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7}=2\omega^{1,2,4,7}+\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,4,7}+2\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7}=2\omega^{1,2,5,7}+\omega^{1,3,4,7}+\omega^{1,3,5,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=\omega^{1,2,4,7}+\omega^{1,2,5,6}+2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7}=\omega^{1,2,5,7}+\omega^{1,3,4,7}+\omega^{1,3,5,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6}=\omega^{1,2,5,7}+\omega^{1,3,4,7}+2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7}=\omega^{1,2,6,7}+3\omega^{1,3,5,7}-\omega^{1,4,5,7}+\omega^{2,3,4,5}$; $\Delta\omega^{1,3,6,7}=2\omega^{1,3,6,7}+\omega^{1,4,5,7}-\omega^{1,4,6,7}+\omega^{2,3,4,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,6}=2\omega^{1,4,5,6}+\omega^{1,4,5,7}+\omega^{2,3,4,5}-\omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,5,7}+\omega^{1,3,6,7}+\omega^{1,4,5,6}+2\omega^{1,4,5,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7}=-\omega^{1,3,6,7}+3\omega^{1,4,6,7}-\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7}=3\omega^{1,5,6,7}-\omega^{2,3,5,7}+\omega^{2,3,6,7}-\omega^{2,4,5,6}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{1,3,5,7}+\omega^{1,4,5,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,2,6,7}+\omega^{1,3,6,7}-\omega^{1,4,5,6}+3\omega^{2,3,4,6}-\omega^{2,3,4,7}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,4,5,7}-\omega^{1,4,6,7}-\omega^{2,3,4,6}+2\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,3,6,7}+\omega^{2,3,4,7}+2\omega^{2,3,5,6}-\omega^{2,3,5,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,5,6,7}-\omega^{2,3,5,6}+3\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6}=\omega^{1,4,6,7}-\omega^{1,5,6,7}+\omega^{2,3,5,6}+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{2,3,6,7}+2\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7}=2\omega^{2,4,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,5,6,7}+\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{2,4,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=3\omega^{3,5,6,7}-\omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7}=-\omega^{3,5,6,7}+3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}-\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,7}=-\omega^{1,2,3,4,6}+2\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}-\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,7}=-\omega^{1,2,3,5,6}+3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,5,6}+3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,3,6,7}+2\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7}=2\omega^{1,2,4,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,2,4,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7}=2\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,3,5,6,7}+2\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-\omega^{1,3,5,6,7}+2\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,4,5,6,7}-\omega^{2,3,4,5,6}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}-\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,5,7}=-\omega^{1,2,3,4,5,6}+2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 4e^0\ \langle\omega^{1,5}+\omega^{1,7}-\omega^{2,3}+\omega^{2,4}+\omega^{3,4},\omega^{1,6}-2\omega^{1,7}-\omega^{2,4}-2\omega^{3,4},\omega^{2,5}-\omega^{3,4},\omega^{2,7}-\omega^{3,6}+\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 6e^0\ \langle 2\omega^{1,2,7}-2\omega^{1,4,5}-2\omega^{1,4,6}+4\omega^{1,4,7}-2\omega^{2,3,4}-2\omega^{2,3,5}-6\omega^{2,3,6}-3\omega^{2,3,7}+6\omega^{2,4,5}+3\omega^{2,4,6}-3\omega^{3,4,5},2\omega^{1,3,6}-4\omega^{1,4,5}-2\omega^{1,4,6}+4\omega^{1,4,7}-2\omega^{2,3,4}-2\omega^{2,3,5}-6\omega^{2,3,6}-3\omega^{2,3,7}+6\omega^{2,4,5}+3\omega^{2,4,6}-3\omega^{3,4,5},\omega^{1,3,7}-\omega^{1,4,6}+3\omega^{1,4,7}-$

$\omega^{2,3,5} - 4\omega^{2,3,6} - 2\omega^{2,3,7} + 4\omega^{2,4,5} + 2\omega^{2,4,6} - 2\omega^{3,4,5}, \omega^{1,5,6} - 2\omega^{2,3,6} - \omega^{2,3,7} + \omega^{2,4,5} + \omega^{2,4,6} - \omega^{3,4,5}, \omega^{1,5,7} - \omega^{2,3,6} - \omega^{2,3,7} + \omega^{2,4,6} - 2\omega^{3,4,5}, 2\omega^{1,6,7} - \omega^{2,3,7} - \omega^{2,4,6} + 2\omega^{2,4,7} - 4\omega^{2,5,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 6e^0 \ \langle(-3\omega^{1,3,6,7} - 9\omega^{1,4,6,7} + 30\omega^{1,5,6,7} - 3\omega^{2,3,4,7} - 3\omega^{2,3,5,6} + 9\omega^{2,3,5,7} + 7\omega^{2,3,6,7} + 21\omega^{2,4,5,6} - 37\omega^{2,4,5,7} + 67\omega^{3,4,5,6})/222, (-\omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6})/6, (-12\omega^{1,3,6,7} + \omega^{1,4,6,7} - 28\omega^{1,5,6,7} - 12\omega^{2,3,4,7} + 25\omega^{2,3,5,6} - \omega^{2,3,5,7} + 28\omega^{2,3,6,7} - 27\omega^{2,4,5,6} - 28\omega^{3,4,5,6})/148, (481\omega^{1,2,6,7} - 74\omega^{1,3,4,7} + 37\omega^{1,3,5,6} + 259\omega^{1,3,5,7} - 88\omega^{1,3,6,7} - 222\omega^{1,4,5,6} + 1221\omega^{1,4,5,7} + 476\omega^{1,4,6,7} - 119\omega^{1,5,6,7} - 37\omega^{2,3,4,5} + 740\omega^{2,3,4,6} + 1873\omega^{2,3,4,7} - 1309\omega^{2,3,5,6} - 476\omega^{2,3,5,7} + 119\omega^{2,3,6,7} + 357\omega^{2,4,5,6} - 119\omega^{3,4,5,6})/4403, (-156\omega^{1,2,6,7} + 24\omega^{1,3,4,7} - 12\omega^{1,3,5,6} + 35\omega^{1,3,5,7} + 80\omega^{1,3,6,7} - 47\omega^{1,4,5,6} - 39\omega^{1,4,5,7} + 12\omega^{2,3,4,5} - 121\omega^{2,3,4,6} - 80\omega^{2,3,4,7})/476, (4\omega^{1,2,6,7} + 36\omega^{1,3,4,7} - 18\omega^{1,3,5,6} - 7\omega^{1,3,5,7} + \omega^{1,3,6,7} - 11\omega^{1,4,5,6} + \omega^{1,4,5,7} + 18\omega^{2,3,4,5} - 3\omega^{2,3,4,6} - \omega^{2,3,4,7})/238\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 4e^0 \ \langle(-\omega^{1,3,5,6,7} - 2\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7} - 2\omega^{2,3,4,6,7})/5, (-5\omega^{1,2,5,6,7} - 5\omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} - 8\omega^{1,4,5,6,7} + 10\omega^{2,3,4,5,6} + 9\omega^{2,3,4,5,7} - 8\omega^{2,3,4,6,7})/40, (-3\omega^{1,2,5,6,7} + 5\omega^{1,3,4,6,7} - \omega^{1,3,5,6,7} - 2\omega^{2,3,4,5,6} - \omega^{2,3,4,5,7})/8, (\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= 2e^0 \ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^0 \ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

5.1.4 Algebra $\mathcal{G}_{7,0.4(\lambda)}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=\lambda x_7+x_6$; $[x_1,x_5]=x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_7$; $[x_2,x_5]=x_6$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ \xi_1^3&\xi_2^3&0&0&0&0&0\\ \xi_1^4&\xi_2^4&\xi_2^3&0&0&0&0\\ \xi_1^5&\xi_2^5&-\xi_1^3&0&0&0&0\\ \xi_1^6&\xi_2^6&\xi_2^4-\xi_1^5&\xi_2^3&-\xi_1^3&0&0\\ \xi_1^7&\xi_2^7&-\xi_1^4+\xi_2^4\lambda+\xi_2^5+\xi_2^6&-\xi_1^3+\xi_2^3\lambda+\xi_2^4-2\xi_1^5&\xi_2^3-\xi_2^5&\xi_2^3&0 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\lambda\delta_1^7+\delta_1^6+\delta_2^7), ad\ x_5=-(\delta_2^6+\delta_1^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0$ $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= e^0 $\langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0$ $\langle \delta_1^4,\delta_2^4,\delta_1^5,\delta_2^5,\delta_1^6,\delta_2^6,\delta_1^3,\delta_1^7,\delta_2^7,\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_1^5]=2\delta_1^7$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^6+\lambda\delta_1^7)$; $[\delta_2^4,\delta_1^5]=2\delta_2^7$; $[\delta_2^4,\delta_1^3]=\delta_2^7+\delta_1^6+\lambda\delta_1^7$; $[\delta_1^5,\delta_2^5]=\delta_1^7$; $[\delta_1^5,\delta_2^3]=-(\delta_1^7+\delta_2^6)$; $[\delta_2^5,\delta_1^3]=\delta_1^7+\delta_2^6$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $4e^0$ $\langle \delta_2^4,\delta_1^5,\delta_1^4,\delta_1^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4,\delta_1^5]=-2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $9e^0$ $\langle x_1\omega^{1,2}-3x_3(\omega^{1,2}-1/3\omega^{2,5})+2x_4(\omega^{1,5}-1/2\omega^{1,6}+1/2\omega^{2,4}), -x_1\omega^{1,2}+x_2(\lambda\omega^{1,2}-\omega^{1,3})+x_3(\lambda\omega^{1,2}-\lambda\omega^{1,3}-\omega^{2,3}-2\omega^{2,5})+x_4(\lambda\omega^{2,5}+\omega^{1,6}-\omega^{3,5})+x_5\omega^{1,4}, 2x_2\omega^{1,2}+3x_3(\lambda\omega^{1,2}-1/3\omega^{1,3})-2x_4(\omega^{1,4}-3/2\omega^{2,5})+x_5\omega^{1,5}, x_1\omega^{1,2}+x_2\lambda\omega^{1,2}+x_3(\lambda\omega^{1,2}-\lambda\omega^{1,3}-4\omega^{1,2}-\omega^{2,3})-2x_4(\lambda\omega^{1,4}-1/2\lambda\omega^{2,5}-\omega^{1,5}+1/2\omega^{1,6}+1/2\omega^{3,5})+x_6\omega^{1,7}, x_4(\lambda\omega^{1,3}+\omega^{1,4}+\omega^{2,3})+x_6\omega^{2,4}, -x_1\omega^{1,2}-x_2(\lambda\omega^{1,2}-\omega^{1,3})-x_3(\lambda\omega^{1,2}-\lambda\omega^{1,3}-4\omega^{1,2}-\omega^{2,3})+2x_4(\lambda\omega^{1,4}-1/2\lambda\omega^{2,5}-\omega^{1,5}+1/2\omega^{1,6}+1/2\omega^{3,5})+x_6\omega^{3,4}, -7x_1(\lambda\omega^{1,2}+8/21\lambda\omega^{1,4}-10/21\lambda\omega^{1,5}-10/21\lambda\omega^{1,6}-4/7\lambda\omega^{2,3}-10/21\lambda\omega^{2,4}-2/21\lambda\omega^{2,5}-2/3\lambda\omega^{2,6}+2/3\lambda\omega^{3,4}-10/21\lambda\omega^{3,5}+3/7\omega^{1,3}-8/21\omega^{1,4}+2/3\omega^{1,5}+4/7\omega^{2,3}+2/3\omega^{2,4}+2/21\omega^{2,5}+2/3\omega^{2,6}-2/3\omega^{3,4})-5x_2(\lambda\omega^{1,2}-\lambda\omega^{1,3}+2/15\lambda\omega^{1,5}+14/15\lambda\omega^{1,6}+2/15\lambda\omega^{2,4}+14/15\lambda\omega^{3,5}+2/5\omega^{1,2}-2/15\omega^{1,5}-14/15\omega^{1,6}-6/5\omega^{2,3}-2/15\omega^{2,4}-7/15\omega^{2,5}-$

$14/15\omega^{3,5}) + 11x_3(\lambda\omega^{1,2} + 5/11\lambda\omega^{1,3} + 4/11\lambda\omega^{1,4} - 10/33\lambda\omega^{1,5} - 10/33\lambda\omega^{1,6} - 3/11\lambda\omega^{2,3} - 10/33\lambda\omega^{2,4} - 56/33\lambda\omega^{2,5} - 26/33\lambda\omega^{2,6} + 10/33\lambda\omega^{2,7} + 2/11\lambda\omega^{3,4} - 10/33\lambda\omega^{3,5} - 14/33\lambda\omega^{4,5} + 13/11\omega^{1,2} + 2/11\omega^{1,3} - 4/11\omega^{1,4} + 8/11\omega^{1,5} + 8/11\omega^{2,3} + \omega^{2,4} + 2/33\omega^{2,5} + 26/33\omega^{2,6} - 2/11\omega^{3,4} - 7/33\omega^{3,5} + 14/33\omega^{4,5}) + 12x_4(\lambda\omega^{1,4} - 1/3\lambda\omega^{1,5} + 5/4\lambda\omega^{1,6} + 5/18\lambda\omega^{1,7} - 5/12\lambda\omega^{2,5} - 7/18\lambda\omega^{2,7} + 1/12\lambda\omega^{3,5} - 1/3\lambda\omega^{3,6} + 5/18\lambda\omega^{3,7} + 7/18\lambda\omega^{5,6} - 5/12\omega^{1,4} - 2/3\omega^{1,5} + 2/3\omega^{1,6} + 35/36\omega^{1,7} + 5/12\omega^{2,5} + 4/3\omega^{2,6} + 7/18\omega^{2,7} + 1/6\omega^{3,4} + 1/3\omega^{3,5} + 1/3\omega^{3,6} + 7/18\omega^{4,5} - 7/18\omega^{5,6}) - 2/3x_5(\lambda\omega^{1,7} + 12\lambda\omega^{2,6} - 5\lambda\omega^{2,7} + 2\lambda\omega^{4,5} + 7\lambda\omega^{4,6} - 12\omega^{1,6} - \omega^{1,7} - 12\omega^{2,6} - 2\omega^{4,5} - 7\omega^{4,6}) - 4x_6(\lambda\omega^{3,7} - 5/6\lambda\omega^{4,7} - 7/4\omega^{2,7} - 1/4\omega^{3,6} - \omega^{3,7}) + 10/3x_7\lambda\omega^{6,7}, x_4\omega^{1,2}, x_1\omega^{1,3} + x_3(\lambda\omega^{2,5} - \omega^{1,3} - \omega^{1,5}) - x_4(\lambda\omega^{1,6} - \omega^{1,4} + \omega^{2,6} - \omega^{3,4}) - x_5(\omega^{1,6} - \omega^{2,4})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$15e^0$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$16e^0$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$12e^0$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$7e^0$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 10, 48, 114, 147, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 4, 9, 15, 16, 12, 7, 2)$

The preceding results are valid for:

$$\lambda \neq 0^1.$$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,5}$; $d\omega^7 = -\omega^{1,4}\lambda - \omega^{1,5} - \omega^{1,6} - \omega^{2,4} - \omega^{3,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,5}$; $d\omega^{1,7} = \omega^{1,2,4} + \omega^{1,3,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,4}\lambda - \omega^{1,2,5} - \omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4} - \omega^{2,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,4}\lambda - \omega^{1,3,5} - \omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{2,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,5} - \omega^{1,4,6} - \omega^{3,4,5}$; $d\omega^{5,6} =$

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[1] This value is excluded by the computer due to the order used in solving systems of equations. Computing again for this value shows that this value is actually nonsingular, i.e. doesn't gives a different cohomology than the generic one, and hence the preceding results are valid for **all** λ .

$\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=\omega^{1,4,5}\lambda-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=\omega^{1,4,6}\lambda-\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}-\omega^{2,5,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,6}=\omega^{1,2,4,5}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}+\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,4}\lambda-\omega^{1,2,3,5}-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,5}-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=\omega^{1,2,4,5}\lambda-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=\omega^{1,2,4,6}\lambda-\omega^{1,2,4,7}+\omega^{1,2,5,6}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,5}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,5}\lambda-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}+\omega^{1,3,4,6}\lambda-\omega^{1,3,4,7}+\omega^{1,3,5,6}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,5,6}\lambda+\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}+\omega^{1,2,3,5,7}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,5}-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=\omega^{1,2,3,4,5}\lambda-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=\omega^{1,2,3,4,6}\lambda-\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,5,6}\lambda+\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}-\omega^{2,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,5,6}\lambda+\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,5,6}\lambda+\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= 3e^0\ \langle\omega^{1,5}+\omega^{2,4},\omega^{1,4},\omega^{2,6}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 4e^0\ \langle\omega^{1,3,7}+\omega^{2,3,6},\omega^{1,2,6},\omega^{2,5,6}+\omega^{3,4,5},\omega^{1,4,7}+\omega^{2,5,7}+\omega^{3,4,5}\lambda-\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 4e^0\ \langle\omega^{1,2,6,7}+\omega^{2,4,5,6},\omega^{1,3,4,7},\omega^{1,3,5,7}+\omega^{2,3,5,6},\omega^{1,4,6,7}-2\omega^{2,5,6,7}+2\omega^{3,4,5,6}\lambda-\omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 3e^0\ \langle\omega^{1,2,5,6,7},\omega^{1,2,4,6,7}-\omega^{2,3,4,6,7},\omega^{1,3,4,6,7}-\omega^{2,3,4,6,7}\lambda-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= 2e^0\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

The preceding results are valid for:

all λ

Comments: $\mathcal{G}_{7,0.4(i_\lambda)}$ is isomorphic to $\mathcal{G}_{7,0.4(i_{\lambda'})}$ if and only if $\lambda = \pm\lambda'$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6 - \omega^7\bar{\lambda}$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}-\omega^{2,6}-\omega^{2,7}\bar{\lambda}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6}-\omega^{2,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} - \omega^{3,7}\bar{\lambda}$; $d^\star\omega^{1,3,5} = \omega^{1,7} - \omega^{3,7} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7} + \omega^{5,6} + \omega^{5,7}\bar{\lambda}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} + \omega^{6,7}\bar{\lambda}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7} + \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,5}$; $d^\star\omega^{2,3,5} = \omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,6} + \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{2,5,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,5} - \omega^{2,3,6} - \omega^{2,3,7}\bar{\lambda}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{1,3,6} - \omega^{2,3,7} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{2,5,7}\bar{\lambda} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7} + \omega^{2,6,7}\bar{\lambda} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = \omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} - \omega^{3,4,7} + \omega^{3,5,6} + \omega^{3,5,7}\bar{\lambda}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{3,6,7}\bar{\lambda}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7} + \omega^{3,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7} + \omega^{4,6,7} - \omega^{5,6,7}\bar{\lambda}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7} - \omega^{3,4,6} + \omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6} + \omega^{2,3,5,7}\bar{\lambda}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,6,7}\bar{\lambda}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{2,5,6,7}\bar{\lambda} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7} + \omega^{3,4,6,7} -$

$\omega^{3,5,6,7}\bar\lambda$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7} - \omega^{2,3,5,6,7}\bar\lambda$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6} + \omega^{7}\bar\lambda$; $\Delta\omega^{7} = \omega^{6}\lambda + \omega^{7}(\lambda\bar\lambda+4)$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}(\lambda\bar\lambda+1) + \omega^{1,5}\bar\lambda + \omega^{1,6}\bar\lambda + \omega^{2,4}\bar\lambda + \omega^{2,5} + \omega^{3,5}\bar\lambda$; $\Delta\omega^{1,5} = \omega^{1,4}\lambda + 2\omega^{1,5} + \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,4}\lambda + \omega^{1,5} + 2\omega^{1,6} + \omega^{2,4} - \omega^{2,7}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} - \omega^{2,7}\bar\lambda - \omega^{3,4} - \omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,4}\lambda + \omega^{1,6} + 2\omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,5} = \omega^{1,4} + \omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{2,7}\bar\lambda + \omega^{3,4}$; $\Delta\omega^{2,7} = -\omega^{1,6} - \omega^{1,7}\lambda + \omega^{2,6}\lambda + \omega^{2,7}(\lambda\bar\lambda + 3) + \omega^{3,4}\lambda + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,6} + \omega^{2,7}\bar\lambda + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4}\lambda + \omega^{1,5} + \omega^{2,4} + \omega^{2,7} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6} + \omega^{3,7}\bar\lambda$; $\Delta\omega^{3,7} = -\omega^{1,7} + \omega^{3,6}\lambda + \omega^{3,7}(\lambda\bar\lambda+4) + \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = 4\omega^{4,7} - \omega^{5,6} - \omega^{5,7}\bar\lambda - \omega^{6,7}\bar\lambda$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6} + \omega^{5,7}\bar\lambda$; $\Delta\omega^{5,7} = -\omega^{4,6} - \omega^{4,7}\lambda + \omega^{5,6}\lambda + \omega^{5,7}(\lambda\bar\lambda+3) - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{4,7}\lambda - \omega^{5,7} + \omega^{6,7}(\lambda\bar\lambda+5)$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}(\lambda\bar\lambda+3) + \omega^{1,2,5}\bar\lambda + \omega^{1,2,6}\bar\lambda + \omega^{1,3,5} - \omega^{2,3,5}\bar\lambda$; $\Delta\omega^{1,2,5} = \omega^{1,2,4}\lambda + 3\omega^{1,2,5} + \omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,4}\lambda + \omega^{1,2,5} + 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,4}\lambda + \omega^{1,3,5} + \omega^{2,3,4} + \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,2,7}\bar\lambda + \omega^{1,3,4}(\lambda\bar\lambda + 1) + \omega^{1,3,5}\bar\lambda + \omega^{1,3,6}\bar\lambda + \omega^{2,3,4}\bar\lambda + \omega^{2,3,5}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4} + \omega^{1,2,7} + \omega^{1,3,4}\lambda + 3\omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = \omega^{1,3,4}\lambda + \omega^{1,3,5} + 3\omega^{1,3,6} + \omega^{2,3,4} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6} - \omega^{2,3,7}\bar\lambda + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}(\lambda\bar\lambda + 3) + \omega^{1,4,6} - \omega^{1,5,6}\bar\lambda + \omega^{2,4,5}\bar\lambda + \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,5} + \omega^{1,4,6}(\lambda\bar\lambda+2) - \omega^{1,4,7}\bar\lambda + \omega^{1,5,6}\bar\lambda - \omega^{1,5,7} + \omega^{2,4,6}\bar\lambda - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,5,6}\bar\lambda$; $\Delta\omega^{1,4,7} = -\omega^{1,4,6}\lambda + 2\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6} + \omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,7}\bar\lambda$; $\Delta\omega^{1,5,6} = -\omega^{1,4,5}\lambda + \omega^{1,4,6}\lambda - \omega^{1,4,7} + 3\omega^{1,5,6} - \omega^{2,4,5} - \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + 2\omega^{1,5,7} - \omega^{2,5,6} - \omega^{2,5,7}\bar\lambda + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,4,7} - \omega^{2,6,7}\bar\lambda + \omega^{3,4,6} - \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} + \omega^{1,3,4}\lambda + \omega^{1,3,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4}\lambda - \omega^{1,2,5} + \omega^{1,3,4} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6} + \omega^{2,3,7}\bar\lambda$; $\Delta\omega^{2,3,7} = \omega^{1,2,7} - \omega^{1,3,6} - \omega^{1,3,7}\lambda + \omega^{2,3,6}\lambda + \omega^{2,3,7}(\lambda\bar\lambda + 3) + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,5}\lambda - \omega^{1,5,6} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,4,6}\lambda - \omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6} - \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{1,6,7} + 4\omega^{2,4,7} - \omega^{2,5,6} - \omega^{2,5,7}\bar\lambda - \omega^{2,6,7}\bar\lambda + \omega^{3,4,5} + \omega^{3,5,7}$; $\Delta\omega^{2,5,6} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{2,5,7}\bar\lambda - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{1,5,7}\lambda - \omega^{2,4,6} - \omega^{2,4,7}\lambda + \omega^{2,5,6}\lambda + \omega^{2,5,7}(\lambda\bar\lambda + 2) - \omega^{2,6,7} - \omega^{3,4,5}\lambda$; $\Delta\omega^{2,6,7} = -\omega^{1,6,7}\lambda - \omega^{2,4,7}\lambda - \omega^{2,5,7} + \omega^{2,6,7}(\lambda\bar\lambda + 3) - \omega^{3,4,6}\lambda + \omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,4,5} + \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6} - \omega^{2,5,7}\bar\lambda + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} - \omega^{2,6,7}\bar\lambda + 2\omega^{3,4,6} - \omega^{3,5,7}$; $\Delta\omega^{3,4,7} = -\omega^{1,4,7} + \omega^{2,6,7} + 3\omega^{3,4,7} - \omega^{3,5,6} - \omega^{3,5,7}\bar\lambda - \omega^{3,6,7}\bar\lambda$;

$\Delta\omega^{3,5,6} = \omega^{1,4,6}\lambda + \omega^{1,5,6} + \omega^{2,4,6} - \omega^{2,6,7} - \omega^{3,4,7} + 3\omega^{3,5,6} + \omega^{3,5,7}\bar\lambda$; $\Delta\omega^{3,5,7} = \omega^{1,4,7}\lambda + \omega^{2,4,7} - \omega^{3,4,6} - \omega^{3,4,7}\lambda + \omega^{3,5,6}\lambda + \omega^{3,5,7}(\lambda\bar\lambda+3) - \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = -\omega^{1,6,7} - \omega^{3,4,7}\lambda - \omega^{3,5,7} + \omega^{3,6,7}(\lambda\bar\lambda+5)$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} - \omega^{4,6,7} + \omega^{5,6,7}\bar\lambda$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 4\omega^{4,6,7} - \omega^{5,6,7}\bar\lambda$; $\Delta\omega^{5,6,7} = \omega^{4,5,7}\lambda - \omega^{4,6,7}\lambda + \omega^{5,6,7}(\lambda\bar\lambda+3)$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}(\lambda\bar\lambda+3) + \omega^{1,2,3,5}\bar\lambda + \omega^{1,2,3,6}\bar\lambda$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4}\lambda + 4\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,4}\lambda + \omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}(\lambda\bar\lambda+5) + \omega^{1,2,4,6} - \omega^{1,2,5,6}\bar\lambda - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + \omega^{1,2,4,5} + \omega^{1,2,4,6}(\lambda\bar\lambda+3) - \omega^{1,2,4,7}\bar\lambda + \omega^{1,2,5,6}\bar\lambda - \omega^{1,2,5,7} + \omega^{1,3,5,6} - \omega^{2,3,5,6}\bar\lambda$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,4,6}\lambda + 3\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,5} + \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,7}\bar\lambda$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,5}\lambda + \omega^{1,2,4,6}\lambda - \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,5} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,6} + 2\omega^{1,2,5,7} - \omega^{1,3,4,5}\lambda - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,4,6}\lambda + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{1,2,5,7}\bar\lambda + \omega^{1,3,4,5}(\lambda\bar\lambda+3) + \omega^{1,3,4,6} - \omega^{1,3,5,6}\bar\lambda + \omega^{2,3,4,5}\bar\lambda$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,6,7}\bar\lambda + \omega^{1,3,4,5} + \omega^{1,3,4,6}(\lambda\bar\lambda+2) - \omega^{1,3,4,7}\bar\lambda + \omega^{1,3,5,6}\bar\lambda - \omega^{1,3,5,7} + \omega^{2,3,4,6}\bar\lambda - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} - \omega^{1,3,4,6}\lambda + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,4,6} - \omega^{1,2,6,7} - \omega^{1,3,4,5}\lambda + \omega^{1,3,4,6}\lambda - \omega^{1,3,4,7} + 4\omega^{1,3,5,6} - \omega^{2,3,4,5} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6} - \omega^{2,3,5,7}\bar\lambda$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7} + \omega^{2,3,4,7} - \omega^{2,3,6,7}\bar\lambda - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + \omega^{1,4,5,6}(\lambda\bar\lambda+3) - \omega^{1,4,5,7}\bar\lambda + \omega^{2,4,5,6}\bar\lambda - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,4,5,6}\lambda + 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,5,6,7} - \omega^{3,4,6,7} + \omega^{3,5,6,7}\bar\lambda$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{2,5,6,7}\bar\lambda - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,4,5} - \omega^{1,2,5,7} + \omega^{1,3,4,5}\lambda - \omega^{1,3,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,6}\lambda - \omega^{1,3,4,7} + 2\omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6} + \omega^{1,3,6,7} + 3\omega^{2,3,4,7} - \omega^{2,3,5,6} - \omega^{2,3,5,7}\bar\lambda - \omega^{2,3,6,7}\bar\lambda$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6}\lambda - \omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + 2\omega^{2,3,5,6} + \omega^{2,3,5,7}\bar\lambda$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,4,7}\lambda + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{1,3,5,7}\lambda - \omega^{2,3,4,6} - \omega^{2,3,4,7}\lambda + \omega^{2,3,5,6}\lambda + \omega^{2,3,5,7}(\lambda\bar\lambda+2) - \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,2,6,7} - \omega^{1,3,6,7}\lambda - \omega^{2,3,4,7}\lambda - \omega^{2,3,5,7} + \omega^{2,3,6,7}(\lambda\bar\lambda+3)$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6}\lambda - \omega^{1,4,5,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6} - \omega^{1,5,6,7} + 3\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{2,5,6,7}\bar\lambda$; $\Delta\omega^{2,4,6,7} = -\omega^{2,4,5,7} + 3\omega^{2,4,6,7} - \omega^{2,5,6,7}\bar\lambda - \omega^{3,4,5,6} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{1,4,6,7} - \omega^{1,5,6,7}\lambda + \omega^{2,4,5,7}\lambda - \omega^{2,4,6,7}\lambda + \omega^{2,5,6,7}(\lambda\bar\lambda+1) + \omega^{3,4,5,6}\lambda - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{2,5,6,7}\bar\lambda + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7} - \omega^{3,4,6,7} + \omega^{3,5,6,7}\bar\lambda$; $\Delta\omega^{3,4,6,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,7} + 3\omega^{3,4,6,7} - \omega^{3,5,6,7}\bar\lambda$; $\Delta\omega^{3,5,6,7} = \omega^{1,4,6,7}\lambda + \omega^{2,4,6,7} + \omega^{3,4,5,7}\lambda - \omega^{3,4,6,7}\lambda + \omega^{3,5,6,7}(\lambda\bar\lambda+3)$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}(\lambda\bar\lambda+5) + \omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}\bar\lambda$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + \omega^{1,2,3,4,6}(\lambda\bar\lambda+3) - \omega^{1,2,3,4,7}\bar\lambda + \omega^{1,2,3,5,6}\bar\lambda - \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7} = -\omega^{1,2,3,4,6}\lambda + 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,5}\lambda + \omega^{1,2,3,4,6}\lambda - \omega^{1,2,3,4,7} + 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = -\omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$;

$\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}(\lambda\bar{\lambda}+4) - \omega^{1,2,4,5,7}\bar{\lambda} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,4,5,6}\lambda + 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} - \omega^{1,3,4,5,6} + \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7} - \omega^{2,3,5,6,7}\bar{\lambda}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,6}\lambda - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7}\bar{\lambda} + \omega^{1,3,4,5,6}(\lambda\bar{\lambda}+3) - \omega^{1,3,4,5,7}\bar{\lambda} + \omega^{2,3,4,5,6}\bar{\lambda} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6}\lambda + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7} + 2\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7} - \omega^{2,3,5,6,7}\bar{\lambda}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,6} + \omega^{1,2,5,6,7} + \omega^{1,3,4,5,6}\lambda - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,5,6} - \omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7} + \omega^{2,3,5,6,7}\bar{\lambda}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7} + 2\omega^{2,3,4,6,7} - \omega^{2,3,5,6,7}\bar{\lambda}$; $\Delta\omega^{2,3,5,6,7} = -\omega^{1,2,4,6,7}\lambda + \omega^{1,3,4,6,7} - \omega^{1,3,5,6,7}\lambda + \omega^{2,3,4,5,7}\lambda - \omega^{2,3,4,6,7}\lambda + \omega^{2,3,5,6,7}(\lambda\bar{\lambda}+1)$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}(\lambda\bar{\lambda}+4) - \omega^{1,2,3,4,5,7}\bar{\lambda}$; $\Delta\omega^{1,2,3,4,5,7} = -\omega^{1,2,3,4,5,6}\lambda + 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^0\ \langle\omega^{1,5}-\omega^{1,6}+\omega^{2,4}-\omega^{3,5}, 2\omega^{1,4}-\omega^{1,6}\bar{\lambda}-2\omega^{2,5}-\omega^{3,5}\bar{\lambda}, \omega^{2,6}-\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= 4e^0\ \langle 3\omega^{1,2,7} - 3\omega^{1,3,6} - 9\omega^{1,3,7}\lambda + 11\omega^{1,4,7} + 12\omega^{1,5,6} - 9\omega^{2,3,7} + 3\omega^{2,4,5} + 21\omega^{2,4,6} - 2\omega^{2,5,6}\lambda + 11\omega^{2,5,7} + 9\omega^{3,4,5}\lambda - 11\omega^{3,5,6}, 3\omega^{1,4,6} + \omega^{1,4,7}\bar{\lambda} + \omega^{2,5,6}(-\lambda\bar{\lambda}-6) + \omega^{2,5,7}\bar{\lambda} - 3\omega^{3,4,5} - \omega^{3,5,6}\bar{\lambda}, 3\omega^{1,2,6} - 6\omega^{1,3,4} + 6\omega^{1,3,6}\bar{\lambda} + 12\omega^{1,3,7}\lambda\bar{\lambda} - 16\omega^{1,4,7}\bar{\lambda} - 18\omega^{1,5,6}\bar{\lambda} + 3\omega^{2,3,5} + 12\omega^{2,3,7}\bar{\lambda} - 6\omega^{2,4,5}\bar{\lambda} - 30\omega^{2,4,6}\bar{\lambda} + 4\omega^{2,5,6}\lambda\bar{\lambda} - 16\omega^{2,5,7}\bar{\lambda} - 12\omega^{3,4,5}\lambda\bar{\lambda} + 16\omega^{3,5,6}\bar{\lambda}, 6\omega^{1,3,7} + 3\omega^{1,4,5} + 2\omega^{1,4,7}\bar{\lambda} + 3\omega^{1,5,6}\bar{\lambda} + 3\omega^{2,3,6} + 3\omega^{2,4,6}\bar{\lambda} + \omega^{2,5,6}(-2\lambda\bar{\lambda}-9) + 2\omega^{2,5,7}\bar{\lambda} - 9\omega^{3,4,5} - 2\omega^{3,5,6}\bar{\lambda}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 4e^0\ \langle(9\omega^{1,2,4,7}(-2\lambda\bar{\lambda}-11) + 9\omega^{1,2,6,7}\bar{\lambda}(2\lambda\bar{\lambda}+11) + 9\omega^{1,3,4,6}(2\lambda\bar{\lambda}+11) + 3\omega^{1,3,5,7}(8\lambda^2\bar{\lambda}^2 + 58\lambda\bar{\lambda} + 63) + \omega^{1,3,6,7}(16\lambda^3\bar{\lambda}^3 + 164\lambda^2\bar{\lambda}^2 + 408\lambda\bar{\lambda} - 27) + 3\omega^{1,4,5,6}(-16\lambda^2\bar{\lambda}^2 - 98\lambda\bar{\lambda} - 27) + 2\omega^{1,4,5,7}\bar{\lambda}(-8\lambda^2\bar{\lambda}^2 - 46\lambda\bar{\lambda} + 3) + 3\omega^{1,4,6,7}\bar{\lambda}(8\lambda^2\bar{\lambda}^2 + 90\lambda\bar{\lambda} + 253) + 12\omega^{2,3,4,7}(-2\lambda^2\bar{\lambda}^2 - 10\lambda\bar{\lambda} + 9) + 9\omega^{2,3,5,6}(2\lambda\bar{\lambda} + 11) + 2\omega^{2,3,6,7}\bar{\lambda}(8\lambda^2\bar{\lambda}^2 + 46\lambda\bar{\lambda} - 3) + \omega^{2,4,5,6}\bar{\lambda}(16\lambda^2\bar{\lambda}^2 + 110\lambda\bar{\lambda} + 93) + \omega^{2,4,5,7}(16\lambda^3\bar{\lambda}^3 + 164\lambda^2\bar{\lambda}^2 + 408\lambda\bar{\lambda} - 27) + 6\omega^{2,5,6,7}\bar{\lambda}(-8\lambda^2\bar{\lambda}^2 - 90\lambda\bar{\lambda} - 253) + \omega^{3,4,5,6}(32\lambda^3\bar{\lambda}^3 + 376\lambda^2\bar{\lambda}^2 + 1110\lambda\bar{\lambda} + 27) + 3\omega^{3,4,5,7}\bar{\lambda}(-8\lambda^2\bar{\lambda}^2 - 90\lambda\bar{\lambda} - 253))/(72(2\lambda\bar{\lambda}+11)(\lambda\bar{\lambda}+6)(\lambda\bar{\lambda}+2)), (-(\lambda^2\omega^{1,3,5,7}\bar{\lambda} + \lambda^2\bar{\lambda}\omega^{2,3,4,7} + 2\lambda\omega^{1,2,4,7} - \lambda\omega^{1,2,6,7}\bar{\lambda} - 2\lambda\omega^{1,3,4,6} - 3\lambda\omega^{1,3,4,7}\bar{\lambda} + 9\lambda\omega^{1,3,5,7} - \lambda\omega^{1,4,5,7}\bar{\lambda} + 2\lambda\bar{\lambda}\omega^{2,3,5,7} + \lambda\bar{\lambda}\omega^{2,3,6,7} - 2\lambda\bar{\lambda}\omega^{2,4,5,6} + 9\lambda\omega^{2,3,4,7} - 2\lambda\omega^{2,3,5,6} - 3\omega^{1,2,6,7} - 21\omega^{1,3,4,7} - 9\omega^{1,4,5,7} + 24\omega^{2,3,5,7} + 9\omega^{2,3,6,7} - 18\omega^{2,4,5,6}))/(3(\lambda\bar{\lambda}+9)(\lambda\bar{\lambda}+7)), (\omega^{1,2,4,7}\lambda - \omega^{1,2,6,7}\lambda\bar{\lambda} - \omega^{1,3,4,6}\lambda + \omega^{1,3,5,7}\lambda + \omega^{1,3,6,7}\lambda(2\lambda\bar{\lambda}+9) - 5\omega^{1,4,5,6}\lambda - 2\omega^{1,4,5,7}\lambda\bar{\lambda} + \omega^{1,4,6,7}(3\lambda\bar{\lambda} + 16) - 4\omega^{2,3,4,7}\lambda - \omega^{2,3,5,6}\lambda + 2\omega^{2,3,6,7}\lambda\bar{\lambda} + \omega^{2,4,5,6}\lambda\bar{\lambda} + \omega^{2,4,5,7}\lambda(2\lambda\bar{\lambda}+9) + 2\omega^{2,5,6,7}(-3\lambda\bar{\lambda} - 16) + \omega^{3,4,5,6}\lambda(4\lambda\bar{\lambda} + 23) + \omega^{3,4,5,7}(-3\lambda\bar{\lambda} - 16))/(24(\lambda\bar{\lambda} + 6)(\lambda\bar{\lambda} + 2)), (3\omega^{1,2,4,7}\lambda(-2\lambda^2\bar{\lambda}^2 - 17\lambda\bar{\lambda} - 33) + 3\omega^{1,2,6,7}(-6\lambda^3\bar{\lambda}^3 - 91\lambda^2\bar{\lambda}^2 -$

$415\lambda\bar\lambda - 528) + 3\omega^{1,3,4,6}\lambda(2\lambda^2\bar\lambda^2 + 17\lambda\bar\lambda + 33) + 3\omega^{1,3,5,7}\lambda(4\lambda^3\bar\lambda^3 + 58\lambda^2\bar\lambda^2 + 255\lambda\bar\lambda + 303) + \omega^{1,3,6,7}\lambda(4\lambda^3\bar\lambda^3 + 48\lambda^2\bar\lambda^2 + 143\lambda\bar\lambda + 21) + 3\omega^{1,4,5,6}\lambda(-2\lambda^2\bar\lambda^2 - 11\lambda\bar\lambda + 21) + 2\omega^{1,4,5,7}(-8\lambda^3\bar\lambda^3 - 105\lambda^2\bar\lambda^2 - 421\lambda\bar\lambda - 528) + 3\omega^{1,4,6,7}\lambda\bar\lambda(2\lambda^2\bar\lambda^2 + 25\lambda\bar\lambda + 77) + 12\omega^{2,3,4,7}\lambda(\lambda^3\bar\lambda^3 + 14\lambda^2\bar\lambda^2 + 61\lambda\bar\lambda + 81) + 3\omega^{2,3,5,6}\lambda(2\lambda^2\bar\lambda^2 + 17\lambda\bar\lambda + 33) + 12\omega^{2,3,5,7}(2\lambda^3\bar\lambda^3 + 27\lambda^2\bar\lambda^2 + 112\lambda\bar\lambda + 132) + 2\omega^{2,3,6,7}(8\lambda^3\bar\lambda^3 + 105\lambda^2\bar\lambda^2 + 421\lambda\bar\lambda + 528) + \omega^{2,4,5,6}(-26\lambda^3\bar\lambda^3 - 387\lambda^2\bar\lambda^2 - 1747\lambda\bar\lambda - 2112) + \omega^{2,4,5,7}\lambda(4\lambda^3\bar\lambda^3 + 48\lambda^2\bar\lambda^2 + 143\lambda\bar\lambda + 21) + 6\omega^{2,5,6,7}\lambda\bar\lambda(-2\lambda^2\bar\lambda^2 - 25\lambda\bar\lambda - 77) + \omega^{3,4,5,6}\lambda(8\lambda^3\bar\lambda^3 + 102\lambda^2\bar\lambda^2 + 319\lambda\bar\lambda - 21) + 3\omega^{3,4,5,7}\lambda\bar\lambda(-2\lambda^2\bar\lambda^2 - 25\lambda\bar\lambda - 77))/(36(2\lambda\bar\lambda + 11)(\lambda\bar\lambda + 7)(\lambda\bar\lambda + 6)(\lambda\bar\lambda + 2))\rangle$.

$\mathrm{ch}(H^5_{\Phi})= 3e^0\ \langle(4\omega^{1,2,4,6,7} + 2\omega^{1,3,4,6,7}\bar\lambda + \omega^{1,3,5,6,7}(-\lambda\bar\lambda - 4) - 4\omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}(-\lambda\bar\lambda - 4) - 2\omega^{2,3,5,6,7}\bar\lambda)/(2(\lambda\bar\lambda + 8)), (\omega^{1,2,4,6,7}\lambda - 4\omega^{1,3,4,6,7} + \omega^{1,3,5,6,7}\lambda - \omega^{2,3,4,5,7}\lambda + \omega^{2,3,4,6,7}\lambda + 4\omega^{2,3,5,6,7})/(2(\lambda\bar\lambda + 8)), (-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\Phi})= 2e^0\ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\Phi})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

The preceding results are valid for:

$$(\lambda) \neq 0.$$

Comments: $\lambda = 0$ is not a singular value, i.e. results are valid in fact for all λ.

5.1.5 Algebra $\mathcal{G}_{7,0.5}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_6 + x_7$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_5] = x_6$; $[x_3, x_5] = x_7$.

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 0 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^4 - \xi_1^5 & \xi_2^3 & -\xi_1^3 & 0 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_2^4 + \xi_2^6 & \xi_2^3 + \xi_2^4 - 2\xi_1^5 & -\xi_2^5 & \xi_2^3 & 0 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -(\delta_1^6 + \delta_1^7), ad\ x_5 = -\delta_2^6, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : T=$\langle\rangle$.

ch($\mathcal{G}$)= $7e^0$ $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= e^0 $\langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0$ $\langle \delta_1^4, \delta_1^6, \delta_1^3, \delta_1^7, \delta_2^7, \delta_2^6, \delta_2^4, \delta_1^5, \delta_2^5, \delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^4] = -\delta_1^7$; $[\delta_1^4, \delta_1^5] = 2\delta_1^7$; $[\delta_1^4, \delta_2^3] = -(\delta_1^7 + \delta_1^6)$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_1^3, \delta_2^6] = -\delta_1^7$; $[\delta_1^3, \delta_2^4] = -(\delta_1^7 + \delta_1^6)$; $[\delta_1^3, \delta_2^5] = -\delta_2^6$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$; $[\delta_2^4, \delta_1^5] = 2\delta_2^7$; $[\delta_1^5, \delta_2^5] = \delta_1^7$; $[\delta_1^5, \delta_2^3] = -\delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $4e^0$ $\langle \delta_2^7, \delta_2^4, \delta_1^5, \delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^5] = 2\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $9e^0$ $\langle -x_2(\omega^{1,2} - 3\omega^{1,3}) + 2x_3(\omega^{1,3} + 3/2\omega^{2,5}) + 3x_4\omega^{3,5} + x_5\omega^{1,5}, -x_1(\omega^{1,5} + \omega^{2,4} + \omega^{2,6} - \omega^{3,4}) + x_2(\omega^{1,6} + 1/2\omega^{2,5} + \omega^{3,5}) - 1/2x_3(\omega^{3,5} - 2\omega^{4,5}) + 5/2x_4(\omega^{1,7} + 2/5\omega^{4,5} - 2/5\omega^{5,6}) + x_5\omega^{4,6}, x_4\omega^{1,4}, x_2(\omega^{1,2} - \omega^{1,3}) + x_3(\omega^{1,2} - \omega^{1,3} - \omega^{2,5}) + x_4(\omega^{2,5} - \omega^{3,5}), x_4\omega^{2,3} + x_5\omega^{1,3} + x_7\omega^{2,4}, x_3\omega^{2,3} + x_4\omega^{2,4} + x_6\omega^{2,6} + x_7\omega^{2,7}, -x_1\omega^{1,2} - 3x_4(\omega^{1,5} + 2/3\omega^{2,4}) - x_5\omega^{1,3} + x_6(\omega^{2,4} - \omega^{2,6}), x_1(\omega^{1,2} - \omega^{1,3}) + x_3\omega^{1,5} + 2x_4(\omega^{1,5} + 1/2\omega^{2,4} + 1/2\omega^{2,6} - 1/2\omega^{3,4}) + x_5\omega^{1,6}, -x_3\omega^{2,5} + x_4\omega^{1,6}\rangle$.

ch($H^3_{\mathcal{G}}$)=$15e^0$.

ch($H^4_{\mathcal{G}}$)=$16e^0$.

ch($H^5_{\mathcal{G}}$)=$12e^0$.

ch($H^6_{\mathcal{G}}$)=$7e^0$.

ch($H^7_{\mathcal{G}}$)=$2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 10, 48, 114, 147, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 4, 9, 15, 16, 12, 7, 2)$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,5}$; $d\omega^7 = -\omega^{1,4} - \omega^{1,6} - \omega^{3,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,5}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,4} - \omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4} - \omega^{2,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,4} - \omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{2,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = \omega^{1,4,5} - \omega^{1,5,6} - \omega^{2,3,7}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{1,4,7} - \omega^{2,5,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,3,6} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,6} = \omega^{1,2,4,5}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7}$; $d\omega^{1,6,7} = \omega^{1,2,5,7} - \omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4} - \omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = \omega^{1,2,4,5} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = \omega^{1,2,4,6} - \omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,5} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,6} - \omega^{1,3,4,7} - \omega^{2,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,5,6} + \omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,4,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,5,6} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6} + \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $2e^0$ $\langle\omega^1, \omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^0\ \langle \omega^{1,5}+\omega^{2,4}, \omega^{2,6}-\omega^{3,4}, \omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 4e^0\ \langle \omega^{1,2,6}, \omega^{1,4,5}-\omega^{2,3,7}+\omega^{2,4,6}, \omega^{2,5,6}+\omega^{3,4,5}, \omega^{1,3,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 4e^0\ \langle \omega^{1,2,6,7}+\omega^{2,4,5,6}, \omega^{1,2,4,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}, \omega^{1,3,5,7}, \omega^{1,4,6,7}-2\omega^{2,5,6,7}+2\omega^{3,4,5,6}-\omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^0\ \langle \omega^{1,2,5,6,7}, \omega^{1,3,5,6,7}, \omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^0\ \langle \omega^{1,3,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^0\ \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^{3}$; $d^\star\omega^{1,3} = -\omega^{4}$; $d^\star\omega^{1,4} = -\omega^{6}-\omega^{7}$; $d^\star\omega^{1,6} = -\omega^{7}$; $d^\star\omega^{2,3} = -\omega^{5}$; $d^\star\omega^{2,5} = -\omega^{6}$; $d^\star\omega^{3,5} = -\omega^{7}$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}-\omega^{2,7}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}-\omega^{3,7}$; $d^\star\omega^{1,3,5} = \omega^{1,7}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}+\omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}+\omega^{6,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,5} = \omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,6}$; $d^\star\omega^{2,5,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5}-\omega^{2,3,6}-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{1,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6}+\omega^{2,5,6}+\omega^{2,5,7}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}+\omega^{2,6,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = \omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}+\omega^{3,5,6}+\omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}+\omega^{3,6,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7}+\omega^{1,3,4,6}+\omega^{2,3,5,6}+\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6}-\omega^{2,3,4,7}+\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,3,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}-\omega^{2,5,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{1,4,6,7}+\omega^{2,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} =$

$-\omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7} - \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7} - \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6 + \omega^7$; $\Delta\omega^7 = \omega^6 + 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = 2\omega^{1,4} + \omega^{1,6} + \omega^{2,5} + \omega^{3,5}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,4} + 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,4} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,6} + 3\omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6} + \omega^{3,7}$; $\Delta\omega^{3,7} = \omega^{3,6} + 3\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{6,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6} + \omega^{5,7}$; $\Delta\omega^{5,7} = \omega^{5,6} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = -\omega^{4,7} + 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4} + \omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,4} + 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,4} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,2,7} + 2\omega^{1,3,4} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,4} + 3\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5} - \omega^{1,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 3\omega^{1,4,6} - \omega^{1,4,7} + \omega^{2,5,6} + \omega^{3,5,6}$; $\Delta\omega^{1,4,7} = -\omega^{1,4,6} + 2\omega^{1,4,7} + \omega^{2,5,7} + \omega^{3,5,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,5} + 2\omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4} + \omega^{1,3,4} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6} + \omega^{2,3,7}$; $\Delta\omega^{2,3,7} = \omega^{2,3,6} + 3\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + 3\omega^{2,4,7} - \omega^{2,6,7}$; $\Delta\omega^{2,5,6} = \omega^{1,4,6} + \omega^{2,5,6} + \omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = \omega^{1,4,7} + \omega^{2,5,6} + 3\omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = -\omega^{2,4,7} + 3\omega^{2,6,7} - \omega^{3,4,6} + \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} - \omega^{2,5,6} - \omega^{2,5,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,6,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 2\omega^{3,4,7} - \omega^{3,6,7}$; $\Delta\omega^{3,5,6} = \omega^{1,4,6} + 3\omega^{3,5,6} + \omega^{3,5,7}$; $\Delta\omega^{3,5,7} = \omega^{1,4,7} + \omega^{3,5,6} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = -\omega^{3,4,7} + 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} + \omega^{5,6,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{4,5,7} + 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,4} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5} - \omega^{1,2,5,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} - \omega^{1,2,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,4,6} + 3\omega^{1,2,4,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,5} + 2\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,4,6} + \omega^{1,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,6} - \omega^{1,2,5,7} + 3\omega^{1,3,4,5} - \omega^{1,3,5,6}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,6,7} + 3\omega^{1,3,4,6} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} - \omega^{1,3,4,6} + \omega^{1,3,4,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,5} + 3\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 3\omega^{1,4,5,6} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,4,5,6} + 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,5,6,7} + \omega^{3,5,6,7}$;

$\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + 2\omega^{2,3,4,7} - \omega^{2,3,6,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6} + \omega^{1,3,4,6} + 2\omega^{2,3,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,4,7} + \omega^{1,3,4,7} + \omega^{2,3,5,6} + 3\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = -\omega^{2,3,4,7} + 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 3\omega^{2,4,5,7} + \omega^{2,5,6,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + 2\omega^{2,5,6,7} + \omega^{3,4,5,6} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{2,5,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7} + \omega^{3,5,6,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7} + 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6} - \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,7} = -\omega^{1,2,3,4,6} + 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,5} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,4,5,6} + 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,6} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,5,6,7} + 3\omega^{1,3,4,5,6} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + 2\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6} - \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,5,7} = -\omega^{1,2,3,4,5,6} + 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}}) = 2e^0\ \langle \omega^1, \omega^2 \rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 3e^0\ \langle \omega^{1,5} + \omega^{2,4}, \omega^{2,6} - \omega^{3,4}, 2\omega^{1,4} - \omega^{1,6} - 2\omega^{2,5} - \omega^{3,5} \rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 4e^0\ \langle 3\omega^{1,2,6} + 4\omega^{1,2,7} - 6\omega^{1,3,4} + 2\omega^{1,3,6} + 3\omega^{2,3,5} - 2\omega^{2,4,5}, \omega^{1,4,5} + 3\omega^{1,5,6} + \omega^{2,3,6} - 2\omega^{2,3,7} + 5\omega^{2,4,6}, \omega^{1,3,7} - \omega^{2,5,6} - \omega^{3,4,5}, 3\omega^{1,4,6} + \omega^{1,4,7} - 7\omega^{2,5,6} + \omega^{2,5,7} - 3\omega^{3,4,5} - \omega^{3,5,6} \rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = 4e^0\ \langle (2\omega^{1,3,6,7} + 3\omega^{1,4,6,7} + 2\omega^{2,4,5,7} - 6\omega^{2,5,6,7} + 4\omega^{3,4,5,6} - 3\omega^{3,4,5,7})/78, (5\omega^{1,3,5,7} - 2\omega^{1,4,5,6} - \omega^{1,4,5,7} + 3\omega^{2,3,4,7} + \omega^{2,3,6,7})/40, (\omega^{1,2,4,7} - 4\omega^{1,2,6,7} - \omega^{1,3,4,6} - \omega^{2,3,5,6} + 3\omega^{2,3,5,7} - 7\omega^{2,4,5,6})/11, (-3\omega^{1,2,4,7} + \omega^{1,2,6,7} + 3\omega^{1,3,4,6} + 11\omega^{1,3,4,7} + 3\omega^{2,3,5,6} - 9\omega^{2,3,5,7} + 10\omega^{2,4,5,6})/110 \rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = 3e^0\ \langle (-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2, (-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2, (\omega^{1,2,4,6,7} - 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7} + 2\omega^{2,3,5,6,7})/10 \rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = 2e^0\ \langle \omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7} \rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^0\ \langle \omega^{1,2,3,4,5,6,7} \rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.1.6 Algebra $\mathcal{G}_{7,0.6}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_7$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 0 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 0 & 0\\ \xi_1^7 & \xi_2^7 & \xi_2^4-\xi_1^5+\xi_2^6 & \xi_2^3-2\xi_1^4+\xi_2^5 & -(\xi_1^3+\xi_2^4) & \xi_2^3 & 0 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\delta_2^6+\delta_1^7), ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0$ $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= e^0 $\langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0$ $\langle \delta_1^6,\delta_1^7,\delta_2^7,\delta_1^5,\delta_2^6,\delta_1^4,\delta_2^5,\delta_1^3,\delta_2^4,\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=\delta_1^7$; $[\delta_1^5,\delta_2^3]=-(\delta_2^7+\delta_1^6)$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-2\delta_2^7$; $[\delta_1^4,\delta_2^3]=-(\delta_1^7+\delta_2^6)$; $[\delta_2^5,\delta_1^3]=\delta_2^7+\delta_1^6$; $[\delta_2^5,\delta_2^4]=2\delta_2^7$; $[\delta_1^3,\delta_2^4]=-(\delta_1^7+\delta_2^6)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $4e^0$ $\langle \delta_1^6,\delta_1^5,\delta_1^4,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^4]=2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $9e^0$ $\langle -2x_1(\omega^{1,2}-\omega^{2,3})+3x_3(\omega^{1,2}-\omega^{2,3})+2x_4(\omega^{1,5}+1/2\omega^{2,4}-\omega^{2,6}+\omega^{3,5}), -x_1\omega^{1,3}-2x_2(\omega^{1,2}+3/2\omega^{1,4}-\omega^{2,3})+2x_3(\omega^{1,3}-3/2\omega^{1,5}-\omega^{2,4})-x_4(\omega^{1,4}-2\omega^{1,6}+2\omega^{2,5})+3x_5(\omega^{1,7}+2\omega^{2,6})+3x_6(\omega^{2,7}+1/3\omega^{3,6}), -x_3\omega^{1,2}+x_5\omega^{1,4}, -x_1(\omega^{1,2}-3\omega^{2,3})+x_2\omega^{1,3}+x_3(\omega^{1,2}-2\omega^{1,4}-3\omega^{2,3})+2x_4(\omega^{1,5}-3/2\omega^{2,6}+3/2\omega^{3,5})-2x_5\omega^{1,6}+x_6\omega^{1,7}, x_1(\omega^{1,2}-3\omega^{2,3})-x_3(\omega^{1,2}-2\omega^{1,4}-3\omega^{2,3})-2x_4(\omega^{1,5}-3/2\omega^{2,6}+3/2\omega^{3,5})+2x_5\omega^{1,6}+x_6\omega^{3,5}, x_4(\omega^{1,2}-\omega^{2,3}), 2x_2(\omega^{1,2}-3/2\omega^{2,3})-x_3(\omega^{1,3}+3\omega^{2,4})-2x_4(\omega^{1,4}-3/2\omega^{2,5}+3/2\omega^{3,4})+x_5\omega^{1,5}, -2x_2(\omega^{1,2}-\omega^{2,3})+2x_3(\omega^{1,3}+\omega^{2,4})+x_4(\omega^{1,4}-2\omega^{2,5}+2\omega^{3,4})+x_6\omega^{3,4}, x_1\omega^{2,3}+x_2\omega^{1,3}-2x_3\omega^{1,4}-x_4(\omega^{2,6}-\omega^{3,5})-x_5(\omega^{1,6}-\omega^{3,4})\rangle$.

ch($H^3_{\mathcal{G}}$)=$15e^0$.

ch$(H^4_{\mathcal{G}})$=$17e^0$.
ch$(H^5_{\mathcal{G}})$=$13e^0$.
ch$(H^6_{\mathcal{G}})$=$7e^0$.
ch$(H^7_{\mathcal{G}})$=$2e^0$.
Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 10, 48, 114, 148, 110, 44, 7)$
Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 4, 9, 15, 17, 13, 7, 2)$
S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 8, 18, 22, 16, 7, 1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 3, 5, 5, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,4} - \omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,4} - \omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,4} - \omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = \omega^{1,4,5} - \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4} - \omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = \omega^{1,2,4,5} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,4,6} - \omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,5} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,6} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,5,6} - \omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,5,6} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$;

$d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^0\ \langle\omega^{1,4},\omega^{1,6}+\omega^{3,4},\omega^{2,6}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 5e^0\ \langle\omega^{1,3,7}+\omega^{2,4,5},\omega^{1,2,7}-\omega^{2,3,7}+\omega^{2,4,6}-\omega^{3,4,5},\omega^{1,4,7}+\omega^{3,4,5},\omega^{1,3,6}+\omega^{2,3,7}-\omega^{2,4,6}+\omega^{3,4,5},\omega^{1,5,7}+\omega^{2,4,7}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 5e^0\ \langle\omega^{1,2,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6},\omega^{1,3,4,7}-\omega^{2,3,5,7}+\omega^{2,4,5,6},\omega^{1,4,6,7}-\omega^{3,4,5,6},\omega^{1,2,5,7}-\omega^{2,3,5,7}+\omega^{2,4,5,6},\omega^{1,3,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^0\ \langle\omega^{1,2,4,6,7},\omega^{1,3,4,6,7}-\omega^{2,3,5,6,7},\omega^{1,2,5,6,7}-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^0\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+2Y^2+3Y^3+2Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,18,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,5,5,3,2,1)$

Total dimension of trivial cohomology: 22.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{2,7}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}-\omega^{3,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}+\omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}+\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}+\omega^{2,5,7}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}+\omega^{2,6,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}-\omega^{3,4,6}+\omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}-\omega^{3,4,7}+\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}+$

$\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{2,5,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 4\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,4} + 2\omega^{1,6} + \omega^{2,5} - \omega^{2,7}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} - \omega^{3,5} - \omega^{3,7}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,4} + \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = -\omega^{1,6} + 3\omega^{2,7} + \omega^{3,4} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,4} + \omega^{2,5} + \omega^{2,7} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = -\omega^{1,7} + 4\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,6} - \omega^{6,7}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,6} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = -\omega^{4,7} + 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4} + \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,4} + 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,4} + \omega^{1,4,5} + \omega^{2,3,5} + \omega^{2,3,7}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,2,7} + 2\omega^{1,3,4} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,4} + 3\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 3\omega^{1,4,5} - \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} - \omega^{1,5,7} + \omega^{2,4,5} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6} + \omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,5} - \omega^{1,4,7} + 3\omega^{1,5,6} - \omega^{2,5,7}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + 3\omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,5,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,5,6} - \omega^{3,6,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,4} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,3,4} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,2,7} - \omega^{1,3,6} + 3\omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6} - \omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + 3\omega^{2,4,7} - \omega^{2,5,6} - \omega^{2,6,7}$; $\Delta\omega^{2,5,6} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,4,7} - \omega^{1,5,6} + \omega^{1,6,7} - \omega^{2,4,6} + 3\omega^{2,5,7} - \omega^{3,4,5} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = -\omega^{2,4,7} + 3\omega^{2,6,7} - \omega^{3,4,6} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,4,5} - \omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6} - \omega^{2,5,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,6} + \omega^{2,5,6} - \omega^{2,6,7} + 3\omega^{3,4,6} - \omega^{3,5,7}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 3\omega^{3,4,7} - \omega^{3,5,6} - \omega^{3,6,7}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{1,5,7} + \omega^{2,6,7} - \omega^{3,4,6} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = -\omega^{1,6,7} - \omega^{3,4,7} + 5\omega^{3,6,7}$;

$\Delta\omega^{4,5,6} = -\omega^{2,6,7}+\omega^{3,5,7}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}+\omega^{5,6,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{4,5,7} + 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,4}+3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}+\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 5\omega^{1,2,4,5} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} - \omega^{1,2,5,7} + \omega^{1,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,5}-\omega^{1,2,4,7} + 3\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,6} + 3\omega^{1,2,5,7} - \omega^{1,3,4,5} + \omega^{1,3,4,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,4,6} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6} + \omega^{2,3,6,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,2,5,7} + 3\omega^{1,3,4,5} - \omega^{1,3,5,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} - \omega^{1,2,6,7} + 3\omega^{1,3,4,6} - \omega^{1,3,5,7} + \omega^{2,3,4,5} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,5} - \omega^{1,3,4,7} + 3\omega^{1,3,5,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7}-\omega^{1,3,4,6}+2\omega^{1,3,5,7}+\omega^{1,4,5,6}-\omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7}+\omega^{1,3,5,7}+3\omega^{1,4,5,6}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{2,5,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}-\omega^{3,5,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,5}+\omega^{1,2,4,7}+\omega^{1,3,4,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,6}-\omega^{1,3,4,7}+3\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,4,6}+3\omega^{2,3,4,7}-\omega^{2,3,5,6}-\omega^{2,3,6,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7}+\omega^{1,3,4,6}-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{1,3,6,7} - \omega^{2,3,4,6} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,2,6,7} - \omega^{2,3,4,7} + 3\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6}+\omega^{1,4,6,7}+3\omega^{2,4,5,7}+\omega^{2,5,6,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + 2\omega^{2,5,6,7} + \omega^{3,4,5,6} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,5,6} + \omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,5,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7} + \omega^{3,5,6,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7} + 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,5}-\omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = -\omega^{1,2,3,4,6}+2\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 4\omega^{1,2,4,5,6} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}+\omega^{1,3,4,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,5,6,7} + 3\omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,5,6}-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,5,6}+\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = 2e^0\ \langle \omega^1, \omega^2 \rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}}) = 3e^0\ \langle \omega^{1,4} - \omega^{2,5}, \omega^{1,6} - 2\omega^{2,5} + \omega^{3,4}, \omega^{2,6} - \omega^{3,5} \rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= 5e^0\ \langle 2\omega^{1,3,7} - 3\omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5} + 3\omega^{2,5,6}, \omega^{1,2,7} - \omega^{2,3,5} - \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}, 3\omega^{1,3,6} - 3\omega^{1,4,5} - \omega^{1,5,6} - 3\omega^{2,3,5} + 6\omega^{2,3,7} - 7\omega^{2,4,6} + 8\omega^{3,4,5}, 3\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}, \omega^{1,5,7} + \omega^{2,4,7} + 3\omega^{2,5,6} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 5e^0\ \langle(-3\omega^{1,2,5,7} - 3\omega^{1,3,4,7} + 3\omega^{1,3,5,6} - 4\omega^{1,3,6,7} - 4\omega^{1,4,5,7} + 3\omega^{2,3,4,6} + 12\omega^{2,3,5,7} - 8\omega^{2,4,5,6})/69, (-11\omega^{1,2,6,7} + 17\omega^{1,3,5,7} - 23\omega^{1,4,5,6} - 77\omega^{1,4,6,7} - 6\omega^{2,3,4,7} - 18\omega^{2,3,6,7} + 18\omega^{2,4,5,7} + 59\omega^{3,4,5,6})/187, (22\omega^{1,2,6,7} - 17\omega^{1,3,5,7} + 12\omega^{1,4,5,6} - 33\omega^{1,4,6,7} - 5\omega^{2,3,4,7} - 15\omega^{2,3,6,7} + 15\omega^{2,4,5,7} + 18\omega^{3,4,5,6})/561, (\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7} + 3\omega^{2,3,5,6})/12, (8\omega^{1,2,5,7} - 15\omega^{1,3,4,7} - 8\omega^{1,3,5,6} + 3\omega^{1,3,6,7} + 3\omega^{1,4,5,7} - 8\omega^{2,3,4,6} - 9\omega^{2,3,5,7} + 6\omega^{2,4,5,6})/69\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 3e^0\ \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7} + 3\omega^{2,3,5,6,7})/4, (\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} - \omega^{2,3,5,6,7})/4, (\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= 2e^0\ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 2Y^2 + 3Y^3 + 2Y^4 + Y^5 + Y^6)(1 + Y)$.

5.1.7 Algebra $\mathcal{G}_{7,0.7}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_7$; $[x_1,x_5]=x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_7$; $[x_2,x_5]=x_6$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 0 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 0 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^5 & 0 & -\xi_1^3 & 0 & 0\\ \xi_1^7 & \xi_2^7 & \xi_2^4+\xi_2^5+\xi_2^6-\xi_1^4 & -2\xi_1^5-\xi_1^3+\xi_2^3 & -\xi_2^5+\xi_2^3 & \xi_2^3 & 0 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\delta_1^7+\delta_2^7), ad\ x_5=-(\delta_2^6+\delta_1^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0$ $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= e^0 $\langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0$ $\langle \delta_1^6,\delta_1^7,\delta_2^7,\delta_1^4,\delta_2^4,\delta_2^6,\delta_1^3,\delta_1^5,\delta_2^5,\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^4,\delta_1^5]=2\delta_1^7$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^7)$; $[\delta_2^4,\delta_1^3]=\delta_2^7+\delta_1^7$; $[\delta_2^4,\delta_1^5]=2\delta_2^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^5]=-(\delta_1^7+\delta_2^6)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$; $[\delta_1^5,\delta_2^5]=\delta_1^7$; $[\delta_1^5,\delta_2^3]=-(\delta_1^7+\delta_2^6)$.

ch($H^1_{\mathcal{G}}$)= $4e^0$ $\langle \delta_1^6,\delta_2^4,\delta_1^5,\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $10e^0$ $\langle x_1(\omega^{1,2}-\omega^{1,3}-2/3\omega^{1,5}-\omega^{2,3}-2/3\omega^{2,4}-8/3\omega^{2,5}-2/3\omega^{2,6})-x_2(\omega^{1,2}-\omega^{1,3}-\omega^{2,3}-1/3\omega^{2,5})-x_3(\omega^{1,2}-\omega^{1,3}+\omega^{1,5}-\omega^{2,3}+\omega^{2,4}-2/3\omega^{2,5}+4/3\omega^{2,6}+1/3\omega^{3,5})+5/3x_4(\omega^{1,7}+6/5\omega^{2,6}+2/5\omega^{2,7}-6/5\omega^{3,6}+2/5\omega^{4,5}-2/5\omega^{5,6})-x_5(\omega^{1,6}+\omega^{2,6})+x_6(\omega^{1,7}+\omega^{2,7}), -x_1(\omega^{1,2}-\omega^{1,3}-2/3\omega^{1,5}-\omega^{2,3}-2/3\omega^{2,4}-8/3\omega^{2,5}-2/3\omega^{2,6})+x_2(\omega^{1,2}-\omega^{2,3}-1/3\omega^{2,5})+x_3(\omega^{1,2}-\omega^{1,3}+\omega^{1,5}-\omega^{2,3}+\omega^{2,4}-2/3\omega^{2,5}+4/3\omega^{2,6}+1/3\omega^{3,5})-5/3x_4(\omega^{1,7}+6/5\omega^{2,6}+2/5\omega^{2,7}-6/5\omega^{3,6}+2/5\omega^{4,5}-2/5\omega^{5,6})+x_5(\omega^{1,6}+\omega^{2,6})-x_6(\omega^{2,7}-\omega^{3,4}), x_1(\omega^{1,2}+\omega^{1,3}-2/3\omega^{1,5}-2/3\omega^{2,4}-8/3\omega^{2,5}-2/3\omega^{2,6})-x_2(\omega^{1,2}-1/3\omega^{2,5})+x_3(\omega^{1,3}-2\omega^{1,5}+2\omega^{2,3}-\omega^{2,4}+8/3\omega^{2,5}-4/3\omega^{2,6}-1/3\omega^{3,5})-x_4(\omega^{1,5}-5/3\omega^{1,7}-3\omega^{2,6}-2/3\omega^{2,7}-2\omega^{3,5}+2\omega^{3,6}-2/3\omega^{4,5}+2/3\omega^{5,6})-x_5(\omega^{1,4}+2\omega^{1,6}+2\omega^{2,6})+x_6(\omega^{2,7}+\omega^{3,6}), x_1\omega^{1,2}-3x_3\omega^{1,2}+2x_4(\omega^{1,5}+1/2\omega^{2,4}+1/2\omega^{2,6}), -x_1\omega^{1,2}+2x_3\omega^{1,2}+x_4(\omega^{1,4}-\omega^{1,5}-\omega^{2,6}), x_1\omega^{1,2}-x_2\omega^{1,2}+x_3(\omega^{1,2}+2\omega^{1,3}+3\omega^{2,3}+3\omega^{2,5})-2x_4(\omega^{1,5}-$

$1/2\omega^{2,6}-3/2\omega^{3,5})+x_5\omega^{1,5}, x_6\omega^{1,4}, x_1\omega^{1,2}-2x_3\omega^{1,2}+x_4(\omega^{1,5}+\omega^{2,6})-x_5\omega^{1,3}+x_6\omega^{2,4}, -x_1\omega^{1,2}+x_2\omega^{1,2}+x_3(\omega^{1,2}-\omega^{1,3}-\omega^{2,3}-\omega^{2,5})-x_4(\omega^{2,6}+\omega^{3,5})+x_6\omega^{1,6}, -x_3\omega^{2,5}+x_4(\omega^{1,6}+\omega^{2,6})\rangle$.

ch($H^3_{\mathcal{G}}$)=$17e^0$.

ch($H^4_{\mathcal{G}}$)=$18e^0$.

ch($H^5_{\mathcal{G}}$)=$13e^0$.

ch($H^6_{\mathcal{G}}$)=$7e^0$.

ch($H^7_{\mathcal{G}}$)=$2e^0$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,10,49,115,148,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,4,10,17,18,13,7,2)$

S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{2,5}$; $d\omega^7=-\omega^{1,4}-\omega^{1,5}-\omega^{1,6}-\omega^{2,4}-\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,5}$; $d\omega^{1,7}=\omega^{1,2,4}+\omega^{1,3,5}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,7}=-\omega^{1,2,4}-\omega^{1,2,5}-\omega^{1,2,6}+\omega^{2,3,5}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{2,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,4}-\omega^{1,3,5}-\omega^{1,3,6}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{2,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,5}-\omega^{1,4,6}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{2,3,6}$; $d\omega^{5,7}=\omega^{1,4,5}-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=\omega^{1,4,6}+\omega^{1,5,6}+\omega^{2,4,6}-\omega^{2,5,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,6}=\omega^{1,2,4,5}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}+\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $d\omega^{2,3,7}=-\omega^{1,2,3,4}-\omega^{1,2,3,5}-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,5}-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,7}=\omega^{1,2,4,5}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=\omega^{1,2,4,6}+\omega^{1,2,5,6}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,5}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,5}-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}+\omega^{1,3,4,6}+\omega^{1,3,5,6}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,5,6}-\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}+\omega^{1,2,3,5,7}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,5}-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=\omega^{1,2,3,4,5}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=$

$\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,6} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^0\ \langle\omega^1, \omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^0\ \langle\omega^{1,4}, \omega^{1,5} + \omega^{2,4}, \omega^{2,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 4e^0\ \langle\omega^{1,2,7} - \omega^{2,4,5}, \omega^{1,4,5} - \omega^{2,3,7} + \omega^{2,4,5} + \omega^{2,4,6}, \omega^{2,5,6}, \omega^{1,3,7} + \omega^{2,3,7} - \omega^{2,4,5} - \omega^{2,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 4e^0\ \langle\omega^{1,2,4,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}, \omega^{1,3,4,7}, \omega^{1,3,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}, \omega^{2,5,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^0\ \langle\omega^{1,3,4,5,7}, \omega^{1,2,4,6,7} + \omega^{2,3,5,6,7}, \omega^{1,3,5,6,7} + \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^0\ \langle\omega^{1,3,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7} - \omega^{2,7} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6} - \omega^{2,7} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}$; $d^\star\omega^{1,3,5} = \omega^{1,7} - \omega^{3,7} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7} + \omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} + \omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7} + \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,5}$; $d^\star\omega^{2,3,5} = \omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,6} + \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{2,5,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,5} - \omega^{2,3,7}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{1,3,6} - \omega^{2,3,7} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7} + \omega^{2,6,7} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{3,4,7}$;

$d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = \omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} - \omega^{3,4,7} + \omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7} + \omega^{3,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7} + \omega^{4,6,7} - \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7} - \omega^{3,4,6} + \omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{2,5,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7} + \omega^{3,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 5\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{1,5} + \omega^{1,6} + \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{1,5} = \omega^{1,4} + 2\omega^{1,5} + \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,4} + \omega^{1,5} + 2\omega^{1,6} + \omega^{2,4} - \omega^{2,7}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,7} - \omega^{3,4} - \omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,4} + \omega^{1,6} + 2\omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,7} = -\omega^{1,6} - \omega^{1,7} + 4\omega^{2,7} + \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4} + \omega^{1,5} + \omega^{2,4} + \omega^{2,7} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{3,7} = -\omega^{1,7} + 5\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = 4\omega^{4,7} - \omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,6} - \omega^{4,7} + 4\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{4,7} - \omega^{5,7} + 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4} + \omega^{1,2,5} + \omega^{1,2,6} + \omega^{1,3,5} - \omega^{2,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,4} + 3\omega^{1,2,5} + \omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,4} + \omega^{1,2,5} + 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,4} + \omega^{1,3,5} + \omega^{2,3,4} + \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,7} + \omega^{1,3,4} + \omega^{1,3,5} + \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4} + \omega^{1,2,7} + \omega^{1,3,4} + 3\omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = \omega^{1,3,4} + \omega^{1,3,5} + 3\omega^{1,3,6} + \omega^{2,3,4} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5} + \omega^{1,4,6} - \omega^{1,5,6} + \omega^{2,4,5} + \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,5} + 3\omega^{1,4,6} + \omega^{1,5,6} - \omega^{1,5,7} + \omega^{2,4,6} - \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{3,4,7} + \omega^{3,5,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,5} + \omega^{1,4,6} + 3\omega^{1,5,6} - \omega^{2,4,5} - \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + 2\omega^{1,5,7} - \omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,4,7} - \omega^{2,6,7} + \omega^{3,4,6} - \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} + \omega^{1,3,4} + \omega^{1,3,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4} - \omega^{1,2,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,2,7} - \omega^{1,3,6} - \omega^{1,3,7} + 4\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{1,5,6} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,4,6} + \omega^{2,3,7} + 2\omega^{2,4,6} - \omega^{2,5,7} +$

$\omega^{3,5,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{1,6,7} + 4\omega^{2,4,7} - \omega^{2,5,7} - \omega^{2,6,7} + \omega^{3,4,5} + \omega^{3,5,7}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} - \omega^{1,5,7} - \omega^{2,4,6} - \omega^{2,4,7} + 3\omega^{2,5,7} - \omega^{2,6,7} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = -\omega^{1,6,7} - \omega^{2,4,7} - \omega^{2,5,7} + 3\omega^{2,6,7} - \omega^{3,4,6} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,4,5} + \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} - \omega^{2,6,7} + 2\omega^{3,4,6} - \omega^{3,5,7}$; $\Delta\omega^{3,4,7} = -\omega^{1,4,7} + 3\omega^{3,4,7} - \omega^{3,5,7} - \omega^{3,6,7}$; $\Delta\omega^{3,5,6} = \omega^{1,4,6} + \omega^{1,5,6} + \omega^{2,4,6} - \omega^{2,6,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,4,7} + \omega^{2,4,7} - \omega^{3,4,6} - \omega^{3,4,7} + 4\omega^{3,5,7} - \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = -\omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,7} + 5\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} - \omega^{4,6,7} + \omega^{5,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 4\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = \omega^{4,5,7} - \omega^{4,6,7} + 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} + \omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 4\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,4} + \omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 5\omega^{1,2,4,5} + \omega^{1,2,4,6} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + \omega^{1,2,4,5} + 4\omega^{1,2,4,6} + \omega^{1,2,5,6} - \omega^{1,2,5,7} + \omega^{1,3,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,5} + \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,5} + \omega^{1,2,4,6} + 3\omega^{1,2,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,6} + 2\omega^{1,2,5,7} - \omega^{1,3,4,5} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,4,6} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,7} + 3\omega^{1,3,4,5} + \omega^{1,3,4,6} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,5} + 3\omega^{1,3,4,6} + \omega^{1,3,5,6} - \omega^{1,3,5,7} + \omega^{2,3,4,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,4,6} - \omega^{1,2,6,7} - \omega^{1,3,4,5} + \omega^{1,3,4,6} + 4\omega^{1,3,5,6} - \omega^{2,3,4,5} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7} + \omega^{2,3,4,7} - \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 4\omega^{1,4,5,6} + \omega^{2,4,5,6} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{3,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{2,5,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,4,5} - \omega^{1,2,5,7} + \omega^{1,3,4,5} - \omega^{1,3,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,6} + 2\omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6} + \omega^{1,3,6,7} + 3\omega^{2,3,4,7} - \omega^{2,3,5,7} - \omega^{2,3,6,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6} - \omega^{1,2,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,4,7} - \omega^{1,3,5,6} - \omega^{1,3,5,7} - \omega^{2,3,4,6} - \omega^{2,3,4,7} + 3\omega^{2,3,5,7} - \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,2,6,7} - \omega^{1,3,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,7} + 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6} - \omega^{1,5,6,7} + 3\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{2,4,5,7} + 3\omega^{2,4,6,7} - \omega^{2,5,6,7} - \omega^{3,4,5,6} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{2,5,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{2,5,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7} - \omega^{3,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,7} + 3\omega^{3,4,6,7} - \omega^{3,5,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{1,4,6,7} + \omega^{2,4,6,7} + \omega^{3,4,5,7} - \omega^{3,4,6,7} + 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + 4\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6} - \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6} + 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = -\omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 5\omega^{1,2,4,5,6} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} - \omega^{1,3,4,5,6} + \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} =$

$-\omega^{1,2,4,6,7}+\omega^{1,2,5,6,7}+4\omega^{1,3,4,5,6}+\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,2,4,6,7}+2\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}-\omega^{2,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,6}+\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,4,5,6}-\omega^{1,3,5,6,7}+2\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{1,2,4,6,7}-\omega^{2,3,4,5,7}+2\omega^{2,3,4,6,7}-\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=5\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= 2e^0\ \langle\omega^1,\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= 3e^0\ \langle\omega^{2,6}, 2\omega^{1,4}-\omega^{1,6}-\omega^{3,5}, \omega^{1,5}-\omega^{1,6}+\omega^{2,4}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= 4e^0\ \langle\omega^{2,5,6}, 5\omega^{1,2,7}-5\omega^{1,3,6}+15\omega^{1,4,5}-23\omega^{1,4,6}+35\omega^{1,5,6}-15\omega^{2,3,7}+5\omega^{2,4,5}+50\omega^{2,4,6}+31\omega^{2,5,7}+8\omega^{3,4,5}-31\omega^{3,5,6}, 5\omega^{1,3,4}-5\omega^{1,3,6}+10\omega^{1,4,5}-16\omega^{1,4,6}+25\omega^{1,5,6}-10\omega^{2,3,7}+5\omega^{2,4,5}+35\omega^{2,4,6}+22\omega^{2,5,7}+6\omega^{3,4,5}-22\omega^{3,5,6}, 5\omega^{1,3,7}+5\omega^{1,4,5}-6\omega^{1,4,6}+5\omega^{1,5,6}+5\omega^{2,4,6}+2\omega^{2,5,7}-4\omega^{3,4,5}-2\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= 4e^0\ \langle-\omega^{1,3,4,7}, (-35\omega^{1,2,4,7}+42\omega^{1,2,6,7}+35\omega^{1,3,4,6}+99\omega^{1,3,5,7}+214\omega^{1,3,6,7}-135\omega^{1,4,5,6}-36\omega^{2,3,4,7}-7\omega^{2,3,5,7}+43\omega^{2,3,6,7}+92\omega^{2,4,5,6}+214\omega^{2,4,5,7}-777\omega^{2,5,6,7}+563\omega^{3,4,5,6})/9380, (-5\omega^{1,2,4,7}+6\omega^{1,2,6,7}+5\omega^{1,3,4,6}-5\omega^{1,3,5,7}-46\omega^{1,3,6,7}+19\omega^{1,4,5,6}+14\omega^{2,3,4,7}-\omega^{2,3,5,7}-13\omega^{2,3,6,7}-6\omega^{2,4,5,6}-46\omega^{2,4,5,7}+157\omega^{2,5,6,7}-111\omega^{3,4,5,6})/1340, (154\omega^{1,2,4,7}-560\omega^{1,2,6,7}-154\omega^{1,3,4,6}-47\omega^{1,3,5,7}+50\omega^{1,3,6,7}+192\omega^{1,4,5,6}+145\omega^{2,3,4,7}+406\omega^{2,3,5,7}+387\omega^{2,3,6,7}-579\omega^{2,4,5,6}+50\omega^{2,4,5,7}+42\omega^{2,5,6,7}-92\omega^{3,4,5,6})/9380\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= 3e^0\ \langle-\omega^{1,3,4,5,7}, (\omega^{1,2,4,6,7}+\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}+\omega^{2,3,4,6,7}+4\omega^{2,3,5,6,7})/10, (2\omega^{1,2,4,6,7}-3\omega^{1,3,5,6,7}-2\omega^{2,3,4,5,7}-3\omega^{2,3,4,6,7}-2\omega^{2,3,5,6,7})/10\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= 2e^0\ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

5.1.8 Algebra $\mathcal{G}_{7,0.8}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_7$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_6$; $[x_2,x_6]=x_7$; $[x_4,x_5]=-x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 0 & 0 & 0 & 0\\ \xi_1^5 & \xi_1^3/2 & 0 & \xi_2^4 & 0 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4 & (-\xi_1^3-2\xi_1^4)/2 & \xi_2^4 & 0 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^3-\xi_1^6 & \xi_1^5 & -\xi_1^4 & 0 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^7, ad\ x_2=-\delta_1^4, ad\ x_3=-(\delta_2^6+\delta_1^7), ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_2^7, ad\ x_7=0$.

Maximal torus : $T=\langle\rangle$.

ch($\mathcal{G}$)= $7e^0\ \langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^0\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $10e^0\ \langle \delta_2^6,\delta_1^3,\delta_2^4,\delta_1^7,\delta_2^7,\delta_3^7,\delta_2^3,\delta_1^6,\delta_1^5,\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^6,\delta_1^4]=\delta_2^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_1^5]=-1/2\delta_2^7$; $[\delta_1^3,\delta_1^4]=1/2\delta_1^6$; $[\delta_2^4,\delta_2^3]=-\delta_2^7$; $[\delta_2^4,\delta_1^6]=\delta_2^7$; $[\delta_2^4,\delta_1^5]=\delta_1^6$; $[\delta_2^4,\delta_1^4]=\delta_2^6+\delta_1^5$; $[\delta_3^7,\delta_2^3]=\delta_2^7$; $[\delta_2^3,\delta_1^4]=\delta_1^7+\delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $4e^0\ \langle \delta_1^3,\delta_2^4,\delta_2^3,\delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^4]=\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $11e^0\ \langle -2x_1\omega^{1,2}+3x_3(\omega^{1,5}-2/3\omega^{2,3})-x_4\omega^{2,4}+x_5\omega^{2,5}, 3/2x_1\omega^{1,5}-3/2x_2(\omega^{1,2}+2/3\omega^{1,6}+2/3\omega^{2,5}-2/3\omega^{3,4})+3x_3(\omega^{1,3}+\omega^{1,4}-1/2\omega^{2,6}+1/2\omega^{4,5})-3/2x_4(\omega^{1,3}+\omega^{1,4}-2/3\omega^{2,6}-2/3\omega^{3,5}+1/3\omega^{4,5})+x_5(\omega^{3,6}+\omega^{4,6}), x_1\omega^{2,4}-x_2\omega^{1,2}+2x_3(\omega^{1,3}+\omega^{1,4}-1/2\omega^{2,6}+1/2\omega^{4,5})-x_4\omega^{1,4}+x_5\omega^{2,3}, -3x_1(\omega^{1,3}+7/6\omega^{1,4})-7/2x_2(\omega^{1,5}-2/7\omega^{2,3})-7/2x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+7/2x_4(\omega^{1,2}+\omega^{1,6}-3/7\omega^{3,4})+3/2x_5\omega^{3,5}+x_6\omega^{3,6}, -x_1(\omega^{1,3}+5/2\omega^{1,4}-\omega^{2,6}+\omega^{4,5})-5/2x_2(\omega^{1,5}-2/5\omega^{2,3})-5/2x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+5/2x_4(\omega^{1,2}+\omega^{1,6}+2/5\omega^{2,7}-3/5\omega^{3,4})+1/2x_5(\omega^{3,5}+2\omega^{4,7})+x_6\omega^{5,7}, -x_1\omega^{2,4}+x_2\omega^{1,2}-2x_3(\omega^{1,4}-1/2\omega^{2,6}+1/2\omega^{4,5})+x_4\omega^{1,4}+x_6\omega^{3,4}, -x_2\omega^{2,4}-x_3(\omega^{1,6}+\omega^{2,5}-\omega^{3,4})+x_4(\omega^{1,2}-\omega^{2,5})+x_5(\omega^{1,3}-\omega^{2,6}), -x_3\omega^{1,5}+x_4\omega^{1,5}, x_3\omega^{2,4}+x_5\omega^{1,2}, x_3\omega^{2,3}+x_6\omega^{1,3}, -x_2\omega^{1,2}+2x_3(\omega^{1,4}-1/2\omega^{2,6}+1/2\omega^{4,5})-x_4\omega^{1,4}+x_6\omega^{2,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$17e^0$.

$\mathrm{ch}(H^4_{\mathcal{G}})=17e^0$.
$\mathrm{ch}(H^5_{\mathcal{G}})=15e^0$.
$\mathrm{ch}(H^6_{\mathcal{G}})=10e^0$.
$\mathrm{ch}(H^7_{\mathcal{G}})=3e^0$.
Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 10, 50, 114, 148, 112, 45, 7)$
Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 4, 11, 17, 17, 15, 10, 3)$
S=0.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 8, 17, 22, 17, 7, 1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 4, 4, 4, 4, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,3} - \omega^{2,6} + \omega^{4,5}$; $d\omega^{1,6} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,3} - \omega^{2,4,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{2,3,6} - \omega^{3,4,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} + \omega^{1,3,4} - \omega^{2,4,6}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = \omega^{1,3,5} - \omega^{1,4,7} - \omega^{2,5,6}$; $d\omega^{6,7} = \omega^{1,3,6} - \omega^{1,5,7} - \omega^{2,3,7} - \omega^{2,4,7} - \omega^{4,5,6}$; $d\omega^{1,2,7} = \omega^{1,2,4,5}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,6} + \omega^{1,3,4,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,6}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,5,6}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} + \omega^{1,2,4,7} + \omega^{1,4,5,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = \omega^{2,3,4,5}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = \omega^{1,2,3,5} - \omega^{1,2,4,7}$; $d\omega^{2,6,7} = \omega^{1,2,3,6} - \omega^{1,2,5,7} + \omega^{2,4,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{2,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,5} - \omega^{2,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,6} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,5,6} - \omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $d\omega^{1,2,3,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7} = \omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,4,6} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,5,6} - \omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,3,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= 3e^0\ \langle\omega^1,\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,5},\omega^{2,3},\omega^{1,6}+\omega^{2,5}-\omega^{3,4},\omega^{1,3}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,4,6}+\omega^{2,4,7},\omega^{1,2,6},\omega^{1,5,6}-\omega^{2,4,6}+\omega^{3,4,5},\omega^{1,3,6}-\omega^{2,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,2,3,7}-\omega^{2,4,5,7},\omega^{1,3,5,6}-\omega^{2,4,5,7},(2\omega^{1,5,6,7}+2\omega^{2,3,5,6}-\omega^{2,3,6,7}-2\omega^{2,4,6,7}+\omega^{3,4,5,7})/2,\omega^{1,2,6,7}+\omega^{1,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,2,3,6,7}-\omega^{2,4,5,6,7},\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7},\omega^{1,2,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= 3e^0\ \langle\omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^7$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,6}=-\omega^7$; $d^\star\omega^{4,5}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}+\omega^{4,7}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}+\omega^{5,7}$; $d^\star\omega^{1,3,6}=\omega^{6,7}$; $d^\star\omega^{1,4,5}=-\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,6}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{5,6}$; $d^\star\omega^{2,3,6}=-\omega^{3,7}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=-\omega^{2,7}+\omega^{5,6}$; $d^\star\omega^{2,4,6}=-\omega^{4,7}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=-\omega^{3,7}$; $d^\star\omega^{4,5,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,6}-\omega^{1,4,6}-\omega^{2,3,5}+\omega^{2,4,7}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,6}-\omega^{2,3,6}+\omega^{2,5,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,3,7}+\omega^{2,6,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,2,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6}=\omega^{1,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,3,7}-\omega^{3,4,6}-\omega^{4,5,7}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,5,6}-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{1,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,3,7}+\omega^{3,5,6}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,3,7}-\omega^{1,3,5,6}+\omega^{1,4,5,6}-\omega^{2,3,4,6}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,4,6}=$

$\omega^{1,3,4,7} - \omega^{2,3,5,6} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,3,5,7} - \omega^{2,5,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,3,6,7} + \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,3,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} + \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 3\omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,6} - \omega^{4,5}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{2,5} - \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{1,3} + 2\omega^{2,6}$; $\Delta\omega^{2,7} = -\omega^{1,6} + 2\omega^{2,7} - \omega^{3,4} - \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5} - \omega^{4,7}$; $\Delta\omega^{3,6} = 2\omega^{3,6} - \omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{1,3} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,5} + 3\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{2,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{3,6} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3} + \omega^{1,2,4} + \omega^{2,4,5}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,4} - \omega^{1,5,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,7} + 2\omega^{1,3,4} - \omega^{2,4,6}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{1,4,7} + \omega^{2,3,4} - \omega^{2,5,6}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,3,5} - \omega^{2,3,7} - \omega^{4,5,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{2,6,7} - \omega^{4,5,7}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} - \omega^{2,4,5} - \omega^{2,4,7}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,5} + 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,2,7} + 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,6} - \omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,3,6} + 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,5} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} + \omega^{2,4,5} - \omega^{2,4,7}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6} - \omega^{2,5,7}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,6} + \omega^{1,5,7} + 2\omega^{2,3,7} + \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,3} - \omega^{1,4,6} + \omega^{2,3,5} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,3,4} + \omega^{1,5,6} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{1,5,7} - \omega^{2,3,5} + \omega^{2,3,7} + 2\omega^{2,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,3,5} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} - \omega^{1,6,7} - \omega^{2,3,6} + 2\omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = \omega^{1,3,7} + 3\omega^{2,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + \omega^{2,5,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,6,7} + 2\omega^{3,4,6} + \omega^{4,5,7}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{2,3,7} + 2\omega^{3,5,6} - \omega^{4,5,6} + \omega^{4,6,7}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,3,6} + \omega^{2,4,7} - \omega^{3,5,6} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{1,3,7} + \omega^{3,4,6} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{3,5,6} - \omega^{3,6,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,4,5} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} + \omega^{1,2,4,7} + \omega^{1,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,5} + \omega^{1,2,3,7} + 2\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,5,7} + \omega^{1,3,4,5}$;

$\Delta\omega^{1,2,6,7}=2\omega^{1,2,6,7}+\omega^{1,3,4,6}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,5,7}+3\omega^{1,3,4,5}+\omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,6}=\omega^{1,2,6,7}+2\omega^{1,3,4,6}+\omega^{1,4,5,7}-\omega^{2,3,4,5}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,7}=2\omega^{1,3,4,7}-\omega^{2,4,6,7}$; $\Delta\omega^{1,3,5,6}=\omega^{1,2,3,7}+2\omega^{1,3,5,6}-\omega^{1,4,5,6}+\omega^{1,4,6,7}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7}=2\omega^{1,3,5,7}+\omega^{2,3,4,7}-\omega^{2,5,6,7}$; $\Delta\omega^{1,3,6,7}=2\omega^{1,3,6,7}-\omega^{1,4,6,7}-\omega^{2,3,5,7}-\omega^{4,5,6,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,4,7}-\omega^{1,3,5,6}+2\omega^{1,4,5,6}-\omega^{2,3,4,6}-\omega^{2,4,5,7}$; $\Delta\omega^{1,4,5,7}=\omega^{1,3,4,6}+2\omega^{1,4,5,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7}=\omega^{1,3,5,6}-\omega^{1,3,6,7}+2\omega^{1,4,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,5,6}-\omega^{1,3,4,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,3,5,6}-\omega^{1,4,5,6}+2\omega^{2,3,4,6}+\omega^{2,4,5,7}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,6}+\omega^{1,3,5,7}-\omega^{1,4,5,7}+2\omega^{2,3,4,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6}=2\omega^{2,3,5,6}+\omega^{2,4,6,7}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,5,6}-\omega^{1,3,6,7}+2\omega^{2,3,5,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,5,6,7}+2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6}=\omega^{1,2,3,6}+\omega^{1,3,4,5}+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,2,3,7}-\omega^{1,4,5,6}-\omega^{1,4,6,7}+\omega^{2,3,4,6}+\omega^{2,3,5,7}+2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=-\omega^{1,3,4,7}+\omega^{1,5,6,7}+\omega^{2,3,5,6}+2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=-\omega^{1,3,5,7}+2\omega^{2,5,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,6}=-\omega^{2,3,4,7}-\omega^{2,5,6,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,5,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}-\omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7}=-\omega^{1,3,6,7}-\omega^{3,5,6,7}+3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=5\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=3\omega^{1,2,3,4,6}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=3\omega^{1,2,3,5,6}+\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,5,7}=3\omega^{1,2,3,5,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,4,5,6}=2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,3,4,6}+\omega^{1,2,3,5,7}+2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,3,5,6}+\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,4,7}-\omega^{1,2,5,6,7}+2\omega^{1,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7}=2\omega^{1,3,4,5,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,5,6}-\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=3\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= 3e^0\ \langle\omega^1,\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,5}-\omega^{2,4},\omega^{2,3}-\omega^{2,4},\omega^{1,6}+\omega^{2,5}-\omega^{3,4},2\omega^{1,3}-\omega^{2,6}+\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= 4e^0\ \langle\omega^{1,2,6}-2\omega^{1,3,5}+\omega^{1,4,5}-\omega^{1,4,7}+2\omega^{2,3,4}-\omega^{2,5,6},\omega^{1,2,7}+2\omega^{1,5,6}-\omega^{2,3,6}-\omega^{2,4,6}+\omega^{3,4,5},3\omega^{1,3,6}+2\omega^{1,5,7}+2\omega^{2,3,7}-3\omega^{2,4,7}+2\omega^{4,5,6},3\omega^{1,4,6}-2\omega^{1,5,7}+3\omega^{2,3,5}-2\omega^{2,3,7}+6\omega^{2,4,7}-2\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= 4e^0\ \langle(\omega^{1,3,4,7}-2\omega^{1,5,6,7}-\omega^{2,3,5,6}+\omega^{2,3,6,7}+2\omega^{2,4,6,7}-\omega^{3,4,5,7})/12,(\omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{1,4,5,7}+2\omega^{2,3,4,7}+\omega^{3,4,5,6})/8,(-4\omega^{1,2,3,7}+\omega^{1,3,5,6}+4\omega^{1,4,5,6}+5\omega^{1,4,6,7}-4\omega^{2,3,4,6}-5\omega^{2,3,5,7}+11\omega^{2,4,5,7})/60,(-\omega^{1,3,5,6}+\omega^{1,4,6,7}-\omega^{2,3,5,7}+\omega^{2,4,5,7})/12\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= 4e^0\ \langle(\omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} - 2\omega^{2,3,4,6,7})/3, (\omega^{1,3,5,6,7} + 2\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7})/3, (-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3, (\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} - 2\omega^{2,4,5,6,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= 3e^0\ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^0\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.2 rank 1 Lie Algebras with a null weight

5.2.1 Algebra $\mathcal{G}_{7,1.01(i)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5+x_7$; $[x_3,x_4]=-x_6$; $[x_3,x_5]=-x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^4 & \xi_2^4 & \xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -(\xi_1^2+\xi_1^5) & \xi_2^5 & \xi_2^4 & \xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & \xi_2^5 & \xi_2^4 & \xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^7, ad\ x_3=-(\delta_2^5+\delta_2^7), ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_1^6+\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_2\rangle$.

ch($\mathcal{G}$)= $2e^0\ \langle x_1,x_3\rangle+5e^{\beta_2}\ \langle x_2,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $6e^{\beta_2}\ \langle \delta_1^4,\delta_1^6,\delta_1^2,\delta_1^5,\delta_1^7,\delta_3^7\rangle+5e^0\ \langle\delta_2^7,\delta_2^6,\delta_2^5,\delta_2^4,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=\delta_3^7-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^5$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=\delta_3^7-\delta_1^6$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^7-\delta_1^4$; $[\delta_1^5,\delta_2^5]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=\delta_3^7-\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $2e^{\beta_2}\ \langle\delta_1^2,\delta_1^4\rangle+3e^0\ \langle\delta_2^6,\delta_2^5,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{\beta_2}\ \langle x_2\omega^{1,3}\rangle+4e^{-\beta_2}\ \langle x_1\omega^{1,2}-2x_3\omega^{2,3}+3x_4\omega^{2,4}+2x_5\omega^{2,5}+x_6\omega^{2,6}, x_7\omega^{2,4}, -2x_1\omega^{1,4}+3x_2\omega^{2,4}+2x_3(\omega^{1,2}+\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,5}-2x_5(\omega^{2,4}+1/2\omega^{4,5})+x_6(\omega^{2,7}-\omega^{4,6})+x_7\omega^{4,7}, -x_1\omega^{1,2}+2x_3\omega^{2,3}-2x_4\omega^{2,4}-x_5\omega^{2,5}+x_7\omega^{2,7}\rangle+8e^0\ \langle x_2(\omega^{1,2}+\omega^{1,5}-\omega^{3,4})-x_3\omega^{1,3}+x_4(\omega^{1,4}-\omega^{3,5}), x_1\omega^{1,3}+x_2(\omega^{1,6}-3\omega^{2,3}-\omega^{3,5})+x_4(\omega^{1,5}+\omega^{3,4}-\omega^{3,6}), -x_1\omega^{1,3}-2x_2(\omega^{1,6}-\omega^{2,3}-\omega^{3,5})-x_4(\omega^{1,5}-2\omega^{3,6})+x_5\omega^{1,6}, -x_2(\omega^{1,7}-\omega^{3,4}-\omega^{3,6})-x_4(\omega^{2,3}-\omega^{3,5}-\omega^{3,7})+x_5\omega^{1,7}, -x_1\omega^{1,3}-x_2(\omega^{1,4}+\omega^{1,6}-2\omega^{2,3}-\omega^{3,5})-x_4(\omega^{1,5}-\omega^{3,6})+x_5\omega^{1,4}, -x_2\omega^{1,2}-x_4\omega^{2,3}+x_5\omega^{1,2}, -x_2(\omega^{1,5}-\omega^{3,4})-x_4(\omega^{2,3}-\omega^{3,5})+x_5\omega^{1,5}, x_2\omega^{2,3}-x_4(\omega^{1,7}-\omega^{3,6})+x_5\omega^{3,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$8e^{-\beta_2}+3e^{-2\beta_2}+e^{-3\beta_2}+5e^0$.

$\mathrm{ch}(H^4_{\mathcal{G}})=4e^{-\beta_2}+6e^{-2\beta_2}+4e^{-3\beta_2}+2e^{-4\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-2\beta_2}+5e^{-3\beta_2}+5e^{-4\beta_2}+2e^{-5\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-3\beta_2}+4e^{-4\beta_2}+4e^{-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-4\beta_2}+2e^{-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,11,51,113,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,5,13,17,16,15,10,3)$

S$=5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}+\omega^{3,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,3}+\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=-\omega^{1,3,4}$; $d\omega^{1,7}=\omega^{1,2,3}-\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}-\omega^{2,3,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}+\omega^{2,3,4}+\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{2,3,5}-\omega^{2,3,7}$; $d\omega^{6,7}=-\omega^{1,5,7}+\omega^{2,3,6}+\omega^{3,4,7}-\omega^{3,5,6}$; $d\omega^{1,2,6}=\omega^{1,2,3,4}$; $d\omega^{1,2,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=-\omega^{1,2,3,4}-\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{1,5,7}=-\omega^{1,2,3,5}+\omega^{1,2,3,7}$; $d\omega^{1,6,7}=-\omega^{1,2,3,6}-\omega^{1,3,4,7}+\omega^{1,3,5,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}-\omega^{2,3,4,5}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}-\omega^{2,3,4,6}-\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,5,6}-\omega^{2,3,6,7}-\omega^{3,4,5,7}$; $d\omega^{1,2,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=\omega^{1,2,3,4,5}-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=\omega^{1,2,3,4,6}+\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,5,6}+\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,6}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,5,6}-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $2e^0\ \langle\omega^1,\omega^3\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^0\ \langle\omega^{1,3}\rangle + e^{-2\beta_2}\ \langle\omega^{2,4}\rangle + 2e^{-\beta_2}\ \langle\omega^{1,4},\omega^{1,7}-\omega^{3,4}-\omega^{3,6}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-\beta_2}\ \langle\omega^{1,3,7}\rangle + e^{-3\beta_2}\ \langle\omega^{2,4,5}\rangle + 2e^{-2\beta_2}\ \langle\omega^{1,4,7}-\omega^{2,3,5}-\omega^{2,3,7}-\omega^{3,4,6},\omega^{1,2,7}-\omega^{3,4,5}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-2\beta_2}\ \langle\omega^{1,3,6,7}\rangle + 2e^{-3\beta_2}\ \langle\omega^{1,2,6,7}+\omega^{2,3,5,7}+2\omega^{3,4,5,6},\omega^{1,2,5,7}\rangle + e^{-4\beta_2}\ \langle\omega^{2,4,5,6}\rangle$.

ch($H^5_{\mathbb{C}}$)= $e^{-3\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + 2e^{-4\beta_2}\ \langle\omega^{1,2,4,5,7},\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}-\omega^{2,3,5,6,7}\rangle + e^{-5\beta_2}\ \langle\omega^{2,4,5,6,7}\rangle$.

ch($H^6_{\mathbb{C}}$)= $2e^{-5\beta_2}\ \langle\omega^{1,2,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle$.

ch($H^7_{\mathbb{C}}$)= $e^{-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5-\omega^7$; $d^\star\omega^{3,4}=\omega^6$; $d^\star\omega^{3,5}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}+\omega^{1,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,6}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,6}+\omega^{4,5}+\omega^{4,7}$; $d^\star\omega^{2,3,5}=-\omega^{2,7}+\omega^{5,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}+\omega^{6,7}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{3,4,7}=\omega^{6,7}$; $d^\star\omega^{3,5,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,6}-\omega^{1,4,5}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,2,7}-\omega^{1,5,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{1,6,7}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,4,7}+\omega^{1,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=\omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{4,5,6}-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{2,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{2,6,7}-\omega^{5,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{4,6,7}$; $d^\star\omega^{3,4,5,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,4,7}-\omega^{1,2,5,6}+$

$\omega^{1,4,5,7}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,4,5,6}+\omega^{1,4,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,2,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,2,6,7}+\omega^{1,5,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7}=\omega^{1,5,6,7}-\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,4,6,7}+\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7}=\omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=-\omega^{1,2,4,6,7}-\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{1,2,5,6,7}-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=-\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^{4}=\omega^{4}$; $\Delta\omega^{5}=2\omega^{5}+\omega^{7}$; $\Delta\omega^{6}=2\omega^{6}$; $\Delta\omega^{7}=\omega^{5}+3\omega^{7}$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,4}=\omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,5}=2\omega^{1,5}+\omega^{1,7}$; $\Delta\omega^{1,6}=2\omega^{1,6}+\omega^{2,3}$; $\Delta\omega^{1,7}=\omega^{1,5}+2\omega^{1,7}+\omega^{3,4}+\omega^{3,6}$; $\Delta\omega^{2,3}=\omega^{1,4}+\omega^{1,6}+2\omega^{2,3}-\omega^{3,5}$; $\Delta\omega^{2,5}=\omega^{2,5}$; $\Delta\omega^{2,6}=2\omega^{2,6}-\omega^{4,7}$; $\Delta\omega^{2,7}=2\omega^{2,7}+\omega^{4,6}-\omega^{5,7}$; $\Delta\omega^{3,4}=\omega^{1,7}+2\omega^{3,4}$; $\Delta\omega^{3,5}=-\omega^{2,3}+2\omega^{3,5}$; $\Delta\omega^{3,6}=\omega^{1,7}+\omega^{3,6}$; $\Delta\omega^{3,7}=\omega^{3,7}$; $\Delta\omega^{4,5}=2\omega^{4,5}+\omega^{4,7}$; $\Delta\omega^{4,6}=\omega^{2,7}+2\omega^{4,6}$; $\Delta\omega^{4,7}=-\omega^{2,6}+\omega^{4,5}+4\omega^{4,7}$; $\Delta\omega^{5,6}=3\omega^{5,6}-\omega^{6,7}$; $\Delta\omega^{5,7}=-\omega^{2,7}+4\omega^{5,7}$; $\Delta\omega^{6,7}=-\omega^{5,6}+4\omega^{6,7}$; $\Delta\omega^{1,2,3}=3\omega^{1,2,3}-\omega^{1,3,5}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=2\omega^{1,2,5}$; $\Delta\omega^{1,2,6}=3\omega^{1,2,6}-\omega^{1,4,7}$; $\Delta\omega^{1,2,7}=2\omega^{1,2,7}+\omega^{1,4,6}-\omega^{1,5,7}-\omega^{2,3,4}-\omega^{2,3,6}$; $\Delta\omega^{1,3,4}=2\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=-\omega^{1,2,3}+2\omega^{1,3,5}$; $\Delta\omega^{1,3,6}=\omega^{1,3,6}$; $\Delta\omega^{1,4,5}=2\omega^{1,4,5}+\omega^{1,4,7}+\omega^{2,3,5}$; $\Delta\omega^{1,4,6}=\omega^{1,2,7}+2\omega^{1,4,6}-\omega^{2,3,4}+\omega^{2,3,6}$; $\Delta\omega^{1,4,7}=-\omega^{1,2,6}+\omega^{1,4,5}+3\omega^{1,4,7}+\omega^{2,3,7}+\omega^{3,4,6}$; $\Delta\omega^{1,5,6}=3\omega^{1,5,6}-\omega^{1,6,7}-\omega^{2,3,5}$; $\Delta\omega^{1,5,7}=-\omega^{1,2,7}+3\omega^{1,5,7}-\omega^{3,4,5}+\omega^{3,5,6}$; $\Delta\omega^{1,6,7}=-\omega^{1,5,6}+3\omega^{1,6,7}+\omega^{2,3,7}-\omega^{3,4,6}$; $\Delta\omega^{2,3,4}=-\omega^{1,2,7}-\omega^{1,4,6}+3\omega^{2,3,4}+\omega^{3,4,5}$; $\Delta\omega^{2,3,5}=\omega^{1,4,5}-\omega^{1,5,6}+3\omega^{2,3,5}-\omega^{2,3,7}$; $\Delta\omega^{2,3,6}=-\omega^{1,2,7}+\omega^{1,4,6}+3\omega^{2,3,6}+\omega^{3,4,7}-\omega^{3,5,6}$; $\Delta\omega^{2,3,7}=\omega^{1,4,7}+\omega^{1,6,7}-\omega^{2,3,5}+2\omega^{2,3,7}-\omega^{3,4,6}$; $\Delta\omega^{2,4,6}=\omega^{2,4,6}$; $\Delta\omega^{2,4,7}=2\omega^{2,4,7}+\omega^{4,5,7}$; $\Delta\omega^{2,5,6}=2\omega^{2,5,6}-\omega^{4,5,7}$; $\Delta\omega^{2,5,7}=2\omega^{2,5,7}+\omega^{4,5,6}$; $\Delta\omega^{2,6,7}=3\omega^{2,6,7}-\omega^{5,6,7}$; $\Delta\omega^{3,4,5}=-\omega^{1,5,7}+\omega^{2,3,4}+3\omega^{3,4,5}$; $\Delta\omega^{3,4,6}=\omega^{1,4,7}-\omega^{1,6,7}-\omega^{2,3,7}+2\omega^{3,4,6}$; $\Delta\omega^{3,4,7}=\omega^{2,3,6}+3\omega^{3,4,7}$; $\Delta\omega^{3,5,6}=\omega^{1,5,7}-\omega^{2,3,6}+2\omega^{3,5,6}$; $\Delta\omega^{3,5,7}=2\omega^{3,5,7}$; $\Delta\omega^{3,6,7}=\omega^{3,6,7}$; $\Delta\omega^{4,5,6}=\omega^{2,5,7}+2\omega^{4,5,6}-\omega^{4,6,7}$; $\Delta\omega^{4,5,7}=\omega^{2,4,7}-\omega^{2,5,6}+4\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=-\omega^{4,5,6}+4\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=-\omega^{2,6,7}+4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4}=4\omega^{1,2,3,4}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,5}=4\omega^{1,2,3,5}-\omega^{1,2,3,7}$; $\Delta\omega^{1,2,3,6}=4\omega^{1,2,3,6}+\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $\Delta\omega^{1,2,3,7}=-\omega^{1,2,3,5}+2\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5}=\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7}=2\omega^{1,2,4,7}+\omega^{1,4,5,7}-\omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6}=3\omega^{1,2,5,6}-\omega^{1,4,5,7}$; $\Delta\omega^{1,2,5,7}=2\omega^{1,2,5,7}+\omega^{1,4,5,6}+\omega^{2,3,4,5}-\omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7}=3\omega^{1,2,6,7}-\omega^{1,5,6,7}+\omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,3,4}+3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,3,7}+2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7}=\omega^{1,2,3,6}+2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6}=-\omega^{1,2,3,6}+2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7}=\omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,5,7}+2\omega^{1,4,5,6}-\omega^{1,4,6,7}+$

$\omega^{2,3,4,5}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}+3\omega^{1,4,5,7}+\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7}=-\omega^{1,4,5,6}+3\omega^{1,4,6,7}-\omega^{2,3,4,7}+\omega^{2,3,6,7}$; $\Delta\omega^{1,5,6,7}=-\omega^{1,2,6,7}+3\omega^{1,5,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,5,7}+\omega^{1,4,5,6}+3\omega^{2,3,4,5}-\omega^{2,3,4,7}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,2,4,7}+\omega^{1,2,6,7}+3\omega^{2,3,4,6}+\omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,4,6,7}-\omega^{2,3,4,5}+3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6}=-\omega^{1,2,5,7}+\omega^{1,4,5,6}+3\omega^{2,3,5,6}+\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{2,3,5,7}=\omega^{1,4,5,7}-\omega^{1,5,6,7}+2\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7}=\omega^{1,4,6,7}+\omega^{2,3,5,6}+2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7}=\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=2\omega^{2,4,6,7}+\omega^{4,5,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{1,4,5,7}+\omega^{1,5,6,7}+\omega^{2,3,4,6}-\omega^{2,3,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{2,3,5,6}+3\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=\omega^{2,4,6,7}+3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}-\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,4,7}=-\omega^{1,2,3,4,5}+3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6}=4\omega^{1,2,3,5,6}+\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,5,6}+2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7}=2\omega^{1,2,4,6,7}+\omega^{1,4,5,6,7}$; $\Delta\omega^{1,2,5,6,7}=2\omega^{1,2,5,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,4,6}-\omega^{1,2,3,5,7}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,2,3,5,6}+2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,2,4,6,7}+2\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,2,4,5,7}-\omega^{1,2,5,6,7}+2\omega^{2,3,4,5,6}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,4,5,6,7}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,5,6}+2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}+\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,5,6}+2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= 2e^{0}\ \langle\omega^1,\omega^3\rangle+e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{0}\ \langle\omega^{1,3}\rangle+e^{-2\beta_2}\ \langle\omega^{2,4}\rangle+2e^{-\beta_2}\ \langle 2\omega^{1,4}+\omega^{1,6}-2\omega^{2,3}-\omega^{3,5},\omega^{1,5}-2\omega^{1,7}+\omega^{3,4}+2\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-\beta_2}\ \langle\omega^{1,3,7}\rangle+e^{-3\beta_2}\ \langle\omega^{2,4,5}\rangle+2e^{-2\beta_2}\ \langle 3\omega^{1,2,6}-3\omega^{1,4,5}+9\omega^{1,4,7}-\omega^{1,5,6}-3\omega^{2,3,5}-11\omega^{2,3,7}-10\omega^{3,4,6},11\omega^{1,2,7}-10\omega^{1,4,6}+3\omega^{1,5,7}+9\omega^{2,3,6}+\omega^{3,4,5}-3\omega^{3,4,7}+3\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-4\beta_2}\ \langle-\omega^{2,4,5,6}\rangle+e^{-2\beta_2}\ \langle-\omega^{1,3,6,7}\rangle+2e^{-3\beta_2}\ \langle(10\omega^{1,2,5,7}-11\omega^{1,4,5,6}-3\omega^{1,4,6,7}-\omega^{2,3,4,7}+9\omega^{2,3,5,6}-3\omega^{2,3,6,7}-3\omega^{3,4,5,7})/330,(3\omega^{1,2,4,7}-3\omega^{1,2,5,6}+\omega^{1,2,6,7}-9\omega^{1,4,5,7}-3\omega^{2,3,4,6}+10\omega^{2,3,5,7}+11\omega^{3,4,5,6})/330\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-5\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle+e^{-3\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle+2e^{-4\beta_2}\ \langle(\omega^{1,2,4,6,7}-2\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}+2\omega^{2,3,5,6,7})/10,(2\omega^{1,2,4,5,7}+\omega^{1,2,5,6,7}+2\omega^{2,3,4,5,6}-\omega^{2,3,4,6,7})/10\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= 2e^{-5\beta_2}\ \langle\omega^{2,3,4,5,6,7},\omega^{1,2,4,5,6,7}\rangle+e^{-4\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

5.2.2 Algebra $\mathcal{G}_{7,1.01(ii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6+x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & \xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -(\xi_1^2+\xi_1^4) & -\xi_1^3+\xi_2^5 & \xi_2^4 & \xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3+\xi_2^6 & -2\xi_1^3+\xi_2^5 & \xi_2^4 & \xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^7, ad\ x_3=-(\delta_2^6+\delta_2^7), ad\ x_4=-\delta_1^5+\delta_3^7, ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_2\rangle$.

ch($\mathcal{G}$)= $2e^0\ \langle x_1,x_3\rangle+5e^{\beta_2}\ \langle x_2,x_4,x_5,x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $6e^{\beta_2}\ \langle \delta_1^5,\delta_1^6,\delta_1^2,\delta_1^4,\delta_1^7,\delta_3^7\rangle+6e^0\ \langle\delta_2^7,\delta_2^6,\delta_1^3,\delta_2^5,\delta_2^4,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^3]=2\delta_1^7$; $[\delta_1^5,\delta_2^5]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^6$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_1^3]=-2\delta_3^7-\delta_1^6-\delta_1^5$; $[\delta_1^2,\delta_2^5]=\delta_3^7-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^7-\delta_1^4$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_1^5$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^5]=-2\delta_2^7$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_2^6)$.

ch($H^1_{\mathcal{G}}$)= $2e^{\beta_2}\ \langle\delta_1^2,\delta_1^4\rangle+4e^0\ \langle\delta_2^6,\delta_1^3,\delta_2^5,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^5]=2\delta_2^6$; $[\delta_1^2,\delta_1^3]=-3\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $e^{\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+5e^{-\beta_2}\ \langle x_1(\omega^{1,2}+\omega^{1,4})-x_2\omega^{2,4}-x_3(\omega^{1,6}+2\omega^{2,3}-\omega^{3,4})+2x_4\omega^{2,4}+x_5(\omega^{2,5}-\omega^{2,6}+\omega^{4,5}), x_3(\omega^{1,2}+\omega^{1,4})+x_5\omega^{2,4}, -2x_1(\omega^{1,2}+\omega^{1,4})+2x_2\omega^{2,4}-x_3(\omega^{1,2}+\omega^{1,4}-\omega^{1,5}-2\omega^{1,6}-2\omega^{2,3}+2\omega^{3,4})-2x_4\omega^{2,4}+2x_5(\omega^{2,6}-\omega^{4,5})+x_6(\omega^{2,5}+\omega^{4,5}), x_3\omega^{1,2}+x_7\omega^{2,5}, x_1\omega^{1,4}-x_2\omega^{2,4}-x_3(\omega^{1,6}-\omega^{2,3}-\omega^{3,4})-x_5(\omega^{2,6}-\omega^{4,5})+x_7\omega^{2,7}\rangle+9e^0\ \langle -x_1\omega^{1,3}-2x_2(\omega^{1,5}-\omega^{2,3})+2x_4(\omega^{1,5}-1/2\omega^{1,6})+x_5\omega^{3,5}, -x_2(\omega^{1,6}+\omega^{2,3}-\omega^{3,4})+x_4(\omega^{1,6}+\omega^{3,5})+x_5\omega^{3,6}, -x_2(\omega^{1,7}+\omega^{2,3}-\omega^{3,4}-\omega^{3,5})+x_4(\omega^{1,7}+\omega^{3,6})+x_5\omega^{3,7}, x_2\omega^{1,2}-x_4\omega^{1,2}+x_5\omega^{2,3}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}, -x_2(\omega^{1,2}+\omega^{1,4})+x_3\omega^{1,3}-x_4\omega^{2,3}+$

$x_5\omega^{3,4}, x_5\omega^{1,5}, x_1\omega^{1,3} + x_2(\omega^{1,5} - 3\omega^{2,3}) - x_4(\omega^{1,5} - \omega^{1,6} - \omega^{3,4}), -x_1\omega^{1,3} - x_2(\omega^{1,5} - 2\omega^{2,3}) + x_4(\omega^{1,5} - \omega^{1,6} - \omega^{2,3}) + x_5\omega^{1,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=10e^{-\beta_2} + 7e^{-2\beta_2} + 2e^{-3\beta_2} + 5e^0$.

$\mathrm{ch}(H^4_{\mathcal{G}})=5e^{-\beta_2} + 11e^{-2\beta_2} + 7e^{-3\beta_2} + 2e^{-4\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=5e^{-2\beta_2} + 8e^{-3\beta_2} + 5e^{-4\beta_2} + 2e^{-5\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=3e^{-3\beta_2} + 4e^{-4\beta_2} + 4e^{-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-4\beta_2} + 2e^{-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 12, 53, 118, 152, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 6, 16, 24, 25, 20, 11, 3)$

S=$5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3} + \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,3} - \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} - \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6} - \omega^{2,3,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,3,7} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6} + \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{a}})= 2e^0 \langle \omega^1, \omega^3\rangle + e^{-\beta_2} \langle \omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^0 \langle \omega^{1,3}\rangle + 2e^{-2\beta_2} \langle \omega^{2,4}, \omega^{2,6} - \omega^{4,5}\rangle + 2e^{-\beta_2} \langle \omega^{1,5}, \omega^{1,7} - \omega^{3,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-\beta_2} \langle \omega^{1,3,7}\rangle + 4e^{-2\beta_2} \langle \omega^{1,2,6}, \omega^{1,4,7} - \omega^{2,3,5}, \omega^{1,5,7} + \omega^{2,3,7} + \omega^{3,4,5} + \omega^{3,4,6}, \omega^{1,6,7} - \omega^{2,3,7} - \omega^{3,4,7} + 2\omega^{3,5,6}\rangle + 2e^{-3\beta_2} \langle \omega^{2,4,5}, \omega^{2,4,7} - \omega^{2,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= 4e^{-3\beta_2} \langle \omega^{1,2,4,7}, \omega^{1,2,6,7} - \omega^{2,3,5,6}, \omega^{1,4,5,7} + \omega^{2,3,4,6}, \omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{3,4,5,6} - \omega^{3,4,5,7}\rangle + 2e^{-2\beta_2} \langle \omega^{1,3,4,7}, \omega^{1,3,6,7}\rangle + e^{-4\beta_2} \langle \omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= 2e^{-3\beta_2} \langle \omega^{1,2,3,6,7}, \omega^{1,3,5,6,7}\rangle + 2e^{-4\beta_2} \langle \omega^{1,2,4,6,7}, \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}\rangle + e^{-5\beta_2} \langle \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= 2e^{-5\beta_2} \langle \omega^{1,2,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_2} \langle \omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-5\beta_2} \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6 - \omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{1,7} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} + \omega^{4,6} + \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,6} + \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} - \omega^{1,4,6} - \omega^{1,4,7} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{1,5,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{4,5,6} - \omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} - \omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} +$

$\omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6 + \omega^7$; $\Delta\omega^7 = \omega^6 + 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{1,7} + \omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,6} + 2\omega^{1,7} + \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{1,6} + 2\omega^{2,3} - \omega^{3,4}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} - \omega^{4,7}$; $\Delta\omega^{3,4} = \omega^{1,7} - \omega^{2,3} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6} + \omega^{4,7}$; $\Delta\omega^{4,7} = -\omega^{2,7} + \omega^{4,6} + 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6} + \omega^{5,7}$; $\Delta\omega^{5,7} = \omega^{5,6} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3} - \omega^{1,3,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,4,7} - \omega^{2,3,4} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,3} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 3\omega^{1,4,6} + \omega^{1,4,7} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = -\omega^{1,2,7} + \omega^{1,4,6} + 2\omega^{1,4,7} + \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{1,5,7} - \omega^{2,3,5} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,6} + 3\omega^{1,5,7} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6} - \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} - \omega^{1,4,5} - \omega^{1,4,6} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} - \omega^{1,5,6} + 3\omega^{2,3,5} - \omega^{3,4,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6} - \omega^{2,3,7} - \omega^{3,4,5} - \omega^{3,4,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{1,6,7} - \omega^{2,3,6} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7} - \omega^{4,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7} - \omega^{1,5,7} - \omega^{2,3,5} - \omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} - \omega^{2,3,6} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6} + \omega^{4,5,7}$; $\Delta\omega^{4,5,7} = -\omega^{2,5,7} + \omega^{2,6,7} + \omega^{4,5,6} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{2,6,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,3,7} - \omega^{1,3,4,5} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,3,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} - \omega^{1,4,5,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{1,4,6,7} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,5} - \omega^{1,2,3,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,6} + 3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6} + \omega^{1,4,5,7} + \omega^{2,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,2,5,7} + \omega^{1,2,6,7} + \omega^{1,4,5,6} + 2\omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,2,6,7} + 3\omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,7} +$

$\omega^{1,2,5,7}+\omega^{1,4,5,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,2,6,7}-\omega^{1,4,5,6}+3\omega^{2,3,4,6}-\omega^{2,3,4,7}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,4,5,7}-\omega^{1,4,6,7}-\omega^{2,3,4,6}+2\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,2,6,7}+\omega^{2,3,4,7}+2\omega^{2,3,5,6}-\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,5,6,7}-\omega^{2,3,5,6}+3\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,5,6,7}+\omega^{2,3,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,7}=\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}-\omega^{4,5,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,4,6,7}+\omega^{1,5,6,7}-\omega^{2,3,5,6}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{2,3,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=-\omega^{2,5,6,7}+3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}-\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,7}=-\omega^{1,2,3,4,6}+2\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}-\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,7}=-\omega^{1,2,3,5,6}+3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7}=2\omega^{1,2,5,6,7}-\omega^{1,4,5,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,5,6}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,2,5,6,7}+2\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,4,6,7}-\omega^{1,2,5,6,7}+2\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,4,5,6,7}-\omega^{2,3,4,5,6}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}-\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,5,7}=-\omega^{1,2,3,4,5,6}+2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{A}})=2e^0\ \langle\omega^1,\omega^3\rangle+e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{A}})=e^0\ \langle\omega^{1,3}\rangle+2e^{-2\beta_2}\ \langle\omega^{2,4},\omega^{2,6}-\omega^{4,5}\rangle+2e^{-\beta_2}\ \langle\omega^{1,5}+\omega^{1,7}-\omega^{2,3}-\omega^{3,4}-\omega^{3,5},\omega^{1,6}-2\omega^{1,7}+\omega^{3,4}+2\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{A}})=e^{-\beta_2}\ \langle\omega^{1,3,7}\rangle+2e^{-3\beta_2}\ \langle\omega^{2,4,5},\omega^{2,4,7}-\omega^{2,5,6}\rangle+4e^{-2\beta_2}\ \langle\omega^{1,2,6}-2\omega^{1,4,5}-\omega^{1,4,6}+2\omega^{1,4,7}+3\omega^{1,6,7}-\omega^{2,3,4}-\omega^{2,3,5}-3\omega^{2,3,6}-3\omega^{2,3,7}-3\omega^{3,4,5}-3\omega^{3,4,7}+6\omega^{3,5,6},\omega^{1,2,7}-\omega^{1,4,6}+3\omega^{1,4,7}+4\omega^{1,6,7}-\omega^{2,3,5}-4\omega^{2,3,6}-4\omega^{2,3,7}-4\omega^{3,4,5}-4\omega^{3,4,7}+8\omega^{3,5,6},\omega^{1,5,6}+2\omega^{1,6,7}-2\omega^{2,3,6}-2\omega^{2,3,7}-\omega^{3,4,5}-2\omega^{3,4,7}+4\omega^{3,5,6},\omega^{1,5,7}+4\omega^{1,6,7}-\omega^{2,3,6}-3\omega^{2,3,7}+\omega^{3,4,6}-4\omega^{3,4,7}+8\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{A}})=e^{-4\beta_2}\ \langle-\omega^{2,4,5,6}\rangle+2e^{-2\beta_2}\ \langle-\omega^{1,3,6,7},(\omega^{1,3,4,7}-\omega^{1,3,5,6})/2\rangle+4e^{-3\beta_2}\ \langle(-\omega^{1,2,6,7}-3\omega^{1,4,6,7}+10\omega^{1,5,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}+3\omega^{2,3,5,7}-20\omega^{2,3,6,7}-7\omega^{3,4,5,6}-10\omega^{3,4,5,7})/67,(-24\omega^{1,2,6,7}-5\omega^{1,4,6,7}-28\omega^{1,5,6,7}-24\omega^{2,3,4,7}+43\omega^{2,3,5,6}+5\omega^{2,3,5,7}+56\omega^{2,3,6,7}+33\omega^{3,4,5,6}+28\omega^{3,4,5,7})/268,(-2\omega^{1,2,4,7}+\omega^{1,2,5,6}+7\omega^{1,2,5,7}-6\omega^{1,4,5,6}+20\omega^{1,4,5,7}+7\omega^{1,4,6,7}-\omega^{1,5,6,7}-\omega^{2,3,4,5}+7\omega^{2,3,4,6}+27\omega^{2,3,4,7}-20\omega^{2,3,5,6}-7\omega^{2,3,5,7}+2\omega^{2,3,6,7}-6\omega^{3,4,5,6}+\omega^{3,4,5,7})/67,(24\omega^{1,2,4,7}-12\omega^{1,2,5,6}-17\omega^{1,2,5,7}+28\omega^{1,2,6,7}+5\omega^{1,4,5,6}-39\omega^{1,4,5,7}+12\omega^{2,3,4,5}-17\omega^{2,3,4,6}-28\omega^{2,3,4,7})/268\rangle$.

$\mathrm{ch}(H^5_{\mathcal{A}})=e^{-5\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle+2e^{-3\beta_2}\ \langle-\omega^{1,3,5,6,7},(-\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7})/2\rangle+2e^{-4\beta_2}\ \langle(-\omega^{1,2,5,6,7}-2\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}-2\omega^{2,3,4,6,7})/5,(-\omega^{1,2,4,6,7}-\omega^{1,4,5,6,7}+\omega^{2,3,4,5,6}+\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7})/5\rangle$.

$\mathrm{ch}(H^6_{\mathcal{A}})=2e^{-5\beta_2}\ \langle\omega^{2,3,4,5,6,7},\omega^{1,2,4,5,6,7}\rangle+e^{-4\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle$.

$$\mathrm{ch}(H^7_{\mathfrak{T}}) = e^{-5\beta_2} \ \langle \omega^{1,2,3,4,5,6,7} \rangle.$$
$$P(Y) \ = \ (1+Y^2)^2(1+Y)^3.$$

5.2.3 Algebra $\mathcal{G}_{7,1.02}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_2,x_6]=x_7$; $[x_3,x_5]=-x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & \xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 2\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & -(\xi_1^5+\xi_1^6) & \xi_1^5 & -\xi_1^3-\xi_1^4+2\xi_2^5 & -\xi_1^3 & 3\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_2^6, ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=-\delta_2^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_2\rangle$.

ch($\mathcal{G}$)= $e^0\ \langle x_1\rangle+3e^{\beta_2}\ \langle x_2,x_3,x_4\rangle+2e^{2\beta_2}\ \langle x_5,x_6\rangle+e^{3\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^0\ \langle\delta_2^4,\delta_2^3,t_2\rangle+4e^{\beta_2}\ \langle\delta_2^6,\delta_1^4,\delta_2^5,\delta_1^3\rangle+e^{3\beta_2}\ \langle\delta_1^7\rangle+3e^{2\beta_2}\ \langle\delta_2^7,\delta_1^6,\delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4,\delta_1^5]=-\delta_2^7$; $[\delta_2^4,\delta_1^3]=\delta_2^6$; $[\delta_2^6,\delta_1^3]=\delta_2^7$; $[\delta_2^3,\delta_1^6]=\delta_2^7$; $[\delta_2^3,\delta_1^5]=\delta_2^7+\delta_1^6$; $[\delta_2^3,\delta_1^4]=\delta_2^6$; $[\delta_2^3,\delta_1^3]=\delta_2^5+\delta_1^4$; $[\delta_1^5,\delta_1^4]=2\delta_1^7$; $[\delta_1^5,\delta_2^5]=-2\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_2^7$; $[\delta_2^5,\delta_1^3]=\delta_2^7+\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $2e^0\ \langle\delta_2^4,t_2\rangle+e^{3\beta_2}\ \langle\delta_1^7\rangle+e^{2\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^4]=2\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_2}\ \langle x_1\omega^{2,5}+x_4\omega^{2,7}\rangle+3e^{-2\beta_2}\ \langle x_4\omega^{2,5},x_3\omega^{2,5}+x_4\omega^{2,6},4/5x_1(\omega^{1,5}-9/4\omega^{1,6}-5/4\omega^{2,3}+\omega^{2,4}-9/4\omega^{3,4})+3/5x_2(\omega^{2,5}+3\omega^{2,6}-3\omega^{3,5})+2x_3(\omega^{2,5}+3/2\omega^{2,6}-4/5\omega^{3,5}-9/10\omega^{4,5})-2x_4(\omega^{1,7}-1/2\omega^{3,6}-1/10\omega^{4,5}+9/10\omega^{4,6})-x_5(\omega^{2,7}-9/5\omega^{5,6})+x_6\omega^{3,7}\rangle+e^{2\beta_2}\ \langle x_7\omega^{1,4}\rangle+3e^0\ \langle x_2\omega^{1,2}+x_3\omega^{1,2}-x_4\omega^{1,4}+x_5\omega^{1,5}+x_7\omega^{1,7},x_2(\omega^{1,2}-\omega^{1,3})+x_3(\omega^{1,2}-\omega^{1,3})-2x_4\omega^{1,4}-x_5(\omega^{1,6}+\omega^{3,4})+x_6\omega^{3,4},x_2(\omega^{1,2}-\omega^{1,3})+3/2x_3(\omega^{1,2}-\omega^{1,3})+1/2x_5(\omega^{1,5}-2\omega^{1,6}+2\omega^{2,4}-2\omega^{3,4})\rangle+e^{\beta_2}\ \langle x_5(\omega^{1,2}-\omega^{1,3})\rangle+2e^{-\beta_2}\ \langle -1/2x_1\omega^{1,2}+1/2x_4(\omega^{1,5}+2\omega^{2,4}),-1/2x_1(\omega^{1,2}-2\omega^{1,3})-x_3(\omega^{1,5}+\omega^{2,3})+1/2x_4\omega^{1,5}+x_5\omega^{3,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-\beta_2}+4e^{-2\beta_2}+4e^{-3\beta_2}+3e^{-4\beta_2}+2e^{-5\beta_2}+e^0$.

ch($H^4_{\mathcal{G}}$)=$e^{-2\beta_2}+3e^{-3\beta_2}+4e^{-4\beta_2}+4e^{-5\beta_2}+3e^{-6\beta_2}+2e^{-7\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_2}+2e^{-5\beta_2}+3e^{-6\beta_2}+3e^{-7\beta_2}+3e^{-8\beta_2}+e^{-9\beta_2}$.
$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_2}+3e^{-8\beta_2}+2e^{-9\beta_2}+e^{-10\beta_2}$.
$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-9\beta_2}+e^{-10\beta_2}$.
Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,11,49,114,148,110,44,7)$
Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,5,11,16,17,13,7,2)$
$\mathrm{S}=10\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,2,8,18,22,16,7,1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,2,3,5,5,3,2,1)$

$d\omega^{3}=-\omega^{1,2}$; $d\omega^{4}=-\omega^{1,3}$; $d\omega^{5}=-\omega^{2,3}$; $d\omega^{6}=-\omega^{1,5}-\omega^{2,4}$; $d\omega^{7}=-\omega^{2,5}-\omega^{2,6}+\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,2,6}-\omega^{1,3,5}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{2,3,5}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{2,3,5}-\omega^{2,3,6}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{2,4,5}-\omega^{2,4,6}+\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{2,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{2,3,7}-\omega^{2,5,6}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}+\omega^{1,2,3,6}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}+\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}+\omega^{1,2,5,6}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,3,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,5}-\omega^{2,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{2,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}+\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,4,5,6}-\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $d\omega^{1,2,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}+\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}+\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{0}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-2\beta_2}\ \langle\omega^{1,6}+\omega^{3,4}\rangle + e^{-3\beta_2}\ \langle\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-2\beta_2}\ \langle\omega^{1,3,4}\rangle + e^{-3\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-6\beta_2}\ \langle\omega^{2,5,7}\rangle + e^{-4\beta_2}\ \langle\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle + e^{-5\beta_2}\ \langle\omega^{1,5,7}+\omega^{2,4,7}+\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= e^{-6\beta_2}\ \langle\omega^{1,2,6,7}-\omega^{2,4,5,6}\rangle + e^{-4\beta_2}\ \langle\omega^{1,3,4,6}\rangle + e^{-5\beta_2}\ \langle\omega^{1,3,4,7}+\omega^{1,4,5,6}+\omega^{2,3,4,6}\rangle + e^{-7\beta_2}\ \langle\omega^{2,3,5,7}\rangle + e^{-8\beta_2}\ \langle\omega^{2,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= e^{-7\beta_2}\ \langle\omega^{1,2,4,6,7}-\omega^{1,3,4,6,7}\rangle + e^{-8\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-9\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-9\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-10\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+2Y^2+3Y^3+2Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{a}}$'s $0\le j\le 7$: $(1,2,8,18,22,16,7,1)$

Dimensions of $H^j_{\mathfrak{a}}$'s $0\le j\le 7$: $(1,2,3,5,5,3,2,1)$

Total dimension of trivial cohomology: 22.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{2,6}=-\omega^7$; $d^\star\omega^{3,5}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,5}=-\omega^{1,7}-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{2,7}-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,6}=-\omega^{4,7}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=-\omega^{5,7}+\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{4,7}$; $d^\star\omega^{3,5,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,6}-\omega^{1,4,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,2,7}+\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=\omega^{1,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,5,7}-\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,5,6}=\omega^{1,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,4,7}-\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{3,4,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{2,6,7}-\omega^{3,5,7}+\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{4,5,7}+\omega^{4,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,4,7}+\omega^{1,3,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,3,4,7}-\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{1,3,6,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,4,5,7}-\omega^{1,4,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=$

$-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,6,7} - \omega^{3,4,5,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{3,4}$; $\Delta\omega^{1,7} = 3\omega^{1,7} - \omega^{2,6} - \omega^{3,5} + \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,5} + 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,7}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} - \omega^{2,5} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{2,7} + 3\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{3,7} - \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{5,7} + 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} + \omega^{1,2,6} - \omega^{1,3,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,5} + 2\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,7} + \omega^{2,3,5} - \omega^{2,4,5}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,5} + 3\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,2,7} + 3\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 3\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} - \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{1,6,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = -\omega^{1,5,7} + 3\omega^{1,6,7} - \omega^{3,4,7} + \omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + 2\omega^{2,3,5} + \omega^{2,3,6}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,5} + 3\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} + 3\omega^{2,4,5} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,5} + 3\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 3\omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,4,7} + 2\omega^{2,5,6} - \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,7} - \omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,4,5} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + \omega^{2,4,7} + 3\omega^{3,4,7} - \omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} - \omega^{2,5,6} - \omega^{3,4,7} + 3\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} - \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = \omega^{2,6,7} - \omega^{3,5,7} + 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5} + \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,5} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 3\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} - \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,6,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,5} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,4,7} + 2\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,7} + 3\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = 3\omega^{1,3,5,7} - \omega^{1,3,6,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + 3\omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 3\omega^{1,4,5,7} - \omega^{1,4,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,4,5,7} + 2\omega^{1,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,2,5,6} + 3\omega^{2,3,4,5} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,6} =$

$-\omega^{1,3,4,7}+\omega^{2,3,4,5}+3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,2,6,7}+2\omega^{2,3,4,7}-\omega^{2,3,5,6}-\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,4,7}+3\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7}=2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6}=\omega^{1,2,6,7}-\omega^{1,4,5,7}-\omega^{2,3,4,7}+3\omega^{2,4,5,6}-\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7}=2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=3\omega^{2,4,6,7}+\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{1,3,6,7}+\omega^{1,4,6,7}-\omega^{2,4,5,6}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{1,5,6,7}+2\omega^{3,4,5,7}-\omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7}=\omega^{2,4,6,7}-\omega^{3,4,5,7}+3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}+\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,5}+2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}-\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=-\omega^{1,2,3,4,7}+4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=-\omega^{1,2,3,4,7}+3\omega^{1,2,4,5,6}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=2\omega^{1,2,4,6,7}+\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,4,5,6}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=2\omega^{1,3,4,5,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7}=2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{0}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_2}\ \langle\omega^{1,6}+\omega^{3,4}\rangle+e^{-3\beta_2}\ \langle 2\omega^{2,5}-\omega^{2,6}+\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_2}\ \langle\omega^{1,3,4}\rangle+e^{-3\beta_2}\ \langle\omega^{1,4,6}\rangle+e^{-6\beta_2}\ \langle\omega^{2,5,7}\rangle+e^{-4\beta_2}\ \langle 3\omega^{1,2,7}-9\omega^{1,3,7}+10\omega^{1,5,6}-3\omega^{2,3,6}-3\omega^{2,4,5}+\omega^{2,4,6}-11\omega^{3,4,5}\rangle+e^{-5\beta_2}\ \langle\omega^{1,5,7}-3\omega^{2,3,7}+\omega^{2,4,7}+3\omega^{2,5,6}+\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-8\beta_2}\ \langle\omega^{2,5,6,7}\rangle+e^{-7\beta_2}\ \langle-\omega^{2,3,5,7}\rangle+e^{-4\beta_2}\ \langle\omega^{1,3,4,6}\rangle+e^{-6\beta_2}\ \langle(-11\omega^{1,2,6,7}+\omega^{1,3,5,7}+3\omega^{1,3,6,7}+3\omega^{1,4,5,7}+10\omega^{2,3,4,7}+9\omega^{2,4,5,6}+3\omega^{3,4,5,6})/330\rangle+e^{-5\beta_2}\ \langle(\omega^{1,2,4,7}-3\omega^{1,3,4,7}-\omega^{1,3,5,6}-3\omega^{1,4,5,6}-\omega^{2,3,4,6})/21\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle+e^{-8\beta_2}\ \langle(\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7})/2\rangle+e^{-7\beta_2}\ \langle(-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,7}+2\omega^{1,3,4,6,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-10\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-9\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+2Y^2+3Y^3+2Y^4+Y^5+Y^6)(1+Y)$.

5.2.4 Algebra $\mathcal{G}_{7,1.03}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_7$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & \xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & \xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3 & 0 & 0 & 2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^4-\xi_1^5+\xi_2^6 & -\xi_1^3+\xi_1^4 & -\xi_1^3 & \xi_2^3+2\xi_2^4 & 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^6), ad\ x_4=-(\delta_1^5+\delta_2^7), ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_2\rangle$.

ch($\mathcal{G}$)= $e^0\ \langle x_1\rangle+4e^{\beta_2}\ \langle x_2,x_3,x_4,x_5\rangle+2e^{2\beta_2}\ \langle x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^0\ \langle\delta_2^5,\delta_2^3,\delta_2^4,t_2\rangle+2e^{2\beta_2}\ \langle\delta_1^6,\delta_1^7\rangle+5e^{\beta_2}\ \langle\delta_2^7,\delta_1^5,\delta_2^6,\delta_1^4,\delta_1^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^6,\delta_2^4]=-2\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_2^4]=-2\delta_2^7$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^5)$; $[\delta_1^4,\delta_2^4]=\delta_2^7$; $[\delta_1^3,\delta_2^3]=-(\delta_2^6+\delta_1^4)$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_1^5)$.

ch($H^1_{\mathcal{G}}$)= $3e^0\ \langle\delta_2^5,\delta_2^4,t_2\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $2e^{-2\beta_2}\ \langle x_5\omega^{2,6},x_1\omega^{2,3}-x_5(\omega^{2,7}-\omega^{3,6})\rangle+4e^{-\beta_2}\ \langle 1/3x_1(\omega^{1,2}-\omega^{1,3})+2x_2\omega^{2,3}+x_3(\omega^{2,3}+5/3\omega^{2,4})-1/3x_4(\omega^{1,6}-\omega^{2,4}-3\omega^{2,5}-2\omega^{3,4})+x_5\omega^{3,5},x_5\omega^{2,3},-x_1(\omega^{1,2}-\omega^{1,3})-2x_3(\omega^{2,3}-1/2\omega^{2,4})+x_4(\omega^{1,6}+\omega^{3,4})+x_5\omega^{3,4},-2/3x_1(\omega^{1,2}-\omega^{1,3})-x_3(\omega^{2,3}-2/3\omega^{2,4})+2/3x_4(\omega^{1,6}+1/2\omega^{2,4}+\omega^{3,4})+x_5\omega^{2,5}\rangle+e^{\beta_2}\ \langle x_6\omega^{1,5}\rangle+4e^0\ \langle x_2(\omega^{1,4}+\omega^{1,5})+x_3(\omega^{1,4}+\omega^{1,5})+x_6(\omega^{1,7}+\omega^{4,5}),-x_2(\omega^{1,2}-\omega^{1,3})-x_3(\omega^{1,2}-\omega^{1,3})+x_4\omega^{1,5},-x_3(\omega^{1,2}-\omega^{1,3})-x_4\omega^{1,4}+x_6\omega^{1,6},-x_3(\omega^{1,2}+\omega^{1,4})-x_4\omega^{1,4}+x_6\omega^{3,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$4e^{-\beta_2}+5e^{-2\beta_2}+6e^{-3\beta_2}+e^{-4\beta_2}+e^0$.

ch($H^4_{\mathcal{G}}$)=$3e^{-2\beta_2}+6e^{-3\beta_2}+5e^{-4\beta_2}+4e^{-5\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$4e^{-4\beta_2}+5e^{-5\beta_2}+3e^{-6\beta_2}+e^{-7\beta_2}$.

ch($H^6_{\mathcal{G}}$)=$e^{-5\beta_2}+3e^{-6\beta_2}+2e^{-7\beta_2}+e^{-8\beta_2}$.

ch($H^7_{\mathcal{G}}$)=$e^{-7\beta_2}+e^{-8\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,11,49,115,148,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,5,11,17,18,13,7,2)$

S=$8\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^{3}=-\omega^{1,2}$; $d\omega^{4}=-\omega^{1,3}$; $d\omega^{5}=-\omega^{1,4}$; $d\omega^{6}=-\omega^{2,3}$; $d\omega^{7}=-\omega^{1,6}-\omega^{2,4}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}+\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,4}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{2,4,5}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{2,3,7}+\omega^{2,4,6}+\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}+\omega^{1,2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}+\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,4,6}-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}+\omega^{2,3,4,6}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}+\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{0}\ \langle\omega^{1}\rangle+e^{-\beta_2}\ \langle\omega^{2}\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-3\beta_2}\ \langle\omega^{2,6}\rangle+e^{-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_2}\ \langle\omega^{1,4,5}\rangle+2e^{-4\beta_2}\ \langle\omega^{2,3,6},\omega^{2,3,7}-\omega^{2,4,6}\rangle+e^{-3\beta_2}\ \langle\omega^{1,4,7}+\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-4\beta_2}\ \langle\omega^{1,2,5,7}+\omega^{1,4,5,6},\omega^{1,4,5,7}\rangle + e^{-5\beta_2}\ \langle\omega^{1,3,6,7}-\omega^{2,3,5,6}-\omega^{2,4,5,6}\rangle + e^{-6\beta_2}\ \langle\omega^{2,3,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_2}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-6\beta_2}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}\rangle + e^{-7\beta_2}\ \langle\omega^{2,3,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 0.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,7}-\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}+\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{5,7}$; $d^\star\omega^{3,4,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}+\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{1,5,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{1,5,7}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}+\omega^{3,6,7}$; $d^\star\omega^{2,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}-\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,2,6,7}-\omega^{1,3,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,6,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}+\omega^{1,5,6,7}-\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=$

$-\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7} - \omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 4\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{1,7} = 3\omega^{1,7} - \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,5} = \omega^{1,6} - \omega^{1,7} + \omega^{2,4} + 2\omega^{2,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6} + \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} - \omega^{1,7} - \omega^{2,4} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{2,7} + 4\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6} - \omega^{5,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,7} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4} + \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,4} + 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} + \omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,4} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,2,7} + 3\omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,2,7} + 3\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{1,5,6} - \omega^{1,5,7} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,7} + 2\omega^{1,5,7} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 4\omega^{1,6,7} + \omega^{2,5,6} + \omega^{2,5,7} + \omega^{3,4,6} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + \omega^{1,2,7} + 3\omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} - \omega^{1,3,7} + \omega^{2,3,4} + 3\omega^{2,3,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} - \omega^{1,4,7} + 3\omega^{2,4,5} - \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} - \omega^{2,3,7} + \omega^{2,4,6} + \omega^{2,4,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{3,5,6} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{1,5,7} - \omega^{2,4,5} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{2,4,6} + \omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 3\omega^{3,4,7} + \omega^{3,5,6} - \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,5,7} - \omega^{3,4,7} + 4\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{2,6,7} + 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7} + 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4} + \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,6} = 4\omega^{1,2,4,6} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,6,7} - \omega^{2,3,4,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,5} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,2,4,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + 2\omega^{1,3,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,6,7} + 3\omega^{1,3,6,7} + \omega^{2,3,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = 3\omega^{1,4,6,7} -$

$\omega^{1,5,6,7}+\omega^{2,4,5,6}+\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=-\omega^{1,4,6,7}+3\omega^{1,5,6,7}-\omega^{3,4,5,6}+\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,2,5,6}-\omega^{1,2,5,7}+\omega^{1,3,4,6}-\omega^{1,3,4,7}+4\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,2,6,7}+2\omega^{2,3,4,6}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,2,6,7}+2\omega^{2,3,4,7}+\omega^{2,3,5,6}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,3,6,7}+\omega^{2,3,4,6}+\omega^{2,3,4,7}+2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=\omega^{1,3,6,7}+3\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6}=\omega^{1,4,6,7}+\omega^{2,3,4,7}+3\omega^{2,4,5,6}-\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,4,6,7}+2\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}+\omega^{3,5,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,5,6,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}+\omega^{2,4,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{1,5,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}-\omega^{3,5,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{2,5,6,7}-\omega^{3,4,6,7}+3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}+\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,4,6}+\omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}+4\omega^{1,2,4,5,6}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7}=2\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=2\omega^{1,2,5,6,7}+\omega^{1,3,5,6,7}+\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}+\omega^{1,2,4,5,7}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=2\omega^{1,3,4,6,7}-\omega^{1,3,5,6,7}+\omega^{2,3,4,5,6}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{0}\ \langle\omega^{1}\rangle+e^{-\beta_2}\ \langle\omega^{2}\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-3\beta_2}\ \langle\omega^{2,6}\rangle+e^{-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,4}-\omega^{2,5}+\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_2}\ \langle\omega^{1,4,5}\rangle+2e^{-4\beta_2}\ \langle\omega^{2,3,6},\omega^{2,3,7}-\omega^{2,4,6}+\omega^{2,5,6}-\omega^{3,4,6}\rangle+e^{-3\beta_2}\ \langle 2\omega^{1,2,7}-6\omega^{1,3,7}-2\omega^{1,4,6}+11\omega^{1,4,7}-14\omega^{1,5,6}-2\omega^{2,3,5}+8\omega^{2,4,5}+11\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_2}\ \langle\omega^{2,3,6,7}\rangle+2e^{-4\beta_2}\ \langle-\omega^{1,4,5,7},(\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6})/4\rangle+e^{-5\beta_2}\ \langle(11\omega^{1,2,6,7}-8\omega^{1,3,6,7}-2\omega^{1,4,6,7}-14\omega^{2,3,4,7}+11\omega^{2,3,5,6}+2\omega^{2,3,5,7}+6\omega^{2,4,5,6}+2\omega^{3,4,5,6})/550\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_2}\ \langle-\omega^{2,3,4,6,7}\rangle+e^{-5\beta_2}\ \langle-\omega^{1,3,4,5,7}\rangle+e^{-6\beta_2}\ \langle(\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7})/4\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-7\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

5.3 rank 1 Lie Algebras without null weight

5.3.1 Algebra $\mathcal{G}_{7,1.1(i_\lambda)}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_2,x_5]=\lambda x_7$; $[x_3,x_4]=x_7(-\lambda+1)$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & 0 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_2^3 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 6\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5\lambda+\xi_2^6 & \xi_1^4(\lambda-2)+\xi_2^5 & -\xi_1^3\lambda & \xi_2^3 & 7\xi_1^1\end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-(\lambda\delta_2^7+\delta_1^6), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+e^{5\beta_1}\ \langle x_5\rangle+e^{6\beta_1}\ \langle x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{3\beta_1}\ \langle\delta_1^4,\delta_2^5\rangle+2e^{4\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{5\beta_1}\ \langle\delta_1^6,\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_1^3\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^3]=-(\delta_2^6+\delta_1^5)$; $[\delta_1^5,\delta_2^3]=-(\delta_1^6+\lambda\delta_2^7)$; $[\delta_2^5,\delta_1^3]=\delta_1^6+\lambda\delta_2^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle\delta_1^4\rangle+e^{4\beta_1}\ \langle\delta_1^5\rangle+e^{5\beta_1}\ \langle\delta_2^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1}\ \langle -10/9x_1(\lambda^2\omega^{1,4}+1/2\lambda^2\omega^{2,3}-8/5\lambda\omega^{1,4}-7/5\lambda\omega^{2,3}+3/5\omega^{1,4}+9/10\omega^{2,3})-4/9x_2(\lambda\omega^{1,5}+\lambda\omega^{2,4}-\omega^{1,5}-\omega^{2,4})-5/3x_3(\lambda^2\omega^{2,5}-1/3\lambda^2\omega^{3,4}-1/3\lambda\omega^{1,6}-8/5\lambda\omega^{2,5}+7/15\lambda\omega^{3,4}-4/15\omega^{1,6}-2/15\omega^{3,4})-10/9x_4(\lambda^2\omega^{3,5}-1/2\lambda\omega^{1,7}-19/10\lambda\omega^{3,5}+1/2\omega^{1,7}-9/10\omega^{2,6}+9/10\omega^{3,5})-10/9x_5(\lambda^2\omega^{4,5}+1/2\lambda\omega^{3,6}-2\lambda\omega^{4,5}-1/2\omega^{3,6}+\omega^{4,5})-5/9x_6\omega^{3,7}(\lambda-1)+5/9x_7\omega^{4,7}(\lambda^2-3\lambda+2)\rangle+2e^{-\beta_1}\ \langle x_3\omega^{1,3}+x_4\omega^{2,3}-x_7(\lambda\omega^{2,6}-\lambda\omega^{3,5}-\omega^{2,6}+\omega^{3,5}),-x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}-x_7(\lambda\omega^{2,6}-2\lambda\omega^{3,5}-2\omega^{2,6}+2\omega^{3,5})\rangle+3e^{-2\beta_1}\ \langle -x_1\lambda\omega^{1,2}+1/2x_2\lambda\omega^{1,3}(\lambda-1)+1/4x_3(\lambda^2\omega^{1,4}-\lambda^2\omega^{2,3}+7\lambda\omega^{1,4}-$

$3\lambda\omega^{2,3} + 4\omega^{1,4}) + 1/2x_4\lambda(\lambda\omega^{1,5} + 1/2\lambda\omega^{2,4} + \omega^{1,5} + 3/2\omega^{2,4}) + x_5\omega^{1,6} - 1/2x_6\lambda\omega^{3,5}(\lambda - 1), -x_1\omega^{1,2} + 1/2x_2\omega^{1,3}(\lambda - 1) + 1/4x_3(\lambda\omega^{1,4} - \lambda\omega^{2,3} + 9\omega^{1,4} - \omega^{2,3}) + 1/2x_4(\lambda\omega^{1,5} + 1/2\lambda\omega^{2,4} + \omega^{1,5} + 5/2\omega^{2,4}) - 1/2x_6\omega^{3,5}(\lambda - 1) + x_7\omega^{2,7}, -1/2x_2\omega^{1,3}(\lambda-1) - 1/4x_3(\lambda\omega^{1,4} - \lambda\omega^{2,3} + \omega^{1,4} + 3\omega^{2,3}) - 1/2x_4(\lambda\omega^{1,5} + 1/2\lambda\omega^{2,4} - \omega^{1,5} + 1/2\omega^{2,4}) + 1/2x_6\omega^{3,5}(\lambda-1) + x_7\omega^{4,5}(\lambda-1)\rangle + e^0\ \langle -x_4\omega^{1,3} + x_6\omega^{2,4} + x_7\omega^{2,5}\rangle + 2e^{-3\beta_1}\ \langle 1/3x_1\lambda\omega^{1,3}(\lambda^2 - 6\lambda + 5) + 2/3x_2(\lambda^3\omega^{1,4} - 13/2\lambda^2\omega^{1,4} - \lambda^2\omega^{2,3} + 11/2\lambda\omega^{1,4} + 5/2\lambda\omega^{2,3} - 3/2\omega^{2,3}) - 2/3x_3(\lambda^3\omega^{1,5} + 1/2\lambda^3\omega^{2,4} - 11/2\lambda^2\omega^{1,5} - 3/2\lambda^2\omega^{2,4} - 5\lambda\omega^{2,4} + 3/2\omega^{2,4}) - 1/3x_4(\lambda^3\omega^{2,5} - \lambda^2\omega^{1,6} - 4\lambda^2\omega^{2,5} + 4\lambda\omega^{1,6} - 6\lambda\omega^{2,5} - 3\omega^{1,6}) + 1/3x_5\lambda\omega^{3,5}(\lambda^2 - 7\lambda + 6) - 1/3x_6(\lambda^2\omega^{3,6} - 3\lambda\omega^{2,7} - \lambda\omega^{3,6} + 3\omega^{2,7}) + x_7\omega^{3,7}(\lambda - 1), -x_2(\lambda\omega^{1,4} - \lambda\omega^{2,3} - \omega^{1,4} + \omega^{2,3}) + x_3(\lambda\omega^{1,5} + 2\lambda\omega^{2,4} - \omega^{2,4}) + x_4(\lambda\omega^{2,5} + \lambda\omega^{3,4} - \omega^{3,4})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{-5\beta_1} + 2e^{-6\beta_1} + 3e^{-7\beta_1} + 4e^{-8\beta_1} + 2e^{-9\beta_1} + e^{-10\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-10\beta_1} + 2e^{-11\beta_1} + 4e^{-12\beta_1} + 3e^{-13\beta_1} + 2e^{-14\beta_1} + 2e^{-15\beta_1} + e^{-16\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-16\beta_1} + 2e^{-17\beta_1} + 3e^{-18\beta_1} + 2e^{-19\beta_1} + e^{-20\beta_1} + e^{-21\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-20\beta_1} + e^{-21\beta_1} + e^{-23\beta_1} + e^{-24\beta_1} + e^{-25\beta_1} + e^{-26\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-26\beta_1} + e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 10, 48, 113, 147, 109, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 4, 9, 14, 15, 11, 6, 2)$

The preceding results are valid for:

$$(\lambda^2 - \lambda + 1)(\lambda + 1)(\lambda - 1)^3(\lambda - 2)(\lambda - 3)^2 \neq 0.$$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5}\lambda + \omega^{3,4}(\lambda - 1)$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,5}\lambda + \omega^{1,3,4}(-\lambda + 1)$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}(-\lambda + 1)$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,5}\lambda$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,4,5}\lambda$; $d\omega^{5,6} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}(-\lambda + 1)$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6}\lambda + \omega^{3,4,6}(-\lambda + 1)$; $d\omega^{1,2,7} = \omega^{1,2,3,4}(\lambda - 1)$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}\lambda$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}\lambda$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} + \omega^{1,3,4,5}(\lambda - 1)$;

$d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6}\lambda + \omega^{1,3,4,6}(\lambda - 1)$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}(\lambda - 1)$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,6}(\lambda - 1)$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}\lambda$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}\lambda$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,4,5,6}\lambda$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}(\lambda - 1)$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}(-\lambda + 1)$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}(-\lambda + 1)$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}\lambda$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}\lambda$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}\lambda$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}(-\lambda + 1)$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}(-\lambda + 1)$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}\lambda$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}(\lambda - 1)$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}\lambda$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{T}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{T}}) = e^{-8\beta_1} \langle\omega^{1,7} + \omega^{2,6}(2\lambda - 1) + \omega^{3,5}(-\lambda + 1)\rangle + e^{-5\beta_1} \langle\omega^{1,4}\rangle + e^{-7\beta_1} \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{T}}) = e^{-10\beta_1} \langle\omega^{1,2,7} + \omega^{2,3,5}(\lambda - 1)\rangle + 2e^{-12\beta_1} \langle\omega^{1,4,7} + \omega^{3,4,5}\lambda, \omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}\rangle + e^{-11\beta_1} \langle\omega^{1,3,7} + \omega^{2,4,5}\lambda\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{T}}) = 2e^{-16\beta_1} \langle\omega^{1,2,6,7} + \omega^{2,3,5,6}(-\lambda + 1), \omega^{1,3,5,7}\rangle + e^{-17\beta_1} \langle\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}(\lambda + 1)\rangle + e^{-18\beta_1} \langle\omega^{1,4,6,7} - \omega^{3,4,5,6}\lambda\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{T}}) = e^{-21\beta_1} \langle\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}(\lambda - 1)\rangle + e^{-20\beta_1} \langle\omega^{2,3,4,5,6}\rangle + e^{-23\beta_1} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{T}}) = e^{-26\beta_1} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-27\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{T}}) = e^{-28\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

The preceding results are valid for:

$$-(\lambda^2 - \lambda + 1)(\lambda - 1)(\lambda - 3) \neq 0.$$

Comments: singular values.

$\mathcal{G}_{7,1.1(i_\lambda)}$ is isomorphic to $\mathcal{G}_{7,1.1(i_{\lambda'})}$ if and only if $\lambda = \lambda'$. The values excluded by the computer are not necessarily singular, i.e. give a different cohomology than the generic one: some of them appear from the order used by the computer in solving systems of equations . Computing again for each of these values shows that only $\lambda^2 - \lambda + 1 = 0$, $\lambda = 1$, $\lambda = 3$ are singular values for the adjoint cohomology and that only $\lambda^2 - \lambda + 1 = 0$, $\lambda = 3$ are singular values for the trivial cohomology.

changes in adjoint cohomology:

case $\lambda = 1$:

The changes are dim($Z^1_\mathcal{G}$)=11 (δ^4_2 enters) and dim($Z^2_\mathcal{G}$)=49 which implies dim($H^1_\mathcal{G}$)=5, dim($H^2_\mathcal{G}$)=11, dim($H^3_\mathcal{G}$)=15,

$$\delta = \begin{pmatrix} \xi^1_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi^1_1 & 0 & 0 & 0 & 0 & 0 \\ \xi^3_1 & \xi^3_2 & 3\xi^1_1 & 0 & 0 & 0 & 0 \\ \xi^4_1 & \xi^4_2 & \xi^3_2 & 4\xi^1_1 & 0 & 0 & 0 \\ \xi^5_1 & \xi^5_2 & -\xi^3_1 + \xi^4_2 & \xi^3_2 & 5\xi^1_1 & 0 & 0 \\ \xi^6_1 & \xi^6_2 & -\xi^4_1 + \xi^5_2 & -\xi^3_1 + \xi^4_2 & \xi^3_2 & 6\xi^1_1 & 0 \\ \xi^7_1 & \xi^7_2 & -\xi^5_1 + \xi^6_2 & -\xi^4_1 + \xi^5_2 & -\xi^3_1 + \xi^4_2 & \xi^3_2 & 7\xi^1_1 \end{pmatrix}$$

change in ch($Z^1_\mathcal{G}$) $e^{2\beta_1}$ $\langle\delta^3_1\rangle$ to $2e^{2\beta_1}$ $\langle\delta^3_1, \delta^4_2\rangle$; add in ch($H^1_\mathcal{G}$) $e^{2\beta_1}$ $\langle\delta^4_2\rangle$.

Comm. relations among the δ^j_i's $i \neq j$ in $Z^1_\mathcal{G}$: add $[\delta^5_1, \delta^4_2] = -\delta^7_1$; $[\delta^4_1, \delta^4_2] = -(\delta^7_2 + \delta^6_1)$; $[\delta^3_1, \delta^4_2] = -(\delta^6_2 + \delta^5_1)$;

Comm. relations among the δ^j_i's $i \neq j$ in $H^1_\mathcal{G}$: .

Add in ch($H^2_\mathcal{G}$) $e^{-5\beta_1}$ $\langle x_3(\omega^{1,7} + \omega^{2,6}) + x_4\omega^{2,7}\rangle$ and $e^{2\beta_1}$ $\langle x_6\omega^{1,3}\rangle$.

Dimensions of $Z^j_\mathcal{G}$'s $0 \leq j \leq 7$: $(1, \underline{11}, \underline{49}, 113, 147, 109, 44, 7)$ [2]

Dimensions of $H^j_\mathcal{G}$'s $0 \leq j \leq 7$: $(1, \underline{5}, \underline{11}, \underline{15}, 15, 11, 6, 2)$

$e^{2\beta_1}$ appears as a new term in ch($H^1_\mathcal{G}$) and ch($H^2_\mathcal{G}$); $e^{-5\beta_1}$ has multiplicity 3 in ch($H^3_\mathcal{G}$); and appears as a new term in ch($H^2_\mathcal{G}$).

case $\lambda = 3$:

Dimensions of $Z^j_\mathcal{G}$'s $0 \leq j \leq 7$: $(1, 10, 48, \underline{114}, 147, \underline{110}, 44, 7)$

Dimensions of $H^j_\mathcal{G}$'s $0 \leq j \leq 7$: $(1, 4, 9, \underline{15}, \underline{16}, \underline{12}, \underline{7}, 2)$

$e^{-9\beta_1}$ (resp. $e^{-19\beta_1}$) has multiplicity 3 in ch($H^3_\mathcal{G}$) (resp. ch($H^5_\mathcal{G}$)) and appears as a new term in ch($H^4_\mathcal{G}$) (resp.ch($H^6_\mathcal{G}$)).

[2] underlined dimensions are those different from the generic ones

case $\lambda^2-\lambda+1=0$**:**

The only change is $\dim(Z^4_{\mathcal{G}})$=148 which implies $\dim(H^4_{\mathcal{G}})$=16, $\dim(H^5_{\mathcal{G}})$=12. $e^{-14\beta_1}$ has multiplicity 3 in $\mathrm{ch}(H^4_{\mathcal{G}})$ and appears as a new term in $\mathrm{ch}(H^5_{\mathcal{G}})$.

changes in trivial cohomology:

case $\lambda=3$**:**

Add in $\mathrm{ch}(H^2_{\mathbb{C}})$ $e^{-9\beta_1}$ $\langle\omega^{2,7}-\omega^{3,6}+\omega^{4,5}\rangle$ and correspondingly in $\mathrm{ch}(H^5_{\mathbb{C}})$ $e^{-19\beta_1}$ $\langle\omega^{1,3,4,5,6}\rangle$; in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-9\beta_1}$ $\langle\omega^{1,2,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-19\beta_1}$ $\langle\omega^{1,5,6,7}+\omega^{2,4,6,7}-2\omega^{3,4,5,7}\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,\underline{9},17,\underline{23},16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,\underline{4},\underline{5},\underline{5},\underline{4},2,1)$

$P(Y)\;=\;(1+Y+Y^2)(1+Y^2)^2(1+Y)$.

case $\lambda^2-\lambda+1=0$**:**

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-14\beta_1}$ $\langle\omega^{1,6,7}+(1-\lambda)\omega^{2,5,7}+\lambda\omega^{3,4,7}-\omega^{3,5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-14\beta_1}$ $\langle\omega^{2,3,4,5}\rangle$

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,\underline{18},22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,\underline{5},\underline{5},3,2,1)$

$P(Y)\;=\;(1+Y+2Y^2+3Y^3+2Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7\bar\lambda$; $d^\star\omega^{3,4}=\omega^7(\bar\lambda-1)$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}\bar\lambda-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}(-\bar\lambda+1)-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}(-\bar\lambda+1)-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}\bar\lambda$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}\bar\lambda+\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}\bar\lambda$; $d^\star\omega^{3,4,5}=\omega^{5,7}(-\bar\lambda+1)$; $d^\star\omega^{3,4,6}=\omega^{6,7}(-\bar\lambda+1)$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}(\bar\lambda-1)+\omega^{1,3,6}-\omega^{1,4,5}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}\bar\lambda-$

$\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}\bar\lambda-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}\bar\lambda-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}(\bar\lambda-1)-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}(\bar\lambda-1)-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}(\bar\lambda-1)-\omega^{3,4,7}\bar\lambda+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}(\bar\lambda-1)+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}\bar\lambda$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}\bar\lambda$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}(\bar\lambda-1)$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}(-\bar\lambda+1)+\omega^{1,3,4,7}\bar\lambda-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}(-\bar\lambda+1)-\omega^{1,4,5,6}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}\bar\lambda-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}\bar\lambda-\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}(-\bar\lambda+1)-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}(-\bar\lambda+1)+\omega^{3,4,6,7}\bar\lambda$; $d^\star\omega^{2,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,2,5,6,7}(\bar\lambda-1)-\omega^{1,3,4,6,7}\bar\lambda-\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=-\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=-\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7}=-\omega^{3,4,5,6,7}$;

$\Delta\omega^3=\omega^3$; $\Delta\omega^4=\omega^4$; $\Delta\omega^5=2\omega^5$; $\Delta\omega^6=2\omega^6$; $\Delta\omega^7=\omega^7(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,4}=\omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,5}=2\omega^{1,5}$; $\Delta\omega^{1,6}=2\omega^{1,6}+\omega^{2,5}(\lambda-1)-\omega^{3,4}\lambda$; $\Delta\omega^{1,7}=\omega^{1,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+1)-\omega^{2,6}\lambda-\omega^{3,5}$; $\Delta\omega^{2,3}=\omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{2,4}=2\omega^{2,4}$; $\Delta\omega^{2,5}=\omega^{1,6}(\bar\lambda-1)+\omega^{2,5}(\lambda\bar\lambda+1)+\omega^{3,4}(-\lambda\bar\lambda+\bar\lambda+1)$; $\Delta\omega^{2,6}=-\omega^{1,7}\bar\lambda+\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{2,7}=\omega^{2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,6}\lambda+\omega^{4,5}(-\lambda+1)$; $\Delta\omega^{3,4}=-\omega^{1,6}\bar\lambda+\omega^{2,5}(-\lambda\bar\lambda+\lambda+1)+\omega^{3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,5}=-\omega^{1,7}+\omega^{2,6}+2\omega^{3,5}$; $\Delta\omega^{3,6}=\omega^{2,7}\bar\lambda+3\omega^{3,6}$; $\Delta\omega^{3,7}=\omega^{3,7}(\lambda\bar\lambda+2)+\omega^{4,6}$; $\Delta\omega^{4,5}=\omega^{2,7}(-\bar\lambda+1)+2\omega^{4,5}$; $\Delta\omega^{4,6}=\omega^{3,7}+2\omega^{4,6}$; $\Delta\omega^{4,7}=\omega^{4,7}(\lambda\bar\lambda+2)+\omega^{5,6}(-\lambda+1)$; $\Delta\omega^{5,6}=\omega^{4,7}(-\bar\lambda+1)+3\omega^{5,6}$; $\Delta\omega^{5,7}=\omega^{5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+4)$; $\Delta\omega^{6,7}=\omega^{6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=3\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=\omega^{1,2,5}(\lambda\bar\lambda+2)+\omega^{1,3,4}(-\lambda\bar\lambda+\bar\lambda+1)$; $\Delta\omega^{1,2,6}=2\omega^{1,2,6}+\omega^{1,3,5}+\omega^{2,3,4}\lambda$; $\Delta\omega^{1,2,7}=\omega^{1,2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,6}\lambda+\omega^{1,4,5}(-\lambda+1)+\omega^{2,3,5}$; $\Delta\omega^{1,3,4}=\omega^{1,2,5}(-\lambda\bar\lambda+\lambda+1)+\omega^{1,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,3,5}=\omega^{1,2,6}+2\omega^{1,3,5}$; $\Delta\omega^{1,3,6}=\omega^{1,2,7}\bar\lambda+3\omega^{1,3,6}+\omega^{2,3,5}(\lambda-1)$; $\Delta\omega^{1,3,7}=\omega^{1,3,7}(\lambda\bar\lambda+1)+\omega^{1,4,6}-\omega^{2,3,6}\lambda$; $\Delta\omega^{1,4,5}=\omega^{1,2,7}(-\bar\lambda+1)+2\omega^{1,4,5}+\omega^{2,3,5}$; $\Delta\omega^{1,4,6}=\omega^{1,3,7}+2\omega^{1,4,6}+\omega^{2,3,6}+\omega^{2,4,5}(\lambda-1)$; $\Delta\omega^{1,4,7}=\omega^{1,4,7}(\lambda\bar\lambda+1)+\omega^{1,5,6}(-\lambda+1)+\omega^{2,3,7}-\omega^{2,4,6}\lambda-\omega^{3,4,5}$; $\Delta\omega^{1,5,6}=\omega^{1,4,7}(-\bar\lambda+1)+3\omega^{1,5,6}+\omega^{3,4,5}\lambda$; $\Delta\omega^{1,5,7}=\omega^{1,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)-$

$\omega^{2,5,6}\lambda$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}(2\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{2,5,7}(\lambda - 1) - \omega^{3,4,7}\lambda + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6}\bar\lambda + \omega^{2,3,4}(\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,3,6}(\bar\lambda - 1) + \omega^{1,4,5} + \omega^{2,3,5}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7}\bar\lambda + \omega^{1,4,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,4,5}(\lambda - 1)$; $\Delta\omega^{2,4,5} = \omega^{1,4,6}(\bar\lambda - 1) + \omega^{2,4,5}(\lambda\bar\lambda + 2)$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7}\bar\lambda + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 3\omega^{2,4,7} + \omega^{2,5,6}(-\lambda + 1) + \omega^{3,4,6}\lambda$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7}\bar\lambda + \omega^{2,4,7}(-\bar\lambda + 1) + \omega^{2,5,6}(\lambda\bar\lambda + 1) + \omega^{3,4,6}(-\lambda\bar\lambda + \bar\lambda + 1)$; $\Delta\omega^{2,5,7} = \omega^{1,6,7}(\bar\lambda - 1) + \omega^{2,5,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{3,4,7}(-\lambda\bar\lambda + \bar\lambda + 1) + \omega^{3,5,6}\lambda$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{3,5,7} + \omega^{4,5,6}(\lambda - 1)$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} + \omega^{1,5,6}\bar\lambda + \omega^{2,3,7}(\bar\lambda - 1) + \omega^{2,4,6} + \omega^{3,4,5}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,6} = \omega^{2,4,7}\bar\lambda + \omega^{2,5,6}(-\lambda\bar\lambda + \lambda + 1) + \omega^{3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7}\bar\lambda + \omega^{2,5,7}(-\lambda\bar\lambda + \lambda + 1) + \omega^{3,4,7}(\lambda\bar\lambda + 2) + \omega^{3,5,6}(-\lambda + 1)$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} + \omega^{2,5,7}\bar\lambda + \omega^{3,4,7}(-\bar\lambda + 1) + 3\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}(\lambda\bar\lambda + 3)$; $\Delta\omega^{4,5,6} = \omega^{2,6,7}(\bar\lambda - 1) + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}(\lambda\bar\lambda + 2)$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 4)$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}(\lambda\bar\lambda - \lambda - \bar\lambda + 4)$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}(\lambda\bar\lambda + 2)$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,3,4,5}(\lambda - 1)$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}(\lambda\bar\lambda + 3)$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 3\omega^{1,2,4,7} + \omega^{1,2,5,6}(-\lambda + 1) + \omega^{1,3,4,6}\lambda + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7}(-\bar\lambda + 1) + \omega^{1,2,5,6}(\lambda\bar\lambda + 2) + \omega^{1,3,4,6}(-\lambda\bar\lambda + \bar\lambda + 1) - \omega^{2,3,4,5}\lambda$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{1,3,4,7}(-\lambda\bar\lambda + \bar\lambda + 1) + \omega^{1,3,5,6}\lambda$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{1,3,5,7} + \omega^{1,4,5,6}(\lambda - 1) + \omega^{2,3,4,7}\lambda - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7}(\bar\lambda - 1) + \omega^{1,2,4,6} + \omega^{1,3,4,5}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7}\bar\lambda + \omega^{1,2,5,6}(-\lambda\bar\lambda + \lambda + 1) + \omega^{1,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{2,3,4,5}(\lambda - 1)$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7}(-\lambda\bar\lambda + \lambda + 1) + \omega^{1,3,4,7}(\lambda\bar\lambda + 1) + \omega^{1,3,5,6}(-\lambda + 1) - \omega^{2,3,4,6}\lambda$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7}\bar\lambda + \omega^{1,3,4,7}(-\bar\lambda + 1) + 3\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6}\lambda$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}(\lambda\bar\lambda + 2) + \omega^{2,3,5,7}(\lambda - 1)$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,6,7}(\bar\lambda - 1) + \omega^{1,3,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}\lambda$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}(\lambda\bar\lambda + 1) + \omega^{2,3,6,7} + \omega^{2,4,5,7}(\lambda - 1) + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{3,4,5,7}\lambda$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6}\bar\lambda + \omega^{1,3,4,6}(\bar\lambda - 1) + \omega^{2,3,4,5}(2\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7}\bar\lambda + \omega^{2,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7}\bar\lambda + 3\omega^{2,3,4,7} + \omega^{2,3,5,6}(-\lambda + 1)$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7}\bar\lambda + \omega^{1,4,5,6} + \omega^{2,3,4,7}(-\bar\lambda + 1) + \omega^{2,3,5,6}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7}(\bar\lambda - 1) + \omega^{1,4,5,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + 2\omega^{2,3,6,7} + \omega^{3,4,5,6}(-\lambda + 1)$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7}\bar\lambda + \omega^{2,3,5,7} + \omega^{2,4,5,6}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7}(\bar\lambda - 1) + 3\omega^{2,4,5,7} + \omega^{3,4,5,6}\lambda$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{3,4,6,7}(-\lambda\bar\lambda + \bar\lambda + 1)$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{2,3,6,7}(-\bar\lambda + 1) + \omega^{2,4,5,7}\bar\lambda + \omega^{3,4,5,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7}\bar\lambda + \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7}(-\lambda\bar\lambda + \lambda + 1) + \omega^{3,4,6,7}(\lambda\bar\lambda + 2)$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}(2\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 4)$;

$\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}(-\lambda+1)$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7}(-\bar\lambda+1) + \omega^{1,2,3,5,6}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}(-\lambda+1)$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}\lambda$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2) + \omega^{1,3,4,6,7}(-\lambda\bar\lambda+\bar\lambda+1) - \omega^{2,3,4,5,7}\lambda$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7}(-\bar\lambda+1) + \omega^{1,2,4,5,7}\bar\lambda + \omega^{1,3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}\lambda$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7}(-\lambda\bar\lambda+\lambda+1) + \omega^{1,3,4,6,7}(\lambda\bar\lambda+1) + \omega^{2,3,4,5,7}(\lambda-1)$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}\bar\lambda + \omega^{2,3,4,5,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7}\bar\lambda + \omega^{1,3,4,6,7}(\bar\lambda-1) + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-8\beta_1}\ \langle\omega^{1,7} + \omega^{2,6}(2\lambda-1) + \omega^{3,5}(-\lambda+1)\rangle + e^{-5\beta_1}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-7\beta_1}\ \langle\omega^{1,6}(2\bar\lambda-1) + \omega^{2,5}(\bar\lambda-2) + \omega^{3,4}(\bar\lambda+1)\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-10\beta_1}\ \langle\omega^{1,2,7}(2\bar\lambda+1) + \omega^{1,3,6}(-\lambda\bar\lambda+\bar\lambda-1) + \omega^{1,4,5}(\lambda\bar\lambda-\bar\lambda+1) + \omega^{2,3,5}(\lambda-3)\rangle + e^{-11\beta_1}\ \langle\omega^{1,3,7}(\bar\lambda+2) + \omega^{1,4,6}(-\lambda\bar\lambda-2) + \omega^{2,3,6}(\lambda\bar\lambda+\lambda+1) + \omega^{2,4,5}(\lambda-1)\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,4,7}(\bar\lambda-3) + \omega^{1,5,6}(\lambda\bar\lambda-\lambda+1) + \omega^{2,3,7}(\lambda\bar\lambda-\lambda+1) + \omega^{3,4,5}(-2\lambda-1), -\omega^{1,5,6}\bar\lambda + \omega^{2,3,7}(-2\bar\lambda+3) + \omega^{2,4,6}(\bar\lambda-3) + 3\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-18\beta_1}\ \langle(\omega^{1,4,6,7}(\bar\lambda-3) + \omega^{2,3,6,7}(\lambda\bar\lambda-\lambda+1) + \omega^{2,4,5,7}(-\lambda\bar\lambda+\lambda-1) + \omega^{3,4,5,6}(2\lambda+1))/((2\lambda\bar\lambda+3)(\lambda\bar\lambda-\lambda-\bar\lambda+4))\rangle + e^{-17\beta_1}\ \langle(\omega^{1,3,6,7}(-\bar\lambda+1) - \omega^{1,4,5,7}(\lambda\bar\lambda+\bar\lambda+1) + \omega^{2,3,5,7}(\lambda\bar\lambda+2) + \omega^{2,4,5,6}(-\lambda-2))/((2\lambda\bar\lambda+\lambda+\bar\lambda+5)(\lambda\bar\lambda+2))\rangle + 2e^{-16\beta_1}\ \langle(\omega^{1,2,6,7}(-\bar\lambda^2+\bar\lambda+3) + \omega^{1,3,5,7}\bar\lambda(\lambda\bar\lambda-\lambda-\bar\lambda+4) + \omega^{1,4,5,6}(-\lambda\bar\lambda-3) + \omega^{2,3,4,7}(\lambda\bar\lambda^2-2\lambda\bar\lambda-\bar\lambda^2+4\bar\lambda-3) + \omega^{2,3,5,6}(2\lambda\bar\lambda-3\lambda-3\bar\lambda+9))/((\lambda\bar\lambda-\lambda-\bar\lambda+4)(\lambda\bar\lambda+3)(\bar\lambda-3)), (-3\omega^{1,2,6,7} + \omega^{1,3,5,7}(2\lambda\bar\lambda+3) + \omega^{1,4,5,6}(-\lambda\bar\lambda-3) + \omega^{2,3,4,7}\lambda\bar\lambda + 3\omega^{2,3,5,6}\lambda)/(3(\lambda\bar\lambda+3)(\bar\lambda-3))\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-20\beta_1}\ \langle(\omega^{1,2,4,6,7}(\bar\lambda-1) + \omega^{1,3,4,5,7}(-2\bar\lambda+1) - \omega^{2,3,4,5,6})/(5\lambda\bar\lambda-3\lambda-3\bar\lambda+3)\rangle + e^{-23\beta_1}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-21\beta_1}\ \langle(\omega^{1,2,5,6,7}(\lambda+1) + \omega^{1,3,4,6,7}(\lambda-2) + \omega^{2,3,4,5,7}(2\lambda-1))/(3(2\lambda\bar\lambda-\lambda-\bar\lambda+2))\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-26\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

The preceding results are valid for: $(\bar\lambda^2-\bar\lambda+1)(\lambda-3\bar\lambda-1)(\lambda^2-\lambda+1)(\bar\lambda+2)(-(\lambda\bar\lambda+\lambda-\bar\lambda^2-2\bar\lambda+1))(-(\bar\lambda-3))(\bar\lambda+2)(-(\lambda-3\bar\lambda-1))(\bar\lambda-3)(2\bar\lambda+1)(\bar\lambda-$

$1)(\bar{\lambda}-1)(-(\bar{\lambda}-3))(-(\lambda\bar{\lambda}-\lambda-1))(-(\bar{\lambda}-3))(\lambda-3)(\bar{\lambda}-1)(\bar{\lambda}-2)(\lambda+1) \neq 0.$

Comments:singular values.

Among the values excluded by the computer, only $\lambda = 3$ and $\lambda^2-\lambda+1 = 0$ are singular values, i.e. give different cohomology than the generic one.

Case $\lambda = 3$

For $\lambda = 3$ the generic eigenvectors for $2e^{-12\beta_1}$ in $\mathrm{ch}(H^3_{\mathbb{C}})$ are proportional to $\varphi = \omega^{1,5,6} + \omega^{2,3,7} - \omega^{3,4,5}$. (This reflects the fact that the denominators for $e^{-16\beta_1}$ in $\mathrm{ch}(H^4_{\mathbb{C}})$ cancel). Change in $\mathrm{ch}(H^3_{\mathbb{C}})$ $2e^{-12\beta_1}$ $\langle ... \rangle$ to $2e^{-12\beta_1}$ $\langle \varphi, \psi \rangle$ with $\psi = 3\omega^{1,4,7} - 7\omega^{2,3,7} + 7\omega^{2,4,6} + 2\omega^{3,4,5}$. (Note that the nonharmonic cocycles computed by the first program in the case $\lambda = 3$ are $\omega^{1,4,7} + 3\omega^{3,4,5}$ which is cohomologous to $-\varphi$, and $\omega^{1,5,6} + \omega^{2,4,6} - 2\omega^{3,4,5}$ which is cohomologous to $(10\varphi + \psi)/8$).
Add to $\mathrm{ch}(H^2_{\mathbb{C}})$ $e^{-9\beta_1}$ $\langle \omega^{2,7} - \omega^{3,6} + \omega^{4,5} \rangle$ and correspondingly add to $\mathrm{ch}(H^5_{\mathbb{C}})$ $e^{-19\beta_1}$ $\langle (\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3 \rangle$.
Add to $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-9\beta_1}$ $\langle 2\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4} \rangle$ and correspondingly add to $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-19\beta_1}$ $\langle (\omega^{1,5,6,7} + \omega^{2,4,6,7} - 2\omega^{3,4,5,7})/6 \rangle$

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, \underline{9}, 17, \underline{23}, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, \underline{4}, \underline{5}, \underline{5}, \underline{4}, 2, 1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 20.

Riemann Hyp.: no

$P(Y) \;=\; (1 + Y + Y^2)(1 + Y^2)^2(1 + Y).$

Case $\lambda^2 - \lambda + 1 = 0$

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-14\beta_1}$ $\langle \omega^{1,6,7} + \omega^{2,5,7}(-\lambda + 1) + \omega^{3,4,7}\lambda - \omega^{3,5,6} \rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-14\beta_1}$ $\langle (-\omega^{1,2,4,7} + \omega^{1,2,5,6}(-\lambda + 1) + \omega^{1,3,4,6}\lambda + \omega^{2,3,4,5})/4 \rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, \underline{18}, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, \underline{5}, \underline{5}, 3, 2, 1)$

Total dimension of trivial cohomology: 22.

Length associated polynomial: 20.

Riemann Hyp.: no

$P(Y) \;=\; (1 + Y + 2Y^2 + 3Y^3 + 2Y^4 + Y^5 + Y^6)(1 + Y).$

5.3.2 Algebra $\mathcal{G}_{7,1.1(ii)}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & 0 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^3 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^5 & 0 & \xi_2^3 & 6\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & \xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_1^2+\xi_2^3 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-\delta_1^4, ad\ x_4=-\delta_1^5, ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+e^{5\beta_1}\ \langle x_5\rangle+e^{6\beta_1}\ \langle x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{5\beta_1}\ \langle\delta_1^6,\delta_2^7\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+2e^{4\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^4,\delta_2^5\rangle+e^{2\beta_1}\ \langle\delta_1^3\rangle+2e^{\beta_1}\ \langle\delta_1^2,\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^5,\delta_2^3]=-(\delta_2^7+\delta_1^6)$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+\delta_1^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^3]=-\delta_1^5$; $[\delta_2^5,\delta_1^3]=\delta_2^7+\delta_1^6$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_1^3,\delta_2^3]=-\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{5\beta_1}\ \langle\delta_1^6\rangle+e^{4\beta_1}\ \langle\delta_2^6\rangle+e^{3\beta_1}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_1^2\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6,\delta_1^2]=2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=2e^{-3\beta_1}\ \langle x_1\omega^{1,3}+2x_2\omega^{2,3}+3x_3\omega^{2,4}+x_4(\omega^{1,6}+\omega^{2,5}+2\omega^{3,4})+x_5\omega^{3,5},-x_1\omega^{1,3}+x_2\omega^{2,3}-2x_4(\omega^{1,6}-1/2\omega^{2,5}+1/2\omega^{3,4})+x_5\omega^{2,6}+x_6\omega^{3,6}\rangle+e^{\beta_1}\ \langle x_4\omega^{1,2}+x_6\omega^{2,3}\rangle+e^0\ \langle -x_3\omega^{1,2}+x_5\omega^{2,3}+x_6\omega^{2,4}\rangle+3e^{-2\beta_1}\ \langle -2/5x_1\omega^{1,2}-1/5x_3\omega^{2,3}+1/5x_4\omega^{2,4}+7/5x_5(\omega^{1,6}+3/7\omega^{2,5})+x_6\omega^{2,6},-1/5x_1\omega^{1,2}-3/5x_3\omega^{2,3}-2/5x_4\omega^{2,4}+1/5x_5(\omega^{1,6}-6\omega^{2,5}+5\omega^{3,4})+x_6\omega^{1,7},x_3\omega^{1,4}\rangle+2e^{-\beta_1}\ \langle -x_3\omega^{1,3}+x_4\omega^{2,3}+x_5\omega^{2,4}+x_6\omega^{3,4},x_2\omega^{1,2}+x_4\omega^{2,3}+x_5\omega^{2,4}+x_6\omega^{2,5}\rangle+e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{-5\beta_1}+2e^{-6\beta_1}+3e^{-7\beta_1}+4e^{-8\beta_1}+2e^{-9\beta_1}+2e^{-10\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-10\beta_1}+2e^{-11\beta_1}+4e^{-12\beta_1}+3e^{-13\beta_1}+2e^{-14\beta_1}+2e^{-15\beta_1}+e^{-16\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$2e^{-16\beta_1}+2e^{-17\beta_1}+3e^{-18\beta_1}+2e^{-19\beta_1}+e^{-20\beta_1}+e^{-21\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-20\beta_1}+e^{-21\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}+e^{-25\beta_1}+e^{-26\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-26\beta_1}+e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,48,114,147,109,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,10,15,16,11,6,2)$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,7}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-5\beta_1}\ \langle\omega^{2,3}\rangle+e^{-7\beta_1}\ \langle\omega^{1,6}\rangle+e^{-8\beta_1}\ \langle\omega^{1,7}+2\omega^{2,6}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-10\beta_1}\ \langle\omega^{1,2,7}+\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle\omega^{2,3,6}-\omega^{2,4,5}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,4,7}+\omega^{3,4,5}, \omega^{2,3,7}-\omega^{2,4,6}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 2e^{-16\beta_1}\ \langle\omega^{1,2,6,7}-\omega^{2,3,5,6}, \omega^{2,3,4,7}-\omega^{2,3,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{1,3,6,7}-\omega^{2,4,5,6}\rangle + e^{-18\beta_1}\ \langle\omega^{2,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-21\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-20\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-26\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=\omega^7$; $d^\star\omega^{1,2,3}=-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,7}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{5,7}$; $d^\star\omega^{3,4,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,2,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}-\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=-\omega^{1,5,6,7}-$

$\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,6} + 2\omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,4,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 3\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$;

$\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-5\beta_1}\ \langle\omega^{2,3}\rangle + e^{-7\beta_1}\ \langle 2\omega^{1,6} - \omega^{2,5} + \omega^{3,4}\rangle + e^{-8\beta_1}\ \langle\omega^{1,7} + 2\omega^{2,6} - \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-10\beta_1}\ \langle\omega^{1,2,7} - 2\omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,3,7} + 2\omega^{2,3,6} - \omega^{2,4,5}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,4,7} - 2\omega^{2,3,7} + 2\omega^{2,4,6} - \omega^{3,4,5}, \omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-18\beta_1}\ \langle(\omega^{1,4,6,7} + 2\omega^{2,3,6,7} - 2\omega^{2,4,5,7} + \omega^{3,4,5,6})/10\rangle + e^{-17\beta_1}\ \langle(\omega^{1,3,6,7} - 2\omega^{1,4,5,7} - \omega^{2,4,5,6})/6\rangle + 2e^{-16\beta_1}\ \langle(\omega^{1,2,6,7} + 3\omega^{1,3,5,7} - 3\omega^{1,4,5,6} - 5\omega^{2,3,4,7} + 4\omega^{2,3,5,6})/15, (-\omega^{1,2,6,7} + 2\omega^{2,3,4,7} - \omega^{2,3,5,6})/3\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-23\beta_1}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-21\beta_1}\ \langle(\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/6\rangle + e^{-20\beta_1}\ \langle(\omega^{1,2,4,6,7} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/6\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-26\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.3.3 Algebra $\mathcal{G}_{7,1.1(iii)}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$; $[x_2, x_5] = -x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_2^3 & 5\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 6\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_1^5 & -\xi_1^4 & \xi_1^3 & 0 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -\delta_1^6 + \delta_2^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{3\beta_1}\ \langle x_3\rangle + e^{4\beta_1}\ \langle x_4\rangle + e^{5\beta_1}\ \langle x_5\rangle + e^{6\beta_1}\ \langle x_6\rangle + e^{7\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle x_6\rangle + e^{7\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{5\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + 2e^{4\beta_1}\ \langle \delta_2^6, \delta_1^5\rangle + 2e^{3\beta_1}\ \langle \delta_2^5, \delta_1^4\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^{6\beta_1}\ \langle \delta_1^7\rangle + e^{2\beta_1}\ \langle \delta_1^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^3] = -\delta_2^7 + \delta_1^6$; $[\delta_2^3, \delta_1^5] = -\delta_2^7 + \delta_1^6$; $[\delta_2^3, \delta_1^4] = \delta_2^6 + \delta_1^5$; $[\delta_2^3, \delta_1^3] = \delta_2^5 + \delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle \delta_1^7\rangle + e^{5\beta_1}\ \langle \delta_1^6\rangle + e^{4\beta_1}\ \langle \delta_1^5\rangle + e^{3\beta_1}\ \langle \delta_1^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $2e^{-3\beta_1}\ \langle -x_1\omega^{1,3} - x_2(\omega^{1,4} - 2\omega^{2,3}) + x_3(\omega^{1,5} + 2\omega^{2,4}) + 2x_4\omega^{2,5} + x_5\omega^{2,6} + 3x_6(\omega^{2,7} + 1/3\omega^{3,6}), -x_2(\omega^{1,4} - \omega^{2,3}) + x_3(\omega^{1,5} + 2\omega^{2,4}) + x_4(\omega^{2,5} + \omega^{3,4})\rangle + e^{-4\beta_1}\ \langle x_1\omega^{2,3} + x_5\omega^{2,7} + x_6\omega^{3,7}\rangle + e^0\ \langle x_3\omega^{1,2} + x_7\omega^{1,6}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + 2x_3\omega^{1,3} - x_4(\omega^{1,4} - 3\omega^{2,3}) + x_7\omega^{1,7}, -x_3\omega^{1,3} + x_4(\omega^{1,4} - \omega^{2,3}) + x_6\omega^{1,6}\rangle + 3e^{-2\beta_1}\ \langle -2x_1\omega^{1,2} + x_2\omega^{1,3} + 7/2x_3(\omega^{1,4} - 3/7\omega^{2,3}) + x_4(\omega^{1,5} + 3/2\omega^{2,4}) + x_6\omega^{1,7}, -x_1\omega^{1,2} + 3/2x_3(\omega^{1,4} - 1/3\omega^{2,3}) + 1/2x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} + x_2\omega^{1,3} + 1/2x_3(\omega^{1,4} - 3\omega^{2,3}) + x_4(\omega^{1,5} + 1/2\omega^{2,4}) + x_5\omega^{3,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-5\beta_1} + 2e^{-6\beta_1} + 3e^{-7\beta_1} + 4e^{-8\beta_1} + 2e^{-9\beta_1} + e^{-10\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-10\beta_1} + 2e^{-11\beta_1} + 4e^{-12\beta_1} + 3e^{-13\beta_1} + 2e^{-14\beta_1} + 2e^{-15\beta_1} + e^{-16\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$2e^{-16\beta_1} + 2e^{-17\beta_1} + 3e^{-18\beta_1} + 2e^{-19\beta_1} + e^{-20\beta_1} + e^{-21\beta_1}$.

ch$(H^6_{\mathcal{G}})$=$e^{-20\beta_1}+e^{-21\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}+e^{-25\beta_1}+e^{-26\beta_1}$.

ch$(H^7_{\mathcal{G}})$=$e^{-26\beta_1}+e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,10,48,113,147,109,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,5,9,14,15,11,6,2)$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}+\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}+\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}-\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=-\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=-\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}+\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}+\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1}\ \langle\omega^{1,4}\rangle + e^{-7\beta_1}\ \langle\omega^{1,6}+\omega^{3,4}\rangle + e^{-8\beta_1}\ \langle\omega^{1,7}-2\omega^{2,6}+\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-10\beta_1}\ \langle\omega^{1,3,6}+\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,4,6}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,4,7}-\omega^{3,4,5},\omega^{1,5,6}+\omega^{2,4,6}-2\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-16\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{2,3,5,6},\omega^{1,4,5,6}-\omega^{2,3,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7}+\omega^{2,4,5,6}\rangle + e^{-18\beta_1}\ \langle\omega^{1,4,6,7}+\omega^{3,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-21\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-26\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=-\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}+\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{4,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=$

$\omega^{1,3,6,7}-\omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6}=\omega^{1,4,6,7}-\omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6}=\omega^{1,5,6,7}$; $d^{\star}\omega^{1,3,4,5,7}=-\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7}=-\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,5,6,7}=-\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6}=\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7}=\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,6,7}=\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7}=-\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7}=-\omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7}=-\omega^{3,4,5,6,7}$;

$\Delta\omega^{3}=\omega^{3}$; $\Delta\omega^{4}=\omega^{4}$; $\Delta\omega^{5}=2\omega^{5}$; $\Delta\omega^{6}=2\omega^{6}$; $\Delta\omega^{7}=2\omega^{7}$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,4}=\omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,5}=2\omega^{1,5}$; $\Delta\omega^{1,6}=\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $\Delta\omega^{1,7}=2\omega^{1,7}+\omega^{2,6}$; $\Delta\omega^{2,3}=\omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{2,4}=2\omega^{2,4}$; $\Delta\omega^{2,5}=-\omega^{1,6}+2\omega^{2,5}$; $\Delta\omega^{2,6}=\omega^{1,7}+\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{2,7}=\omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{3,4}=-\omega^{1,6}+2\omega^{3,4}$; $\Delta\omega^{3,5}=\omega^{2,6}+2\omega^{3,5}$; $\Delta\omega^{3,6}=-\omega^{2,7}+3\omega^{3,6}$; $\Delta\omega^{3,7}=2\omega^{3,7}$; $\Delta\omega^{4,5}=\omega^{2,7}+2\omega^{4,5}$; $\Delta\omega^{4,6}=2\omega^{4,6}$; $\Delta\omega^{4,7}=2\omega^{4,7}+\omega^{5,6}$; $\Delta\omega^{5,6}=\omega^{4,7}+3\omega^{5,6}$; $\Delta\omega^{5,7}=3\omega^{5,7}$; $\Delta\omega^{6,7}=4\omega^{6,7}$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=3\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=3\omega^{1,2,5}$; $\Delta\omega^{1,2,6}=\omega^{1,2,6}+\omega^{1,3,5}+\omega^{2,3,4}$; $\Delta\omega^{1,2,7}=2\omega^{1,2,7}-\omega^{1,3,6}+\omega^{1,4,5}$; $\Delta\omega^{1,3,4}=2\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=\omega^{1,2,6}+2\omega^{1,3,5}$; $\Delta\omega^{1,3,6}=-\omega^{1,2,7}+2\omega^{1,3,6}-\omega^{2,3,5}$; $\Delta\omega^{1,3,7}=2\omega^{1,3,7}+\omega^{2,3,6}$; $\Delta\omega^{1,4,5}=\omega^{1,2,7}+2\omega^{1,4,5}+\omega^{2,3,5}$; $\Delta\omega^{1,4,6}=\omega^{1,4,6}+\omega^{2,3,6}-\omega^{2,4,5}$; $\Delta\omega^{1,4,7}=2\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,3,7}+\omega^{2,4,6}$; $\Delta\omega^{1,5,6}=\omega^{1,4,7}+2\omega^{1,5,6}+\omega^{3,4,5}$; $\Delta\omega^{1,5,7}=3\omega^{1,5,7}+\omega^{2,5,6}$; $\Delta\omega^{1,6,7}=3\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,4,7}$; $\Delta\omega^{2,3,4}=\omega^{1,2,6}+3\omega^{2,3,4}$; $\Delta\omega^{2,3,5}=-\omega^{1,3,6}+\omega^{1,4,5}+2\omega^{2,3,5}$; $\Delta\omega^{2,3,6}=\omega^{1,3,7}+\omega^{1,4,6}+2\omega^{2,3,6}$; $\Delta\omega^{2,3,7}=\omega^{1,4,7}+\omega^{2,3,7}-\omega^{3,4,5}$; $\Delta\omega^{2,4,5}=-\omega^{1,4,6}+3\omega^{2,4,5}$; $\Delta\omega^{2,4,6}=\omega^{1,4,7}+2\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{2,4,7}=2\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,5,6}=\omega^{1,5,7}+\omega^{2,4,7}+2\omega^{2,5,6}$; $\Delta\omega^{2,5,7}=-\omega^{1,6,7}+2\omega^{2,5,7}-\omega^{3,5,6}$; $\Delta\omega^{2,6,7}=2\omega^{2,6,7}+\omega^{3,5,7}-\omega^{4,5,6}$; $\Delta\omega^{3,4,5}=\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,6}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6}=-\omega^{2,4,7}+3\omega^{3,4,6}$; $\Delta\omega^{3,4,7}=-\omega^{1,6,7}+2\omega^{3,4,7}+\omega^{3,5,6}$; $\Delta\omega^{3,5,6}=-\omega^{2,5,7}+\omega^{3,4,7}+3\omega^{3,5,6}$; $\Delta\omega^{3,5,7}=\omega^{2,6,7}+2\omega^{3,5,7}$; $\Delta\omega^{3,6,7}=4\omega^{3,6,7}$; $\Delta\omega^{4,5,6}=-\omega^{2,6,7}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7}=2\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=3\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4}=4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6}=2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7}=2\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5}=4\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=2\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7}=3\omega^{1,2,4,7}+\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,4,7}+2\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7}=3\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7}=2\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5}=-\omega^{1,2,3,7}+\omega^{1,2,4,6}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,4,7}+2\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7}=2\omega^{1,3,4,7}+\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}+2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7}=\omega^{1,2,6,7}+2\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7}=3\omega^{1,3,6,7}-\omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6}=-\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7}=2\omega^{1,4,5,7}+\omega^{2,3,5,7}+\omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7}=2\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}$;

$\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,4,7}+3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7}+2\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + 2\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} - \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-5\beta_1} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-7\beta_1} \langle 2\omega^{1,6} + \omega^{2,5} + \omega^{3,4}\rangle + e^{-8\beta_1} \langle\omega^{1,7} - 2\omega^{2,6} + \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-10\beta_1} \langle\omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,5}\rangle + e^{-11\beta_1} \langle\omega^{1,3,7} + 3\omega^{1,4,6} - 2\omega^{2,3,6}+\omega^{2,4,5}\rangle+2e^{-12\beta_1} \langle\omega^{1,4,7}-2\omega^{2,3,7}-\omega^{3,4,5}, \omega^{1,5,6}-2\omega^{2,3,7}+\omega^{2,4,6}-2\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-18\beta_1} \langle(\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}+\omega^{3,4,5,6})/4\rangle+e^{-17\beta_1} \langle(-\omega^{1,3,6,7}+ 2\omega^{1,4,5,7} - 3\omega^{2,3,5,7} - \omega^{2,4,5,6})/15\rangle + 2e^{-16\beta_1} \langle(\omega^{1,2,6,7} - 3\omega^{1,3,5,7} - 4\omega^{1,4,5,6} - 3\omega^{2,3,4,7} + 5\omega^{2,3,5,6})/12, (-\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6})/4\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-23\beta_1} \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-21\beta_1} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/6\rangle + e^{-20\beta_1} \langle(-\omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/6\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-27\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-26\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-28\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.3.4 Algebra $\mathcal{G}_{7,1.1(iv)}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 5\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 6\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & -2\xi_1^4+\xi_2^5 & -(\xi_1^3+\xi_2^4) & \xi_2^3 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_2^6, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T{=}\langle t_1\rangle$.

ch$(\mathcal{G})$= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{3\beta_1}\ \langle x_3\rangle + e^{4\beta_1}\ \langle x_4\rangle + e^{5\beta_1}\ \langle x_5\rangle + e^{6\beta_1}\ \langle x_6\rangle + e^{7\beta_1}\ \langle x_7\rangle$.

ch$(H^0_{\mathcal{G}})$= $e^{7\beta_1}\ \langle x_7\rangle$.

ch$(Z^1_{\mathcal{G}})$= $2e^{5\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + e^{6\beta_1}\ \langle \delta_1^7\rangle + 2e^{4\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + 2e^{3\beta_1}\ \langle \delta_1^4, \delta_2^5\rangle + 2e^{2\beta_1}\ \langle \delta_1^3, \delta_2^4\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = \delta_1^7$; $[\delta_1^5, \delta_2^3] = -(\delta_2^7 + \delta_1^6)$; $[\delta_2^6, \delta_1^3] = \delta_1^7$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -2\delta_2^7$; $[\delta_1^4, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_1^3] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_2^4] = 2\delta_2^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$.

ch$(H^1_{\mathcal{G}})$= $e^{5\beta_1}\ \langle \delta_1^6\rangle + e^{4\beta_1}\ \langle \delta_1^5\rangle + e^{3\beta_1}\ \langle \delta_1^4\rangle + e^{2\beta_1}\ \langle \delta_2^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^4] = 2\delta_1^6$.

ch$(H^2_{\mathcal{G}})$= $e^0\ \langle -x_3\omega^{1,2} + x_5\omega^{1,4}\rangle + 2e^{-3\beta_1}\ \langle -x_1\omega^{1,3} - 3x_2(\omega^{1,4} - 2/3\omega^{2,3}) - 3x_3(\omega^{1,5} + 2/3\omega^{2,4}) + 2x_4(\omega^{1,6} - \omega^{2,5}) + 3x_5(\omega^{1,7} + 2\omega^{2,6}) + 3x_6(\omega^{2,7} + 1/3\omega^{3,6}), x_2\omega^{2,3} + x_3\omega^{2,4} - x_4(\omega^{2,5} - \omega^{3,4})\rangle + e^{2\beta_1}\ \langle x_5\omega^{1,2}\rangle + 2e^{-\beta_1}\ \langle x_4\omega^{2,3}, -2/3x_2\omega^{1,2} + 4/3x_3\omega^{1,3} - 1/3x_4\omega^{1,4} + 2/3x_5\omega^{1,5} + x_6\omega^{3,4}\rangle + 3e^{-2\beta_1}\ \langle 2x_1\omega^{1,2} + x_2\omega^{1,3} - 2x_3(\omega^{1,4} - 3/4\omega^{2,3}) - x_4(\omega^{1,5} + 3/2\omega^{2,4}) - 2x_5\omega^{1,6} + x_6\omega^{1,7}, -2x_1\omega^{1,2} + 2x_3(\omega^{1,4} - 3/4\omega^{2,3}) + x_4(\omega^{1,5} + 3/2\omega^{2,4}) + 2x_5\omega^{1,6} + x_6\omega^{3,5}, x_1\omega^{1,2} + x_2\omega^{1,3} - 2x_3(\omega^{1,4} - 3/4\omega^{2,3}) - x_4(\omega^{1,5} + 1/2\omega^{2,4}) - x_5(\omega^{1,6} - \omega^{3,4})\rangle + e^{-4\beta_1}\ \langle x_1\omega^{2,3} - x_4(\omega^{2,6} - \omega^{3,5})\rangle$.

ch$(H^3_{\mathcal{G}})$=$2e^{-5\beta_1} + 2e^{-6\beta_1} + 3e^{-7\beta_1} + 4e^{-8\beta_1} + 2e^{-9\beta_1} + e^{-10\beta_1} + e^{-11\beta_1}$.

ch$(H^4_{\mathcal{G}})$=$e^{-10\beta_1} + 3e^{-11\beta_1} + 4e^{-12\beta_1} + 4e^{-13\beta_1} + 2e^{-14\beta_1} + 3e^{-15\beta_1} + e^{-16\beta_1}$.

ch$(H^5_{\mathcal{G}})$=$e^{-13\beta_1}+e^{-15\beta_1}+2e^{-16\beta_1}+2e^{-17\beta_1}+3e^{-18\beta_1}+2e^{-19\beta_1}+e^{-20\beta_1}+e^{-21\beta_1}+e^{-22\beta_1}$.

ch$(H^6_{\mathcal{G}})$=$e^{-20\beta_1}+e^{-21\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}+e^{-25\beta_1}+e^{-26\beta_1}$.

ch$(H^7_{\mathcal{G}})$=$e^{-26\beta_1}+e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,48,114,149,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,10,15,18,14,7,2)$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,19,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,6,6,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{2,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{2,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-5\beta_1}\ \langle\omega^{1,4}\rangle + e^{-7\beta_1}\ \langle\omega^{1,6}+\omega^{3,4}\rangle + e^{-8\beta_1}\ \langle\omega^{2,6}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-10\beta_1}\ \langle\omega^{1,2,7}-\omega^{1,4,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,4,5}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,4,7}+\omega^{3,4,5},\omega^{1,5,6}+\omega^{2,4,6}-2\omega^{3,4,5}\rangle + e^{-13\beta_1}\ \langle\omega^{1,5,7}+\omega^{2,4,7}-\omega^{3,4,6}\rangle + e^{-15\beta_1}\ \langle\omega^{2,6,7}-\omega^{3,5,7}+\omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= 2e^{-16\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7},\omega^{1,3,5,7}\rangle + e^{-13\beta_1}\ \langle\omega^{1,2,3,7}\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,4,7}+\omega^{2,3,4,6}\rangle + e^{-18\beta_1}\ \langle\omega^{1,4,6,7}-\omega^{3,4,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{2,3,5,7}-\omega^{2,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-21\beta_1}\ \langle\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-23\beta_1}\ \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-26\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1-Y+3Y^2-Y^3+Y^4)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,2,8,19,22,16,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,2,3,6,6,3,2,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 20.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,6}$;

$d^\star\omega^{2,3,4,6} = -\omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7}-\omega^{1,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 3\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 3\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 3\omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 3\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 3\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$;

$\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,7} + 3\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + 3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 3\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + 3\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{2,3,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{X}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{X}}) = e^{-5\beta_1}\ \langle\omega^{1,4}\rangle + e^{-7\beta_1}\ \langle\omega^{1,6} - 2\omega^{2,5} + \omega^{3,4}\rangle + e^{-8\beta_1}\ \langle\omega^{2,6} - \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{X}}) = e^{-10\beta_1}\ \langle 2\omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - 3\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle 2\omega^{1,3,7} - 3\omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5}\rangle + 2e^{-12\beta_1}\ \langle 3\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-13\beta_1}\ \langle\omega^{1,5,7} + \omega^{2,4,7} + 3\omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-15\beta_1}\ \langle\omega^{2,6,7} - \omega^{3,5,7} + \omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{X}}) = e^{-18\beta_1}\ \langle(-3\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6})/15\rangle + e^{-17\beta_1}\ \langle(-\omega^{1,3,6,7} - \omega^{1,4,5,7} + 3\omega^{2,3,5,7} - 2\omega^{2,4,5,6})/15\rangle + 2e^{-16\beta_1}\ \langle(\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7} + 3\omega^{2,3,5,6})/12, (\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle + e^{-15\beta_1}\ \langle(\omega^{1,2,5,7} - 3\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/12\rangle + e^{-13\beta_1}\ \langle(-\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5})/3\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{X}}) = e^{-23\beta_1}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-21\beta_1}\ \langle(\omega^{1,2,5,6,7} - 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/6\rangle + e^{-20\beta_1}\ \langle(\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{X}}) = e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-26\beta_1}\ \langle -\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{X}}) = e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 - Y + 3Y^2 - Y^3 + Y^4)(1+Y)^3.$

5.3.5 Algebra $\mathcal{G}_{7,1.1(v)}$

Adjoint cohomology

$[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & 0 & \xi_3^4 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_1^4 & \xi_3^5 & \xi_3^4 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_1^5 & \xi_3^6 & \xi_3^5 & \xi_3^4 & 6\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_1^4+\xi_3^6 & \xi_3^5 & \xi_3^4 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_3^4, ad\ x_2=\delta_3^5, ad\ x_3=-\delta_1^4, ad\ x_4=-\delta_1^5+\delta_3^7, ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+e^{5\beta_1}\ \langle x_5\rangle+e^{6\beta_1}\ \langle x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{5\beta_1}\ \langle \delta_1^6,\delta_2^7\rangle+2e^{4\beta_1}\ \langle\delta_1^5,\delta_3^7\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+2e^{3\beta_1}\ \langle\delta_1^4,\delta_3^6\rangle+e^{2\beta_1}\ \langle\delta_3^5\rangle+e^{\beta_1}\ \langle\delta_3^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_3^4]=-\delta_1^7$; $[\delta_1^5,\delta_3^5]=-\delta_1^7$; $[\delta_1^5,\delta_3^4]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^4,\delta_3^6]=-\delta_1^7$; $[\delta_1^4,\delta_3^5]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^4,\delta_3^4]=\delta_3^7-\delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{5\beta_1}\ \langle\delta_1^6\rangle+e^{4\beta_1}\ \langle\delta_1^5\rangle+e^{3\beta_1}\ \langle\delta_3^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= 3e^{-3\beta_1}\ \langle -x_1\omega^{1,3}-x_2(\omega^{1,4}+\omega^{2,3})+x_3(\omega^{1,5}+\omega^{2,4})-2x_4(\omega^{1,6}+1/2\omega^{2,5})-2x_5\omega^{2,6}+x_6\omega^{3,6}, -x_2(\omega^{1,4}-\omega^{2,3})+x_3(\omega^{1,5}+\omega^{2,4})+x_4(\omega^{2,5}+\omega^{3,4}), x_2(\omega^{1,4}-\omega^{2,3})-x_3(\omega^{1,5}+\omega^{2,4})-x_4\omega^{2,5}+x_5(\omega^{1,7}+\omega^{2,6})+x_6\omega^{2,7}\rangle+2e^{-4\beta_1}\ \langle x_1(\omega^{1,4}-\omega^{2,3})+x_2(\omega^{1,5}+\omega^{2,4})-x_3(\omega^{1,6}+\omega^{2,5}+2\omega^{3,4})-x_4(\omega^{1,7}+2\omega^{2,6})-x_5(\omega^{2,7}-\omega^{3,6})+x_6\omega^{3,7}, -x_1(\omega^{1,4}-\omega^{2,3})-x_2(\omega^{1,5}+\omega^{2,4})+x_3(\omega^{1,6}+\omega^{2,5}+2\omega^{3,4})+x_4(\omega^{2,6}+\omega^{3,5})\rangle+e^0\ \langle x_3\omega^{1,2}+x_5\omega^{1,4}\rangle+2e^{-\beta_1}\ \langle -x_2\omega^{1,2}-x_3\omega^{1,3}+x_4(\omega^{1,4}-3\omega^{2,3})+x_5\omega^{1,5}, -x_3\omega^{1,3}-x_4\omega^{2,3}+x_6\omega^{3,4}\rangle+3e^{-2\beta_1}\ \langle 2x_1\omega^{1,2}-x_2\omega^{1,3}-11/2x_3(\omega^{1,4}-3/11\omega^{2,3})-x_4(\omega^{1,5}+3/2\omega^{2,4})-2x_5\omega^{1,6}+x_6\omega^{1,7}, 2x_1\omega^{1,2}-x_2\omega^{1,3}-6x_3(\omega^{1,4}-1/3\omega^{2,3})-2x_4(\omega^{1,5}+\omega^{2,4})-2x_5\omega^{1,6}+x_6\omega^{3,5}, -x_1\omega^{1,2}+x_2\omega^{1,3}+5/2x_3(\omega^{1,4}-3/5\omega^{2,3})+x_4(\omega^{1,5}+1/2\omega^{2,4})+x_5(\omega^{1,6}+\omega^{3,4})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-3\beta_1}+e^{-4\beta_1}+2e^{-5\beta_1}+2e^{-6\beta_1}+3e^{-7\beta_1}+4e^{-8\beta_1}+2e^{-9\beta_1}+e^{-10\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-10\beta_1}+2e^{-11\beta_1}+4e^{-12\beta_1}+3e^{-13\beta_1}+2e^{-14\beta_1}+2e^{-15\beta_1}+e^{-16\beta_1}+e^{-17\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-16\beta_1}+3e^{-17\beta_1}+3e^{-18\beta_1}+2e^{-19\beta_1}+e^{-20\beta_1}+e^{-21\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-20\beta_1}+e^{-21\beta_1}+e^{-22\beta_1}+2e^{-23\beta_1}+2e^{-24\beta_1}+2e^{-25\beta_1}+e^{-26\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-25\beta_1}+e^{-26\beta_1}+e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,10,50,113,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,4,11,16,16,15,10,3)$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,3,7}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}+\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,6}$; $d\omega^{3,4,6}=-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=$

$-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1}\ \langle\omega^{1,2}\rangle+e^{-5\beta_1}\ \langle\omega^{1,4}\rangle+e^{-7\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}\rangle+e^{-8\beta_1}\ \langle\omega^{1,7}+\omega^{2,6}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-10\beta_1}\ \langle\omega^{1,2,7}+\omega^{2,3,5}\rangle+e^{-11\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,4,5}\rangle+2e^{-12\beta_1}\ \langle\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,4,6},\omega^{1,5,6}+2\omega^{2,3,7}-\omega^{2,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-16\beta_1}\ \langle\omega^{1,2,6,7}-\omega^{2,3,5,6},\omega^{1,3,5,7}\rangle+e^{-17\beta_1}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7}-2\omega^{2,4,5,6}\rangle+e^{-18\beta_1}\ \langle\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-21\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}\rangle+e^{-20\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle+e^{-23\beta_1}\ \langle\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle+e^{-25\beta_1}\ \langle\omega^{3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-25\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle+e^{-26\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle+e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 20.

Riemann Hyp.: no

$d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{5,7}$; $d^\star\omega^{3,4,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=$

$\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 4\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 3\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,4,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 3\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 3\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} - \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} =$

$2\omega^{1,2,3,7}+\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5}=3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=\omega^{1,2,3,7}+2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7}=2\omega^{1,2,4,7}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6}=2\omega^{1,2,5,6}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7}=2\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,3,7}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=\omega^{1,2,4,7}-\omega^{1,2,5,6}+3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7}=-\omega^{1,2,5,7}+2\omega^{1,3,4,7}-\omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6}=\omega^{1,2,5,7}+3\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7}=2\omega^{1,3,5,7}+\omega^{1,4,5,6}-\omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7}=3\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,6,7}+\omega^{1,3,5,7}+2\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7}=2\omega^{1,4,5,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7}=2\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7}=3\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,3,4,7}+3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,2,6,7}+3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6}=\omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6}+2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=\omega^{1,4,5,7}+2\omega^{2,3,5,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7}=\omega^{1,4,6,7}+2\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6}=-\omega^{1,4,5,7}+\omega^{2,3,5,7}+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=3\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7}=2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=-\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6}=3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,5,7}+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=2\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=-\omega^{1,2,5,6,7}+2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7}=2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,2,5,6,7}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{A}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{A}})= e^{-3\beta_1}\ \langle\omega^{1,2}\rangle+e^{-5\beta_1}\ \langle\omega^{1,4}-\omega^{2,3}\rangle+e^{-7\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}+2\omega^{3,4}\rangle+e^{-8\beta_1}\ \langle\omega^{1,7}+\omega^{2,6}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{A}})= e^{-10\beta_1}\ \langle 3\omega^{1,2,7}-\omega^{1,3,6}+\omega^{1,4,5}+\omega^{2,3,5}\rangle+e^{-11\beta_1}\ \langle\omega^{1,3,7}-\omega^{1,4,6}+\omega^{2,3,6}\rangle+2e^{-12\beta_1}\ \langle\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,4,6},\omega^{1,5,6}+2\omega^{2,3,7}-\omega^{2,4,6}-3\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{A}})= e^{-18\beta_1}\ \langle(\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}+3\omega^{3,4,5,6})/12\rangle+e^{-17\beta_1}\ \langle(-\omega^{1,4,5,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6})/3\rangle+2e^{-16\beta_1}\ \langle(-3\omega^{1,2,6,7}+4\omega^{1,3,5,7}-3\omega^{1,4,5,6}+\omega^{2,3,4,7}+5\omega^{2,3,5,6})/12,(-3\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,4,7}+\omega^{2,3,5,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{A}})= e^{-25\beta_1}\ \langle\omega^{3,4,5,6,7}\rangle+e^{-23\beta_1}\ \langle(-\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7})/2\rangle+e^{-21\beta_1}\ \langle(2\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/6\rangle+e^{-20\beta_1}\ \langle(\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{A}})= e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-26\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-25\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{L}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$
$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3.$

5.3.6 Algebra $\mathcal{G}_{7,1.1(vi)}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & 0 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_2^3 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & 0 & 0 & 0 & 6\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & \xi_1^4 & -\xi_1^3 & \xi_6^7 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3+\delta_6^7, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+e^{5\beta_1}\ \langle x_5\rangle+e^{6\beta_1}\ \langle x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{3\beta_1}\ \langle\delta_2^5,\delta_1^4\rangle+2e^{\beta_1}\ \langle\delta_2^3,\delta_6^7\rangle+2e^{5\beta_1}\ \langle\delta_1^6,\delta_2^7\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+2e^{4\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_1^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_2^3,\delta_1^5]=\delta_2^7$; $[\delta_2^3,\delta_2^6]=-\delta_2^7$; $[\delta_2^3,\delta_1^4]=\delta_1^5$; $[\delta_2^3,\delta_1^3]=\delta_2^5+\delta_1^4$; $[\delta_1^6,\delta_6^7]=-\delta_1^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_6^7]=-\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{5\beta_1}\ \langle\delta_1^6\rangle+e^{4\beta_1}\ \langle\delta_2^6\rangle+e^{3\beta_1}\ \langle\delta_1^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})=3e^{-2\beta_1}\ \langle 2x_2\omega^{1,3}+x_3(\omega^{1,4}-\omega^{2,3})+2x_4(\omega^{1,5}+1/2\omega^{2,4}), x_1\omega^{1,2}+x_2\omega^{1,3}-x_3\omega^{1,4}+x_4\omega^{1,5}+x_6\omega^{2,6}, x_2\omega^{1,3}-x_3(\omega^{1,4}+\omega^{2,3})+x_4\omega^{1,5}+x_5\omega^{3,4}\rangle+e^{-6\beta_1}\ \langle 8/5x_1(\omega^{1,6}+7/8\omega^{2,5}-7/8\omega^{3,4})+2/5x_2\omega^{2,6}+7/5x_3(\omega^{2,7}+3/7\omega^{3,6})+7/5x_4(\omega^{3,7}+4/7\omega^{4,6})+7/5x_5(\omega^{4,7}+5/7\omega^{5,6})\rangle+e^{\beta_1}\ \langle x_4\omega^{1,2}+x_6\omega^{2,3}\rangle+e^0\ \langle -x_3\omega^{1,2}+x_6(\omega^{1,5}+\omega^{2,4})\rangle+2e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3}-2x_4\omega^{1,4}-x_6(\omega^{2,5}-\omega^{3,4}), x_6\omega^{1,6}\rangle+2e^{-3\beta_1}\ \langle -x_1\omega^{1,3}-x_2(\omega^{1,4}-\omega^{2,3})+x_3(\omega^{1,5}+\omega^{2,4})-x_4(\omega^{1,6}-\omega^{2,5}), x_1\omega^{1,3}+x_3\omega^{2,4}+x_4(\omega^{1,6}+\omega^{3,4})\rangle+2e^{-4\beta_1}\ \langle 3/2x_1(\omega^{1,4}-\omega^{2,3})-x_2(\omega^{1,5}+\omega^{2,4})-3/2x_3(\omega^{1,6}-2/3\omega^{2,5}+1/3\omega^{3,4})-3/2x_4(\omega^{2,6}-2/3\omega^{3,5}), -1/2x_1(\omega^{1,4}-3\omega^{2,3})+x_2(\omega^{1,5}+\omega^{2,4})+1/2x_3(\omega^{1,6}+\omega^{3,4})+3/2x_4\omega^{2,6}+x_5(\omega^{2,7}+\omega^{4,5})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-4\beta_1}+2e^{-5\beta_1}+3e^{-6\beta_1}+3e^{-7\beta_1}+4e^{-8\beta_1}+2e^{-9\beta_1}+3e^{-10\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$3e^{-10\beta_1}+2e^{-11\beta_1}+4e^{-12\beta_1}+3e^{-13\beta_1}+3e^{-14\beta_1}+3e^{-15\beta_1}+e^{-16\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-14\beta_1}+e^{-15\beta_1}+3e^{-16\beta_1}+2e^{-17\beta_1}+3e^{-18\beta_1}+2e^{-19\beta_1}+2e^{-20\beta_1}+2e^{-21\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-16\beta_1}+2e^{-20\beta_1}+2e^{-21\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}+e^{-25\beta_1}+e^{-26\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-22\beta_1}+e^{-26\beta_1}+e^{-27\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,50,115,149,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,12,18,19,16,10,3)$

S=$28\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,3,7}-\omega^{3,4,5}$; $d\omega^{6,7}=\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}+\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{a}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-6\beta_1}\ \langle\omega^6\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{-5\beta_1}\ \langle\omega^{1,4}\rangle + e^{-6\beta_1}\ \langle\omega^{1,5}+\omega^{2,4}\rangle + e^{-8\beta_1}\ \langle\omega^{2,6}\rangle + e^{-7\beta_1}\ \langle\omega^{1,6}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-10\beta_1}\ \langle\omega^{1,4,5}-\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,4,5}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,5,6}+\omega^{2,4,6}, \omega^{1,4,7}+\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{a}})= 2e^{-16\beta_1}\ \langle\omega^{1,3,5,7}, \omega^{1,4,5,6}-\omega^{2,3,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{1,4,5,7}-\omega^{2,3,5,7}+\omega^{2,4,5,6}\rangle + e^{-18\beta_1}\ \langle\omega^{1,4,6,7}-\omega^{3,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{a}})= e^{-20\beta_1}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-21\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}\rangle + e^{-22\beta_1}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-22\beta_1}\ \langle\omega^{1,2,3,4,5,7}\rangle + e^{-26\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-28\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 20.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7}-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7}-$

$\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,6} + 2\omega^{2,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 4\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,3,7}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 2\omega^{1,5,6} - \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} - \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 3\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + 2\omega^{2,3,4,6}$;

$\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,4,5,6} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1}\ \langle \omega^1 \rangle + e^{-2\beta_1}\ \langle \omega^2 \rangle + e^{-6\beta_1}\ \langle \omega^6 \rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-5\beta_1}\ \langle \omega^{1,4} - \omega^{2,3} \rangle + e^{-6\beta_1}\ \langle \omega^{1,5} + \omega^{2,4} \rangle + e^{-8\beta_1}\ \langle \omega^{2,6} \rangle + e^{-7\beta_1}\ \langle 2\omega^{1,6} - \omega^{2,5} + \omega^{3,4} \rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-10\beta_1}\ \langle \omega^{1,2,7} + 2\omega^{1,4,5} - \omega^{2,3,5} \rangle + e^{-11\beta_1}\ \langle \omega^{1,3,7} - 2\omega^{1,4,6} + 2\omega^{2,3,6} + \omega^{2,4,5} \rangle + 2e^{-12\beta_1}\ \langle \omega^{1,5,6} - \omega^{2,3,7} + 2\omega^{2,4,6}, \omega^{1,4,7} - 2\omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5} \rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-18\beta_1}\ \langle (-\omega^{1,4,6,7} + 2\omega^{2,3,6,7} + \omega^{3,4,5,6})/6 \rangle + e^{-17\beta_1}\ \langle (-\omega^{1,3,6,7} - 2\omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{2,4,5,6})/10 \rangle + 2e^{-16\beta_1}\ \langle (-3\omega^{1,2,6,7} + 4\omega^{1,3,5,7} + \omega^{1,4,5,6} + 5\omega^{2,3,4,7} - 3\omega^{2,3,5,6})/12, (\omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,6})/4 \rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-23\beta_1}\ \langle (-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2 \rangle + e^{-22\beta_1}\ \langle (-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2 \rangle + e^{-20\beta_1}\ \langle -\omega^{1,3,4,5,7} \rangle + e^{-21\beta_1}\ \langle (\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/6 \rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-27\beta_1}\ \langle \omega^{2,3,4,5,6,7} \rangle + e^{-26\beta_1}\ \langle -\omega^{1,3,4,5,6,7} \rangle + e^{-22\beta_1}\ \langle -\omega^{1,2,3,4,5,7} \rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-28\beta_1}\ \langle \omega^{1,2,3,4,5,6,7} \rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.3.7 Algebra $\mathcal{G}_{7,1.2(i_\lambda)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_6$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=\lambda x_5$; $[x_2,x_4]=x_6$; $[x_2,x_6]=x_7$; $[x_3,x_4]=x_7(-\lambda+1)$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^3(-\lambda+2)-\xi_1^4\lambda & -\xi_1^3\lambda+\xi_2^4 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_1^3\lambda(-2\lambda+1)+\xi_2^4 & \xi_2^3-\xi_1^4 & 0 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5-\xi_1^6 & a & b & 4\xi_1^1 \end{pmatrix}$$

with $a=\xi_1^3(-2\lambda+1)+\xi_2^4$, $b=\xi_2^3(-\lambda+2)-\xi_1^4$.

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-(\lambda\delta_2^5+\delta_1^6), ad\ x_4=\lambda\delta_3^7-\delta_1^5-\delta_2^6-\delta_3^7, ad\ x_5=-\delta_1^7, ad\ x_6=-\delta_2^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $2e^{\beta_1}\ \langle x_1,x_2\rangle+2e^{2\beta_1}\ \langle x_3,x_4\rangle+2e^{3\beta_1}\ \langle x_5,x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{\beta_1}\ \langle\delta_1^3,\delta_2^3,\delta_1^4,\delta_2^4\rangle+5e^{2\beta_1}\ \langle\delta_1^5,\delta_2^5,\delta_1^6,\delta_2^6,\delta_3^7\rangle+2e^{3\beta_1}\ \langle\delta_1^7,\delta_2^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^3]=2\delta_3^7-2\delta_1^5-7\lambda\delta_3^7+\lambda\delta_2^6+\lambda\delta_1^5+7\lambda^2\delta_3^7-2\lambda^2\delta_2^6-2\lambda^3\delta_3^7$; $[\delta_1^3,\delta_2^4]=-(\delta_1^6+\lambda\delta_2^5)$; $[\delta_1^3,\delta_1^5]=\delta_1^7(1-2\lambda)$; $[\delta_1^3,\delta_2^5]=\delta_2^7(1-2\lambda)$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_2^3,\delta_1^4]=\delta_1^6+\lambda\delta_2^5$; $[\delta_2^3,\delta_1^6]=\delta_1^7(2-\lambda)$; $[\delta_2^3,\delta_2^6]=\delta_2^7(2-\lambda)$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^4,\delta_2^4]=-\delta_3^7-\delta_2^6-\delta_1^5+\lambda\delta_3^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_2^7$; $[\delta_2^4,\delta_1^5]=\delta_1^7$; $[\delta_2^4,\delta_1^6]=\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $2e^{\beta_1}\ \langle\delta_1^3,\delta_2^3\rangle+3e^{2\beta_1}\ \langle\delta_1^5,\delta_2^5,\delta_3^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^3]=2\delta_3^7-8\delta_3^7\lambda+10\delta_3^7\lambda^2-4\delta_3^7\lambda^3-2\delta_1^5+2\delta_1^5\lambda^2$.

ch($H^2_{\mathcal{G}}$)= $8e^{-\beta_1}\ \langle x_3\omega^{1,3}+2x_4\omega^{2,3}(\lambda-1/2)-x_5\omega^{3,4}(\lambda-1), x_3\omega^{1,4}+x_4\lambda\omega^{2,4}+x_7\lambda\omega^{3,6}(\lambda-1), -x_4\lambda\omega^{2,3}+x_5\omega^{1,5}, -x_4\omega^{2,4}+x_5\omega^{1,6}-x_7\omega^{3,6}(\lambda-1), x_1\omega^{1,2}+x_3\omega^{2,3}+2x_4\omega^{2,4}+x_5\omega^{2,5}+3x_7\omega^{3,6}(\lambda-1), x_2\omega^{1,2}+x_3\omega^{2,4}+x_4\omega^{2,3}(\lambda+1)+x_5\omega^{2,6}, x_3\omega^{2,3}+x_4\omega^{1,3}+x_7\omega^{3,6}(\lambda-1), x_3\omega^{2,4}+x_4(\lambda\omega^{2,3}+\omega^{1,4}-\omega^{2,3})-x_5\omega^{3,4}(\lambda-1)\rangle+7e^{-2\beta_1}\ \langle x_1(\lambda^3\omega^{1,3}-5/2\lambda^2\omega^{1,3}+3/2\lambda^2\omega^{2,4}+5/2\lambda\omega^{1,3}-7/2\lambda\omega^{2,4}-\omega^{1,3}+2\omega^{2,4})-1/2x_2(\lambda^3\omega^{2,3}-3\lambda^2\omega^{1,4}-5\lambda^2\omega^{2,3}+7\lambda\omega^{1,4}+6\lambda\omega^{2,3}-4\omega^{1,4}-2\omega^{2,3})-3/2x_3(\lambda\omega^{1,6}+\lambda\omega^{2,5}-4/3\omega^{1,6}-4/3\omega^{2,5})-1/2x_4(\lambda^3\omega^{3,4}+$

$\lambda^2\omega^{2,6}-2\lambda^2\omega^{3,4}+3\lambda\omega^{1,5}-4\lambda\omega^{2,6}-\lambda\omega^{3,4}-4\omega^{1,5}+2\omega^{2,6}+2\omega^{3,4})+x_5(\lambda^3\omega^{3,5}-\lambda^2\omega^{1,7}-\lambda^2\omega^{3,5}-\lambda\omega^{3,5}+\omega^{1,7}+\omega^{3,5}), x_1(\lambda^2\omega^{1,3}-\lambda^2\omega^{2,4}-5/2\lambda\omega^{1,3}+5/2\lambda\omega^{2,4}+3/2\omega^{1,3}-3/2\omega^{2,4})+x_2(\lambda^3\omega^{1,4}-2\lambda^2\omega^{1,4}-3/2\lambda^2\omega^{2,3}+3/2\lambda\omega^{1,4}+5/2\lambda\omega^{2,3}-1/2\omega^{1,4}-\omega^{2,3})+x_3(\lambda\omega^{1,6}+\lambda\omega^{2,5}-3/2\omega^{1,6}-3/2\omega^{2,5})-1/2x_4(\lambda^2\omega^{3,4}-2\lambda\omega^{1,5}+3\lambda\omega^{2,6}+3\omega^{1,5}-2\omega^{2,6}-\omega^{3,4})+x_5(\lambda^3\omega^{1,7}+\lambda^3\omega^{4,6}-\lambda^2\omega^{4,6}-\lambda\omega^{1,7}-\lambda\omega^{4,6}+\omega^{4,6}), 1/3x_1(\lambda^2\omega^{2,3}-\lambda\omega^{1,4}+2\omega^{1,4}-\omega^{2,3})-4/3x_2(\lambda\omega^{1,3}-1/2\lambda\omega^{2,4}-1/2\omega^{1,3}+1/4\omega^{2,4})-x_3(\lambda\omega^{3,4}-\omega^{1,5})+1/3x_4\omega^{2,5}(\lambda+1)-2/3x_5(\lambda\omega^{2,7}+2\lambda\omega^{3,6}-1/2\omega^{2,7}-\omega^{3,6})-2/3x_7\omega^{4,7}(\lambda-1/2), x_1(\lambda\omega^{2,3}+\omega^{1,4}-\omega^{2,3})+x_2\omega^{1,3}+x_4\omega^{2,5}+x_5(\omega^{3,6}+\omega^{4,5}), -x_1(\lambda^4\omega^{1,3}-7/2\lambda^3\omega^{1,3}+1/2\lambda^3\omega^{2,4}+4\lambda^2\omega^{1,3}-3\lambda^2\omega^{2,4}-2\lambda\omega^{1,3}+5\lambda\omega^{2,4}+1/2\omega^{1,3}-5/2\omega^{2,4})+3/2x_2(\lambda^4\omega^{2,3}-\lambda^3\omega^{1,4}-2\lambda^3\omega^{2,3}+8/3\lambda^2\omega^{1,4}+2\lambda^2\omega^{2,3}-8/3\lambda\omega^{1,4}-5/3\lambda\omega^{2,3}+\omega^{1,4}+2/3\omega^{2,3})+3/2x_3(\lambda^2\omega^{1,6}+\lambda^2\omega^{2,5}-5/3\lambda\omega^{1,6}-5/3\lambda\omega^{2,5}+\omega^{1,6}+\omega^{2,5})+1/2x_4(\lambda^4\omega^{3,4}-2\lambda^3\omega^{1,5}+3\lambda^3\omega^{2,6}-3\lambda^3\omega^{3,4}+5\lambda^2\omega^{1,5}-3\lambda^2\omega^{2,6}+2\lambda^2\omega^{3,4}-3\lambda\omega^{1,5}+3\lambda\omega^{2,6}+3\lambda\omega^{3,4}+\omega^{1,5}-2\omega^{2,6}-3\omega^{3,4})-x_5\omega^{1,7}(\lambda^2-1)+x_6\omega^{4,5}(\lambda^3-\lambda^2-\lambda+1), -1/3x_1(\lambda^2\omega^{2,3}-\lambda\omega^{1,4}-3\lambda\omega^{2,3}+2\omega^{1,4}+2\omega^{2,3})+1/3x_2(\lambda\omega^{1,3}-2\lambda\omega^{2,4}+\omega^{1,3}+\omega^{2,4})+x_3(\omega^{2,6}+\omega^{3,4})-1/3x_4(\lambda\omega^{2,5}-3\omega^{1,6}-2\omega^{2,5})-1/3x_5(\lambda\omega^{2,7}+2\lambda\omega^{3,6}-2\omega^{2,7}-4\omega^{3,6})-1/3x_7\omega^{4,7}(\lambda-2), -x_1(\lambda^2\omega^{1,3}-3/2\lambda\omega^{1,3}+3/2\lambda\omega^{2,4}+1/2\omega^{1,3}-3/2\omega^{2,4})+1/2x_2(\lambda^2\omega^{2,3}-3\lambda\omega^{1,4}-3\lambda\omega^{2,3}+3\omega^{1,4}+2\omega^{2,3})+3/2x_3(\omega^{1,6}+\omega^{2,5})+1/2x_4(\lambda^2\omega^{3,4}+3\lambda\omega^{2,6}+3\omega^{1,5}-\omega^{3,4})+x_7\omega^{3,7}(\lambda^2-1)\rangle+e^0\ \langle -x_5\omega^{2,3}+x_7\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-3\beta_1}+10e^{-4\beta_1}+12e^{-5\beta_1}+e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=6e^{-6\beta_1}+12e^{-7\beta_1}+8e^{-8\beta_1}+4e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=4e^{-9\beta_1}+11e^{-10\beta_1}+6e^{-11\beta_1}+2e^{-12\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-11\beta_1}+3e^{-12\beta_1}+4e^{-13\beta_1}+3e^{-14\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-14\beta_1}+2e^{-15\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,53,121,154,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,16,27,30,23,12,3)$

The preceding results are valid for:

$$(\lambda^2-\lambda+1)(\lambda-1/2)(\lambda+1)^2(\lambda-1)^4\lambda\neq 0.$$

S=$16\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,18,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,7,7,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}\lambda$; $d\omega^6=-\omega^{1,3}-\omega^{2,4}$; $d\omega^7=-\omega^{1,5}-\omega^{2,6}+\omega^{3,4}(\lambda-1)$; $d\omega^{1,5}=\omega^{1,2,3}\lambda$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,6}+\omega^{1,3,4}(-\lambda+$

1) ; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5} + \omega^{2,3,4}(-\lambda + 1)$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5} - \omega^{2,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{2,3,4}\lambda$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,4,6}$; $d\omega^{5,6} = \omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,6}\lambda + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{2,3,7}\lambda - \omega^{2,5,6} + \omega^{3,4,5}(-\lambda + 1)$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7} + \omega^{3,4,6}(-\lambda + 1)$; $d\omega^{1,2,7} = \omega^{1,2,3,4}(\lambda - 1)$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,6}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}\lambda$; $d\omega^{1,4,7} = \omega^{1,2,4,6}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}\lambda - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7}\lambda + \omega^{1,2,5,6} + \omega^{1,3,4,5}(\lambda - 1)$; $d\omega^{1,6,7} = \omega^{1,2,4,7} + \omega^{1,3,4,6}(\lambda - 1)$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,5}(\lambda - 1)$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{2,3,4,6}(\lambda - 1)$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5} - \omega^{2,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,5} + \omega^{2,3,4,6}\lambda$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,7}\lambda - \omega^{2,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,6,7}\lambda + \omega^{2,4,5,7} + \omega^{3,4,5,6}(\lambda - 1)$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}(-\lambda + 1)$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}(-\lambda + 1)$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}\lambda$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}\lambda + \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7}\lambda - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}(-\lambda + 1)$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}(-\lambda + 1)$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,6,7}\lambda$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}(\lambda - 1)$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}\lambda$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= 2e^{-\beta_1}\ \langle\omega^1, \omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= 2e^{-3\beta_1}\ \langle\omega^{2,3}, \omega^{1,3}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}, \omega^{1,5}-\omega^{3,4}\lambda\rangle + 2e^{-5\beta_1}\ \langle\omega^{2,7} + \omega^{3,6}(-2\lambda + 1) - \omega^{4,5}, \omega^{1,7} + \omega^{3,5}(-\lambda + 2) + \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 5e^{-6\beta_1}\ \langle\omega^{1,3,6} - \omega^{2,4,6}, \omega^{1,2,7} + \omega^{2,4,6}(-\lambda + 1), \omega^{1,4,6}, \omega^{2,3,5} + \omega^{2,4,6}, \omega^{1,3,5}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,3,7} - \omega^{3,4,6}, \omega^{2,3,7} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 5e^{-10\beta_1}\ \langle\omega^{1,3,5,7} - \omega^{3,4,5,6}, \omega^{1,3,6,7} + \omega^{2,3,5,7}, \omega^{1,4,6,7}, \omega^{2,3,6,7} - \omega^{3,4,5,6}, \omega^{1,4,5,7} - \omega^{2,3,5,7}(\lambda + 1) - \omega^{2,4,6,7}\rangle + 2e^{-9\beta_1}\ \langle\omega^{1,2,6,7} + \omega^{2,3,5,6}(-\lambda + 1), \omega^{1,3,5,6} - \omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-11\beta_1}\ \langle\omega^{1,2,3,6,7}, \omega^{1,2,3,5,7}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}(-\lambda + 1), \omega^{1,3,4,6,7}\rangle + 2e^{-13\beta_1}\ \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\lambda, \omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + 2e^{-15\beta_1}\ \langle\omega^{1,3,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle$.

ch($H^7_{\mathcal{G}}$)= $e^{-16\beta_1}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

The preceding results are valid for:

$$(\lambda^2-\lambda+1)(\lambda-1)\neq 0.$$

Comments: singular values.

For different λ,λ' , there exists an isomorphism

$$F:\mathcal{G}_{7,1.2(i_\lambda)}\to\mathcal{G}_{7,1.2(i_{\lambda'})}$$

if and only if $\lambda\lambda'=1$ and one may then choose $F(x_1)=x_2$, $F(x_2)=x_1$, $F(x_3)=-\lambda x_3$. The values excluded by the computer are not necessarily singular, i.e. give a different cohomology than the generic one: some of them appear from the order used by the computer in solving systems of equations. Computing again for each of these values shows that only $\lambda^2-\lambda+1=0$, $\lambda=1$, $\lambda=-1$ are singular values for both adjoint and trivial cohomologies.

changes in adjoint cohomology:

case $\lambda=1$ or $\lambda^2-\lambda+1=0$:

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,53,\underline{122},\underline{155},114,45,7)$ [3]

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,16,\underline{28},\underline{32},\underline{24},12,3)$

The multiplicity of $e^{-6\beta_1}$ in ch($H^3_{\mathcal{G}}$) is 2; the multiplicity of $e^{-8\beta_1}$ and $e^{-6\beta_1}$ in ch($H^4_{\mathcal{G}}$) are respectively 9 and 7, and $e^{-8\beta_1}$ appears as a new term in ch($H^5_{\mathcal{G}}$).

case $\lambda=-1$:

$$\delta=\begin{pmatrix}
\xi^1_1 & \xi^1_2 & 0 & 0 & 0 & 0 & 0\\
-\xi^1_2 & \xi^1_1 & 0 & 0 & 0 & 0 & 0\\
\xi^3_1 & \xi^3_2 & 2\xi^1_1 & 0 & 0 & 0 & 0\\
\xi^4_1 & \xi^4_2 & 0 & 2\xi^1_1 & 0 & 0 & 0\\
\xi^5_1 & \xi^5_2 & 3\xi^3_2+\xi^4_1 & \xi^3_1+\xi^4_2 & 3\xi^1_1 & \xi^1_2 & 0\\
\xi^6_1 & \xi^6_2 & -3\xi^3_1+\xi^4_2 & \xi^3_2-\xi^4_1 & -\xi^1_2 & 3\xi^1_1 & 0\\
\xi^7_1 & \xi^7_2 & \xi^7_3 & \xi^5_2-\xi^6_1 & 3\xi^3_1+\xi^4_2 & 3\xi^3_2-\xi^4_1 & 4\xi^1_1
\end{pmatrix}$$

All the change is that δ^1_2 enters: δ^1_2 is semisimple and commutes with the torus $\langle t_1\rangle$. The algebra $\mathcal{G}$ has rank 2 but doesn't split over $\boldsymbol{R}$*since the*

[3]underlined dimensions are those different from the generic ones

spectrum of δ_2^1 *is* $\{0, i, -i\}$ *; over* $\mathbb{R}$*the action of* δ_2^1 *on the cohomology splits the eigenspaces of* t_1 *into 2-dimensional subspaces and eventually some line in the kernel. Over* $\mathbb{C}$ *, the algebra* $\mathcal{G}$ *is isomorphic to* $\mathcal{G}_{2.37}$*.*

Comm. relations among the δ_i^j*'s* $i \neq j$ *in* $Z^1_{\mathcal{G}}$ *: add* $[\delta_1^5, \delta_2^1] = \delta_1^6 + \delta_2^5$; $[\delta_2^6, \delta_2^1] = -(\delta_1^6 + \delta_2^5)$; $[\delta_2^1, \delta_1^7] = -\delta_2^7$; $[\delta_2^1, \delta_2^7] = \delta_1^7$; $[\delta_2^1, \delta_2^5] = -\delta_2^6 + \delta_1^5$; $[\delta_2^1, \delta_1^6] = -\delta_2^6 + \delta_1^5$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^1, \delta_2^4] = \delta_1^4$; $[\delta_2^1, \delta_2^3] = \delta_1^3$; $[\delta_2^1, \delta_1^4] = -\delta_2^4$.

Comm. relations among the δ_i^j*'s* $i \neq j$ *in* $H^1_{\mathcal{G}}$ *: add* $[\delta_2^1, \delta_1^5] = -2\delta_2^5$; $[\delta_2^1, \delta_2^6] = 2\delta_2^5$; $[\delta_2^1, \delta_2^5] = -\delta_2^6 + \delta_1^5$; $[\delta_2^1, \delta_1^3] = -\delta_2^3$; $[\delta_2^1, \delta_2^3] = \delta_1^3$.

The multiplicity of 0 in ch($H^2_{\mathcal{G}}$*) is 2 with the new term* $x_5\omega^{1,3}$.

Dimensions of $Z^j_{\mathcal{G}}$*'s* $0 \leq j \leq 7$*:* $(1, \underline{13}, 53, 121, 154, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$*'s* $0 \leq j \leq 7$*:* $(1, \underline{7}, \underline{17}, 27, 30, 23, 12, 3)$

changes in trivial cohomology:

case $\lambda = 1$**:** Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-8\beta_1}$ $\langle \omega^{1,6,7} + \omega^{2,5,7} \rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-8\beta_1}$ $\langle \omega^{2,3,4,5} \rangle$

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, \underline{19}, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, \underline{8}, \underline{8}, 6, 3, 1)$

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

case $\lambda^2 - \lambda + 1 = 0$**:**

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-8\beta_1}$ $\langle \omega^{1,5,7} + (\lambda - 1)\omega^{2,6,7} + \omega^{3,4,7} + \lambda\omega^{4,5,6} \rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-8\beta_1}$ $\langle \omega^{2,3,4,6} \rangle$

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, \underline{19}, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, \underline{8}, \underline{8}, 6, 3, 1)$

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

case $\lambda = -1$**:**

The only change is that the action of δ_2^1 is not described.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5\bar{\lambda}$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7(\bar{\lambda} - 1)$; $d^\star\omega^{1,2,3} = \omega^{1,5}\bar{\lambda} -$

$\omega^{2,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}(-\bar\lambda+1)-\omega^{3,5}+\omega^{4,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{6,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,7}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}(-\bar\lambda+1)-\omega^{3,6}+\omega^{4,5}\bar\lambda$; $d^\star\omega^{2,3,6}=-\omega^{3,7}-\omega^{5,6}\bar\lambda$; $d^\star\omega^{2,3,7}=-\omega^{5,7}\bar\lambda$; $d^\star\omega^{2,4,5}=\omega^{5,6}$; $d^\star\omega^{2,4,6}=-\omega^{4,7}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}(-\bar\lambda+1)$; $d^\star\omega^{3,4,6}=\omega^{6,7}(-\bar\lambda+1)$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}(\bar\lambda-1)+\omega^{1,3,6}-\omega^{1,4,5}\bar\lambda-\omega^{2,3,5}+\omega^{2,4,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,3,7}+\omega^{2,5,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,3,7}+\omega^{1,5,6}\bar\lambda+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}\bar\lambda-\omega^{2,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,6}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6}=\omega^{1,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,5,7}+\omega^{2,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}(\bar\lambda-1)-\omega^{3,4,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}(\bar\lambda-1)-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}+\omega^{4,6,7}$; $d^\star\omega^{1,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}(\bar\lambda-1)+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}(\bar\lambda-1)-\omega^{3,4,7}+\omega^{4,5,6}\bar\lambda$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}\bar\lambda$; $d^\star\omega^{2,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}\bar\lambda$; $d^\star\omega^{2,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}(\bar\lambda-1)$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}(-\bar\lambda+1)-\omega^{1,3,5,6}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}(-\bar\lambda+1)+\omega^{1,3,4,7}-\omega^{1,4,5,6}\bar\lambda-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}\bar\lambda-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,3,5,7}+\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=\omega^{2,5,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}\bar\lambda+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,4,5,7}+\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}(-\bar\lambda+1)+\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}(-\bar\lambda+1)-\omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7}=\omega^{4,5,6,7}\bar\lambda$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,2,5,6,7}(\bar\lambda-1)+\omega^{1,3,4,5,7}+\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=-\omega^{1,4,5,6,7}\bar\lambda-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^4=\omega^4$; $\Delta\omega^5=\omega^5(\lambda\bar\lambda+1)$; $\Delta\omega^6=2\omega^6$; $\Delta\omega^7=\omega^7(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{1,4}=\omega^{1,4}+\omega^{2,3}\lambda$; $\Delta\omega^{1,5}=\omega^{1,5}(\lambda\bar\lambda+1)+\omega^{2,6}(-\lambda+1)+\omega^{3,4}$; $\Delta\omega^{1,6}=\omega^{1,6}-\omega^{2,5}$; $\Delta\omega^{1,7}=\omega^{1,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,5}(\lambda-1)-\omega^{4,6}\lambda$; $\Delta\omega^{2,3}=\omega^{1,4}\bar\lambda+\omega^{2,3}\lambda\bar\lambda$; $\Delta\omega^{2,4}=\omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{2,5}=-\omega^{1,6}+\omega^{2,5}$; $\Delta\omega^{2,6}=\omega^{1,5}(-\bar\lambda+1)+2\omega^{2,6}-\omega^{3,4}\lambda$; $\Delta\omega^{2,7}=\omega^{2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,6}(\lambda-1)+\omega^{4,5}(-\lambda\bar\lambda+\bar\lambda+1)$; $\Delta\omega^{3,4}=\omega^{1,5}-\omega^{2,6}\bar\lambda+\omega^{3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,5}=\omega^{1,7}(\bar\lambda-1)+\omega^{3,5}-\omega^{4,6}$; $\Delta\omega^{3,6}=\omega^{2,7}(\bar\lambda-1)+\omega^{3,6}-\omega^{4,5}\bar\lambda$; $\Delta\omega^{3,7}=2\omega^{3,7}+\omega^{5,6}(\bar\lambda-1)$; $\Delta\omega^{4,5}=\omega^{2,7}(-\lambda\bar\lambda+\lambda+1)-\omega^{3,6}\lambda+\omega^{4,5}(\lambda\bar\lambda+1)$; $\Delta\omega^{4,6}=-\omega^{1,7}\bar\lambda-\omega^{3,5}+2\omega^{4,6}$; $\Delta\omega^{4,7}=3\omega^{4,7}$; $\Delta\omega^{5,6}=\omega^{3,7}(\lambda-1)+\omega^{5,6}(\lambda\bar\lambda+3)$; $\Delta\omega^{5,7}=\omega^{5,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{6,7}=\omega^{6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+4)$; $\Delta\omega^{1,2,3}=\omega^{1,2,3}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,4}=2\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=2\omega^{1,2,5}-\omega^{2,3,4}$; $\Delta\omega^{1,2,6}=2\omega^{1,2,6}-\omega^{1,3,4}\lambda$; $\Delta\omega^{1,2,7}=\omega^{1,2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,6}(\lambda-1)+\omega^{1,4,5}(-\lambda\bar\lambda+$

$\bar\lambda+1)+\omega^{2,3,5}(-\lambda+1)+\omega^{2,4,6}\lambda$; $\Delta\omega^{1,3,4}=-\omega^{1,2,6}\bar\lambda+\omega^{1,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,3,5}=2\omega^{1,3,5}-\omega^{1,4,6}+\omega^{2,3,6}(-\lambda+1)+\omega^{2,4,5}$; $\Delta\omega^{1,3,6}=\omega^{1,2,7}(\bar\lambda-1)+\omega^{1,3,6}-\omega^{1,4,5}\bar\lambda-\omega^{2,3,5}+\omega^{2,4,6}$; $\Delta\omega^{1,3,7}=2\omega^{1,3,7}+\omega^{1,5,6}(\bar\lambda-1)+\omega^{2,4,7}+\omega^{3,4,6}\lambda$; $\Delta\omega^{1,4,5}=\omega^{1,2,7}(-\lambda\bar\lambda+\lambda+1)-\omega^{1,3,6}\lambda+\omega^{1,4,5}(\lambda\bar\lambda+1)+\omega^{2,3,5}\lambda+\omega^{2,4,6}(-\lambda+1)$; $\Delta\omega^{1,4,6}=-\omega^{1,3,5}+\omega^{1,4,6}+\omega^{2,3,6}\lambda-\omega^{2,4,5}$; $\Delta\omega^{1,4,7}=2\omega^{1,4,7}+\omega^{2,3,7}\lambda+\omega^{3,4,5}(\lambda-1)$; $\Delta\omega^{1,5,6}=\omega^{1,3,7}(\lambda-1)+\omega^{1,5,6}(\lambda\bar\lambda+2)+\omega^{3,4,6}$; $\Delta\omega^{1,5,7}=\omega^{1,5,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{2,6,7}(-\lambda+1)+\omega^{3,4,7}-\omega^{4,5,6}\lambda$; $\Delta\omega^{1,6,7}=\omega^{1,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)-\omega^{2,5,7}+\omega^{3,5,6}(-\lambda+1)$; $\Delta\omega^{2,3,4}=-\omega^{1,2,5}+\omega^{2,3,4}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,5}=\omega^{1,2,7}(-\bar\lambda+1)-\omega^{1,3,6}+\omega^{1,4,5}\bar\lambda+\omega^{2,3,5}-\omega^{2,4,6}$; $\Delta\omega^{2,3,6}=\omega^{1,3,5}(-\bar\lambda+1)+\omega^{1,4,6}\bar\lambda+\omega^{2,3,6}(\lambda\bar\lambda+1)-\omega^{2,4,5}\bar\lambda$; $\Delta\omega^{2,3,7}=\omega^{1,4,7}\bar\lambda+\omega^{2,3,7}(\lambda\bar\lambda+1)+\omega^{2,5,6}(\bar\lambda-1)+\omega^{3,4,5}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{2,4,5}=\omega^{1,3,5}-\omega^{1,4,6}-\omega^{2,3,6}\lambda+\omega^{2,4,5}$; $\Delta\omega^{2,4,6}=\omega^{1,2,7}\bar\lambda+\omega^{1,3,6}+\omega^{1,4,5}(-\bar\lambda+1)-\omega^{2,3,5}+2\omega^{2,4,6}$; $\Delta\omega^{2,4,7}=\omega^{1,3,7}+2\omega^{2,4,7}+\omega^{3,4,6}(\lambda-1)$; $\Delta\omega^{2,5,6}=\omega^{2,3,7}(\lambda-1)+3\omega^{2,5,6}+\omega^{3,4,5}\lambda$; $\Delta\omega^{2,5,7}=-\omega^{1,6,7}+\omega^{2,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,5,6}(\lambda-1)$; $\Delta\omega^{2,6,7}=\omega^{1,5,7}(-\bar\lambda+1)+\omega^{2,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)-\omega^{3,4,7}\lambda+\omega^{4,5,6}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{3,4,5}=\omega^{1,4,7}(\bar\lambda-1)+\omega^{2,3,7}(\lambda\bar\lambda-\lambda-1)+\omega^{2,5,6}\bar\lambda+\omega^{3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,6}=\omega^{1,3,7}\bar\lambda+\omega^{1,5,6}+\omega^{2,4,7}(\bar\lambda-1)+\omega^{3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,7}=\omega^{1,5,7}-\omega^{2,6,7}\bar\lambda+3\omega^{3,4,7}+\omega^{4,5,6}(-\bar\lambda+1)$; $\Delta\omega^{3,5,6}=\omega^{1,6,7}(-\bar\lambda+1)+\omega^{2,5,7}(\bar\lambda-1)+2\omega^{3,5,6}$; $\Delta\omega^{3,5,7}=2\omega^{3,5,7}-\omega^{4,6,7}$; $\Delta\omega^{3,6,7}=2\omega^{3,6,7}-\omega^{4,5,7}\bar\lambda$; $\Delta\omega^{4,5,6}=-\omega^{1,5,7}\bar\lambda+\omega^{2,6,7}(\lambda\bar\lambda-\lambda-1)+\omega^{3,4,7}(-\lambda+1)+\omega^{4,5,6}(\lambda\bar\lambda+2)$; $\Delta\omega^{4,5,7}=-\omega^{3,6,7}\lambda+\omega^{4,5,7}(\lambda\bar\lambda+2)$; $\Delta\omega^{4,6,7}=-\omega^{3,5,7}+3\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=\omega^{5,6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+4)$; $\Delta\omega^{1,2,3,4}=\omega^{1,2,3,4}(2\lambda\bar\lambda-\lambda-\bar\lambda+4)$; $\Delta\omega^{1,2,3,5}=3\omega^{1,2,3,5}-\omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6}=\omega^{1,2,3,6}(\lambda\bar\lambda+2)-\omega^{1,2,4,5}\bar\lambda$; $\Delta\omega^{1,2,3,7}=\omega^{1,2,3,7}(\lambda\bar\lambda+2)+\omega^{1,2,5,6}(\bar\lambda-1)+\omega^{1,3,4,5}(\lambda\bar\lambda-\bar\lambda-1)-\omega^{2,3,4,6}\lambda$; $\Delta\omega^{1,2,4,5}=-\omega^{1,2,3,6}\lambda+2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=-\omega^{1,2,3,5}+2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7}=2\omega^{1,2,4,7}+\omega^{1,3,4,6}(\lambda-1)+\omega^{2,3,4,5}(-\lambda+1)$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,3,7}(\lambda-1)+3\omega^{1,2,5,6}+\omega^{1,3,4,5}\lambda-\omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7}=\omega^{1,2,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,5,6}(\lambda-1)-\omega^{2,3,4,7}+\omega^{2,4,5,6}\lambda$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)-\omega^{1,3,4,7}\lambda+\omega^{1,4,5,6}(\lambda\bar\lambda-\bar\lambda-1)+\omega^{2,3,5,6}(\lambda-1)$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,3,7}(\lambda\bar\lambda-\lambda-1)+\omega^{1,2,5,6}\bar\lambda+\omega^{1,3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+3)+\omega^{2,3,4,6}(-\lambda+1)$; $\Delta\omega^{1,3,4,6}=\omega^{1,2,4,7}(\bar\lambda-1)+\omega^{1,3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7}=-\omega^{1,2,6,7}\bar\lambda+3\omega^{1,3,4,7}+\omega^{1,4,5,6}(-\bar\lambda+1)$; $\Delta\omega^{1,3,5,6}=\omega^{1,2,5,7}(\bar\lambda-1)+2\omega^{1,3,5,6}+\omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7}=2\omega^{1,3,5,7}-\omega^{1,4,6,7}+\omega^{2,3,6,7}(-\lambda+1)+\omega^{2,4,5,7}+\omega^{3,4,5,6}\lambda$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}\bar\lambda-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,6,7}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4,7}(-\lambda+1)+\omega^{1,4,5,6}(\lambda\bar\lambda+1)+\omega^{2,3,5,6}\lambda$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,6,7}\lambda+\omega^{1,4,5,7}(\lambda\bar\lambda+1)+\omega^{2,3,5,7}\lambda+\omega^{2,4,6,7}(-\lambda+1)$; $\Delta\omega^{1,4,6,7}=-\omega^{1,3,5,7}+\omega^{1,4,6,7}+\omega^{2,3,6,7}\lambda-\omega^{2,4,5,7}+\omega^{3,4,5,6}(-\lambda+1)$; $\Delta\omega^{1,5,6,7}=\omega^{1,5,6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,4,7}(-\bar\lambda+1)-\omega^{1,3,4,6}+\omega^{2,3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,4,6}=-\omega^{1,2,3,7}\bar\lambda-\omega^{1,2,5,6}+\omega^{1,3,4,5}(-\bar\lambda+1)+\omega^{2,3,4,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,4,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,7}(\lambda\bar\lambda+2)+\omega^{2,4,5,6}(-\bar\lambda+1)$; $\Delta\omega^{2,3,5,6}=\omega^{1,2,6,7}(\bar\lambda-$

$1)+\omega^{1,4,5,6}\bar\lambda+2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}\bar\lambda+\omega^{2,3,5,7}-\omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,3,5,7}(-\bar\lambda+1)+\omega^{1,4,6,7}\bar\lambda+\omega^{2,3,6,7}(\lambda\bar\lambda+1)-\omega^{2,4,5,7}\bar\lambda+\omega^{3,4,5,6}(-\lambda\bar\lambda+\bar\lambda+1)$; $\Delta\omega^{2,4,5,6}=\omega^{1,2,5,7}\bar\lambda+\omega^{1,3,5,6}+\omega^{2,3,4,7}(-\lambda+1)+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,3,5,7}-\omega^{1,4,6,7}-\omega^{2,3,6,7}\lambda+\omega^{2,4,5,7}+\omega^{3,4,5,6}(\lambda-1)$; $\Delta\omega^{2,4,6,7}=\omega^{1,3,6,7}+\omega^{1,4,5,7}(-\bar\lambda+1)-\omega^{2,3,5,7}+2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)+\omega^{3,4,5,7}\lambda$; $\Delta\omega^{3,4,5,6}=\omega^{1,3,5,7}\bar\lambda+\omega^{1,4,6,7}(-\bar\lambda+1)+\omega^{2,3,6,7}(-\lambda\bar\lambda+\lambda+1)+\omega^{2,4,5,7}(\bar\lambda-1)+\omega^{3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,5,7}=\omega^{2,5,6,7}\bar\lambda+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{1,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=\omega^{4,5,6,7}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,3,4,5}=\omega^{1,2,3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+4)$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3,4,7}=\omega^{1,2,3,4,7}(\lambda\bar\lambda+3)+\omega^{1,2,4,5,6}(-\bar\lambda+1)$; $\Delta\omega^{1,2,3,5,6}=3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}\lambda$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}(\lambda\bar\lambda+1)-\omega^{1,2,4,5,7}\bar\lambda+\omega^{1,3,4,5,6}(-\lambda\bar\lambda+\bar\lambda+1)$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}(-\lambda+1)+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,6,7}\lambda+\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}(\lambda-1)$; $\Delta\omega^{1,2,4,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}(\lambda-1)$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,4,5,7}\lambda-\omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,6,7}(-\lambda\bar\lambda+\lambda+1)+\omega^{1,2,4,5,7}(\bar\lambda-1)+\omega^{1,3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,2,5,6,7}\bar\lambda+2\omega^{1,3,4,5,7}+\omega^{2,3,4,6,7}(-\lambda+1)$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}\lambda\bar\lambda+\omega^{2,3,5,6,7}\lambda$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,2,3,5,7}\bar\lambda+\omega^{1,2,4,6,7}(\bar\lambda-1)+\omega^{2,3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}(-\bar\lambda+1)+\omega^{2,3,4,6,7}(\lambda\bar\lambda+1)$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}\bar\lambda+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}(\lambda\bar\lambda+1)$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})=2e^{-\beta_1}\ \langle\omega^1,\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})=2e^{-4\beta_1}\ \langle\omega^{1,5}(\bar\lambda-2)+\omega^{2,6}(\lambda\bar\lambda-\lambda+1)+\omega^{3,4}(\lambda+1),\omega^{1,6}+\omega^{2,5}\rangle+2e^{-3\beta_1}\ \langle-\omega^{1,4}\bar\lambda+\omega^{2,3},\omega^{1,3}-\omega^{2,4}\rangle+2e^{-5\beta_1}\ \langle\omega^{2,7}+\omega^{3,6}(-2\lambda+1)-\omega^{4,5},\omega^{1,7}+\omega^{3,5}(-\lambda+2)+\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})=5e^{-6\beta_1}\ \langle\omega^{1,3,6}+\omega^{2,3,5},\omega^{1,4,6}+\omega^{2,4,5},\omega^{1,3,5}-\omega^{2,3,6}-\omega^{2,4,5}(\bar\lambda+1),\omega^{1,2,7}+\omega^{2,3,5}(\lambda-2)-\omega^{2,4,6},\omega^{1,4,5}-\omega^{2,3,5}(\lambda+1)-\omega^{2,4,6}\rangle+2e^{-7\beta_1}\ \langle\omega^{1,3,7}(\lambda\bar\lambda-\lambda+2)+\omega^{1,5,6}(-\bar\lambda+2)+\omega^{2,4,7}(\bar\lambda-2)+\omega^{3,4,6}(-\lambda-2),\omega^{1,4,7}(2\lambda\bar\lambda^2-3\lambda\bar\lambda+\lambda+4\bar\lambda-2)+\omega^{2,3,7}(\lambda\bar\lambda^2-4\lambda\bar\lambda+3\lambda+2\bar\lambda-6)+\omega^{2,5,6}(2\lambda\bar\lambda^2-3\lambda\bar\lambda+\lambda+4\bar\lambda-2)+\omega^{3,4,5}(-\lambda\bar\lambda^2-\lambda\bar\lambda+2\lambda-2\bar\lambda-4)\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})=5e^{-10\beta_1}\ \langle(-\omega^{1,3,5,7}+\omega^{1,4,6,7}+\omega^{2,3,6,7}\lambda+\omega^{2,4,5,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+3)+\omega^{3,4,5,6}(-\lambda+1))/(2\lambda\bar\lambda-\lambda-\bar\lambda+4),(-(\lambda\omega^{2,3,5,7}\bar\lambda+\lambda\omega^{2,3,5,7}-\omega^{1,4,5,7}\bar\lambda-\omega^{1,4,5,7}+\omega^{2,3,5,7}\bar\lambda+3\omega^{2,3,5,7}+\omega^{2,4,6,7}\bar\lambda+\omega^{2,4,6,7}+2\omega^{1,3,6,7}))/(\lambda\bar\lambda+\lambda+\bar\lambda+5),(-(\lambda\omega^{2,3,5,7}-\lambda\omega^{1,3,6,7}-2\omega^{1,4,5,7}+\omega^{2,3,5,7}+2\omega^{2,4,6,7}-\omega^{1,3,6,7}))/(\lambda\bar\lambda+\lambda+\bar\lambda+5),(\omega^{1,3,5,7}(-2\lambda\bar\lambda+\lambda-2\bar\lambda-1)+3\omega^{1,4,6,7}(\bar\lambda-1)+\omega^{2,3,6,7}(\lambda\bar\lambda-2\lambda+\bar\lambda-4)+$

$3\omega^{2,4,5,7}(-\bar\lambda+1)+\omega^{3,4,5,6}(\lambda\bar\lambda+\lambda+\bar\lambda+5))/(3(2\lambda\bar\lambda-\lambda-\bar\lambda+4)),(\omega^{1,3,5,7}(-2\lambda\bar\lambda+\lambda+4\bar\lambda-4)-3\omega^{1,4,6,7}\bar\lambda+\omega^{2,3,6,7}(\lambda\bar\lambda-2\lambda-2\bar\lambda+8)+3\omega^{2,4,5,7}\bar\lambda+\omega^{3,4,5,6}(\lambda\bar\lambda+\lambda-2\bar\lambda-4))/(3(2\lambda\bar\lambda-\lambda-\bar\lambda+4))\rangle+2e^{-9\beta_1}\ \langle(-(\lambda\omega^{2,3,4,7}+\lambda\omega^{2,4,5,6}\bar\lambda+\lambda\omega^{1,3,5,6}-2\omega^{2,3,4,7}-\omega^{2,4,5,6}\bar\lambda+2\omega^{2,4,5,6}-\bar\lambda\omega^{1,2,5,7}-2\omega^{1,2,5,7}-2\omega^{1,3,5,6}))/((\lambda\bar\lambda-\lambda-\bar\lambda+4)(\lambda\bar\lambda+4)),(\omega^{1,2,6,7}(-\lambda-2)+\omega^{1,3,4,7}(-2\lambda+1)+\omega^{1,4,5,6}(\lambda-3)+\omega^{2,3,5,6}(2\lambda-1))/(5(2\lambda\bar\lambda-\lambda-\bar\lambda+3)(\lambda\bar\lambda-\lambda+2))\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=\ 2e^{-12\beta_1}\ \langle(\omega^{1,2,5,6,7}(\bar\lambda+1)+\omega^{1,3,4,5,7}(-\lambda\bar\lambda+\bar\lambda-1)+\omega^{2,3,4,6,7}(-\lambda+2))/(\lambda^2\bar\lambda^2-\lambda^2\bar\lambda-\lambda\bar\lambda^2+5\lambda\bar\lambda-2\lambda-2\bar\lambda+6),(\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/2\rangle+2e^{-13\beta_1}\ \langle(\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\lambda)/(\lambda\bar\lambda+1),(\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7})/2\rangle+2e^{-11\beta_1}\ \langle(-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}(-2\bar\lambda+1)+\omega^{1,3,4,5,6})/(4\lambda\bar\lambda-2\lambda-2\bar\lambda+3),(-\omega^{1,2,3,5,7}+\omega^{1,2,4,6,7}(\bar\lambda-2)-\omega^{2,3,4,5,6})/(\lambda\bar\lambda-2\lambda-2\bar\lambda+6)\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=2e^{-15\beta_1}\ \langle\omega^{2,3,4,5,6,7},-\omega^{1,3,4,5,6,7}\rangle+e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})=e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

The preceding results are valid for: $(-(\bar\lambda^2-\bar\lambda+1))(\lambda\bar\lambda-\bar\lambda^2+\bar\lambda+2)(\lambda^2-\lambda+1)(-(\bar\lambda-1))(\lambda-1)(\lambda\bar\lambda+\lambda-\bar\lambda^2-2\bar\lambda+1)(\bar\lambda+2)(\lambda\bar\lambda-\lambda+2)(-(\lambda\bar\lambda-\lambda-\bar\lambda-1))(\bar\lambda-3)(\lambda+2)(\lambda\bar\lambda-\bar\lambda^2+\bar\lambda+2)(\bar\lambda-2)(\bar\lambda+1)\neq 0$.

Comments: singular values. Only $\lambda=1,\lambda^2-\lambda+1,\lambda=-1$ are singular values.

Case $\lambda=1$: Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-8\beta_1}\ \langle\omega^{1,6,7}+\omega^{2,5,7}\rangle$ and add in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-8\beta_1}\ \langle(\omega^{1,3,4,6}+\omega^{2,3,4,5})/2\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,\underline{19},24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,\underline{8},\underline{8},6,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 34.

Riemann Hyp.: no

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

Case $\lambda^2-\lambda+1=0$: Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-8\beta_1}\ \langle\omega^{1,5,7}(-\lambda+1)+\omega^{2,6,7}\lambda+\omega^{3,4,7}(\lambda-1)+\omega^{4,5,6}\rangle$ and in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-8\beta_1}\ \langle(-(\lambda\omega^{2,3,4,6}+\lambda\omega^{1,2,5,6}-\lambda\omega^{1,3,4,5}+\omega^{1,2,3,7}+\omega^{1,3,4,5}))/4\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,\underline{19},24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,\underline{8},\underline{8},6,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 34.
Riemann Hyp.: no
$P(Y) \ = \ (1+Y+Y^2)^2(1+Y^2)(1+Y)$.
Case $\lambda = -1$: see previous comments

5.3.8 Algebra $\mathcal{G}_{7,1.2(ii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_2^3 & \xi_2^4 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^3-\xi_1^4 & -(\xi_1^3+\xi_1^4) & 0 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5+\xi_2^5+\xi_2^6 & -2\xi_1^3-\xi_1^4+\xi_2^4 & -\xi_2^3+\xi_2^4 & 4\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_2^6, ad\ x_4=-\delta_1^5-\delta_2^6+\delta_3^7, ad\ x_5=-(\delta_1^7+\delta_2^7), ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $2e^{\beta_1}\ \langle x_1,x_2\rangle+2e^{2\beta_1}\ \langle x_3,x_4\rangle+2e^{3\beta_1}\ \langle x_5,x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $5e^{2\beta_1}\ \langle\delta_1^6,\delta_3^7,\delta_1^5,\delta_2^5,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^7,\delta_2^7\rangle+4e^{\beta_1}\ \langle\delta_1^3,\delta_1^4,\delta_2^3,\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_2^3]=\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_3^7,\delta_2^3]=\delta_2^7$; $[\delta_1^5,\delta_1^3]=2\delta_1^7$; $[\delta_1^5,\delta_2^4]=-(\delta_2^7+\delta_1^7)$; $[\delta_2^5,\delta_1^3]=2\delta_2^7$; $[\delta_2^5,\delta_1^4]=\delta_2^7+\delta_1^7$; $[\delta_2^6,\delta_1^4]=\delta_1^7$; $[\delta_2^6,\delta_2^3]=\delta_2^7$; $[\delta_1^3,\delta_2^3]=2\delta_3^7-\delta_1^6+\delta_1^5$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$; $[\delta_1^4,\delta_2^3]=-\delta_2^6$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_2^6-\delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $3e^{2\beta_1}\ \langle\delta_1^6,\delta_1^5,\delta_2^5\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^3]=-\delta_1^6+3\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $8e^{-\beta_1}\ \langle x_1\omega^{1,2}-x_2\omega^{1,2}+x_3(\omega^{1,3}+2\omega^{1,4}-\omega^{2,3})-x_4(\omega^{1,4}-\omega^{2,4})+x_5\omega^{2,6}, x_3\omega^{2,4}, x_2\omega^{1,2}-x_3(\omega^{1,3}+2\omega^{1,4}+\omega^{2,3})+x_4(\omega^{1,4}+\omega^{2,4})+x_5\omega^{2,5}, x_6\omega^{1,5}, x_3(\omega^{1,3}+2\omega^{1,4})-x_4\omega^{1,4}+x_6\omega^{1,6}, -x_3(\omega^{1,3}+\omega^{1,4})+x_4(\omega^{1,3}+\omega^{1,4}), x_3(\omega^{1,4}-\omega^{2,3})-x_4(\omega^{1,4}+\omega^{2,3})+x_5\omega^{3,4}, x_3(\omega^{1,3}+2\omega^{1,4})-2x_4\omega^{1,4}+x_6\omega^{3,4}\rangle+7e^{-2\beta_1}\ \langle -x_1(\omega^{1,3}+2\omega^{1,4}+\omega^{2,3}+4\omega^{2,4})+2x_2(\omega^{1,3}+\omega^{1,4}+\omega^{2,3}+3/2\omega^{2,4})+x_3(\omega^{1,6}+\omega^{2,5}+3\omega^{2,6}-\omega^{3,4})-x_4(\omega^{1,6}+\omega^{2,6})+x_5(\omega^{3,5}-\omega^{4,6}), x_1(\omega^{1,3}+\omega^{1,4}+\omega^{2,3}+\omega^{2,4})-x_2(\omega^{1,3}+\omega^{1,4}+\omega^{2,3}+\omega^{2,4})-x_5(\omega^{1,7}+\omega^{2,7}-\omega^{3,6}-\omega^{4,6}), -2/3x_1(\omega^{1,3}+4\omega^{1,4}+3\omega^{2,3}+6\omega^{2,4})+2/3x_2(\omega^{1,3}+5/2\omega^{1,4}+3\omega^{2,3}+5\omega^{2,4})+2/3x_3(\omega^{1,6}+3/2\omega^{1,5}+\omega^{2,5}+3\omega^{2,6}-\omega^{3,4})-2/3x_4(\omega^{1,6}+3\omega^{2,6}+\omega^{3,4})-2x_5\omega^{4,6}+x_6\omega^{1,7}, -2/3x_1(\omega^{1,3}+4\omega^{1,4}+3\omega^{2,3}+6\omega^{2,4})+5/3x_2(\omega^{1,3}+$

$8/5\omega^{1,4} + 6/5\omega^{2,3} + 2\omega^{2,4}) + 2/3x_3(\omega^{1,6} + \omega^{2,5} + 3\omega^{2,6} - \omega^{3,4}) - 2/3x_4(\omega^{1,6} + 3\omega^{2,6} + \omega^{3,4}) - 2x_5\omega^{4,6} + x_6\omega^{3,5}, 2/3x_1(\omega^{1,3} + \omega^{1,4} + 3/2\omega^{2,3}) - 2/3x_2(\omega^{1,3} + \omega^{1,4} + 3/2\omega^{2,3} + 1/2\omega^{2,4}) + 1/3x_3(\omega^{1,6} + \omega^{2,5} + 3\omega^{2,6} - \omega^{3,4}) - 1/3x_4(\omega^{1,6} - 3\omega^{2,6} - 2\omega^{3,4}) + x_5\omega^{4,6} + x_6\omega^{3,6}, x_2(\omega^{1,3} + \omega^{1,4}) - x_3\omega^{1,5} + x_4\omega^{1,5}, x_1(\omega^{1,4} + 3\omega^{2,4}) - x_2(\omega^{2,3} + 2\omega^{2,4}) - x_3(\omega^{1,6} + \omega^{1,5} + \omega^{2,5} + 3\omega^{2,6} - \omega^{3,4}) - x_4(\omega^{2,5} - \omega^{3,4}) + x_5\omega^{1,7} + x_6\omega^{2,7}\rangle + e^0 \langle x_4\omega^{1,2} + x_5\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-3\beta_1} + 10e^{-4\beta_1} + 12e^{-5\beta_1} + e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=6e^{-6\beta_1} + 12e^{-7\beta_1} + 8e^{-8\beta_1} + 4e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=4e^{-9\beta_1} + 11e^{-10\beta_1} + 6e^{-11\beta_1} + 2e^{-12\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-11\beta_1} + 3e^{-12\beta_1} + 4e^{-13\beta_1} + 3e^{-14\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-14\beta_1} + 2e^{-15\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 12, 53, 121, 154, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 6, 16, 27, 30, 23, 12, 3)$

$\mathrm{S}=16\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{2,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5} - \omega^{1,2,6} - \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5} - \omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} - \omega^{3,4,5}$; $d\omega^{6,7} = \omega^{1,5,6} - \omega^{2,3,7} - \omega^{2,4,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,7} -$

$\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1} \langle\omega^1, \omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-3\beta_1} \langle\omega^{1,3}, \omega^{2,3}\rangle + 2e^{-4\beta_1} \langle\omega^{2,6}, \omega^{1,5}\rangle + 2e^{-5\beta_1} \langle\omega^{1,7} - \omega^{3,5} + \omega^{4,5}, \omega^{2,7} - 2\omega^{3,6} - \omega^{4,5} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 5e^{-6\beta_1} \langle\omega^{1,4,5}, \omega^{1,2,7} + \omega^{2,3,5}, \omega^{1,3,5}, \omega^{2,3,6}, \omega^{2,4,6}\rangle + 2e^{-7\beta_1} \langle\omega^{1,3,7} - \omega^{3,4,5}, \omega^{1,4,7} + \omega^{2,4,7} - \omega^{2,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-9\beta_1} \langle\omega^{1,2,6,7} - \omega^{2,3,5,6}, \omega^{1,2,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}\rangle + 5e^{-10\beta_1} \langle\omega^{1,3,5,7}, \omega^{1,3,6,7} + \omega^{2,3,5,7} + 2\omega^{3,4,5,6}, \omega^{1,4,5,7}, \omega^{2,3,6,7} - \omega^{3,4,5,6}, \omega^{1,4,6,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-11\beta_1} \langle\omega^{1,2,3,6,7}, \omega^{1,2,3,5,7}\rangle + 2e^{-12\beta_1} \langle\omega^{1,3,4,5,7}, \omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}\rangle + 2e^{-13\beta_1} \langle\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}, \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-14\beta_1} \langle\omega^{1,2,4,5,6,7}\rangle + 2e^{-15\beta_1} \langle\omega^{1,3,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-16\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 3Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7} + \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} - \omega^{3,6} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$;

$d^{\star}\omega^{3,4,5} = -\omega^{5,7}$; $d^{\star}\omega^{3,4,6} = -\omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^{\star}\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,4,6}$; $d^{\star}\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^{\star}\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,7}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^{\star}\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} + \omega^{2,6,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,7}$; $d^{\star}\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{3,5,7} + \omega^{3,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{4,5,7} + \omega^{4,6,7}$; $d^{\star}\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6} - \omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^{\star}\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^{\star}\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 4\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,5} + 3\omega^{1,6}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,7} + \omega^{3,5} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,5} + 2\omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,7} = -\omega^{1,7} + 3\omega^{2,7} + \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{1,5} - \omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + \omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{5,7} + 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} + \omega^{1,2,6} - \omega^{1,3,4} + \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,5} + 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,5} + 2\omega^{1,3,6} - \omega^{1,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + \omega^{1,4,5} + 3\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,7} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 4\omega^{1,5,6} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,4,7}$; $\Delta\omega^{1,6,7} = 4\omega^{1,6,7} - \omega^{2,6,7} - \omega^{3,5,6} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,5} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + \omega^{1,3,5} + 3\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,7} + 3\omega^{2,3,7} + \omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,5} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,7} + \omega^{2,3,7} + 3\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{1,5,6} - \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7} - \omega^{2,6,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = -\omega^{1,6,7} - \omega^{2,5,7} +$

$2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,5,6} + \omega^{2,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,5,7} - \omega^{2,5,7} + 4\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} - \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = -\omega^{3,5,7} + 3\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} - \omega^{2,6,7} + \omega^{3,4,7} - \omega^{3,5,6} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} - \omega^{4,5,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,5} + 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{1,3,4,5} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 3\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,5} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,3,7} + 2\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 4\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,5} + 3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 3\omega^{1,3,5,6} - \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} + 3\omega^{1,4,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 3\omega^{1,4,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,4,7} + \omega^{1,3,4,5} + 4\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + 4\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} + \omega^{1,3,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 3\omega^{2,3,5,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,5,7} + 2\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} - \omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{1,5,6,7} - \omega^{2,5,6,7} - \omega^{3,4,5,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{3,5,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + 3\omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,6,7} + \omega^{1,3,4,5,6} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 3\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1}\ \langle\omega^1, \omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-3\beta_1}\ \langle\omega^{1,3}, \omega^{2,3} - \omega^{2,4}\rangle + 2e^{-4\beta_1}\ \langle\omega^{2,6}, 3\omega^{1,5} - \omega^{1,6} - \omega^{2,5} + \omega^{3,4}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,7} - \omega^{3,5} + \omega^{4,5}, \omega^{2,7} - 2\omega^{3,6} - \omega^{4,5} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= 5e^{-6\beta_1}\ \langle\omega^{2,3,6},\omega^{1,2,7}-3\omega^{1,4,5}+\omega^{1,4,6}+\omega^{2,4,5},\omega^{1,3,5}-3\omega^{1,4,5}+\omega^{1,4,6}-\omega^{2,3,5}+2\omega^{2,4,5},\omega^{1,3,6}-5\omega^{1,4,5}+2\omega^{1,4,6}-\omega^{2,3,5}+3\omega^{2,4,5},\omega^{2,4,6}\rangle+2e^{-7\beta_1}\ \langle 7\omega^{1,3,7}+2\omega^{1,5,6}+3\omega^{2,3,7}-2\omega^{2,4,7}+2\omega^{2,5,6}-2\omega^{3,4,5}+3\omega^{3,4,6},\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,5}-\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= 5e^{-10\beta_1}\ \langle-\omega^{1,4,5,7},(\omega^{1,3,6,7}+\omega^{1,4,6,7}-\omega^{2,3,6,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6})/4,(-\omega^{1,3,6,7}-2\omega^{1,4,6,7}+\omega^{2,3,5,7}-3\omega^{2,4,5,7}-5\omega^{2,4,6,7})/8,(-\omega^{1,3,6,7}-\omega^{2,3,5,7}+3\omega^{2,4,5,7}+3\omega^{2,4,6,7}-2\omega^{3,4,5,6})/8,\omega^{1,3,5,7}\rangle+2e^{-9\beta_1}\ \langle(-5\omega^{1,2,5,7}-10\omega^{1,2,6,7}-2\omega^{1,3,4,7}+2\omega^{1,3,5,6}+5\omega^{1,4,5,6}+6\omega^{2,3,4,7}+4\omega^{2,3,5,6}-21\omega^{2,4,5,6})/217,(59\omega^{1,2,5,7}-99\omega^{1,2,6,7}+67\omega^{1,3,4,7}-67\omega^{1,3,5,6}-59\omega^{1,4,5,6}+16\omega^{2,3,4,7}+83\omega^{2,3,5,6}-56\omega^{2,4,5,6})/434\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= 2e^{-13\beta_1}\ \langle-\omega^{2,4,5,6,7},(\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7})/2\rangle+2e^{-12\beta_1}\ \langle-\omega^{1,3,4,5,7},(\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}-3\omega^{2,3,4,6,7})/12\rangle+2e^{-11\beta_1}\ \langle(\omega^{1,2,3,5,7}+6\omega^{1,2,3,6,7}-2\omega^{1,2,4,5,7}+7\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}-7\omega^{2,3,4,5,6})/20,(3\omega^{1,2,3,5,7}-2\omega^{1,2,3,6,7}-6\omega^{1,2,4,5,7}+\omega^{1,2,4,6,7}+3\omega^{1,3,4,5,6}-\omega^{2,3,4,5,6})/20\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= 2e^{-15\beta_1}\ \langle\omega^{2,3,4,5,6,7},-\omega^{1,3,4,5,6,7}\rangle+e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y).$

5.3.9 Algebra $\mathcal{G}_{7,1.2(iii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_6$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -(\xi_2^3+\xi_1^4) & \xi_2^4 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^3-\xi_1^4 & 0 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_2^4 & -\xi_2^3 & 4\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^6, ad\ x_2=-\delta_1^4, ad\ x_3=-(\delta_2^5+\delta_1^6), ad\ x_4=-\delta_1^5-\delta_2^6+\delta_3^7, ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=2e^{\beta_1}\ \langle x_1,x_2\rangle+2e^{2\beta_1}\ \langle x_3,x_4\rangle+2e^{3\beta_1}\ \langle x_5,x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{3\beta_1}\ \langle x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}})=5e^{2\beta_1}\ \langle\delta_1^5,\delta_1^6,\delta_2^6,\delta_3^7,\delta_2^5\rangle+4e^{\beta_1}\ \langle\delta_3^6,\delta_1^4,\delta_2^4,\delta_2^3\rangle+2e^{3\beta_1}\ \langle\delta_1^7,\delta_2^7\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^6,\delta_2^3]=\delta_1^7$; $[\delta_2^6,\delta_2^3]=\delta_2^7$; $[\delta_3^6,\delta_2^3]=\delta_3^7+\delta_2^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_2^6-\delta_1^5$; $[\delta_1^4,\delta_2^3]=-(\delta_1^6+\delta_2^5)$; $[\delta_3^7,\delta_2^3]=\delta_2^7$; $[\delta_2^4,\delta_2^3]=-(\delta_3^7+\delta_2^6)$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{3\beta_1}\ \langle\delta_2^7\rangle+3e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6,\delta_2^5\rangle+2e^{\beta_1}\ \langle\delta_2^4,\delta_2^3\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6,\delta_2^3]=\delta_2^7$; $[\delta_2^4,\delta_2^3]=-\delta_2^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=7e^{-2\beta_1}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})-2x_2(\omega^{1,3}-\omega^{2,4})+x_3(\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,5}, 2x_1\omega^{1,4}+3x_2(\omega^{1,3}-2/3\omega^{2,4})-x_3(\omega^{1,5}-\omega^{3,4})+x_5\omega^{4,5}, x_1\omega^{1,3}+5/2x_2(\omega^{1,4}-1/5\omega^{2,3})-1/2x_4\omega^{3,4}+x_5\omega^{3,5}, x_2\omega^{1,4}+x_5\omega^{1,7}, -x_1\omega^{1,3}-5/2x_2(\omega^{1,4}-1/5\omega^{2,3})-x_4(\omega^{1,5}-3/2\omega^{3,4})+x_6\omega^{2,7}, -x_2\omega^{1,3}+x_6\omega^{1,7}, -1/2x_2(\omega^{1,4}+\omega^{2,3})+1/2x_4\omega^{3,4}+x_6\omega^{3,6}\rangle+8e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_3\omega^{2,4}+x_5\omega^{1,5}, -x_1\omega^{1,2}+3x_3(\omega^{1,4}-1/3\omega^{2,3})+2x_4\omega^{2,4}+x_7\omega^{2,7}, x_2\omega^{1,2}+x_3\omega^{2,4}+x_4\omega^{2,3}, x_2\omega^{1,2}-x_3(\omega^{1,3}-2\omega^{2,4})+x_4\omega^{1,4}, x_3\omega^{1,4}+x_5\omega^{1,6}, -x_3(\omega^{1,4}-\omega^{2,3})-x_4\omega^{2,4}+x_6\omega^{2,6}, -x_3(\omega^{1,4}-\omega^{2,3})+x_4(\omega^{1,3}-\omega^{2,4}), -x_3\omega^{2,4}+x_6\omega^{1,6}\rangle+e^0\ \langle -x_4\omega^{1,2}+x_7\omega^{2,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-3\beta_1}+10e^{-4\beta_1}+12e^{-5\beta_1}+2e^{-6\beta_1}.$

ch($H^4_{\mathcal{G}}$)=$7e^{-6\beta_1} + 12e^{-7\beta_1} + 8e^{-8\beta_1} + 4e^{-9\beta_1}$.
ch($H^5_{\mathcal{G}}$)=$4e^{-9\beta_1} + 11e^{-10\beta_1} + 6e^{-11\beta_1} + 2e^{-12\beta_1}$.
ch($H^6_{\mathcal{G}}$)=$2e^{-11\beta_1} + 3e^{-12\beta_1} + 4e^{-13\beta_1} + 3e^{-14\beta_1}$.
ch($H^7_{\mathcal{G}}$)=$e^{-14\beta_1} + 2e^{-15\beta_1}$.
Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(2, 12, 53, 122, 154, 114, 45, 7)$
Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(2, 7, 16, 28, 31, 23, 12, 3)$
S=$16\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5} + \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = -\omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5} - \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = \omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,5} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $2e^{-\beta_1}\ \langle\omega^1, \omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $2e^{-3\beta_1}\ \langle\omega^{1,4}, \omega^{1,3}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,6} + \omega^{2,5}, \omega^{2,6} + \omega^{3,4}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,7} - \omega^{3,5}, \omega^{2,7} - 2\omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= 5e^{-6\beta_1}\ \langle\omega^{1,4,6},\omega^{2,3,6},\omega^{1,3,6}-\omega^{2,4,6},\omega^{1,2,7}-\omega^{2,4,6},\omega^{2,3,5}+\omega^{2,4,6}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,4,7},\omega^{1,3,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= 5e^{-10\beta_1}\ \langle\omega^{1,3,5,7},\omega^{1,3,6,7}+\omega^{2,3,5,7},\omega^{1,4,5,7}-\omega^{2,3,5,7},\omega^{1,4,6,7},\omega^{2,3,6,7}-\omega^{3,4,5,6}\rangle + 2e^{-9\beta_1}\ \langle\omega^{1,2,6,7}-\omega^{2,3,5,6},\omega^{1,2,5,7}-\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= 2e^{-11\beta_1}\ \langle\omega^{1,2,3,6,7},\omega^{1,2,4,6,7}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,3,4,6,7},\omega^{1,2,5,6,7}-\omega^{2,3,4,6,7}\rangle + 2e^{-13\beta_1}\ \langle\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7},\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + 2e^{-15\beta_1}\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,10,18,24,17,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,6,7,7,6,3,1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^6$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{3,4}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,7}-\omega^{3,5}+\omega^{4,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{6,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,7}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{5,7}$; $d^\star\omega^{3,4,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}-\omega^{2,3,5}+\omega^{2,4,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,3,7}+\omega^{2,5,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,6}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{2,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}-\omega^{3,4,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}+\omega^{4,6,7}$; $d^\star\omega^{1,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=\omega^{2,5,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=-\omega^{1,5,6,7}+\omega^{3,4,6,7}$;

$d^{\star}\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = 2\omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5} - \omega^{2,6}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{3,5} - \omega^{4,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,5} + \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{2,6} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5} - \omega^{4,6}$; $\Delta\omega^{3,6} = \omega^{2,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{1,7} - \omega^{3,5} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 4\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,6} + 3\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,5} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 3\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} - \omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,5} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} + 2\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7} + 2\omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = -\omega^{1,5,7} + 3\omega^{2,6,7} - \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7} + \omega^{2,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,3,7} + \omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,6,7} + 2\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,6,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,5,7} + \omega^{3,4,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,5,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 5\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 5\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,5,6} + 3\omega^{1,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + 2\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 2\omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,4,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,6,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,5,7} + \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}$;

$\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,3,4,5} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 3\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7} = -\omega^{1,3,5,7} + \omega^{1,4,6,7} + 2\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 3\omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,5,7} - \omega^{1,4,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 4\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1}\ \langle\omega^1, \omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-3\beta_1}\ \langle\omega^{1,4} - \omega^{2,3}, \omega^{1,3} - \omega^{2,4}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,6} + \omega^{2,5}, \omega^{1,5} + 2\omega^{2,6} + \omega^{3,4}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,7} - \omega^{3,5}, \omega^{2,7} - 2\omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 5e^{-6\beta_1}\ \langle\omega^{1,4,6} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}, \omega^{1,2,7} - \omega^{1,4,5} - 2\omega^{2,4,6}, \omega^{1,3,6} - \omega^{2,4,6}, \omega^{2,3,5} + \omega^{2,4,6}\rangle + 2e^{-7\beta_1}\ \langle 2\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}, 3\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 5e^{-10\beta_1}\ \langle(-\omega^{1,3,6,7} + \omega^{1,4,5,7} - 2\omega^{2,3,5,7})/3, (-\omega^{1,3,6,7} - 2\omega^{1,4,5,7} + \omega^{2,3,5,7})/3, (-2\omega^{1,3,5,7} + 2\omega^{1,4,6,7} - 3\omega^{2,3,6,7} - 2\omega^{2,4,5,7} + 3\omega^{3,4,5,6})/10, (-\omega^{1,3,5,7} + \omega^{1,4,6,7} + \omega^{2,3,6,7} + 4\omega^{2,4,5,7} - \omega^{3,4,5,6})/5, (\omega^{1,3,5,7} + 4\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6})/5\rangle + 2e^{-9\beta_1}\ \langle(-\omega^{1,2,6,7} - \omega^{1,4,5,6} + 2\omega^{2,3,5,6})/6, (\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,3,4,7} - 3\omega^{2,4,5,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-13\beta_1}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2, (\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + 2e^{-12\beta_1}\ \langle(\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2, (\omega^{1,2,5,6,7} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,6,7})/6\rangle + 2e^{-11\beta_1}\ \langle(\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2, (-\omega^{1,2,3,6,7} - 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/6\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = 2e^{-15\beta_1}\ \langle\omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 3Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y)$.

5.3.10 Algebra $\mathcal{G}_{7,1.2(iv)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_5]=-x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_1^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^3 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_1^4+\xi_3^5 & \xi_2^4 & 0 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5-\xi_2^5+\xi_2^6 & -\xi_2^4+\xi_3^5 & 2\xi_1^3+\xi_2^4 & 4\xi_1^1\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-(\delta_1^6+\delta_3^7), ad\ x_5=\delta_1^7-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=2e^{\beta_1}\ \langle x_1,x_2\rangle+2e^{2\beta_1}\ \langle x_3,x_4\rangle+2e^{3\beta_1}\ \langle x_5,x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{4\beta_1}\ \langle x_7\rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}})=5e^{2\beta_1}\ \langle \delta_1^6,\delta_3^7,\delta_1^5,\delta_2^5,\delta_2^6\rangle+4e^{\beta_1}\ \langle\delta_1^4,\delta_3^5,\delta_1^3,\delta_2^4\rangle+2e^{3\beta_1}\ \langle\delta_1^7,\delta_2^7\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6,\delta_1^3]=-2\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_1^5]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_1^3]=-2\delta_3^7+\delta_2^6+\delta_1^6+\delta_1^5$; $[\delta_1^4,\delta_2^4]=-(\delta_3^7+\delta_1^6)$; $[\delta_3^7,\delta_1^3]=\delta_2^7+\delta_1^7$; $[\delta_1^5,\delta_3^5]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_2^7+\delta_1^7$; $[\delta_2^5,\delta_3^5]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=-2\delta_2^7$; $[\delta_3^5,\delta_1^3]=-2\delta_3^7+\delta_2^6+\delta_1^6+\delta_2^5+\delta_1^5$; $[\delta_1^3,\delta_2^4]=-\delta_2^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})=3e^{2\beta_1}\ \langle\delta_1^6,\delta_1^5,\delta_2^6\rangle+2e^{\beta_1}\ \langle\delta_1^4,\delta_1^3\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^3]=\delta_2^6+3\delta_1^6+\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})=8e^{-\beta_1}\ \langle x_3\omega^{2,4}, x_1\omega^{1,2}+x_2\omega^{1,2}-x_3(\omega^{1,3}+2\omega^{2,3})+x_4(\omega^{1,4}+2\omega^{2,4})+x_6\omega^{2,6}, -x_2\omega^{1,2}+2x_3\omega^{1,3}-2x_4\omega^{1,4}+x_5\omega^{1,5}, -x_1\omega^{1,2}-x_2\omega^{1,2}+x_3(\omega^{1,3}+\omega^{2,3})-x_4(\omega^{1,4}+\omega^{2,4})+x_5\omega^{1,6}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}+x_6\omega^{2,5}, x_4\omega^{1,3}, x_4(\omega^{1,4}-\omega^{2,3})+x_6\omega^{3,4}, x_3(\omega^{1,3}+\omega^{2,3})-2x_4\omega^{1,4}+x_5\omega^{3,4}\rangle+7e^{-2\beta_1}\ \langle x_1\omega^{1,4}-x_2(\omega^{1,4}+\omega^{2,4})-x_3(\omega^{1,5}-\omega^{1,6}+2\omega^{2,5}-\omega^{3,4})+x_5(\omega^{1,7}-\omega^{2,7}+\omega^{4,5}), -x_1(\omega^{1,4}-\omega^{2,4})-x_2(\omega^{1,4}-\omega^{2,4})+x_5(\omega^{1,7}-\omega^{2,7}+\omega^{4,6}), x_1\omega^{2,3}+2x_2(\omega^{1,3}+1/2\omega^{1,4}+\omega^{2,3}+1/2\omega^{2,4})+x_3(\omega^{1,5}+\omega^{2,5}-\omega^{3,4})-2x_4(\omega^{1,5}+\omega^{1,6}+1/2\omega^{2,5}-1/2\omega^{2,6})+x_5(\omega^{1,7}+2\omega^{3,6})+x_6\omega^{2,7}, x_2\omega^{1,3}+x_6\omega^{3,5}, -x_1\omega^{1,3}-2x_2\omega^{1,3}+x_4(\omega^{1,5}+2\omega^{1,6})+x_6\omega^{3,6}, x_1\omega^{1,3}+$

$3x_2\omega^{1,3}-2x_4(\omega^{1,5}+1/2\omega^{1,6}-1/2\omega^{3,4}), x_1\omega^{1,3}+3x_2(\omega^{1,3}+1/3\omega^{2,3})-2x_4(\omega^{1,5}+1/2\omega^{1,6}+1/2\omega^{2,5})+x_6\omega^{1,7}\rangle + e^0\ \langle -x_3\omega^{1,2}+x_5\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-3\beta_1}+10e^{-4\beta_1}+12e^{-5\beta_1}+2e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=7e^{-6\beta_1}+12e^{-7\beta_1}+8e^{-8\beta_1}+4e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=4e^{-9\beta_1}+11e^{-10\beta_1}+6e^{-11\beta_1}+2e^{-12\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-11\beta_1}+3e^{-12\beta_1}+4e^{-13\beta_1}+3e^{-14\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-14\beta_1}+2e^{-15\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,53,122,154,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,16,28,31,23,12,3)$

S=$16\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,18,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,7,7,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=\omega^{1,5}-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=\omega^{1,2,5}-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=\omega^{1,3,5}-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}+\omega^{1,4,5}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=\omega^{1,2,3,5}-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=\omega^{1,2,4,5}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}+\omega^{1,3,4,5}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,6}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=\omega^{1,2,3,4,5}-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= 2e^{-\beta_1}\ \langle\omega^1,\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= 2e^{-3\beta_1}\ \langle\omega^{1,3},\omega^{2,4}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,6},\omega^{1,5}-\omega^{3,4}\rangle + 2e^{-5\beta_1}\ \langle\omega^{3,5},(2\omega^{1,7}-\omega^{2,7}+2\omega^{3,6}+\omega^{4,5}+\omega^{4,6})/2\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 5e^{-6\beta_1}\ \langle\omega^{1,3,5},\omega^{1,3,6},\omega^{1,4,6},\omega^{1,2,7}-\omega^{2,3,6},\omega^{2,4,6}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,3,7}-\omega^{3,4,5},\omega^{1,4,7}-\omega^{2,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 2e^{-9\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7}-\omega^{2,3,5,6},\omega^{1,2,5,7}-\omega^{2,3,5,6}\rangle + 5e^{-10\beta_1}\ \langle\omega^{1,3,5,7},\omega^{1,3,6,7}+\omega^{3,4,5,6},\omega^{1,4,6,7}-\omega^{2,4,6,7},\omega^{2,3,5,7}+\omega^{3,4,5,6},\omega^{2,4,5,7}-\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-11\beta_1}\ \langle\omega^{1,2,4,6,7},\omega^{1,2,3,5,7}\rangle + 2e^{-12\beta_1}\ \langle\omega^{1,3,4,6,7}-\omega^{2,3,4,6,7},\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}\rangle + 2e^{-13\beta_1}\ \langle\omega^{1,3,5,6,7},\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-14\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + 2e^{-15\beta_1}\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,10,18,24,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,6,7,7,6,3,1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,5}=\omega^7$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}$; $d^\star\omega^{1,2,5}=\omega^{1,7}+\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5}=\omega^{3,7}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}+\omega^{2,3,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}+\omega^{2,4,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{2,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}+\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}+\omega^{1,3,4,7}+$

$\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}-\omega^{2,3,5,7}-\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 4\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = 2\omega^{1,5} - \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{1,6} = -\omega^{1,5} + \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{2,7} - \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,5} + \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{1,7} + 3\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} + \omega^{6,7}$; $\Delta\omega^{6,7} = \omega^{5,7} + 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} - \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = -\omega^{1,2,5} + 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,6} = -\omega^{1,3,5} + \omega^{1,3,6}+\omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}+\omega^{2,3,7}+\omega^{3,4,5}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5}-\omega^{1,4,6}-\omega^{2,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7}-\omega^{1,4,5}+\omega^{1,4,6}+\omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,7}-\omega^{3,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 3\omega^{1,5,6} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{2,6,7} + \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,5} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,3,7} + 3\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} - \omega^{1,4,5} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{1,4,7} + 2\omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,6} + \omega^{2,4,7} + 2\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7}+2\omega^{2,5,7}+\omega^{2,6,7}+\omega^{3,4,7}+\omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{1,6,7}+\omega^{2,5,7}+3\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} - \omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 4\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} + \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = \omega^{3,5,7} + 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} + \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = \omega^{4,5,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = -\omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 4\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = 3\omega^{1,3,4,5} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,5} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 2\omega^{1,3,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7} + 2\omega^{1,3,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} + \omega^{2,3,6,7} -$

$\omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 3\omega^{1,4,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,5} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + 3\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} + \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} + 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + \omega^{3,4,5,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = -\omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 3\omega^{1,3,4,5,6} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} + 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1}\ \langle \omega^1, \omega^2\rangle + e^{-2\beta_1}\ \langle \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-3\beta_1}\ \langle \omega^{1,3}, \omega^{2,4}\rangle + 2e^{-4\beta_1}\ \langle \omega^{1,6} - \omega^{2,5}, \omega^{1,5} + 2\omega^{2,5} - \omega^{3,4}\rangle + 2e^{-5\beta_1}\ \langle \omega^{3,5}, 2\omega^{1,7} - \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5} + \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 5e^{-6\beta_1}\ \langle \omega^{1,3,5} + \omega^{2,3,5}, \omega^{1,3,6} - \omega^{2,3,5}, 2\omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,3,6} - 3\omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,4,5}, \omega^{2,4,6}\rangle + 2e^{-7\beta_1}\ \langle 5\omega^{1,3,7} - \omega^{1,5,6} - 2\omega^{2,3,7} - 3\omega^{3,4,5} - \omega^{3,4,6}, 5\omega^{1,4,7} - 4\omega^{1,5,6} + 2\omega^{2,3,7} - 5\omega^{2,5,6} - 2\omega^{3,4,5} + 6\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 5e^{-10\beta_1}\ \langle (\omega^{1,4,6,7} + \omega^{2,4,5,7} - 2\omega^{2,4,6,7})/3, (-\omega^{1,4,6,7} + 2\omega^{2,4,5,7} - \omega^{2,4,6,7})/3, (3\omega^{1,3,6,7} + 2\omega^{1,4,5,7} + 3\omega^{2,3,5,7} - 2\omega^{2,3,6,7} + 4\omega^{3,4,5,6})/21, (2\omega^{1,3,6,7} - \omega^{1,4,5,7} - 5\omega^{2,3,5,7} + \omega^{2,3,6,7} - 2\omega^{3,4,5,6})/7, \omega^{1,3,5,7}\rangle + 2e^{-9\beta_1}\ \langle (\omega^{1,2,5,7} - 3\omega^{1,2,6,7} - 2\omega^{1,4,5,6} - \omega^{2,3,4,7} - 5\omega^{2,4,5,6})/40, (-6\omega^{1,2,5,7} - 2\omega^{1,2,6,7} + 5\omega^{1,3,4,7} + 2\omega^{1,4,5,6} - 4\omega^{2,3,4,7} + 5\omega^{2,3,5,6})/110\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-13\beta_1}\ \langle -\omega^{2,4,5,6,7}, -\omega^{1,3,5,6,7}\rangle + 2e^{-12\beta_1}\ \langle (-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 3\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7})/4, (-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7} - \omega^{2,3,4,6,7})/4\rangle + 2e^{-11\beta_1}\ \langle -\omega^{1,2,4,6,7}, (-\omega^{1,2,3,5,7} + \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6} - 2\omega^{2,3,4,5,6})/11\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = 2e^{-15\beta_1}\ \langle \omega^{2,3,4,5,6,7}, -\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1}\ \langle \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-16\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

5.3.11 Algebra $\mathcal{G}_{7,1.3(i_\lambda)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=\lambda x_7$; $[x_2,x_5]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -2\xi_1^2-\xi_1^3 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & a & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^4 & -\xi_1^2-\xi_1^3 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4\lambda-\xi_1^5+\xi_2^6 & -\xi_1^5+\xi_3^6 & \xi_1^2\lambda-\xi_1^3+\xi_2^4 & 5\xi_1^1\end{pmatrix}$$

with $a=3\xi_1^2\lambda+\xi_1^3(\lambda-1)+\xi_2^4+\xi_2^5$.

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=-(\lambda\delta_2^7+\delta_1^6), ad\ x_5=-(\delta_2^7+\delta_3^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+2e^{2\beta_1}\ \langle x_2,x_3\rangle+2e^{3\beta_1}\ \langle x_4,x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{\beta_1}\ \langle\delta_1^2,\delta_1^3,\delta_2^4,\delta_2^5\rangle+4e^{2\beta_1}\ \langle\delta_1^4,\delta_1^5,\delta_2^6,\delta_3^6\rangle+3e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7,\delta_3^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3]=-\delta_3^6-2\delta_1^4+2\lambda\delta_3^6+3\lambda\delta_1^5$; $[\delta_1^2,\delta_1^4]=-2\lambda\delta_3^7$; $[\delta_1^2,\delta_2^4]=\delta_3^6-\delta_1^4$; $[\delta_1^2,\delta_1^5]=-2\delta_3^7-\delta_1^6+3\lambda\delta_3^7$; $[\delta_1^2,\delta_2^5]=-(\delta_3^6+\delta_2^6+\delta_1^5)$; $[\delta_1^2,\delta_1^6]=\lambda\delta_1^7$; $[\delta_1^2,\delta_2^6]=2\delta_3^7-\delta_1^6+\lambda\delta_2^7$; $[\delta_1^2,\delta_3^6]=-2\lambda\delta_3^7$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^3,\delta_1^4]=-(\delta_1^6+\lambda\delta_3^7)$; $[\delta_1^3,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_1^3,\delta_1^5]=-2\delta_3^7-\delta_1^6+\lambda\delta_3^7$; $[\delta_1^3,\delta_2^5]=-(\delta_3^6+\delta_2^6+\delta_1^5)$; $[\delta_1^3,\delta_1^6]=-\delta_1^7$; $[\delta_1^3,\delta_2^6]=\delta_3^7-\delta_2^7$; $[\delta_1^3,\delta_3^6]=-(\delta_1^6+\lambda\delta_3^7)$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_1^6+\lambda\delta_2^7)$; $[\delta_1^4,\delta_1^5]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_2^4,\delta_1^5]=\delta_3^7+\delta_2^7$; $[\delta_2^4,\delta_1^6]=\delta_1^7$; $[\delta_1^5,\delta_2^5]=-(\delta_3^7+\delta_2^7)$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_2^5,\delta_3^6]=-(\delta_3^7+\delta_2^7)$.

ch($H^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle\delta_1^2,\delta_1^3,\delta_2^5\rangle+2e^{2\beta_1}\ \langle\delta_1^5,\delta_1^4\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3]=3\delta_1^5\lambda-3\delta_1^4+2\delta_1^4\lambda$; $[\delta_1^2,\delta_2^5]=-\delta_1^4$; $[\delta_1^2,\delta_1^5]=2\delta_2^7-2\delta_2^7\lambda$; $[\delta_1^2,\delta_1^4]=2\delta_2^7\lambda$; $[\delta_1^3,\delta_2^5]=-\delta_1^4$; $[\delta_1^3,\delta_1^5]=2\delta_2^7$; $[\delta_1^3,\delta_1^4]=2\delta_2^7\lambda$.

ch($H^2_{\mathcal{G}}$)= $5e^{-\beta_1}\ \langle x_2\omega^{1,3}-x_3\omega^{1,3}-x_4\omega^{2,3}(\lambda+1)+x_6\lambda\omega^{3,4},-x_2\omega^{1,2}+x_3\omega^{1,2}-x_4\omega^{2,3}(\lambda+1)+x_5\lambda\omega^{1,4}+x_6\lambda\omega^{3,4},-x_2\lambda\omega^{1,2}-x_4(\lambda^2\omega^{2,3}+\lambda\omega^{2,3}-\omega^{1,5})+x_5\lambda^2\omega^{1,4}+x_6\lambda^2\omega^{3,4},-x_2\omega^{1,2}-x_3\omega^{1,3}-x_4\omega^{2,3}(\lambda+2)+x_5(\lambda\omega^{1,4}+\omega^{1,5})+$

$x_6\lambda\omega^{3,4}, x_4(\lambda\omega^{2,3}+\omega^{1,4})-x_5\lambda\omega^{1,4}-x_6\lambda\omega^{3,4}\rangle+9e^{-2\beta_1}\ \langle -x_1\lambda\omega^{1,2}-x_2(\lambda\omega^{1,4}+\lambda\omega^{2,3}+\omega^{2,3})+2x_3\lambda(\lambda\omega^{2,3}+1/2\omega^{1,4})+x_4(\lambda\omega^{3,4}+\omega^{1,6})-2x_5(\lambda^2\omega^{3,4}-\lambda^2\omega^{3,5}+1/2\lambda\omega^{3,4}+1/2\lambda\omega^{3,5}+1/2\omega^{3,5}), x_2\omega^{2,3}-x_3(\lambda\omega^{2,3}-\omega^{1,5})+x_4\omega^{2,5}+x_5(\lambda\omega^{3,4}-\lambda\omega^{3,5}+\omega^{3,5}), -x_2(\lambda\omega^{1,4}+\lambda\omega^{2,3}+\omega^{1,5})+2x_3\lambda^2\omega^{2,3}+x_4(\lambda\omega^{3,4}+\omega^{3,5})-2x_5\lambda(\lambda\omega^{3,4}-\lambda\omega^{3,5}+\omega^{3,5}), -x_1(\omega^{1,2}+\omega^{1,3})-x_2(\omega^{1,4}-\omega^{2,3})-x_3(\lambda\omega^{2,3}-\omega^{1,4}+2\omega^{2,3})+x_5(\lambda\omega^{3,4}-\lambda\omega^{3,5}+\omega^{1,6}-\omega^{3,5}), -x_3\omega^{2,3}+x_5(\omega^{2,4}+\omega^{3,4}-\omega^{3,5}), x_3\omega^{2,3}+x_5(\omega^{2,5}+2\omega^{3,5}), x_2(\omega^{1,4}+2\omega^{2,3})-4x_3(\lambda\omega^{2,3}+1/4\omega^{1,5})-2x_4\omega^{3,4}+4x_5(\lambda\omega^{3,4}-\lambda\omega^{3,5}+1/2\omega^{3,5})+x_6\omega^{4,5}, -x_1\omega^{1,2}-x_2(\omega^{1,4}+\omega^{2,3})+2x_3(\lambda\omega^{2,3}+1/2\omega^{1,4}-1/2\omega^{2,3})+x_4\omega^{3,4}-2x_5(\lambda\omega^{3,4}-\lambda\omega^{3,5}+3/2\omega^{3,5})+x_7\omega^{2,7}, -x_2(\omega^{1,4}+\omega^{2,3})+2x_3(\lambda\omega^{2,3}+1/2\omega^{1,4})+x_4(\omega^{2,4}+\omega^{3,4})-2x_5(\lambda\omega^{3,4}-\lambda\omega^{3,5}+1/2\omega^{3,5})\rangle+4e^{-3\beta_1}\ \langle -4/3x_1(\lambda^2\omega^{1,4}-7/4\lambda^2\omega^{2,3}-1/2\lambda\omega^{1,5}+1/4\omega^{1,5})+4/3x_2(\lambda^2\omega^{2,4}-\lambda\omega^{1,6}+\lambda\omega^{2,4}-3/4\lambda\omega^{2,5}+1/4\lambda\omega^{3,4}-1/4\omega^{2,4}-1/4\omega^{2,5}-1/4\omega^{3,4}-3/4\omega^{3,5})-8/3x_3(\lambda^2\omega^{2,4}-7/8\lambda\omega^{1,6}+1/2\lambda\omega^{2,4}-3/8\lambda\omega^{2,5}+1/2\lambda\omega^{3,4}-1/8\lambda\omega^{3,5}+1/8\omega^{1,6}-1/8\omega^{2,4}-1/8\omega^{3,4}-1/8\omega^{3,5})+7/3x_4(\lambda\omega^{2,6}-1/7\lambda\omega^{4,5}+3/7\omega^{1,7}-1/7\omega^{2,6}+1/7\omega^{4,5})-11/3x_5(\lambda^2\omega^{2,6}+4/11\lambda^2\omega^{3,6}+3/11\lambda\omega^{1,7}+4/11\lambda\omega^{2,6}+4/11\lambda\omega^{3,6}-1/11\omega^{2,6}-1/11\omega^{3,6})-x_6\lambda(\omega^{3,7}+1/3\omega^{5,6}), -x_1\lambda(\omega^{1,4}-\omega^{2,3})+x_2(\lambda\omega^{2,4}-\omega^{1,6}+\omega^{2,4}+\omega^{3,4})-x_3(\lambda\omega^{2,4}-\omega^{1,6}+\omega^{2,4}+\omega^{3,4})+x_4(\omega^{2,6}+\omega^{3,6})-x_5(\lambda\omega^{2,6}+\lambda\omega^{3,6}+\omega^{2,6}+\omega^{3,6}), x_1\omega^{2,3}+x_3(\omega^{2,4}+\omega^{2,5}+\omega^{3,4})+x_5(\omega^{2,6}+2\omega^{3,6}+\omega^{4,5}), -4/3x_1(\lambda\omega^{1,4}-7/4\lambda\omega^{2,3}+3/4\omega^{1,4}-1/2\omega^{1,5}-3/2\omega^{2,3})+4/3x_2(\lambda\omega^{2,4}-\omega^{1,6}+5/2\omega^{2,4}-3/4\omega^{2,5}+1/4\omega^{3,4})-8/3x_3(\lambda\omega^{2,4}-7/8\omega^{1,6}+13/8\omega^{2,4}-3/8\omega^{2,5}+1/2\omega^{3,4}-1/8\omega^{3,5})+7/3x_4(\omega^{2,6}-1/7\omega^{4,5})-11/3x_5(\lambda\omega^{2,6}+4/11\lambda\omega^{3,6}+3/11\omega^{1,7}+13/11\omega^{2,6}+7/11\omega^{3,6})-x_6(\omega^{3,7}+1/3\omega^{5,6})+x_7\omega^{4,7}\rangle+e^0\ \langle x_7\omega^{2,4}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=5e^{-4\beta_1}+11e^{-5\beta_1}+10e^{-6\beta_1}+4e^{-7\beta_1}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-7\beta_1}+7e^{-8\beta_1}+13e^{-9\beta_1}+8e^{-10\beta_1}+3e^{-11\beta_1}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-11\beta_1}+7e^{-12\beta_1}+8e^{-13\beta_1}+4e^{-14\beta_1}+e^{-15\beta_1}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=3e^{-15\beta_1}+5e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=2e^{-18\beta_1}+e^{-19\beta_1}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,13,55,122,155,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,7,19,30,32,23,11,3)$

The preceding results are valid for:

$$-(\lambda-1)\neq 0.$$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1,3,5,7,7,5,3,1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,4}\lambda - \omega^{2,5} - \omega^{3,5}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4}\lambda + \omega^{1,2,5} + \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,4}\lambda - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,4,5}\lambda$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6}\lambda + \omega^{2,5,6} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}\lambda + \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5} + \omega^{1,3,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}\lambda$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6}\lambda - \omega^{1,2,5,6} - \omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6} + \omega^{2,3,4,5}\lambda$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}\lambda + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}\lambda$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}\lambda$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}\lambda - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6} - \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}\lambda$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}\lambda$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}\lambda$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + 2e^{-2\beta_1} \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = 2e^{-4\beta_1} \langle\omega^{1,5}, \omega^{1,4}\rangle + 3e^{-5\beta_1} \langle\omega^{2,4}, \omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = 3e^{-9\beta_1} \langle\omega^{1,5,7} + \omega^{2,5,6}\lambda + \omega^{3,4,6}\lambda, \omega^{2,4,6}, \omega^{1,4,7} - \omega^{2,5,6} - \omega^{3,4,6} - \omega^{3,5,6}\rangle + e^{-7\beta_1} \langle\omega^{1,2,6}\rangle + 3e^{-8\beta_1} \langle\omega^{2,3,6} + \omega^{2,4,5}, \omega^{1,2,7} + \omega^{3,4,5}, \omega^{1,3,7} - \omega^{2,4,5}\lambda - \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = 3e^{-11\beta_1} \langle\omega^{1,2,5,7}, \omega^{1,3,5,7} + \omega^{2,3,5,6}\lambda, \omega^{2,3,4,6}\rangle + 3e^{-12\beta_1} \langle\omega^{1,3,6,7} + \omega^{2,4,5,6}\lambda + \omega^{3,4,5,6}, \omega^{1,4,5,7}, \omega^{1,2,6,7} - \omega^{3,4,5,6}\rangle + e^{-13\beta_1} \langle\omega^{1,4,6,7} - \omega^{1,5,6,7} + \omega^{2,3,6,7}(-\lambda - 2) - \omega^{2,4,5,7}(\lambda + 1) - \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = 3e^{-15\beta_1} \langle\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}, \omega^{1,2,5,6,7}, \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}\lambda\rangle + 2e^{-16\beta_1} \langle\omega^{2,3,4,6,7}, \omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

ch($H^6_{\mathcal{G}}$)= $2e^{-18\beta_1}$ $\langle \omega^{1,2,4,5,6,7}, \omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1}$ $\langle \omega^{2,3,4,5,6,7}\rangle$.
ch($H^7_{\mathcal{G}}$)= $e^{-20\beta_1}$ $\langle \omega^{1,2,3,4,5,6,7}\rangle$.
$P(Y) \ = \ (1+Y^2)^2(1+Y)^3$.
The preceding results are valid for:

$$-(\lambda-1) \neq 0.$$

Comments:
$\mathcal{G}_{7,1.3(i_\lambda)}$ is isomorphic to $\mathcal{G}_{7,1.3(i_{\lambda'})}$ if and only if $\lambda = \lambda'$.
singular value $\lambda = 1$.
changes in adjoint cohomology:
case $\lambda = 1$**:** The only changes is dim($Z^4_{\mathcal{G}}$)=156 which implies dim($H^4_{\mathcal{G}}$)=33, dim($H^5_{\mathcal{G}}$)=24. $e^{-10\beta_1}$ has multiplicity 9 in ch($H^4_{\mathcal{G}}$) and appears as a new term in ch($H^5_{\mathcal{G}}$).
changes in trivial cohomology:
case $\lambda = 1$**:**
Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1,3,9,\underline{20},23,17,7,1)$ [4]
Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1,3,5,\underline{8},\underline{8},5,3,1)$
Add in ch($H^3_{\mathcal{G}}$) $e^{-10\beta_1}$ $\langle \omega^{1,6,7} - \omega^{2,4,7} - \omega^{3,4,7} - \omega^{3,5,7} - \omega^{4,5,6}\rangle$ and correspondingly in ch($H^4_{\mathcal{G}}$) $e^{-10\beta_1}$ $\langle \omega^{1,2,3,7}\rangle$.
$P(Y) \ = \ (1+2Y+3Y^2+5Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.
Trivial cohomology (via harmonic cocycles and Poincaré Duality):
Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1,3,9,19,23,17,7,1)$
Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1,3,5,7,7,5,3,1)$
Total dimension of trivial cohomology: 32.
Length associated polynomial: 32.
Riemann Hyp.: yes
$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7\bar{\lambda}$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}\bar{\lambda} - \omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}\bar{\lambda} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{2,7} - \omega^{3,7} + \omega^{5,6}$;

[4] underlined dimensions are those different from the generic ones

$d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,7}\bar{\lambda}$; $d^\star\omega^{2,4,6} = \omega^{6,7}\bar{\lambda}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7}\bar{\lambda} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,7}\bar{\lambda} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}\bar{\lambda} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7} - \omega^{3,4,7} + \omega^{3,5,7}\bar{\lambda} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}\bar{\lambda}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7} + \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7} - \omega^{5,6,7}\bar{\lambda}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} + \omega^{1,3,4,7} - \omega^{1,3,5,7}\bar{\lambda} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7}\bar{\lambda} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} - \omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} + \omega^{1,5,6,7}\bar{\lambda} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} + \omega^{3,4,6,7} - \omega^{3,5,6,7}\bar{\lambda}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{1,3,4,6,7} + \omega^{1,3,5,6,7}\bar{\lambda} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7(\lambda\bar{\lambda} + 3)$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,4}\lambda + \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7}(\lambda\bar{\lambda} + 2) - \omega^{2,6}\lambda - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,6}\bar{\lambda} + \omega^{2,4}\lambda\bar{\lambda} + \omega^{2,5}\bar{\lambda} + \omega^{3,5}\bar{\lambda}$; $\Delta\omega^{2,5} = \omega^{2,4}\lambda + 2\omega^{2,5} - \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{2,6} = -\omega^{1,7}\bar{\lambda} + \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} - \omega^{3,7} + \omega^{4,6} + \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{2,4}\lambda + \omega^{2,5} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = -\omega^{2,7} + \omega^{3,7}(\lambda\bar{\lambda} + 2) - \omega^{4,6}\lambda$; $\Delta\omega^{4,5} = -\omega^{1,7} - \omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} - \omega^{3,7}\bar{\lambda} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7} - \omega^{5,7}\bar{\lambda}$; $\Delta\omega^{5,6} = \omega^{2,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,7}\lambda + \omega^{5,7}(\lambda\bar{\lambda} + 2)$; $\Delta\omega^{6,7} = \omega^{6,7}(\lambda\bar{\lambda} + 4)$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}(\lambda\bar{\lambda} + 1) + \omega^{1,2,5}\bar{\lambda} + \omega^{1,3,5}\bar{\lambda}$; $\Delta\omega^{1,2,5} = \omega^{1,2,4}\lambda + 2\omega^{1,2,5} - \omega^{1,3,4} + \omega^{1,3,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,7} + \omega^{1,4,6} + \omega^{1,5,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4}\lambda + \omega^{1,2,5} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,4}\lambda$; $\Delta\omega^{1,3,7} = -\omega^{1,2,7} + \omega^{1,3,7}(\lambda\bar{\lambda} + 2) - \omega^{1,4,6}\lambda - \omega^{2,3,6}\lambda + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} - \omega^{1,3,7}\bar{\lambda} + 2\omega^{1,4,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 3\omega^{1,4,7} - \omega^{1,5,7}\bar{\lambda} + \omega^{2,3,7} - \omega^{2,4,6}\lambda$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + 2\omega^{1,5,6} - \omega^{2,4,5}\lambda - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,7}\lambda + \omega^{1,5,7}\lambda\bar{\lambda} - \omega^{2,5,6}\lambda$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}(\lambda\bar{\lambda} + 3) + \omega^{2,4,7}\lambda + \omega^{3,4,7} + \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,6}\bar{\lambda} + \omega^{2,3,4}(\lambda\bar{\lambda} + 1) + \omega^{2,3,5}\bar{\lambda}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{1,4,5} + \omega^{2,3,4}\lambda + 3\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7}\bar{\lambda} + \omega^{1,4,6} + \omega^{2,3,6} -$

$\omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{2,4,6}\lambda - \omega^{3,4,6} - \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} - \omega^{1,5,6}\bar{\lambda} - \omega^{2,3,6} + \omega^{2,4,5}(\lambda\bar{\lambda}+2) + \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7}\bar{\lambda} - \omega^{2,3,7}\bar{\lambda} + \omega^{2,4,6}\lambda\bar{\lambda} + \omega^{2,5,6}\bar{\lambda} + \omega^{3,5,6}\bar{\lambda}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7}\bar{\lambda} + 2\omega^{2,4,7} - \omega^{3,4,7} + \omega^{3,5,7}\bar{\lambda} - \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7}\bar{\lambda} + \omega^{2,4,6}\lambda + 3\omega^{2,5,6} - \omega^{3,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,4,6} - \omega^{1,5,6} + \omega^{2,4,5} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,4,7} - \omega^{2,5,7} + 3\omega^{3,4,7} - \omega^{3,5,7}\bar{\lambda}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7} + \omega^{2,4,6}\lambda + \omega^{2,5,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{2,4,7}\lambda - \omega^{3,4,7}\lambda + \omega^{3,5,7}(\lambda\bar{\lambda}+1) - \omega^{4,5,6}\lambda$; $\Delta\omega^{3,6,7} = -\omega^{2,6,7} + \omega^{3,6,7}(\lambda\bar{\lambda}+2) - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} - \omega^{2,4,7} + \omega^{2,5,7} - \omega^{3,5,7}\bar{\lambda} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 4\omega^{4,6,7} - \omega^{5,6,7}\bar{\lambda}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7}\lambda + \omega^{5,6,7}(\lambda\bar{\lambda}+3)$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}(\lambda\bar{\lambda}+3) + \omega^{1,2,3,5}\bar{\lambda}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4}\lambda + 4\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6}\lambda - \omega^{1,3,4,6} - \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}(\lambda\bar{\lambda}+2) + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7}\bar{\lambda} + \omega^{1,2,4,6}(\lambda\bar{\lambda}+1) + \omega^{1,2,5,6}\bar{\lambda} + \omega^{1,3,5,6}\bar{\lambda} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,3,4,7} + \omega^{1,3,5,7}\bar{\lambda} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,6}\lambda + 3\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,5} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,7} + 3\omega^{1,3,4,7} - \omega^{1,3,5,7}\bar{\lambda} - \omega^{2,3,4,6}\lambda$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6}\lambda + \omega^{1,2,5,6} + 2\omega^{1,3,5,6} - \omega^{2,3,4,5}\lambda$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7}\lambda - \omega^{1,3,4,7}\lambda + \omega^{1,3,5,7}\lambda\bar{\lambda} - \omega^{1,4,5,6}\lambda - \omega^{2,3,5,6}\lambda$; $\Delta\omega^{1,3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,6,7}(\lambda\bar{\lambda}+2) - \omega^{1,4,5,7} + \omega^{2,3,4,7}\lambda - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} + \omega^{1,2,5,7} - \omega^{1,3,5,7}\bar{\lambda} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}\lambda$; $\Delta\omega^{1,4,6,7} = 3\omega^{1,4,6,7} - \omega^{1,5,6,7}\bar{\lambda} + \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,4,6,7}\lambda + \omega^{1,5,6,7}(\lambda\bar{\lambda}+1) - \omega^{2,4,5,7}\lambda - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,2,5,6} - \omega^{1,3,5,6}\bar{\lambda} + \omega^{2,3,4,5}(\lambda\bar{\lambda}+3)$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7}\bar{\lambda} + \omega^{2,3,4,6}\lambda\bar{\lambda} + \omega^{2,3,5,6}\bar{\lambda}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{1,3,6,7}\bar{\lambda} + 2\omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7}\bar{\lambda} + \omega^{1,4,5,6} + \omega^{2,3,4,6}\lambda + 3\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{2,4,5,6}\lambda - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,7}\bar{\lambda} - \omega^{2,3,5,7}\bar{\lambda} + \omega^{2,4,5,6}(\lambda\bar{\lambda}+2) + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7}\bar{\lambda} - \omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,6,7} + \omega^{3,5,6,7}\bar{\lambda}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{2,3,4,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} - \omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,4,6,7} - \omega^{2,5,6,7} + 2\omega^{3,4,6,7} - \omega^{3,5,6,7}\bar{\lambda}$; $\Delta\omega^{3,5,6,7} = \omega^{2,4,6,7}\lambda - \omega^{3,4,6,7}\lambda + \omega^{3,5,6,7}(\lambda\bar{\lambda}+1)$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}(\lambda\bar{\lambda}+4)$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}(\lambda\bar{\lambda}+2) + \omega^{1,2,3,5,6}\bar{\lambda}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,6}\lambda + 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}\lambda - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7}\bar{\lambda} + \omega^{1,2,4,5,6}(\lambda\bar{\lambda}+2) + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,6,7} +$

$\omega^{1,3,5,6,7}\bar\lambda - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}\lambda$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{1,3,5,6,7}\bar\lambda$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7}\lambda - \omega^{1,3,4,6,7}\lambda + \omega^{1,3,5,6,7}\lambda\bar\lambda - \omega^{2,3,4,5,7}\lambda$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}\bar\lambda + \omega^{2,3,4,5,6}(\lambda\bar\lambda + 2)$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} - \omega^{1,3,5,6,7}\bar\lambda + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}(\lambda\bar\lambda + 3)$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-2\beta_1}\ \langle\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= 2e^{-4\beta_1}\ \langle\omega^{1,5},\omega^{1,4} - \omega^{2,3}\rangle + 3e^{-5\beta_1}\ \langle\omega^{2,4} - \omega^{3,5}\bar\lambda,\omega^{1,6} - \omega^{3,4} - \omega^{3,5},\omega^{2,5} + \omega^{3,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 3e^{-8\beta_1}\ \langle\omega^{1,2,7}(\lambda\bar\lambda + \lambda + 2) + 2\omega^{1,3,7} - 2\omega^{1,5,6}(\lambda\bar\lambda + 1) - 2\omega^{2,4,5}\lambda + \omega^{3,4,5}(-\lambda\bar\lambda + \lambda - 2),\omega^{1,2,7}(-\bar\lambda - 2) + 2\omega^{1,4,6} + 2\omega^{1,5,6}\bar\lambda + 2\omega^{2,4,5} + \omega^{3,4,5}(\bar\lambda - 2),-\omega^{1,2,7}(\bar\lambda + 1) + 2\omega^{1,5,6}\bar\lambda + 2\omega^{2,3,6} + 2\omega^{2,4,5} + \omega^{3,4,5}(\bar\lambda - 1)\rangle + 3e^{-9\beta_1}\ \langle\omega^{1,5,7} + \omega^{2,3,7}\lambda + \omega^{2,5,6}\lambda + 2\omega^{3,4,6}\lambda,\omega^{2,3,7}\bar\lambda + \omega^{2,4,6} + \omega^{3,4,6}\bar\lambda,\omega^{1,4,7} - 3\omega^{2,3,7} - \omega^{2,5,6} - 4\omega^{3,4,6} - \omega^{3,5,6}\rangle + e^{-7\beta_1}\ \langle\omega^{1,2,6} - \omega^{1,3,6}(\bar\lambda + 1) + \omega^{1,4,5}(-\bar\lambda - 2) + \omega^{2,3,4} + \omega^{2,3,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 3e^{-12\beta_1}\ \langle(-(\lambda\bar\lambda^2\omega^{1,2,6,7} + 3\lambda\bar\lambda^2\omega^{1,4,5,7} + 2\lambda\bar\lambda^2\omega^{2,3,5,7} + \lambda\bar\lambda^2\omega^{3,4,5,6} + 3\lambda\bar\lambda\omega^{1,2,6,7} + \lambda\bar\lambda\omega^{2,3,4,7} + 5\lambda\bar\lambda\omega^{2,4,5,6} - 3\lambda\bar\lambda\omega^{3,4,5,6} + 4\bar\lambda\omega^{1,2,6,7} + 4\bar\lambda\omega^{1,3,6,7} + 8\bar\lambda\omega^{1,4,5,7} + 4\bar\lambda\omega^{2,3,5,7} + 4\bar\lambda\omega^{3,4,5,6} + 8\omega^{1,2,6,7} + 8\omega^{2,3,4,7} + 8\omega^{2,4,5,6} - 8\omega^{3,4,5,6}))/(2(3\lambda\bar\lambda+8)(\lambda\bar\lambda+4)),(-(3\lambda^2\bar\lambda^2\omega^{1,2,6,7} + 6\lambda^2\bar\lambda^2\omega^{2,3,5,7} + 3\lambda^2\bar\lambda^2\omega^{3,4,5,6} + 3\lambda^2\bar\lambda\omega^{1,2,6,7} + 6\lambda^2\bar\lambda\omega^{2,4,5,6} - 3\lambda^2\bar\lambda\omega^{3,4,5,6} + 18\lambda\bar\lambda\omega^{1,2,6,7} + 6\lambda\bar\lambda\omega^{1,3,6,7} - 12\lambda\bar\lambda\omega^{1,4,5,7} + 30\lambda\bar\lambda\omega^{2,3,5,7} + 18\lambda\bar\lambda\omega^{3,4,5,6} + 8\lambda\omega^{1,2,6,7} + 4\lambda\omega^{2,3,4,7} + 12\lambda\omega^{2,4,5,6} - 8\lambda\omega^{3,4,5,6} + 24\omega^{1,2,6,7} + 8\omega^{1,3,6,7} - 32\omega^{1,4,5,7} + 40\omega^{2,3,5,7} + 24\omega^{3,4,5,6}))/(6(3\lambda\bar\lambda + 8)(\lambda\bar\lambda + 4)),(-(3\lambda\bar\lambda\omega^{1,4,5,7} + \lambda\omega^{1,2,6,7} - \lambda\omega^{2,3,4,7} + 3\lambda\omega^{2,4,5,6} - \lambda\omega^{3,4,5,6} + 4\omega^{1,3,6,7} + 8\omega^{1,4,5,7} - 4\omega^{2,3,5,7}))/(6(\lambda\bar\lambda+4))\rangle + 3e^{-11\beta_1}\ \langle(-(3\bar\lambda\omega^{1,2,4,7} + 2\bar\lambda\omega^{1,2,5,7} + 4\bar\lambda\omega^{1,3,4,7} + 2\bar\lambda\omega^{1,4,5,6} - 3\bar\lambda\omega^{2,3,5,6} + 7\omega^{2,3,4,6}))/(6\lambda\bar\lambda+7),(-(\lambda\omega^{1,2,4,7} - \lambda\omega^{1,3,4,7} - \lambda\omega^{1,4,5,6} - \lambda\omega^{2,3,5,6} - 4\omega^{1,3,5,7}))/(\lambda\bar\lambda + 4),(-(3\lambda^2\bar\lambda^2\omega^{1,2,4,7} - 3\lambda^2\bar\lambda^2\omega^{2,3,5,6} + 3\lambda^2\bar\lambda\omega^{2,3,4,6} - 6\lambda\bar\lambda^2\omega^{1,3,5,7} + 8\lambda\bar\lambda\omega^{1,2,4,7} - \lambda\bar\lambda\omega^{1,2,5,7} + 4\lambda\bar\lambda\omega^{1,3,4,7} + 5\lambda\bar\lambda\omega^{1,4,5,6} - 8\lambda\bar\lambda\omega^{2,3,5,6} + 12\lambda\omega^{2,3,4,6} - 7\bar\lambda\omega^{1,3,5,7} + \omega^{1,2,4,7} - 4\omega^{1,2,5,7} - \omega^{1,3,4,7} + 3\omega^{1,4,5,6} - \omega^{2,3,5,6}))/((6\lambda\bar\lambda + 7)(\lambda\bar\lambda + 4))\rangle + e^{-13\beta_1}\ \langle(-(\lambda\omega^{2,3,6,7} + \lambda\omega^{2,4,5,7} - \omega^{1,4,6,7} + \omega^{1,5,6,7} + 2\omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,7}))/(2\lambda\bar\lambda + 3\lambda + 3\bar\lambda + 8)\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-16\beta_1}\ \langle-\omega^{2,3,4,6,7},(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + 3e^{-15\beta_1}\ \langle(\omega^{1,2,4,6,7}\lambda - \omega^{1,3,4,6,7}\lambda - 3\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}\lambda)/(\lambda\bar\lambda + 3),(3\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7}(-\lambda\bar\lambda - 3) + \omega^{1,3,4,6,7}\lambda\bar\lambda + 3\omega^{1,3,5,6,7}\bar\lambda + \omega^{2,3,4,5,7}(2\lambda\bar\lambda + 3))/(3(\lambda\bar\lambda + 3)),(3\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7}(\lambda\bar\lambda + 3) + \omega^{1,3,4,6,7}(2\lambda\bar\lambda + 3) + 3\omega^{1,3,5,6,7}\bar\lambda + \omega^{2,3,4,5,7}\lambda\bar\lambda)/(3(\lambda\bar\lambda + 3))\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + 2e^{-18\beta_1}\ \langle-\omega^{1,3,4,5,6,7},\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.
$P(Y)\ =\ (1+Y^2)^2(1+Y)^3$.
The preceding results are valid for:

$$(-(\bar\lambda-1))(\lambda-1)(\lambda+1)(-(\lambda\bar\lambda+2\bar\lambda+2))(-(\lambda+1))(\bar\lambda-2)\neq 0.$$

Comments: the only singular value is $\lambda=1$.
Case $\lambda=1$: Add in $\mathrm{ch}(H^3_{\mathbb{C}})\ \ e^{-10\beta_1}\ \langle\omega^{1,6,7}-\omega^{2,4,7}-\omega^{3,4,7}-\omega^{3,5,7}-\omega^{4,5,6}\rangle$ and in $\mathrm{ch}(H^4_{\mathbb{C}})\ \ e^{-10\beta_1}\ \langle(\omega^{1,2,3,7}+\omega^{1,2,4,6}-\omega^{1,2,5,6}+\omega^{1,3,5,6}+\omega^{2,3,4,5})/5\rangle$.
Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,\underline{20},23,17,7,1)$
Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,\underline{8},\underline{8},5,3,1)$
Total dimension of trivial cohomology: 34.
Length associated polynomial: 32.
Riemann Hyp.: no
$P(Y)\ =\ (1+2Y+3Y^2+5Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

5.3.12 Algebra $\mathcal{G}_{7,1.3(ii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_7$; $[x_2,x_5]=x_7/2$; $[x_3,x_4]=(-x_7)/2$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & (-3\xi_1^2)/2 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & a & \xi_2^3 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^4 & (-\xi_1^2)/2 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & b & (\xi_1^4+2\xi_3^6)/2 & c & 5\xi_1^1 \end{pmatrix}$$

with $a=5\xi_1^2-3\xi_1^3+\xi_2^4$, $b=(-2\xi_1^4-\xi_1^5+2\xi_2^6)/2$, $c=(2\xi_1^2-3\xi_1^3+2\xi_2^4)/2$.

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=(-2\delta_1^6-2\delta_2^7+\delta_3^7)/2, ad\ x_5=(-\delta_2^7)/2, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+2e^{2\beta_1}\ \langle x_2,x_3\rangle+2e^{3\beta_1}\ \langle x_4,x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{\beta_1}\ \langle\delta_2^5,\delta_1^2,\delta_1^3,\delta_2^4\rangle+2e^0\ \langle\delta_2^3,t_1\rangle+3e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7,\delta_3^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+4e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6,\delta_1^4,\delta_3^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^4]=-1/2\delta_2^7$; $[\delta_2^5,\delta_3^6]=-\delta_2^7$; $[\delta_2^5,\delta_1^2]=1/2\delta_2^6+\delta_1^5$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_2^3,\delta_1^4]=\delta_1^5$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_1^2]=-5\delta_2^5+3/2\delta_2^4+\delta_1^3$; $[\delta_2^3,\delta_1^3]=3\delta_2^5$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_1^6,\delta_1^3]=3/2\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^5,\delta_1^4]=-\delta_1^7$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_1^5,\delta_1^2]=3/4\delta_3^7+1/2\delta_1^6$; $[\delta_1^5,\delta_2^4]=-1/2\delta_2^7$; $[\delta_2^6,\delta_1^4]=\delta_1^7$; $[\delta_2^6,\delta_1^2]=-3/2\delta_3^7-\delta_2^7+\delta_1^6$; $[\delta_2^6,\delta_1^3]=3/2\delta_2^7$; $[\delta_1^4,\delta_1^2]=4\delta_3^7$; $[\delta_1^4,\delta_1^3]=-3/2\delta_3^7+\delta_1^6$; $[\delta_1^4,\delta_2^4]=1/2\delta_3^7-\delta_2^7-\delta_1^6$; $[\delta_3^6,\delta_1^2]=4\delta_3^7$; $[\delta_3^6,\delta_1^3]=-3/2\delta_3^7+\delta_1^6$; $[\delta_1^2,\delta_1^3]=5\delta_1^5-3/2\delta_1^4$; $[\delta_1^2,\delta_2^4]=\delta_3^6-\delta_1^4$; $[\delta_1^3,\delta_2^4]=-(\delta_2^6+\delta_1^5)$.

ch($H^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle\delta_2^5,\delta_1^2,\delta_1^3\rangle+2e^0\ \langle\delta_2^3,t_1\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_1^5,\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=1/2\delta_1^5$; $[\delta_2^3,\delta_1^4]=\delta_1^5$; $[\delta_2^3,\delta_1^2]=-5\delta_2^5+\delta_1^3$; $[\delta_2^3,\delta_1^3]=3\delta_2^5$; $[\delta_1^5,\delta_1^2]=2\delta_1^6$; $[\delta_1^4,\delta_1^2]=8\delta_1^6$; $[\delta_1^4,\delta_1^3]=-2\delta_1^6$; $[\delta_1^2,\delta_1^3]=5\delta_1^5-3/2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $4e^{-3\beta_1}\ \langle x_3\omega^{2,4}+x_5\omega^{2,6}, x_1\omega^{2,3}+x_3(\omega^{2,5}+\omega^{3,4})+2x_5(\omega^{3,6}+1/2\omega^{4,5}), -2/3x_1(\omega^{1,4}-3/2\omega^{1,5}+8\omega^{2,3})+14x_2(\omega^{2,4}+3/14\omega^{2,5}+3/14\omega^{3,4})+4/3x_3(\omega^{1,6}-18\omega^{2,4}-9\omega^{2,5}-10\omega^{3,4}-3/4\omega^{3,5})-12x_4(\omega^{1,7}-1/9\omega^{2,6}-$

$7/12\omega^{3,6}+1/6\omega^{4,5})+24x_5(\omega^{1,7}-13/12\omega^{3,6})+6x_6(\omega^{3,7}+1/6\omega^{5,6}), -1/2x_1\omega^{2,3}+x_2\omega^{2,4}-2x_3(\omega^{2,4}+1/2\omega^{2,5}+1/2\omega^{3,4})+x_4\omega^{2,6}+2x_5(\omega^{1,7}-\omega^{3,6})+x_6\omega^{2,7}\rangle + 5e^{-\beta_1}\ \langle -2x_2(\omega^{1,2}-1/2\omega^{1,3})-4x_3(\omega^{1,2}-1/2\omega^{1,3})+x_4\omega^{1,5}, -x_2\omega^{1,2}-4x_3(\omega^{1,2}-1/4\omega^{1,3})-x_4\omega^{1,4}+x_5\omega^{1,5}, x_5\omega^{2,3}, -2x_3(\omega^{1,2}-1/2\omega^{1,3})-x_4(\omega^{1,4}-\omega^{2,3}), -6x_3(\omega^{1,2}-1/6\omega^{1,3})-3x_4\omega^{1,4}+x_6\omega^{3,4}\rangle + 2e^{0}\ \langle -1/2x_4\omega^{1,3}+x_5\omega^{1,3}, x_4(\omega^{1,2}-1/2\omega^{1,3})\rangle + 9e^{-2\beta_1}\ \langle 2x_1(\omega^{1,2}-1/2\omega^{1,3})-2x_2(\omega^{1,4}+1/2\omega^{1,5}-\omega^{2,3})-2x_3(\omega^{1,4}-\omega^{2,3})-2x_4(\omega^{1,6}-1/2\omega^{3,4}-1/2\omega^{3,5}), x_5\omega^{2,4}, -x_3(\omega^{1,4}-\omega^{2,3})-x_4\omega^{2,4}+x_5\omega^{2,5}, -1/2x_1\omega^{1,2}+2x_3(\omega^{1,4}-3/4\omega^{2,3})+3/2x_4\omega^{2,4}+x_5\omega^{1,6}, -x_1(\omega^{1,2}-1/2\omega^{1,3})+x_2(\omega^{1,4}+3/4\omega^{1,5}-3/2\omega^{2,3})+1/2x_3(\omega^{1,5}-2\omega^{2,3})+x_4(\omega^{1,6}-2\omega^{2,4}-3/4\omega^{3,5})+x_6\omega^{1,7}, -x_3\omega^{2,3}+x_5\omega^{3,4}, -x_1(\omega^{1,2}-1/2\omega^{1,3})+1/2x_2(\omega^{1,4}+3/2\omega^{1,5}-\omega^{2,3})+x_3(\omega^{1,4}+\omega^{1,5}-\omega^{2,3})+x_4(\omega^{1,6}+\omega^{2,5}-3/4\omega^{3,5}), x_1(\omega^{1,2}-1/2\omega^{1,3})-1/2x_2(\omega^{1,4}+1/2\omega^{1,5}-3\omega^{2,3})+x_3(\omega^{1,4}+\omega^{2,3})-x_4(\omega^{1,6}-2\omega^{2,4}-1/4\omega^{3,5})+x_5\omega^{3,5}, 4x_1(\omega^{1,2}-1/2\omega^{1,3})-3x_2(\omega^{1,4}+\omega^{1,5}-2\omega^{2,3})+4x_3(\omega^{1,4}-1/4\omega^{1,5}+\omega^{2,3})-4x_4(\omega^{1,6}-2\omega^{2,4}-3/4\omega^{3,5})+x_6\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=5e^{-4\beta_1}+11e^{-5\beta_1}+11e^{-6\beta_1}+4e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-6\beta_1}+e^{-7\beta_1}+7e^{-8\beta_1}+13e^{-9\beta_1}+8e^{-10\beta_1}+3e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-11\beta_1}+7e^{-12\beta_1}+8e^{-13\beta_1}+5e^{-14\beta_1}+e^{-15\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-14\beta_1}+3e^{-15\beta_1}+5e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=2e^{-18\beta_1}+e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,14,55,123,155,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,8,20,31,33,24,12,3)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,19,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,8,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}-\omega^{2,3}$; $d\omega^7=(-2\omega^{1,6}-2\omega^{2,4}-\omega^{2,5}+\omega^{3,4})/2$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=(2\omega^{1,2,4}+\omega^{1,2,5}-\omega^{1,3,4})/2$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=(-2\omega^{1,2,6}-\omega^{2,3,4})/2$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=(-2\omega^{1,3,6}-2\omega^{2,3,4}-\omega^{2,3,5})/2$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{4,7}=(-2\omega^{1,2,7}-2\omega^{1,4,6}-\omega^{2,4,5})/2$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}+\omega^{2,3,5}$; $d\omega^{5,7}=(-2\omega^{1,3,7}-2\omega^{1,5,6}+2\omega^{2,4,5}-\omega^{3,4,5})/2$; $d\omega^{6,7}=(-2\omega^{1,4,7}-2\omega^{2,3,7}+2\omega^{2,4,6}+\omega^{2,5,6}-\omega^{3,4,6})/2$; $d\omega^{1,2,7}=\omega^{1,2,3,4}/2$; $d\omega^{1,3,7}=(2\omega^{1,2,3,4}+\omega^{1,2,3,5})/2$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}/2$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=(-2\omega^{1,2,4,5}+\omega^{1,3,4,5})/2$; $d\omega^{1,6,7}=(2\omega^{1,2,3,7}-2\omega^{1,2,4,6}-\omega^{1,2,5,6}+\omega^{1,3,4,6})/2$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$;

$d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = (-2\omega^{1,2,3,7} - 2\omega^{1,2,5,6} + \omega^{2,3,4,5})/2$; $d\omega^{2,6,7} = (-2\omega^{1,2,4,7} + \omega^{2,3,4,6})/2$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = (2\omega^{1,2,3,7} - 2\omega^{1,3,4,6} - \omega^{2,3,4,5})/2$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = (-2\omega^{1,3,4,7} + 2\omega^{2,3,4,6} + \omega^{2,3,5,6})/2$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = (-2\omega^{1,2,6,7} + 2\omega^{2,3,4,7} + \omega^{2,4,5,6})/2$; $d\omega^{5,6,7} = (-2\omega^{1,3,6,7} + 2\omega^{1,4,5,7} + 2\omega^{2,3,5,7} - 2\omega^{2,4,5,6} + \omega^{3,4,5,6})/2$; $d\omega^{1,2,5,7} = (-\omega^{1,2,3,4,5})/2$; $d\omega^{1,2,6,7} = (-\omega^{1,2,3,4,6})/2$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}/2$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = (-2\omega^{1,2,3,4,6} - \omega^{1,2,3,5,6})/2$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = (-2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6})/2$; $d\omega^{1,5,6,7} = (-2\omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6} - \omega^{1,3,4,5,6})/2$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = (-2\omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = (2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6})/2$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}/2$; $d\omega^{1,3,4,6,7} = (-\omega^{1,2,3,4,5,6})/2$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + 2e^{-2\beta_1} \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-4\beta_1} \langle\omega^{1,5}, \omega^{1,4}\rangle + 3e^{-5\beta_1} \langle\omega^{2,4}, \omega^{1,6} - \omega^{3,4}, \omega^{3,5}\rangle + e^{-6\beta_1} \langle(2\omega^{1,7} + 2\omega^{2,6} + \omega^{4,5})/2\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-7\beta_1} \langle\omega^{1,3,6}\rangle + 3e^{-8\beta_1} \langle(2\omega^{1,2,7} - \omega^{2,4,5})/2, \omega^{2,3,6} + \omega^{2,4,5}, (2\omega^{1,3,7} - 2\omega^{2,4,5} - \omega^{3,4,5})/2\rangle + 3e^{-9\beta_1} \langle(2\omega^{1,4,7} - \omega^{2,5,6} - \omega^{3,4,6})/2, \omega^{2,3,7} + \omega^{3,4,6}, (2\omega^{1,5,7} + 2\omega^{2,5,6} + 2\omega^{3,4,6} - \omega^{3,5,6})/2\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-11\beta_1} \langle(2\omega^{1,2,5,7} + \omega^{2,3,5,6})/2, \omega^{1,3,5,7} + \omega^{2,3,5,6}, \omega^{1,2,4,7}\rangle + 3e^{-12\beta_1} \langle(2\omega^{1,3,6,7} + 2\omega^{2,4,5,6} + \omega^{3,4,5,6})/2, \omega^{1,4,5,7}, (2\omega^{1,2,6,7} + \omega^{2,4,5,6})/2\rangle + e^{-13\beta_1} \langle(2\omega^{1,4,6,7} - 3\omega^{2,3,6,7} - \omega^{2,4,5,7})/2\rangle + e^{-14\beta_1} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^{-15\beta_1} \langle\omega^{1,2,4,6,7}, (2\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7})/2, \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}\rangle + e^{-14\beta_1} \langle\omega^{1,2,3,6,7}\rangle + 2e^{-16\beta_1} \langle\omega^{2,3,4,6,7}, \omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^{-18\beta_1} \langle\omega^{1,2,4,5,6,7}, \omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.
Length associated polynomial: 32.
Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{2,5} = (-\omega^7)/2$; $d^\star\omega^{3,4} = \omega^7/2$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7} - \omega^{2,6}$; $d^\star\omega^{1,2,5} = (\omega^{1,7} - 2\omega^{4,5})/2$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = (-\omega^{1,7} - 2\omega^{3,6} + 2\omega^{4,5})/2$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = (-\omega^{2,7} - 2\omega^{3,7} + 2\omega^{4,6})/2$; $d^\star\omega^{2,3,5} = (-\omega^{3,7} + 2\omega^{5,6})/2$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = (-\omega^{4,7} + 2\omega^{5,7})/2$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}/2$; $d^\star\omega^{3,4,5} = (-\omega^{5,7})/2$; $d^\star\omega^{3,4,6} = (-\omega^{6,7})/2$; $d^\star\omega^{1,2,3,4} = (\omega^{1,2,7} + 2\omega^{1,3,7} - 2\omega^{1,4,6} - 2\omega^{2,3,6} + 2\omega^{2,4,5})/2$; $d^\star\omega^{1,2,3,5} = (\omega^{1,3,7} - 2\omega^{1,5,6} + 2\omega^{3,4,5})/2$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = (\omega^{1,4,7} - 2\omega^{1,5,7} + 2\omega^{2,5,6})/2$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = (-\omega^{1,6,7} - 2\omega^{2,5,7} - 2\omega^{4,5,6})/2$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = (\omega^{1,5,7} + 2\omega^{3,5,6})/2$; $d^\star\omega^{1,3,4,6} = (\omega^{1,6,7} - 2\omega^{3,4,7} + 2\omega^{4,5,6})/2$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = (\omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{3,5,7} - 2\omega^{4,5,6})/2$; $d^\star\omega^{2,3,4,6} = (\omega^{2,6,7} + 2\omega^{3,6,7})/2$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}/2$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = (\omega^{4,6,7} - 2\omega^{5,6,7})/2$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}/2$; $d^\star\omega^{1,2,3,4,5} = (-\omega^{1,2,5,7} + \omega^{1,3,4,7} - 2\omega^{1,3,5,7} + 2\omega^{1,4,5,6} + 2\omega^{2,3,5,6})/2$; $d^\star\omega^{1,2,3,4,6} = (-\omega^{1,2,6,7} - 2\omega^{1,3,6,7} - 2\omega^{2,3,4,7} + 2\omega^{2,4,5,6})/2$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = (-\omega^{1,3,6,7} - 2\omega^{2,3,5,7} + 2\omega^{3,4,5,6})/2$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = (-\omega^{1,4,6,7} + 2\omega^{1,5,6,7} - 2\omega^{2,4,5,7})/2$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = (-\omega^{1,5,6,7} - 2\omega^{3,4,5,7})/2$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = (-\omega^{2,5,6,7} + \omega^{3,4,6,7} - 2\omega^{3,5,6,7})/2$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = (\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{1,3,5,6,7} - 2\omega^{2,3,4,5,7})/2$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = (5\omega^7)/2$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = (4\omega^{1,6} + 2\omega^{2,4} - \omega^{2,5} + \omega^{3,4})/2$; $\Delta\omega^{1,7} = (3\omega^{1,7} - 2\omega^{2,6} + \omega^{3,6} - 2\omega^{4,5})/2$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = (2\omega^{1,6} + 2\omega^{2,4} + \omega^{2,5} - \omega^{3,4})/2$; $\Delta\omega^{2,5} = (-2\omega^{1,6} + 2\omega^{2,4} + 5\omega^{2,5} - 5\omega^{3,4})/4$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = (5\omega^{2,7} + 2\omega^{3,7} + 2\omega^{4,6})/4$; $\Delta\omega^{3,4} = (2\omega^{1,6} - 2\omega^{2,4} - 5\omega^{2,5} + 5\omega^{3,4})/4$; $\Delta\omega^{3,6} = (\omega^{1,7} + 2\omega^{3,6} - 2\omega^{4,5})/2$; $\Delta\omega^{3,7} = (2\omega^{2,7} + 9\omega^{3,7} - 4\omega^{4,6} + 2\omega^{5,6})/4$; $\Delta\omega^{4,5} = -\omega^{1,7} - \omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} =$

$(\omega^{2,7} - 2\omega^{3,7} + 4\omega^{4,6})/2$; $\Delta\omega^{4,7} = (9\omega^{4,7} - 2\omega^{5,7})/4$; $\Delta\omega^{5,6} = (\omega^{3,7} + 6\omega^{5,6})/2$; $\Delta\omega^{5,7} = (-2\omega^{4,7} + 13\omega^{5,7})/4$; $\Delta\omega^{6,7} = (7\omega^{6,7})/2$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = (4\omega^{1,2,4} + \omega^{1,2,5} - \omega^{1,3,4})/2$; $\Delta\omega^{1,2,5} = (2\omega^{1,2,4} + 5\omega^{1,2,5} - 5\omega^{1,3,4})/4$; $\Delta\omega^{1,2,6} = (4\omega^{1,2,6} - \omega^{2,3,4})/2$; $\Delta\omega^{1,2,7} = (5\omega^{1,2,7} + 2\omega^{1,3,7} + 2\omega^{1,4,6} - 2\omega^{2,3,6} + 4\omega^{2,4,5})/4$; $\Delta\omega^{1,3,4} = (-2\omega^{1,2,4} - 5\omega^{1,2,5} + 9\omega^{1,3,4})/4$; $\Delta\omega^{1,3,6} = (4\omega^{1,3,6} - 2\omega^{1,4,5} + 2\omega^{2,3,4} - \omega^{2,3,5})/2$; $\Delta\omega^{1,3,7} = (2\omega^{1,2,7} + 9\omega^{1,3,7} - 4\omega^{1,4,6} + 2\omega^{1,5,6} - 4\omega^{2,3,6} + 4\omega^{3,4,5})/4$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = (\omega^{1,2,7} - 2\omega^{1,3,7} + 4\omega^{1,4,6} + 2\omega^{2,3,6} - \omega^{2,4,5})/2$; $\Delta\omega^{1,4,7} = (5\omega^{1,4,7} - 2\omega^{1,5,7} + 4\omega^{2,3,7} - 4\omega^{2,4,6} + 2\omega^{3,4,6})/4$; $\Delta\omega^{1,5,6} = (\omega^{1,3,7} + 4\omega^{1,5,6} - 2\omega^{2,4,5} - \omega^{3,4,5})/2$; $\Delta\omega^{1,5,7} = (-2\omega^{1,4,7} + 5\omega^{1,5,7} - 4\omega^{2,5,6} + 2\omega^{3,5,6})/4$; $\Delta\omega^{1,6,7} = (5\omega^{1,6,7} + 2\omega^{2,4,7} - \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{4,5,6})/2$; $\Delta\omega^{2,3,4} = (-2\omega^{1,2,6} + 4\omega^{1,3,6} + 9\omega^{2,3,4} + 2\omega^{2,3,5})/4$; $\Delta\omega^{2,3,5} = (-2\omega^{1,3,6} + 4\omega^{1,4,5} + 2\omega^{2,3,4} + 5\omega^{2,3,5})/4$; $\Delta\omega^{2,3,6} = (-\omega^{1,2,7} - 2\omega^{1,3,7} + 2\omega^{1,4,6} + 2\omega^{2,3,6} - 2\omega^{2,4,5})/2$; $\Delta\omega^{2,3,7} = (2\omega^{1,4,7} + 4\omega^{2,3,7} - 2\omega^{2,4,6} + \omega^{2,5,6} - \omega^{3,4,6})/2$; $\Delta\omega^{2,4,5} = (4\omega^{1,2,7} - 2\omega^{1,4,6} - 4\omega^{1,5,6} - 4\omega^{2,3,6} + 9\omega^{2,4,5} - 2\omega^{3,4,5})/4$; $\Delta\omega^{2,4,6} = (-2\omega^{1,4,7} - 2\omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{2,5,6} - \omega^{3,4,6})/2$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = (-4\omega^{1,5,7} + 2\omega^{2,3,7} + 2\omega^{2,4,6} + 9\omega^{2,5,6} - 5\omega^{3,4,6})/4$; $\Delta\omega^{2,5,7} = (-2\omega^{1,6,7} + 9\omega^{2,5,7} - 5\omega^{3,4,7} + 2\omega^{3,5,7} + 2\omega^{4,5,6})/4$; $\Delta\omega^{2,6,7} = (5\omega^{2,6,7} + 2\omega^{3,6,7})/4$; $\Delta\omega^{3,4,5} = (4\omega^{1,3,7} - 2\omega^{1,5,6} - 2\omega^{2,4,5} + 5\omega^{3,4,5})/4$; $\Delta\omega^{3,4,6} = (2\omega^{1,4,7} - 2\omega^{2,3,7} - 2\omega^{2,4,6} - 5\omega^{2,5,6} + 5\omega^{3,4,6})/4$; $\Delta\omega^{3,4,7} = (2\omega^{1,6,7} - 5\omega^{2,5,7} + 9\omega^{3,4,7} - 2\omega^{3,5,7} - 2\omega^{4,5,6})/4$; $\Delta\omega^{3,5,6} = (\omega^{1,5,7} + 2\omega^{3,5,6})/2$; $\Delta\omega^{3,5,7} = (\omega^{2,5,7} - \omega^{3,4,7} + 4\omega^{3,5,7} - 2\omega^{4,5,6})/2$; $\Delta\omega^{3,6,7} = (2\omega^{2,6,7} + 9\omega^{3,6,7} - 4\omega^{4,5,7})/4$; $\Delta\omega^{4,5,6} = (2\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7} - 2\omega^{3,5,7} + 6\omega^{4,5,6})/2$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = (9\omega^{4,6,7} - 2\omega^{5,6,7})/4$; $\Delta\omega^{5,6,7} = (-2\omega^{4,6,7} + 17\omega^{5,6,7})/4$; $\Delta\omega^{1,2,3,4} = (17\omega^{1,2,3,4} + 2\omega^{1,2,3,5})/4$; $\Delta\omega^{1,2,3,5} = (2\omega^{1,2,3,4} + 9\omega^{1,2,3,5})/4$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = (6\omega^{1,2,3,7} - 2\omega^{1,2,4,6} + \omega^{1,2,5,6} - \omega^{1,3,4,6} - 2\omega^{2,3,4,5})/2$; $\Delta\omega^{1,2,4,5} = (-4\omega^{1,2,3,6} + 9\omega^{1,2,4,5} - 2\omega^{1,3,4,5})/4$; $\Delta\omega^{1,2,4,6} = (-2\omega^{1,2,3,7} + 4\omega^{1,2,4,6} + \omega^{1,2,5,6} - \omega^{1,3,4,6})/2$; $\Delta\omega^{1,2,4,7} = (2\omega^{1,2,4,7} - \omega^{2,3,4,6})/2$; $\Delta\omega^{1,2,5,6} = (2\omega^{1,2,3,7} + 2\omega^{1,2,4,6} + 9\omega^{1,2,5,6} - 5\omega^{1,3,4,6} + 2\omega^{2,3,4,5})/4$; $\Delta\omega^{1,2,5,7} = (5\omega^{1,2,5,7} - 5\omega^{1,3,4,7} + 2\omega^{1,3,5,7} + 2\omega^{1,4,5,6} - 2\omega^{2,3,5,6})/4$; $\Delta\omega^{1,2,6,7} = (5\omega^{1,2,6,7} + 2\omega^{1,3,6,7} - 2\omega^{2,3,4,7} - 4\omega^{2,4,5,6})/4$; $\Delta\omega^{1,3,4,5} = (-2\omega^{1,2,4,5} + 5\omega^{1,3,4,5})/4$; $\Delta\omega^{1,3,4,6} = (-2\omega^{1,2,3,7} - 2\omega^{1,2,4,6} - 5\omega^{1,2,5,6} + 9\omega^{1,3,4,6} - 2\omega^{2,3,4,5})/4$; $\Delta\omega^{1,3,4,7} = (-5\omega^{1,2,5,7} + 9\omega^{1,3,4,7} - 2\omega^{1,3,5,7} - 2\omega^{1,4,5,6} - 4\omega^{2,3,4,6})/4$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = (\omega^{1,2,5,7} - \omega^{1,3,4,7} + 2\omega^{1,3,5,7} - 2\omega^{1,4,5,6} - 2\omega^{2,3,5,6})/2$; $\Delta\omega^{1,3,6,7} = (2\omega^{1,2,6,7} + 9\omega^{1,3,6,7} - 4\omega^{1,4,5,7} + 4\omega^{2,3,4,7} - 2\omega^{2,3,5,7} - 4\omega^{3,4,5,6})/4$; $\Delta\omega^{1,4,5,6} = (\omega^{1,2,5,7} - \omega^{1,3,4,7} - 2\omega^{1,3,5,7} + 4\omega^{1,4,5,6} + 2\omega^{2,3,5,6})/2$; $\Delta\omega^{1,4,5,7} = (-2\omega^{1,3,6,7} + 2\omega^{1,4,5,7} + 2\omega^{2,3,5,7} - 2\omega^{2,4,5,6} + \omega^{3,4,5,6})/2$; $\Delta\omega^{1,4,6,7} = (5\omega^{1,4,6,7} - 2\omega^{1,5,6,7} + 4\omega^{2,3,6,7} - 2\omega^{2,4,5,7})/4$; $\Delta\omega^{1,5,6,7} = (-2\omega^{1,4,6,7} + 9\omega^{1,5,6,7} - 4\omega^{2,4,5,7} - 2\omega^{3,4,5,7})/4$; $\Delta\omega^{2,3,4,5} = (-2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6} - 2\omega^{1,3,5,6} + 5\omega^{2,3,4,5})/2$; $\Delta\omega^{2,3,4,6} = (-2\omega^{1,2,4,7} - 4\omega^{1,3,4,7} + 5\omega^{2,3,4,6} + 2\omega^{2,3,5,6})/4$;

$\Delta\omega^{2,3,4,7} = (-\omega^{1,2,6,7} + 2\omega^{1,3,6,7} + 4\omega^{2,3,4,7} - \omega^{2,4,5,6})/2$; $\Delta\omega^{2,3,5,6} = (-2\omega^{1,2,5,7} - 4\omega^{1,3,5,7} + 4\omega^{1,4,5,6} + 2\omega^{2,3,4,6} + 5\omega^{2,3,5,6})/4$; $\Delta\omega^{2,3,5,7} = (-\omega^{1,3,6,7} + 2\omega^{1,4,5,7} + 4\omega^{2,3,5,7} - 2\omega^{2,4,5,6} - \omega^{3,4,5,6})/2$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = (-4\omega^{1,2,6,7} - 4\omega^{1,4,5,7} - 2\omega^{2,3,4,7} - 4\omega^{2,3,5,7} + 9\omega^{2,4,5,6} - 2\omega^{3,4,5,6})/4$; $\Delta\omega^{2,4,5,7} = (-\omega^{1,4,6,7} - 2\omega^{1,5,6,7} - 2\omega^{2,3,6,7} + 4\omega^{2,4,5,7})/2$; $\Delta\omega^{2,5,6,7} = (9\omega^{2,5,6,7} - 5\omega^{3,4,6,7} + 2\omega^{3,5,6,7})/4$; $\Delta\omega^{3,4,5,6} = (-4\omega^{1,3,6,7} + 2\omega^{1,4,5,7} - 2\omega^{2,3,5,7} - 2\omega^{2,4,5,6} + 5\omega^{3,4,5,6})/4$; $\Delta\omega^{3,4,5,7} = (-\omega^{1,5,6,7} + 4\omega^{3,4,5,7})/2$; $\Delta\omega^{3,4,6,7} = (-5\omega^{2,5,6,7} + 5\omega^{3,4,6,7} - 2\omega^{3,5,6,7})/4$; $\Delta\omega^{3,5,6,7} = (\omega^{2,5,6,7} - \omega^{3,4,6,7} + 4\omega^{3,5,6,7})/2$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = (7\omega^{1,2,3,4,5})/2$; $\Delta\omega^{1,2,3,4,6} = (13\omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,6})/4$; $\Delta\omega^{1,2,3,4,7} = (6\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6})/2$; $\Delta\omega^{1,2,3,5,6} = (2\omega^{1,2,3,4,6} + 9\omega^{1,2,3,5,6})/4$; $\Delta\omega^{1,2,3,5,7} = (4\omega^{1,2,3,5,7} - 2\omega^{1,2,4,5,6} - \omega^{1,3,4,5,6})/2$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = (-2\omega^{1,2,3,4,7} - 4\omega^{1,2,3,5,7} + 9\omega^{1,2,4,5,6} - 2\omega^{1,3,4,5,6})/4$; $\Delta\omega^{1,2,4,5,7} = (-2\omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2$; $\Delta\omega^{1,2,5,6,7} = (5\omega^{1,2,5,6,7} - 5\omega^{1,3,4,6,7} + 2\omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,7})/4$; $\Delta\omega^{1,3,4,5,6} = (-2\omega^{1,2,3,5,7} - 2\omega^{1,2,4,5,6} + 5\omega^{1,3,4,5,6})/4$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = (-5\omega^{1,2,5,6,7} + 5\omega^{1,3,4,6,7} - 2\omega^{1,3,5,6,7} - 2\omega^{2,3,4,5,7})/4$; $\Delta\omega^{1,3,5,6,7} = (\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{1,3,5,6,7} - 2\omega^{2,3,4,5,7})/2$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = (2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} - 2\omega^{1,3,4,5,7} + 3\omega^{2,3,4,5,6})/2$; $\Delta\omega^{2,3,4,5,7} = (\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - 2\omega^{1,3,5,6,7} + 4\omega^{2,3,4,5,7})/2$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = (5\omega^{1,2,3,4,5,6})/2$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + 2e^{-2\beta_1} \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-4\beta_1} \langle\omega^{1,5}, \omega^{1,4} - \omega^{2,3}\rangle + 3e^{-5\beta_1} \langle 2\omega^{1,6} - 3\omega^{2,4} - 2\omega^{3,4}, \omega^{2,5} + \omega^{3,4}, \omega^{3,5}\rangle + e^{-6\beta_1} \langle 2\omega^{1,7} + 2\omega^{2,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-7\beta_1} \langle\omega^{1,3,6} + 3\omega^{1,4,5} - 2\omega^{2,3,5}\rangle + 3e^{-8\beta_1} \langle 6\omega^{1,2,7} - 8\omega^{1,5,6} - 9\omega^{2,3,6} - 12\omega^{2,4,5} - 8\omega^{3,4,5}, 2\omega^{1,3,7} - \omega^{1,5,6} + 2\omega^{2,3,6} - 2\omega^{3,4,5}, 3\omega^{1,4,6} - 4\omega^{1,5,6} - 9\omega^{2,3,6} - 6\omega^{2,4,5} - 4\omega^{3,4,5}\rangle + 3e^{-9\beta_1} \langle 2\omega^{1,4,7} + 2\omega^{2,4,6} - \omega^{2,5,6} - \omega^{3,4,6}, 2\omega^{2,3,7} + 3\omega^{2,4,6} + 2\omega^{3,4,6}, 2\omega^{1,5,7} + 2\omega^{2,5,6} + 2\omega^{3,4,6} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-14\beta_1} \langle -\omega^{2,4,6,7}\rangle + e^{-13\beta_1} \langle(-2\omega^{1,4,6,7} + 3\omega^{2,3,6,7} + \omega^{2,4,5,7})/14\rangle + 3e^{-12\beta_1} \langle(-72\omega^{1,2,6,7} + 2358\omega^{1,3,6,7} - 1017\omega^{1,4,5,7} - 822\omega^{2,3,4,7} + 2857\omega^{2,3,5,7} + 1500\omega^{2,4,5,6} + 4036\omega^{3,4,5,6})/50511, (-2810\omega^{1,2,6,7} - 576\omega^{1,3,6,7} - 1808\omega^{1,4,5,7} - 1213\omega^{2,3,4,7} - 1212\omega^{2,3,5,7} - 3194\omega^{2,4,5,6} - 1500\omega^{3,4,5,6})/16837, (-1500\omega^{1,2,6,7} - 1386\omega^{1,3,6,7} + 4068\omega^{1,4,5,7} - 288\omega^{2,3,4,7} - 5021\omega^{2,3,5,7} - 2424\omega^{2,4,5,6} - 5714\omega^{3,4,5,6})/50511\rangle + 3e^{-11\beta_1} \langle(-84\omega^{1,2,4,7} + 173\omega^{1,2,5,7} + 37\omega^{1,3,4,7} + 206\omega^{1,3,5,7} - 136\omega^{1,4,5,6} - 168\omega^{2,3,4,6} + 410\omega^{2,3,5,6})/1442, (28\omega^{1,2,4,7} - 92\omega^{1,2,5,7} + 22\omega^{1,3,4,7} + 103\omega^{1,3,5,7} + 114\omega^{1,4,5,6} + 56\omega^{2,3,4,6} - 68\omega^{2,3,5,6})/721, (-11\omega^{1,2,4,7} - 8\omega^{1,2,5,7} - 16\omega^{1,3,4,7} - 8\omega^{1,4,5,6} - 22\omega^{2,3,4,6} + 12\omega^{2,3,5,6})/103\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= 2e^{-16\beta_1} \langle -\omega^{2,3,4,6,7}, (-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + 3e^{-15\beta_1} \langle (-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 3\omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,7})/15, (13\omega^{1,2,5,6,7} + 17\omega^{1,3,4,6,7} + 6\omega^{1,3,5,6,7} + 4\omega^{2,3,4,5,7})/30, -\omega^{1,2,4,6,7}\rangle + e^{-14\beta_1} \langle (\omega^{1,2,3,6,7} - 2\omega^{1,3,4,5,7} - 2\omega^{2,3,4,5,6})/9\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-19\beta_1} \langle \omega^{2,3,4,5,6,7}\rangle + 2e^{-18\beta_1} \langle -\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-20\beta_1} \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

5.3.13 Algebra $\mathcal{G}_{7,1.3(iii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_4]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^3 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & -\xi_1^3 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & -\xi_1^3 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4+\xi_2^6 & -\xi_1^5+\xi_3^6 & \xi_1^2+\xi_2^4 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+2e^{2\beta_1}\ \langle x_2,x_3\rangle+2e^{3\beta_1}\ \langle x_4,x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{\beta_1}\ \langle\delta_3^5,\delta_1^3,\delta_1^2,\delta_2^4\rangle+3e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7,\delta_3^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+4e^{2\beta_1}\ \langle\delta_1^4,\delta_2^6,\delta_1^5,\delta_3^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_1^3]=\delta_3^6+\delta_1^5$; $[\delta_3^5,\delta_1^5]=\delta_3^7$; $[\delta_3^5,\delta_3^6]=-\delta_3^7$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_1^4]=-\delta_3^7$; $[\delta_1^3,\delta_2^6]=\delta_3^7$; $[\delta_1^3,\delta_1^5]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^3,\delta_3^6]=\delta_2^7-\delta_1^6$; $[\delta_1^3,\delta_1^2]=-\delta_1^5$; $[\delta_1^3,\delta_2^4]=\delta_3^6$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^7+\delta_1^6)$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+\delta_1^6$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_3^6,\delta_1^2]=-\delta_3^7$; $[\delta_3^6,\delta_2^4]=-\delta_3^7$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle\delta_1^3,\delta_1^2,\delta_2^4\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_2^6,\delta_3^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_3^6]=-2\delta_1^6$; $[\delta_1^3,\delta_2^4]=\delta_3^6$; $[\delta_2^6,\delta_1^2]=2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $9e^{-2\beta_1}\ \langle x_3\omega^{1,5},-2x_1\omega^{1,2}-x_3\omega^{2,3}+2x_4(\omega^{1,6}-\omega^{2,4})+x_5\omega^{2,5},x_4(\omega^{1,6}+\omega^{2,4})+x_6\omega^{2,6},-2/3x_1\omega^{1,3}+1/3x_2\omega^{2,3}+1/3x_4\omega^{3,4}+5/3x_5\omega^{1,6}+x_6\omega^{3,6},-x_1\omega^{1,2}-x_3\omega^{2,3}+x_4(\omega^{1,6}-2\omega^{2,4}-\omega^{3,5})+x_6\omega^{1,7},1/3x_1\omega^{1,3}-2/3x_2\omega^{2,3}+1/3x_4\omega^{3,4}-1/3x_5(\omega^{1,6}-3\omega^{2,4}),x_5\omega^{3,5},x_2\omega^{1,4},x_2\omega^{1,5}+x_3\omega^{1,4}\rangle+4e^{-3\beta_1}\ \langle-2x_1\omega^{1,4}-2x_2(\omega^{1,6}-\omega^{2,4})+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5},-1/3x_1(\omega^{1,5}+3\omega^{2,3})+2/3x_2(\omega^{2,5}+\omega^{3,4})-1/3x_3(\omega^{1,6}-3\omega^{2,4}+2\omega^{3,5})+1/3x_4(\omega^{4,5}+3\omega^{3,6})+x_5(\omega^{1,7}+2\omega^{2,6})+x_6\omega^{3,7},x_1\omega^{1,4}+x_2(\omega^{1,6}-\omega^{2,4})+x_5\omega^{3,6},-x_1\omega^{1,5}+x_2(\omega^{2,5}+\omega^{3,4})-2x_3(\omega^{1,6}-1/2\omega^{2,4}-1/2\omega^{3,5})+x_4\omega^{3,6}+x_5\omega^{2,6}+x_6\omega^{5,6}\rangle+e^0\ \langle x_4\omega^{1,3}+$

$x_6\omega^{2,3}\rangle + 5e^{-\beta_1}\ \langle x_3\omega^{1,2} + x_4\omega^{2,3} + x_6\omega^{2,5}, x_3\omega^{1,3} + x_6\omega^{3,5}, x_5\omega^{2,3}, x_2\omega^{1,2} + x_6\omega^{2,4}, -x_3\omega^{1,2} + x_4\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$5e^{-4\beta_1} + 11e^{-5\beta_1} + 10e^{-6\beta_1} + 4e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-7\beta_1} + 7e^{-8\beta_1} + 13e^{-9\beta_1} + 8e^{-10\beta_1} + 3e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$3e^{-11\beta_1} + 7e^{-12\beta_1} + 8e^{-13\beta_1} + 4e^{-14\beta_1} + e^{-15\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$3e^{-15\beta_1} + 5e^{-16\beta_1} + 2e^{-17\beta_1} + e^{-18\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$2e^{-18\beta_1} + e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 13, 55, 122, 155, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(1, 7, 19, 30, 32, 23, 11, 3)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,4} - \omega^{3,5}$; $d\omega^{1,7} = \omega^{1,2,4} + \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6}+\omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7}-\omega^{1,5,6}+\omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7}+\omega^{2,4,6}+\omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,4,6} - \omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-2\beta_1}\ \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-4\beta_1}\ \langle \omega^{1,5},\omega^{2,3}\rangle + 3e^{-5\beta_1}\ \langle \omega^{1,6},\omega^{2,4},\omega^{2,5}+\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1}\ \langle \omega^{1,3,6}\rangle + 3e^{-8\beta_1}\ \langle \omega^{1,3,7}-\omega^{2,4,5},\omega^{2,3,6}+\omega^{2,4,5},\omega^{1,2,7}+\omega^{3,4,5}\rangle + 3e^{-9\beta_1}\ \langle \omega^{1,5,7}+\omega^{2,5,6}+\omega^{3,4,6},\omega^{2,3,7}+\omega^{3,4,6},\omega^{1,4,7}-\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-11\beta_1}\ \langle \omega^{1,2,5,7},\omega^{1,3,5,7}+\omega^{2,3,5,6},\omega^{1,3,4,7}\rangle + 3e^{-12\beta_1}\ \langle \omega^{1,3,6,7}+\omega^{2,4,5,6},\omega^{2,3,4,7}+\omega^{2,4,5,6},\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{-13\beta_1}\ \langle \omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{2,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^{-15\beta_1}\ \langle \omega^{1,2,4,6,7}+\omega^{2,3,4,5,7},\omega^{1,2,5,6,7},\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7}\rangle + 2e^{-16\beta_1}\ \langle \omega^{1,4,5,6,7},\omega^{2,3,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^{-18\beta_1}\ \langle \omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1}\ \langle \omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1}\ \langle \omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y^2)^2(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,7,7,5,3,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^5$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{3,5}=-\omega^7$; $d^\star\omega^{1,2,3}=-\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,5}=-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5}=\omega^{1,7}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{5,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}$; $d^\star\omega^{2,3,5}=\omega^{2,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{4,7}$; $d^\star\omega^{3,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,3,7}-\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}+\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,4,7}+\omega^{3,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,4,7}-\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}-\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,2,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,5,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7}=\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=-\omega^{1,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7}=\omega^{4,5,6,7}$;

$d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{2,4}+\omega^{3,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7}-\omega^{2,6}$; $\Delta\omega^{2,4} = \omega^{1,6} + \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}+\omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6}+\omega^{2,4}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}-\omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7}+\omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6}+2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7}+\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}+\omega^{1,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}-\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}-\omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}+\omega^{1,4,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5}+2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4}+\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}+\omega^{2,4,7}+\omega^{3,5,7}$; $\Delta\omega^{2,3,4} = \omega^{1,3,6}+\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6}+\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,5,6} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + 2\omega^{2,4,7} + \omega^{3,5,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{2,4,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,4,6} + 2\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} - \omega^{1,3,5,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7}+\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$;

$\Delta\omega^{3,5,6,7} = \omega^{2,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-2\beta_1}\ \langle\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= 2e^{-4\beta_1}\ \langle\omega^{1,5},\omega^{2,3}\rangle + 3e^{-5\beta_1}\ \langle\omega^{2,5}+\omega^{3,4},\omega^{1,6}-\omega^{3,5},\omega^{2,4}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-7\beta_1}\ \langle\omega^{1,3,6}+\omega^{1,4,5}-\omega^{2,3,4}\rangle+3e^{-8\beta_1}\ \langle\omega^{1,3,7}+2\omega^{2,3,6}+\omega^{2,4,5},\omega^{1,5,6}+\omega^{2,3,6}+\omega^{2,4,5},\omega^{1,2,7}-2\omega^{1,4,6}+\omega^{3,4,5}\rangle + 3e^{-9\beta_1}\ \langle\omega^{1,5,7}+\omega^{2,5,6}+\omega^{3,4,6},\omega^{2,3,7}+\omega^{3,4,6},\omega^{1,4,7}+2\omega^{2,4,6}-\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-13\beta_1}\ \langle(\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{2,4,5,7})/3\rangle + 3e^{-12\beta_1}\ \langle(-\omega^{1,4,5,7}-\omega^{2,3,4,7}-\omega^{2,4,5,6})/3,(-\omega^{1,3,6,7}+2\omega^{2,3,4,7}+\omega^{2,4,5,6})/3,(\omega^{1,2,6,7}+2\omega^{2,3,5,7}+\omega^{3,4,5,6})/6\rangle + 3e^{-11\beta_1}\ \langle(-\omega^{1,2,5,7}-2\omega^{1,3,4,7}-\omega^{1,4,5,6}-2\omega^{2,3,4,6})/5,(-2\omega^{1,2,5,7}+\omega^{1,3,4,7}+3\omega^{1,4,5,6}+\omega^{2,3,4,6})/5,(-\omega^{1,2,4,7}+2\omega^{1,3,5,7}+\omega^{2,3,5,6})/6\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= 2e^{-16\beta_1}\ \langle-\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}\rangle + 3e^{-15\beta_1}\ \langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2,(\omega^{1,2,4,6,7}+\omega^{1,3,5,6,7}+2\omega^{2,3,4,5,7})/3,(\omega^{1,2,4,6,7}-2\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + 2e^{-18\beta_1}\ \langle-\omega^{1,3,4,5,6,7},\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

5.3.14 Algebra $\mathcal{G}_{7,1.3(iv)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & -\xi_1^2+\xi_3^4 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & \xi_1^2+\xi_3^4 & 4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4 & -\xi_1^5 & \xi_1^2 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_3^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+2e^{2\beta_1}\ \langle x_2,x_3\rangle+2e^{3\beta_1}\ \langle x_4,x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{\beta_1}\ \langle \delta_3^5,\delta_2^4,\delta_3^4,\delta_1^2\rangle+3e^{3\beta_1}\ \langle \delta_1^6,\delta_2^7,\delta_3^7\rangle+4e^{2\beta_1}\ \langle \delta_2^6,\delta_3^6,\delta_1^4,\delta_1^5\rangle+e^{4\beta_1}\ \langle \delta_1^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_3^4]=-\delta_3^6$; $[\delta_3^5,\delta_1^5]=\delta_3^7$; $[\delta_3^5,\delta_1^2]=-\delta_3^6$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+\delta_1^6$; $[\delta_3^6,\delta_1^2]=-\delta_3^7$; $[\delta_2^4,\delta_3^4]=\delta_3^6$; $[\delta_2^4,\delta_1^4]=\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_3^4,\delta_1^4]=\delta_3^7$; $[\delta_3^4,\delta_1^5]=\delta_2^7+\delta_1^6$; $[\delta_3^4,\delta_1^2]=-2\delta_2^6+\delta_1^5$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^5,\delta_1^2]=\delta_2^7-\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle \delta_2^4,\delta_3^4,\delta_1^2\rangle+e^{4\beta_1}\ \langle \delta_1^7\rangle+e^{3\beta_1}\ \langle \delta_1^6\rangle+2e^{2\beta_1}\ \langle \delta_1^4,\delta_1^5\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4,\delta_2^4]=-\delta_1^4$; $[\delta_3^4,\delta_1^2]=3\delta_1^5$; $[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_1^5,\delta_1^2]=-2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $9e^{-2\beta_1}\ \langle -2x_1\omega^{1,3}-3x_2(\omega^{1,4}-1/3\omega^{2,3})+x_4\omega^{3,4}, -2x_1\omega^{1,2}-x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5}, x_1\omega^{1,3}+x_2\omega^{1,4}+x_3\omega^{1,5}+x_4\omega^{2,5}, x_1\omega^{1,2}-x_2\omega^{1,5}-x_3(\omega^{1,4}-\omega^{2,3})+x_4\omega^{3,5}, -x_2\omega^{1,5}-x_3\omega^{1,4}-2x_4\omega^{2,4}+x_6\omega^{1,7}, x_1\omega^{1,3}+x_2(\omega^{1,4}-\omega^{2,3})+x_5\omega^{2,4}, x_5\omega^{3,5}, x_2\omega^{1,4}+x_6\omega^{3,6}, x_4\omega^{2,4}+x_6\omega^{2,6}\rangle+4e^{-3\beta_1}\ \langle -2x_1(\omega^{1,4}-3/2\omega^{2,3})+2x_2\omega^{2,4}+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5}, x_1(\omega^{1,4}-\omega^{2,3})-x_2\omega^{2,4}+x_5\omega^{3,6}, -x_1\omega^{1,5}+x_2(\omega^{2,5}+\omega^{3,4})+x_3(\omega^{2,4}+\omega^{3,5})+x_4\omega^{3,6}+x_5\omega^{2,6}-3x_6(\omega^{2,7}-1/3\omega^{5,6}), -x_1\omega^{2,3}+x_6\omega^{3,7}\rangle+5e^{-\beta_1}\ \langle x_2\omega^{1,2}-x_3\omega^{1,3}-x_4\omega^{1,4}+x_5\omega^{1,5}, -x_2\omega^{1,2}+x_3\omega^{1,3}+2x_4\omega^{1,4}+x_7\omega^{1,7}, x_2\omega^{1,2}-x_4\omega^{1,4}+x_6\omega^{1,6}, -x_3\omega^{1,2}+x_4\omega^{1,5}, x_5\omega^{2,3}\rangle+$

$2e^{-4\beta_1}\ \langle x_1\omega^{2,4}+x_5\omega^{3,7}, -x_1(\omega^{2,5}+\omega^{3,4})+x_4\omega^{3,7}+x_5\omega^{2,7}+x_6\omega^{5,7}\rangle+e^0\ \langle x_4\omega^{1,3}+x_7\omega^{1,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$7e^{-4\beta_1}+11e^{-5\beta_1}+10e^{-6\beta_1}+4e^{-7\beta_1}+e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-7\beta_1}+8e^{-8\beta_1}+13e^{-9\beta_1}+10e^{-10\beta_1}+3e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-10\beta_1}+3e^{-11\beta_1}+7e^{-12\beta_1}+8e^{-13\beta_1}+4e^{-14\beta_1}+e^{-15\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$3e^{-15\beta_1}+5e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$2e^{-18\beta_1}+e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,13,57,123,157,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,8,21,33,35,25,11,3)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}-\omega^{2,3}$; $d\omega^7=-\omega^{2,4}-\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}+\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,4,6}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,4,6}-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}+\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{2,3,4,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7}=-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+2e^{-2\beta_1}\ \langle\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-4\beta_1}\ \langle\omega^{1,5},\omega^{1,4}\rangle + 3e^{-5\beta_1}\ \langle\omega^{1,6}-\omega^{3,4},\omega^{2,5}+\omega^{3,4},\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1}\ \langle\omega^{1,3,6}\rangle+3e^{-9\beta_1}\ \langle\omega^{2,3,7},\omega^{1,5,7}+\omega^{2,5,6}+\omega^{3,4,6},\omega^{1,4,7}-\omega^{3,5,6}\rangle+ 3e^{-8\beta_1}\ \langle\omega^{1,4,6}+\omega^{2,4,5},\omega^{2,3,6}+\omega^{2,4,5},\omega^{1,5,6}+\omega^{3,4,5}\rangle+2e^{-10\beta_1}\ \langle\omega^{2,5,7}+\omega^{3,4,7},\omega^{2,4,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-10\beta_1}\ \langle\omega^{1,2,5,6},\omega^{1,2,4,6}\rangle + 3e^{-11\beta_1}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6},\omega^{1,4,5,6}-\omega^{2,3,5,6},\omega^{1,2,5,7}\rangle + 3e^{-12\beta_1}\ \langle\omega^{1,4,5,7},\omega^{1,3,6,7}+\omega^{2,4,5,6},\omega^{1,2,6,7}-\omega^{3,4,5,6}\rangle + e^{-13\beta_1}\ \langle\omega^{2,3,6,7}+\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^{-15\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,6,7},\omega^{1,2,4,6,7}-\omega^{1,3,5,6,7}\rangle + 2e^{-16\beta_1}\ \langle\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^{-18\beta_1}\ \langle\omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 32.

Riemann Hyp.: no

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^5$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{3,5}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,5}=-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5}=\omega^{1,7}$; $d^\star\omega^{1,3,6}=-\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{5,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{2,7}+\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{4,7}$; $d^\star\omega^{3,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,2,7}-\omega^{1,5,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{1,6,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,4,7}+\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,4,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,5,7}=\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=$

$-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{2,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,5,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,4,6} - \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7} + \omega^{2,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,4,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,4,7} - \omega^{3,5,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6} - \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$;

$\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,4,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{2,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{a}}) = e^{-\beta_1} \langle\omega^1\rangle + 2e^{-2\beta_1} \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}}) = 2e^{-4\beta_1} \langle\omega^{1,5}, \omega^{1,4} - \omega^{2,3}\rangle + 3e^{-5\beta_1} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}, \omega^{2,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}}) = e^{-7\beta_1} \langle\omega^{1,3,6} + \omega^{1,4,5}\rangle + 3e^{-8\beta_1} \langle\omega^{1,4,6} - \omega^{2,3,6}, \omega^{1,3,7} + 2\omega^{2,3,6} + \omega^{2,4,5}, \omega^{1,2,7} - 2\omega^{1,5,6} - \omega^{3,4,5}\rangle + 3e^{-9\beta_1} \langle\omega^{2,3,7} + \omega^{2,4,6}, \omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}, \omega^{1,4,7} + 2\omega^{2,4,6} - \omega^{3,5,6}\rangle + 2e^{-10\beta_1} \langle\omega^{2,5,7} + \omega^{3,4,7}, \omega^{2,4,7} - \omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}}) = e^{-13\beta_1} \langle(\omega^{2,3,6,7} + \omega^{2,4,5,7})/2\rangle + 3e^{-12\beta_1} \langle(-\omega^{1,3,6,7} + \omega^{1,4,5,7} - 3\omega^{2,3,5,7} - \omega^{2,4,5,6})/4, (-\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6})/4, (-\omega^{1,2,6,7} - 2\omega^{2,3,4,7} + \omega^{3,4,5,6})/6\rangle + 3e^{-11\beta_1} \langle(\omega^{1,2,4,7} + \omega^{1,3,5,7} + 3\omega^{1,4,5,6} - \omega^{2,3,5,6})/4, (-\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{2,3,4,6})/3, (-\omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,5,6})/4\rangle + 2e^{-10\beta_1} \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6})/2, (\omega^{1,2,4,6} - \omega^{1,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}}) = 2e^{-16\beta_1} \langle-\omega^{2,3,4,6,7}, (-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + 3e^{-15\beta_1} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3, (\omega^{1,2,4,6,7} - \omega^{1,3,5,6,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}}) = e^{-19\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + 2e^{-18\beta_1} \langle-\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}}) = e^{-20\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 3Y^2 + 6Y^3 + 3Y^4 + 2Y^5 + Y^6)(1 + Y)$.

5.3.15 Algebra $\mathcal{G}_{7,1.3(v)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_7$; $[x_3,x_4]=-x_7$; $[x_3,x_5]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & 2(\xi_1^2-\xi_1^3) & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -(\xi_1^5+\xi_1^4) & -\xi_1^3+\xi_2^4 & \xi_1^3 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6-\xi_1^4 & \xi_5^7 & \xi_1^2-2\xi_1^3+\xi_2^4 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-(\delta_2^6+\delta_5^7), ad\ x_4=-\delta_1^6-\delta_2^7+\delta_3^7, ad\ x_5=\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+2e^{2\beta_1}\ \langle x_2,x_3\rangle+2e^{3\beta_1}\ \langle x_4,x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 4e^{\beta_1}\ \langle \delta_3^5,\delta_1^2,\delta_1^3,\delta_2^4\rangle+3e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7,\delta_3^7\rangle+4e^{2\beta_1}\ \langle\delta_1^5,\delta_1^4,\delta_2^6,\delta_5^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_5^7]=-\delta_3^7$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_1^6,\delta_1^3]=2\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_5^7]=-\delta_1^7$; $[\delta_1^5,\delta_1^2]=\delta_3^7$; $[\delta_1^5,\delta_1^3]=-2(\delta_3^7+\delta_1^6)$; $[\delta_1^5,\delta_2^4]=\delta_3^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_1^2]=2\delta_3^7$; $[\delta_1^4,\delta_1^3]=-2\delta_3^7$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_2^7-\delta_1^6$; $[\delta_2^6,\delta_1^2]=-\delta_3^7-\delta_2^7+\delta_1^6$; $[\delta_2^6,\delta_1^3]=2\delta_2^7$; $[\delta_5^7,\delta_1^2]=2\delta_2^7$; $[\delta_5^7,\delta_1^3]=-2\delta_2^7$; $[\delta_1^2,\delta_1^3]=\delta_5^7-2\delta_2^6+2\delta_1^5-\delta_1^4$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^3,\delta_2^4]=-(\delta_5^7+\delta_2^6)$.

$\mathrm{ch}(H^1_{\mathcal{G}})= 3e^{\beta_1}\ \langle\delta_3^5,\delta_1^2,\delta_1^3\rangle+2e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3]=-3\delta_2^6+2\delta_1^5$; $[\delta_1^2,\delta_2^6]=-2\delta_1^6$; $[\delta_1^5,\delta_1^3]=-2\delta_1^6$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_1^3,\delta_2^6]=2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 5e^{-3\beta_1}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})-x_2(\omega^{1,6}-\omega^{2,4}-\omega^{3,4})+x_4\omega^{3,6}, -x_1(\omega^{1,4}+\omega^{1,5}-\omega^{2,3})+2x_2(\omega^{2,4}+\omega^{2,5})+x_3(\omega^{1,6}+3\omega^{2,5}-\omega^{3,4}-2\omega^{3,5})+x_4(\omega^{2,6}+\omega^{4,5}), -2x_1(\omega^{1,4}+1/2\omega^{1,5}-\omega^{2,3})+4x_2(\omega^{2,4}+1/4\omega^{2,5})+2x_3(\omega^{1,6}+1/2\omega^{2,5}-\omega^{3,4}-\omega^{3,5})+2x_4\omega^{2,6}+x_6\omega^{5,6}, x_3(\omega^{2,4}+\omega^{2,5}), -x_1(\omega^{1,4}-\omega^{2,3})-x_2(\omega^{1,6}-\omega^{2,4}-\omega^{3,4}-\omega^{3,5})+x_3\omega^{3,5}+x_4(\omega^{1,7}+\omega^{2,6})+x_6(\omega^{2,7}-\omega^{3,7})\rangle+e^0\ \langle x_5\omega^{1,3}\rangle+5e^{-\beta_1}\ \langle x_2(\omega^{1,2}-2\omega^{1,3})-x_3\omega^{1,3}-x_4\omega^{1,4}+x_5\omega^{1,5}, x_5\omega^{2,3}, x_2(\omega^{1,2}-\omega^{1,3})+x_3(\omega^{1,2}-\omega^{1,3})+x_4\omega^{1,5}, x_2(\omega^{1,2}-\omega^{1,3})-x_4(\omega^{1,4}-\omega^{2,3}), x_2\omega^{1,3}+x_4\omega^{1,4}+$

$x_6\omega^{2,4}\rangle + 9e^{-2\beta_1}\ \langle -x_1(\omega^{1,2}-\omega^{1,3}) + x_2(\omega^{1,4}-\omega^{1,5}-2\omega^{2,3}) + x_3(\omega^{1,4}-2\omega^{2,3}) + x_4(\omega^{1,6}+\omega^{3,5}) + x_5\omega^{2,5}, 2x_1(\omega^{1,2}-\omega^{1,3}) - 3x_2(\omega^{1,4}-2/3\omega^{1,5}-\omega^{2,3}) - 2x_3(\omega^{1,4}-\omega^{2,3}) - 2x_4(\omega^{1,6}-1/2\omega^{3,4}+\omega^{3,5}), -2x_1(\omega^{1,2}-\omega^{1,3}) + 3x_2(\omega^{1,4}-\omega^{1,5}-4/3\omega^{2,3}) + 3x_3(\omega^{1,4}-4/3\omega^{2,3}) + 2x_4(\omega^{1,6}+2\omega^{3,5}) + x_6\omega^{1,7}, x_1(\omega^{1,2}-\omega^{1,3}) - x_2(\omega^{1,4}-\omega^{1,5}-2\omega^{2,3}) - x_3(\omega^{1,4}-\omega^{2,3}) - x_4(\omega^{1,6}+\omega^{3,5}) + x_5\omega^{2,4}, -x_1\omega^{1,2} - x_2(\omega^{1,5}+\omega^{2,3}) + x_3(\omega^{1,4}-\omega^{2,3}) + x_4(\omega^{1,6}+\omega^{3,5}) - x_5(\omega^{1,6}-\omega^{3,4}), x_5\omega^{3,5}, -x_2\omega^{1,5} + x_3(\omega^{1,4}-\omega^{2,3}) + x_4(\omega^{2,4}+\omega^{3,5}), x_2\omega^{1,5} + x_3\omega^{1,5} + x_4(\omega^{2,5}-\omega^{3,5}), -2x_1(\omega^{1,2}-\omega^{1,3}) + 2x_2(\omega^{1,4}-2\omega^{1,5}-2\omega^{2,3}) + 2x_3(\omega^{1,4}-1/2\omega^{1,5}-2\omega^{2,3}) + 2x_4(\omega^{1,6}+2\omega^{3,5}) + x_6\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{-3\beta_1} + 5e^{-4\beta_1} + 11e^{-5\beta_1} + 10e^{-6\beta_1} + 4e^{-7\beta_1} + e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-7\beta_1} + 8e^{-8\beta_1} + 13e^{-9\beta_1} + 8e^{-10\beta_1} + 3e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = 3e^{-11\beta_1} + 7e^{-12\beta_1} + 8e^{-13\beta_1} + 5e^{-14\beta_1} + 3e^{-15\beta_1} + e^{-16\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-14\beta_1} + 5e^{-15\beta_1} + 6e^{-16\beta_1} + 3e^{-17\beta_1} + e^{-18\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-17\beta_1} + 2e^{-18\beta_1} + e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 56, 123, 155, 117, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 20, 32, 33, 27, 16, 4)$

$S = 20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 19, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 7, 7, 6, 4, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,4} + \omega^{3,4} + \omega^{3,5}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4} - \omega^{1,3,4} - \omega^{1,3,5}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} - \omega^{2,3,4} - \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{3,4,5}$; $d\omega^{5,6} = \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,5,6} + \omega^{2,4,5} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,6} - \omega^{3,5,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4} + \omega^{1,2,3,5}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,3,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,3,4,6} + \omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,6}$; $d\omega^{1,2,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}$;

$d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-2\beta_1}\ \langle\omega^2,\omega^3\rangle + e^{-3\beta_1}\ \langle\omega^5\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,5},\omega^{1,4}\rangle + 3e^{-5\beta_1}\ \langle\omega^{2,4},\omega^{2,5},\omega^{1,6}-\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1}\ \langle\omega^{1,2,6}\rangle + 3e^{-8\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,3,6},\omega^{1,2,7}+\omega^{2,3,6}-\omega^{3,4,5},\omega^{1,5,6}+\omega^{3,4,5}\rangle + 3e^{-9\beta_1}\ \langle\omega^{2,3,7}+\omega^{3,4,6},\omega^{1,4,7}+\omega^{3,5,6},\omega^{1,5,7}+\omega^{2,5,6}-\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-11\beta_1}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6},\omega^{1,2,5,7}+\omega^{2,3,5,6},\omega^{1,2,4,7}-\omega^{2,3,5,6}\rangle + 3e^{-12\beta_1}\ \langle\omega^{1,3,6,7}-\omega^{2,3,4,7},\omega^{1,4,5,7},\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{-13\beta_1}\ \langle(2\omega^{1,4,6,7}+\omega^{1,5,6,7}-2\omega^{2,3,6,7}+\omega^{2,4,5,7}+\omega^{3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= 3e^{-15\beta_1}\ \langle\omega^{1,2,4,6,7}-\omega^{2,3,4,5,7},\omega^{1,3,4,6,7},\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7}\rangle + 2e^{-16\beta_1}\ \langle\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle + e^{-17\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-17\beta_1}\ \langle\omega^{1,2,3,4,6,7}\rangle + 2e^{-18\beta_1}\ \langle\omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+2Y+2Y^3+Y^4)(1+Y+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,9,19,23,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,6,7,7,6,4,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 32.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{3,5} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5} = -\omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7}-\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{2,7}+\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} =$

$\omega^{4,7} - \omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{3,5,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} + \omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{1,2,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,4,7} + \omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,4,7} + \omega^{2,5,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} + \omega^{3,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,6,7} + \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,3,6,7} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,6,7} - \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,6,7} - \omega^{2,5,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 4\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,4} - \omega^{3,5}$; $\Delta\omega^{1,7} = 3\omega^{1,7} - \omega^{2,6} + \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,6} + \omega^{2,4} - \omega^{3,4} - \omega^{3,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = 3\omega^{2,7} + \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{3,4} = -\omega^{2,4} + 2\omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{3,5} = -\omega^{1,6} - \omega^{2,4} + \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{2,7} + 2\omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,7}$; $\Delta\omega^{5,6} = -\omega^{2,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,7} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} - \omega^{1,3,4} - \omega^{1,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 3\omega^{1,2,7} + \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,4} + 2\omega^{1,3,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,4} + \omega^{1,3,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = \omega^{1,2,7} + \omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,2,7} + 2\omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,7} + 2\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 4\omega^{1,6,7} + \omega^{2,4,7} - \omega^{3,5,7}$; $\Delta\omega^{2,3,4} = \omega^{1,3,6} + 3\omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{1,2,6} + \omega^{1,4,5} + \omega^{2,3,4} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,7} - \omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,5,6} + \omega^{2,4,5} - \omega^{3,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,6} - \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + 2\omega^{2,4,7} - \omega^{2,5,7} - \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{2,4,7} + 2\omega^{2,5,7} + \omega^{3,5,7}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7} + \omega^{3,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,6} - \omega^{2,4,5} + 3\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,7} - \omega^{2,4,6} + 2\omega^{3,4,6} + \omega^{3,5,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{1,6,7} - \omega^{2,4,7} + \omega^{2,5,7} + 2\omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = \omega^{2,6,7} + 2\omega^{3,6,7}$;

$\Delta\omega^{4,5,6} = \omega^{2,4,7} - \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7} + 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4} + \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,6} - \omega^{1,3,4,6} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,7} + \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 3\omega^{1,2,6,7} + \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,5} + 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,3,4,6} + 2\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,4,7} + \omega^{1,2,5,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,6,7} + \omega^{1,3,6,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,4,7} - \omega^{1,3,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,4,6,7} + 3\omega^{1,5,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,6} - \omega^{1,3,5,6} + 4\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,7} - \omega^{1,3,4,7} + 2\omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,6,7} + 2\omega^{2,3,4,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,5,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{2,5,6,7} - \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = -\omega^{2,4,6,7} + 2\omega^{2,5,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6} + 3\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = -\omega^{2,4,6,7} + \omega^{2,5,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,6} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,2,5,6,7} - \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = -\omega^{1,2,4,6,7} + 2\omega^{1,2,5,6,7} + \omega^{1,3,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6} + 3\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,7} + 3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,4,6,7} - \omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-2\beta_1}\ \langle\omega^2, \omega^3\rangle + e^{-3\beta_1}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-4\beta_1}\ \langle\omega^{1,5}, \omega^{1,4} - \omega^{2,3}\rangle + 3e^{-5\beta_1}\ \langle\omega^{2,5}, \omega^{1,6} - \omega^{3,4} + 2\omega^{3,5}, \omega^{2,4} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-7\beta_1}\ \langle\omega^{1,2,6} - \omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,4} - 2\omega^{2,3,5}\rangle + 3e^{-8\beta_1}\ \langle\omega^{1,3,7} + \omega^{2,3,6}, \omega^{1,2,7} + 2\omega^{1,5,6} + \omega^{2,3,6} + 3\omega^{2,4,5} + \omega^{3,4,5}, \omega^{1,4,6} + \omega^{1,5,6} - \omega^{2,3,6} + 2\omega^{2,4,5} + \omega^{3,4,5}\rangle + 3e^{-9\beta_1}\ \langle\omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,6}, \omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{3,5,6}, \omega^{1,5,7} - \omega^{2,4,6} +$

$\omega^{2,5,6} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-13\beta_1} \langle(-2\omega^{1,4,6,7} - \omega^{1,5,6,7} + 2\omega^{2,3,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,7})/11\rangle + 3e^{-12\beta_1} \langle(\omega^{1,2,6,7} - 3\omega^{1,4,5,7} - \omega^{2,3,4,7} - 3\omega^{2,3,5,7} - 8\omega^{2,4,5,6} - 2\omega^{3,4,5,6})/11, (-2\omega^{1,2,6,7} - 11\omega^{1,3,6,7} - 16\omega^{1,4,5,7} + 13\omega^{2,3,4,7} + 17\omega^{2,3,5,7} + 16\omega^{2,4,5,6} + 15\omega^{3,4,5,6})/88, (14\omega^{1,2,6,7} - 11\omega^{1,3,6,7} + 24\omega^{1,4,5,7} - 3\omega^{2,3,4,7} - 31\omega^{2,3,5,7} - 24\omega^{2,4,5,6} - 17\omega^{3,4,5,6})/88\rangle + 3e^{-11\beta_1} \langle(-2\omega^{1,2,4,7} - 3\omega^{1,2,5,7} + 2\omega^{1,3,5,7} + 3\omega^{1,4,5,6} - 2\omega^{2,3,5,6})/10, (\omega^{1,2,4,7} + \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,6} + 2\omega^{2,3,5,6})/5, (-\omega^{1,2,4,7} - 2\omega^{1,3,4,7} - 2\omega^{2,3,4,6} + \omega^{2,3,5,6})/5\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-17\beta_1} \langle -\omega^{2,4,5,6,7}\rangle + 2e^{-16\beta_1} \langle -\omega^{2,3,4,6,7}, (-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + 3e^{-15\beta_1} \langle \omega^{1,3,4,6,7}, (-\omega^{1,2,4,6,7} - \omega^{1,2,5,6,7} + \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7})/4, (-\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} - 3\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7})/4\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-19\beta_1} \langle \omega^{2,3,4,5,6,7}\rangle + 2e^{-18\beta_1} \langle -\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle + e^{-17\beta_1} \langle \omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-20\beta_1} \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 2Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.3.16 Algebra $\mathcal{G}_{7,1.4}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 3\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 5\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 6\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^5 & \xi_1^2+\xi_2^4 & \xi_2^3 & 7\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^4+\xi_2^6 & -\xi_1^3+\xi_2^5 & 2\xi_1^2+\xi_2^4 & \xi_2^3 & 8\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^6), ad\ x_4=-(\delta_1^5+\delta_2^7), ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{3\beta_1}\ \langle x_2\rangle+e^{4\beta_1}\ \langle x_3\rangle+e^{5\beta_1}\ \langle x_4\rangle+e^{6\beta_1}\ \langle x_5\rangle+e^{7\beta_1}\ \langle x_6\rangle+e^{8\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{8M(1)}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{5\beta_1}\ \langle\delta_1^5,\delta_2^7\rangle+e^{6\beta_1}\ \langle\delta_1^6\rangle+e^{7\beta_1}\ \langle\delta_1^7\rangle+2e^{4\beta_1}\ \langle\delta_1^4,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+2e^{2\beta_1}\ \langle\delta_1^2,\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^2]=-2\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_1^6$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_1^2]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^5)$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^3,\delta_2^5]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_1^5)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^6+\delta_1^4)$; $[\delta_2^5,\delta_1^2]=-2\delta_2^7+\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_2^6-\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle\delta_1^5\rangle+e^{4\beta_1}\ \langle\delta_1^4\rangle+e^{3\beta_1}\ \langle\delta_2^5\rangle+2e^{2\beta_1}\ \langle\delta_1^2,\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=3\delta_1^5$; $[\delta_1^2,\delta_2^4]=-2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $4e^{-3\beta_1}\ \langle -x_1\omega^{1,2}+2x_3(\omega^{1,5}-\omega^{2,3})+x_4\omega^{1,6}+x_5\omega^{2,5}, x_1\omega^{1,2}-x_3(\omega^{1,5}-3\omega^{2,3})-x_4(\omega^{1,6}-\omega^{2,4}), x_1\omega^{1,2}-x_3(\omega^{1,5}-2\omega^{2,3})-x_4\omega^{1,6}+x_5\omega^{1,7}, -x_1\omega^{1,2}-x_2\omega^{1,4}+x_3(\omega^{1,5}-2\omega^{2,3})+x_4\omega^{1,6}+x_5\omega^{3,4}\rangle+2e^{-4\beta_1}\ \langle x_3(\omega^{1,6}+\omega^{2,4})+x_4\omega^{2,5}+x_5\omega^{2,6}, -2x_1\omega^{1,3}-6x_2\omega^{2,3}+x_3(\omega^{1,6}-7\omega^{2,4})-x_4(\omega^{1,7}+3\omega^{2,5}+5\omega^{3,4})-3x_5\omega^{3,5}-x_6\omega^{3,6}+x_7\omega^{3,7}\rangle+e^{-5\beta_1}\ \langle x_3(\omega^{1,7}+\omega^{2,5})+x_4\omega^{2,6}+x_5\omega^{2,7}\rangle+3e^{-\beta_1}\ \langle x_3\omega^{1,3}+x_5\omega^{2,3}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, x_5\omega^{1,5}\rangle+e^{\beta_1}\ \langle x_5\omega^{1,3}\rangle+2e^{-2\beta_1}\ \langle x_3\omega^{1,4}+x_4\omega^{2,3}+x_5\omega^{2,4}, -x_2\omega^{1,3}-x_4\omega^{1,5}+x_7\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$4e^{-7\beta_1}+3e^{-8\beta_1}+6e^{-9\beta_1}+2e^{-10\beta_1}+4e^{-11\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-12\beta_1}+e^{-13\beta_1}+5e^{-14\beta_1}+4e^{-15\beta_1}+4e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}+e^{-19\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-18\beta_1}+e^{-19\beta_1}+4e^{-20\beta_1}+e^{-21\beta_1}+4e^{-22\beta_1}+e^{-23\beta_1}+2e^{-24\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-23\beta_1}+2e^{-25\beta_1}+e^{-27\beta_1}+e^{-28\beta_1}+e^{-30\beta_1}+e^{-32\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-31\beta_1}+e^{-33\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,50,116,149,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,13,19,20,14,7,2)$

S=$34\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,18,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,6,6,4,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}$; $d\omega^7=-\omega^{1,6}-\omega^{2,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{2,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}+\omega^{2,4,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,3,4,7}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-3\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-7\beta_1}\ \langle\omega^{1,5}\rangle + 2e^{-9\beta_1}\ \langle\omega^{1,7}+\omega^{3,4},\omega^{2,5}-\omega^{3,4}\rangle + e^{-11\beta_1}\ \langle\omega^{2,7}-\omega^{3,6}+\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 2e^{-12\beta_1}\ \langle\omega^{1,2,7},\omega^{1,3,6}\rangle + 2e^{-14\beta_1}\ \langle\omega^{1,4,7},\omega^{2,3,6}-\omega^{2,4,5}\rangle + e^{-16\beta_1}\ \langle\omega^{1,6,7}+\omega^{2,4,7}-2\omega^{2,5,6}\rangle + e^{-15\beta_1}\ \langle\omega^{1,5,7}-\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-18\beta_1}\ \langle\omega^{1,3,4,7}\rangle + 2e^{-20\beta_1}\ \langle\omega^{1,3,6,7}-\omega^{2,3,5,6},\omega^{1,4,5,7}\rangle + e^{-19\beta_1}\ \langle\omega^{1,2,6,7}\rangle + 2e^{-22\beta_1}\ \langle\omega^{1,5,6,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6},\omega^{2,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-25\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-31\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-33\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-34\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+3Y^2+3Y^3+3Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,18,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,6,6,4,2,1)$

Total dimension of trivial cohomology: 26.

Length associated polynomial: 26.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,6}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{2,3,5,7}=$

$\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,7} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 3\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 3\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,7} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,7} + \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 3\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,7} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 4\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 4\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 3\omega^{1,3,5,7}$;

$\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7}+\omega^{1,4,5,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}+\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,7}-\omega^{1,3,4,7}+2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 3\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,6,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,5,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-3\beta_1}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-7\beta_1}\ \langle\omega^{1,5}-\omega^{2,3}\rangle+2e^{-9\beta_1}\ \langle\omega^{1,7}+\omega^{3,4},\omega^{2,5}-\omega^{3,4}\rangle+e^{-11\beta_1}\ \langle\omega^{2,7}-\omega^{3,6}+\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{-12\beta_1}\ \langle\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,4},\omega^{1,3,6}-2\omega^{1,4,5}-\omega^{2,3,4}\rangle + 2e^{-14\beta_1}\ \langle\omega^{1,4,7}-\omega^{2,3,6}+\omega^{2,4,5},\omega^{1,5,6}-2\omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-16\beta_1}\ \langle\omega^{1,6,7}+\omega^{2,4,7}-2\omega^{2,5,6}\rangle + e^{-15\beta_1}\ \langle 2\omega^{1,5,7}-\omega^{2,3,7}+\omega^{2,4,6}-3\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= 2e^{-22\beta_1}\ \langle(\omega^{1,5,6,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6})/3,(-\omega^{2,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6})/3\rangle + 2e^{-20\beta_1}\ \langle(-\omega^{1,3,6,7}-\omega^{2,3,4,7}+2\omega^{2,3,5,6})/3,(\omega^{1,4,5,7}+\omega^{2,3,4,7}-\omega^{2,3,5,6})/3\rangle + e^{-18\beta_1}\ \langle(2\omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{2,3,4,5})/6\rangle + e^{-19\beta_1}\ \langle(-3\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{1,4,5,6}-2\omega^{2,3,4,6})/15\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-27\beta_1}\ \langle(-\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7})/2\rangle+2e^{-25\beta_1}\ \langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-2\omega^{2,3,4,5,6})/3,(-\omega^{1,2,5,6,7}+2\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6})/3\rangle+e^{-23\beta_1}\ \langle(\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6})/3\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-33\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-31\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-34\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y).$

5.3.17 Algebra $\mathcal{G}_{7,1.5}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_5] = -x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 3\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_1^2/2 & \xi_2^3 & 5\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_1^2/2 & \xi_2^3 & 6\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^5 & (3\xi_1^2)/2 & \xi_2^3 & 7\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_1^5 & -\xi_1^4 & \xi_1^3 & -\xi_1^2 & 9\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^6), ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6 + \delta_2^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{3\beta_1}\ \langle x_2\rangle + e^{4\beta_1}\ \langle x_3\rangle + e^{5\beta_1}\ \langle x_4\rangle + e^{6\beta_1}\ \langle x_5\rangle + e^{7\beta_1}\ \langle x_6\rangle + e^{9\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_6\rangle + e^{9\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{6\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + 2e^{4\beta_1}\ \langle \delta_2^6, \delta_1^4\rangle + 2e^{3\beta_1}\ \langle \delta_2^5, \delta_1^3\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^{8\beta_1}\ \langle \delta_1^7\rangle + e^{5\beta_1}\ \langle \delta_1^5\rangle + e^{2\beta_1}\ \langle \delta_1^2\rangle + e^{0}\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_1^2] = \delta_1^7$; $[\delta_2^6, \delta_1^2] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^3] = -\delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^3, \delta_1^5] = -\delta_2^7 + \delta_1^6$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_1^3] = \delta_2^6 + \delta_1^4$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_1^4, \delta_1^2] = -1/2\delta_2^7 - 3/2\delta_1^6$; $[\delta_1^3, \delta_1^2] = -1/2\delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle \delta_2^5\rangle + e^{8\beta_1}\ \langle \delta_1^7\rangle + e^{6\beta_1}\ \langle \delta_1^6\rangle + e^{4\beta_1}\ \langle \delta_1^4\rangle + e^{2\beta_1}\ \langle \delta_1^2\rangle + e^{0}\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^6, \delta_1^2] = \delta_1^7$; $[\delta_1^4, \delta_1^2] = -2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $3e^{-4\beta_1}\ \langle -x_1\omega^{1,3} - 2x_2(\omega^{1,5} - \omega^{2,3}) + x_3\omega^{2,4} + x_4\omega^{2,5} + x_5\omega^{3,5}, x_1\omega^{1,3} + x_2\omega^{1,5} + x_3\omega^{2,4} + x_4\omega^{3,4}, x_2(\omega^{1,5} + \omega^{2,3}) + x_3\omega^{2,4} + x_4\omega^{2,5} + x_5\omega^{2,6} + x_6\omega^{3,6}\rangle + e^{\beta_1}\ \langle -x_4\omega^{1,2} + x_7\omega^{1,6}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_7\omega^{1,7}, x_3\omega^{1,3} - x_4\omega^{1,4} + x_5\omega^{2,3}, x_3\omega^{1,3} - x_4\omega^{1,4} + x_6\omega^{1,6}\rangle + 2e^{-3\beta_1}\ \langle -4/5x_1\omega^{1,2} - x_2\omega^{1,4} + x_3(\omega^{1,5} - 2/5\omega^{2,3}) + 2/5x_4\omega^{2,4} + 6/5x_5\omega^{2,5} + x_6\omega^{1,7}, -2/5x_1\omega^{1,2} - 1/5x_3\omega^{2,3} + 1/5x_4\omega^{2,4} + 3/5x_5\omega^{2,5} + x_6\omega^{2,6}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3} + x_4\omega^{1,5}\rangle + e^{-6\beta_1}\ \langle x_1\omega^{2,3} + x_5\omega^{2,7} + x_6\omega^{3,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1} + 2e^{-7\beta_1} + 3e^{-8\beta_1} + 3e^{-9\beta_1} + 2e^{-10\beta_1} + 4e^{-11\beta_1} + 2e^{-13\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-12\beta_1}+4e^{-14\beta_1}+2e^{-15\beta_1}+3e^{-16\beta_1}+3e^{-17\beta_1}+2e^{-18\beta_1}+e^{-20\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-19\beta_1}+3e^{-21\beta_1}+2e^{-22\beta_1}+e^{-23\beta_1}+3e^{-24\beta_1}+e^{-26\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-25\beta_1}+e^{-27\beta_1}+e^{-28\beta_1}+e^{-29\beta_1}+e^{-31\beta_1}+e^{-33\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-32\beta_1}+e^{-34\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,11,49,115,147,109,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,6,11,17,17,11,6,2)$

S$=35\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}$; $d\omega^7=\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}+\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,3,7}+\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,4,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}-\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,7}=-\omega^{1,2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=-\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}+\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}+\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}+\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-3\beta_1} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-7\beta_1} \langle\omega^{1,5}\rangle + e^{-8\beta_1} \langle\omega^{1,6}+\omega^{2,4}\rangle + e^{-10\beta_1} \langle\omega^{1,7}-2\omega^{2,6}+\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-12\beta_1} \langle\omega^{1,3,6}\rangle + 2e^{-14\beta_1} \langle\omega^{1,5,6}-\omega^{2,4,5},\omega^{2,3,6}-\omega^{2,4,5}\rangle + e^{-16\beta_1} \langle\omega^{1,5,7}+\omega^{2,5,6}-\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-19\beta_1} \langle\omega^{1,4,5,6}+\omega^{2,3,4,6}\rangle + 2e^{-21\beta_1} \langle\omega^{1,4,5,7},\omega^{1,3,6,7}+\omega^{2,4,5,6}\rangle + e^{-23\beta_1} \langle\omega^{2,3,6,7}-\omega^{2,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-25\beta_1} \langle\omega^{1,2,4,6,7}\rangle + e^{-27\beta_1} \langle\omega^{1,3,5,6,7}\rangle + e^{-28\beta_1} \langle\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-32\beta_1} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-34\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-35\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,5}=\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{4,7}$; $d^\star\omega^{2,5,6}=-\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,3,7}-\omega^{1,5,6}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,4,7}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}+\omega^{3,4,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{3,6,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,6,7}-\omega^{2,3,5,7}$;

$d^\star\omega^{1,2,3,5,6} = \omega^{1,3,6,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,4,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,6} + \omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}+\omega^{4,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6}+2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}+\omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7}+2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + 2\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7}-\omega^{2,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,6}-\omega^{1,4,5}+2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 3\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,3,7}+\omega^{1,5,6}+\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7}+\omega^{2,3,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,5,6} + \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,6,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{1,5,7} + \omega^{2,3,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}+\omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7}+3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7}+2\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 3\omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,3,5,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} =$

$\omega^{1,3,4,7} - \omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 2\omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,5,7} - \omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-3\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-7\beta_1} \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-8\beta_1} \langle\omega^{1,6} + \omega^{2,4}\rangle + e^{-10\beta_1} \langle\omega^{1,7} - 2\omega^{2,6} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-12\beta_1} \langle\omega^{1,3,6} - \omega^{1,4,5}\rangle + 2e^{-14\beta_1} \langle\omega^{1,5,6} - \omega^{2,3,6}, \omega^{1,3,7} - 2\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-16\beta_1} \langle\omega^{1,5,7} - 3\omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-23\beta_1} \langle(-\omega^{2,3,6,7} + \omega^{2,4,5,7})/2\rangle + 2e^{-21\beta_1} \langle(\omega^{1,3,6,7} + \omega^{1,4,5,7} + 3\omega^{2,3,4,7} + \omega^{2,4,5,6})/4, (-\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6})/4\rangle + e^{-19\beta_1} \langle(\omega^{1,2,5,7} - \omega^{1,3,4,7} - 3\omega^{1,4,5,6} - \omega^{2,3,4,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-28\beta_1} \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-27\beta_1} \langle(-\omega^{1,3,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-25\beta_1} \langle(-\omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/6\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-34\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-32\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-35\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.3.18 Algebra $\mathcal{G}_{7,1.6}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 4\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 5\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 6\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 7\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 8\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^3+\xi_2^6 & \xi_1^2+\xi_2^5 & \xi_2^4 & \xi_2^3 & 9\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^7), ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{4\beta_1}\ \langle x_2\rangle+e^{5\beta_1}\ \langle x_3\rangle+e^{6\beta_1}\ \langle x_4\rangle+e^{7\beta_1}\ \langle x_5\rangle+e^{8\beta_1}\ \langle x_6\rangle+e^{9\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{9\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{5\beta_1}\ \langle\delta_1^4,\delta_2^7\rangle+e^{6\beta_1}\ \langle\delta_1^5\rangle+e^{7\beta_1}\ \langle\delta_1^6\rangle+e^{8\beta_1}\ \langle\delta_1^7\rangle+2e^{4\beta_1}\ \langle\delta_1^3,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^2,\delta_2^5\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^2]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^3]=-\delta_1^5$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_1^6$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^3,\delta_2^6]=-\delta_1^7$; $[\delta_1^3,\delta_2^5]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_1^5$; $[\delta_1^3,\delta_2^3]=-(\delta_2^7+\delta_1^4)$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_2^7-\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle\delta_1^4\rangle+e^{4\beta_1}\ \langle\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^2,\delta_2^5\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^4]=-2\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $4e^{-4\beta_1}\ \langle x_1\omega^{1,2}-x_3(\omega^{1,6}-3\omega^{2,3})+2x_4\omega^{2,4}+x_5\omega^{2,5},-x_1\omega^{1,2}+2x_3(\omega^{1,6}-\omega^{2,3})-x_4\omega^{2,4}+x_6\omega^{2,6},-x_3\omega^{2,3}+x_4\omega^{1,7},x_2\omega^{1,5}+x_7\omega^{4,5}\rangle+3e^{-5\beta_1}\ \langle x_1\omega^{1,3}+x_2(\omega^{1,6}+2\omega^{2,3})+3x_3\omega^{2,4}+x_4(\omega^{2,5}+2\omega^{3,4})+x_5\omega^{3,5},x_3(\omega^{1,7}+\omega^{2,4})+x_4\omega^{2,5}+x_5\omega^{2,6}+x_6\omega^{2,7},-x_1\omega^{1,3}-2x_2(\omega^{1,6}-1/2\omega^{2,3})+x_4(\omega^{2,5}-\omega^{3,4})+x_5\omega^{2,6}+x_6\omega^{3,6}\rangle+3e^{-2\beta_1}\ \langle x_3\omega^{1,4}+x_5\omega^{2,3}+x_6\omega^{2,4},x_4\omega^{1,5},x_2\omega^{1,3}\rangle+2e^{-3\beta_1}\ \langle -x_2\omega^{1,4}+x_4\omega^{2,3}+x_5\omega^{2,4}+x_6\omega^{3,4},x_4\omega^{1,6}\rangle+2e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_6\omega^{2,3},x_4\omega^{1,4}\rangle+e^{-6\beta_1}\ \langle -2x_1\omega^{1,4}-1/5x_2(\omega^{1,7}+\omega^{2,4})-7/5x_3(\omega^{2,5}+4/7\omega^{3,4})+7/5x_4(\omega^{2,6}-2\omega^{3,5})+6/5x_5(\omega^{2,7}+1/6\omega^{3,6}-5/2\omega^{4,5})+6/5x_6(\omega^{3,7}-5/6\omega^{4,6})+x_7\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-7\beta_1}+2e^{-8\beta_1}+4e^{-9\beta_1}+5e^{-10\beta_1}+5e^{-11\beta_1}+3e^{-12\beta_1}+2e^{-13\beta_1}+e^{-14\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-14\beta_1}+3e^{-15\beta_1}+4e^{-16\beta_1}+4e^{-17\beta_1}+5e^{-18\beta_1}+3e^{-19\beta_1}+e^{-20\beta_1}+e^{-21\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-22\beta_1}+3e^{-23\beta_1}+2e^{-24\beta_1}+2e^{-25\beta_1}+3e^{-26\beta_1}+e^{-27\beta_1}+e^{-29\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-27\beta_1}+e^{-29\beta_1}+e^{-30\beta_1}+e^{-31\beta_1}+e^{-32\beta_1}+e^{-35\beta_1}+e^{-38\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-36\beta_1}+e^{-39\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,52,118,149,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,15,23,22,14,7,2)$

S=$40\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,18,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,6,6,4,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}$; $d\omega^7=-\omega^{1,6}-\omega^{2,3}$; $d\omega^{1,7}=\omega^{1,2,3}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,3,5}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,4,6}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{2,3,5}$; $d\omega^{6,7}=-\omega^{1,5,7}+\omega^{2,3,6}$; $d\omega^{1,4,7}=-\omega^{1,2,3,4}$; $d\omega^{1,5,7}=-\omega^{1,2,3,5}$; $d\omega^{1,6,7}=-\omega^{1,2,3,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,4,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,5,6}$; $d\omega^{1,4,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-4\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-9\beta_1} \left\langle\omega^{1,6}\right\rangle + e^{-10\beta_1} \left\langle\omega^{1,7} + \omega^{2,4}\right\rangle + e^{-11\beta_1} \left\langle\omega^{2,5} - \omega^{3,4}\right\rangle + e^{-13\beta_1} \left\langle\omega^{2,7} - \omega^{3,6} + \omega^{4,5}\right\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-14\beta_1} \left\langle\omega^{1,2,7}\right\rangle + e^{-15\beta_1} \left\langle\omega^{1,3,7}\right\rangle + e^{-16\beta_1} \left\langle\omega^{1,4,7} - \omega^{2,3,5}\right\rangle + e^{-17\beta_1} \left\langle\omega^{1,5,7} - \omega^{2,4,5}\right\rangle + 2e^{-18\beta_1} \left\langle\omega^{1,6,7} - \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\right\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-22\beta_1} \left\langle\omega^{1,2,6,7}, \omega^{1,4,5,6}\right\rangle + e^{-23\beta_1} \left\langle\omega^{1,3,6,7}\right\rangle + e^{-24\beta_1} \left\langle\omega^{2,3,4,7} - \omega^{2,3,5,6}\right\rangle + e^{-25\beta_1} \left\langle\omega^{1,5,6,7} + \omega^{2,4,5,6}\right\rangle + e^{-26\beta_1} \left\langle\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}\right\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-27\beta_1} \left\langle\omega^{1,2,3,6,7}\right\rangle + e^{-29\beta_1} \left\langle\omega^{1,2,5,6,7}\right\rangle + e^{-30\beta_1} \left\langle\omega^{1,3,5,6,7}\right\rangle + e^{-31\beta_1} \left\langle\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}\right\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-36\beta_1} \left\langle\omega^{1,3,4,5,6,7}\right\rangle + e^{-39\beta_1} \left\langle\omega^{2,3,4,5,6,7}\right\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-40\beta_1} \left\langle\omega^{1,2,3,4,5,6,7}\right\rangle$.

$P(Y) = (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 9, 18, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 4, 6, 6, 4, 2, 1)$

Total dimension of trivial cohomology: 26.

Length associated polynomial: 26.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,7} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,7} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7} - \omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} - \omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$;

$d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,4}$; $\Delta\omega^{2,3} = \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,7} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{1,5,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,3,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,5,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,5,7} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,6,7} + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,6,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,5,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{2,4,5,7}$;

$\Delta\omega^{2,4,5,6} = -\omega^{1,5,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-4\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-9\beta_1} \langle\omega^{1,6} - \omega^{2,3}\rangle + e^{-10\beta_1} \langle\omega^{1,7} + \omega^{2,4}\rangle + e^{-11\beta_1} \langle\omega^{2,5} - \omega^{3,4}\rangle + e^{-13\beta_1} \langle\omega^{2,7} - \omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-14\beta_1} \langle\omega^{1,2,7} - \omega^{1,3,6} + \omega^{1,4,5}\rangle + e^{-15\beta_1} \langle\omega^{1,3,7} + \omega^{2,3,4}\rangle + e^{-16\beta_1} \langle\omega^{1,4,7} - 2\omega^{1,5,6} - \omega^{2,3,5}\rangle + e^{-17\beta_1} \langle\omega^{1,5,7} + \omega^{2,3,6} - 2\omega^{2,4,5}\rangle + 2e^{-18\beta_1} \langle\omega^{1,6,7} - \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-26\beta_1} \langle(\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6})/3\rangle + e^{-25\beta_1} \langle(-\omega^{1,5,6,7} - \omega^{2,4,5,6})/2\rangle + e^{-24\beta_1} \langle(-\omega^{1,4,6,7} - 2\omega^{2,3,4,7} + \omega^{2,3,5,6})/6\rangle + e^{-23\beta_1} \langle(2\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6})/6\rangle + 2e^{-22\beta_1} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} - 2\omega^{1,4,5,6} + 3\omega^{2,3,4,5})/5, (\omega^{1,2,6,7} - \omega^{1,3,5,7} + 3\omega^{1,4,5,6} - 2\omega^{2,3,4,5})/5\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-31\beta_1} \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-30\beta_1} \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-29\beta_1} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-27\beta_1} \langle(\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-39\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-36\beta_1} \langle -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-40\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

5.3.19 Algebra $\mathcal{G}_{7,1.7}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_1, x_5] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & (3\xi_1^1)/2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_1^2+\xi_2^3 & (5\xi_1^1)/2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4-\xi_1^3 & 2\xi_1^2+\xi_2^3 & (7\xi_1^1)/2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4+\xi_2^5 & \xi_3^5 & \xi_1^2+\xi_2^3 & 4\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4 + \delta_3^6, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = -(\delta_1^6 + \delta_2^7), ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T{=}\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{(3\beta_1)/2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + e^{(5\beta_1)/2}\ \langle x_4\rangle + e^{3\beta_1}\ \langle x_5\rangle + e^{(7\beta_1)/2}\ \langle x_6\rangle + e^{4\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{(7\beta_1)/2}\ \langle x_6\rangle + e^{4\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{2\beta_1}\ \langle \delta_1^5, \delta_2^6, \delta_3^7\rangle + 2e^{(5\beta_1)/2}\ \langle \delta_1^6, \delta_2^7\rangle + 3e^{(3\beta_1)/2}\ \langle \delta_3^6, \delta_1^4, \delta_2^5\rangle + 3e^{\beta_1}\ \langle \delta_1^3, \delta_2^4, \delta_3^5\rangle + e^{3\beta_1}\ \langle \delta_1^7\rangle + 2e^{\beta_1/2}\ \langle \delta_1^2, \delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_1^5, \delta_1^2] = -2\delta_1^6$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = -\delta_2^7 + \delta_1^6$; $[\delta_2^6, \delta_2^3] = -\delta_2^7$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_3^6, \delta_1^2] = -\delta_3^7$; $[\delta_3^6, \delta_2^3] = -\delta_3^7 + \delta_2^6$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_1^2] = -(\delta_3^6 + \delta_1^4)$; $[\delta_1^3, \delta_2^3] = -(\delta_3^6 + \delta_1^4)$; $[\delta_2^4, \delta_1^4] = \delta_2^7 + \delta_1^6$; $[\delta_2^4, \delta_2^5] = -\delta_2^7$; $[\delta_2^4, \delta_1^2] = \delta_3^6 + \delta_1^4$; $[\delta_2^4, \delta_2^3] = \delta_3^6 - \delta_2^5$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_1^2] = -\delta_3^7$; $[\delta_1^4, \delta_2^3] = -(\delta_3^7 + \delta_1^5)$; $[\delta_2^5, \delta_3^5] = -\delta_2^7$; $[\delta_2^5, \delta_1^2] = \delta_3^7 - 2\delta_2^6 + \delta_1^5$; $[\delta_2^5, \delta_2^3] = \delta_3^7 - \delta_2^6$; $[\delta_3^5, \delta_1^2] = -2\delta_3^6$; $[\delta_3^5, \delta_2^3] = -\delta_3^6 + \delta_2^5$; $[\delta_1^2, \delta_2^3] = -\delta_3^5 + \delta_2^4 - \delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $2e^{2\beta_1}\ \langle \delta_1^5, \delta_3^7\rangle + 2e^{\beta_1}\ \langle \delta_1^3, \delta_2^4\rangle + e^{(5\beta_1)/2}\ \langle \delta_1^6\rangle + 2e^{(3\beta_1)/2}\ \langle \delta_1^4, \delta_2^5\rangle + 2e^{\beta_1/2}\ \langle \delta_1^2, \delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_1^3] = 2\delta_1^4$; $[\delta_1^2, \delta_2^5] = -\delta_3^7 - 3\delta_1^5$; $[\delta_1^2, \delta_2^3] = 2\delta_2^4 - \delta_1^3$; $[\delta_1^2, \delta_1^5] = 2\delta_1^6$; $[\delta_1^2, \delta_1^4] = \delta_3^7$; $[\delta_1^2, \delta_2^4] = -2\delta_1^4$; $[\delta_1^3, \delta_2^3] = -2\delta_1^4$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$; $[\delta_1^3, \delta_2^4] = \delta_1^5$; $[\delta_2^5, \delta_2^3] = \delta_3^7 + \delta_1^5$; $[\delta_2^5, \delta_2^4] = -\delta_1^6$; $[\delta_2^3, \delta_1^5] = \delta_1^6$; $[\delta_2^3, \delta_1^4] = \delta_3^7 + \delta_1^5$; $[\delta_2^3, \delta_3^7] = \delta_1^6$; $[\delta_2^3, \delta_2^4] = \delta_2^5 - \delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $6e^{-2\beta_1}\ \langle -x_2(\omega^{1,4} - \omega^{2,3}) + 2x_3\omega^{1,5} + 2x_4(\omega^{2,5} + 1/2\omega^{3,4}), x_3(\omega^{1,5} + \omega^{2,4}) + x_4(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{2,6}, -x_3\omega^{2,4} + x_5\omega^{1,7}, x_1\omega^{1,3} + x_2\omega^{1,4} + x_3\omega^{1,5} + x_4\omega^{2,5} +$

$x_5\omega^{3,5}, -x_1\omega^{1,3} - x_2(\omega^{1,4}+\omega^{2,3}) - 2x_3\omega^{1,5} - 2x_4\omega^{2,5} + x_7\omega^{3,7}, x_2\omega^{1,4} + x_6\omega^{3,6}\rangle +$ $4e^{-5/2\beta_1}\ \langle -x_1(\omega^{1,4}-\omega^{2,3}) - x_2\omega^{1,5} - x_3(\omega^{1,6}-\omega^{3,4}) - x_4(\omega^{2,6}-\omega^{3,5}), x_3(\omega^{1,6}+$ $\omega^{2,5}) + x_4(\omega^{1,7}+\omega^{2,6}) + x_5\omega^{2,7}, x_1\omega^{1,4} + 2x_2\omega^{2,4} + 3x_3(\omega^{1,6}+1/3\omega^{2,5}-2/3\omega^{3,4})+$ $3x_4\omega^{2,6} + x_5\omega^{4,5}, x_1\omega^{1,4} + x_2(\omega^{1,5}+\omega^{2,4}) + x_3(\omega^{1,6}-\omega^{3,4}) + 2x_4\omega^{2,6} + x_6(\omega^{3,7}+$ $\omega^{4,6})\rangle + 7e^{-3/2\beta_1}\ \langle x_1\omega^{1,2} - x_3(\omega^{1,4}-2\omega^{2,3}) + x_5\omega^{2,5}, -x_3\omega^{2,3} + x_5\omega^{1,6}, -x_1\omega^{1,2}+$ $x_3(\omega^{1,4}-\omega^{2,3}) + x_4\omega^{1,5}, x_3\omega^{1,4} + x_6\omega^{1,7}, -x_2\omega^{1,3} + x_7\omega^{3,6}, x_3\omega^{1,4} + x_4\omega^{2,4}, x_2\omega^{1,3}-$ $x_3\omega^{2,3} + x_5\omega^{3,4}\rangle + e^{-3\beta_1}\ \langle 5/3x_1(\omega^{1,5} + 3/5\omega^{2,4}) + x_2(\omega^{1,6} + 2/3\omega^{2,5} -$ $1/3\omega^{3,4}) - x_3(\omega^{1,7}+\omega^{2,6}+5/3\omega^{3,5}) + 2x_4(\omega^{2,7}+1/6\omega^{4,5}) - x_5\omega^{4,6} + 3x_6(\omega^{4,7}+$ $1/3\omega^{5,6})\rangle + 2e^{-1/2\beta_1}\ \langle x_3\omega^{1,2}, x_4\omega^{1,3} + x_5\omega^{2,3}\rangle + 5e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3\omega^{1,3} +$ $x_5\omega^{2,4}, x_5\omega^{1,5}, x_3\omega^{1,3} + x_6\omega^{1,6}, x_3\omega^{1,3} + x_4\omega^{2,3}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=3e^{-7\beta_1/2} + 10e^{-9\beta_1/2} + 7e^{-11\beta_1/2} + 7e^{-4\beta_1} + 10e^{-5\beta_1} + 3e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-13\beta_1/2} + 10e^{-15\beta_1/2} + 7e^{-17\beta_1/2} + e^{-19\beta_1/2} + e^{-6\beta_1} + 7e^{-7\beta_1} +$ $10e^{-8\beta_1} + 4e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-19\beta_1/2} + 6e^{-21\beta_1/2} + 5e^{-23\beta_1/2} + e^{-25\beta_1/2} + e^{-9\beta_1} + 4e^{-10\beta_1} +$ $7e^{-11\beta_1} + 3e^{-12\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-25\beta_1/2} + 2e^{-27\beta_1/2} + 2e^{-29\beta_1/2} + e^{-31\beta_1/2} + e^{-12\beta_1} + 2e^{-13\beta_1} +$ $2e^{-14\beta_1} + e^{-15\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-31\beta_1/2} + e^{-33\beta_1/2} + e^{-16\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 59, 128, 159, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 25, 40, 42, 29, 13, 3)$

S=$(35\beta_1)/2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$;

$d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{a}})= e^{-\beta_1} \langle\omega^1\rangle + e^{(-3\beta_1)/2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{(-7\beta_1)/2} \langle\omega^{1,4}\rangle + e^{-4\beta_1} \langle\omega^{1,5}\rangle + 2e^{-5\beta_1} \langle\omega^{1,7} + \omega^{2,6}, \omega^{3,5}\rangle + 2e^{(-9\beta_1)/2} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{(-11\beta_1)/2} \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= 2e^{(-13\beta_1)/2} \langle\omega^{1,3,6}, \omega^{1,2,7}\rangle + 3e^{(-15\beta_1)/2} \langle\omega^{1,4,7}, \omega^{1,5,6} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-6\beta_1} \langle\omega^{1,2,6}\rangle + 2e^{-7\beta_1} \langle\omega^{1,4,6} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle + 2e^{-8\beta_1} \langle\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}, \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{(-17\beta_1)/2} \langle\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= 3e^{-10\beta_1} \langle\omega^{1,2,6,7}, \omega^{1,3,5,7} + \omega^{2,3,5,6}, \omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle + e^{-9\beta_1} \langle\omega^{1,2,4,7}\rangle + 2e^{(-19\beta_1)/2} \langle\omega^{1,2,5,7}, \omega^{1,3,5,6}\rangle + 2e^{(-21\beta_1)/2} \langle\omega^{1,4,5,7}, \omega^{1,3,6,7} + \omega^{2,4,5,6}\rangle + 2e^{-11\beta_1} \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}, \omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}\rangle + e^{(-23\beta_1)/2} \langle\omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= 2e^{(-25\beta_1)/2} \langle\omega^{1,2,4,6,7}, \omega^{1,3,4,5,7}\rangle + 2e^{-13\beta_1} \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-12\beta_1} \langle\omega^{1,2,3,6,7}\rangle + e^{(-27\beta_1)/2} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-14\beta_1} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{(-31\beta_1)/2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-16\beta_1} \langle\omega^{1,3,4,5,6,7}\rangle + e^{(-33\beta_1)/2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{(-35\beta_1)/2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 44.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7} - \omega^{2,6}$;

$d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} - \omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} - \omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,4,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} +$

$2\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} - \omega^{3,5,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + 2\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{(-3\beta_1)/2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{(-7\beta_1)/2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-4\beta_1}\ \langle\omega^{1,5} - \omega^{2,4}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,7} + \omega^{2,6}, \omega^{3,5}\rangle + 2e^{(-9\beta_1)/2}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{(-11\beta_1)/2}\ \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-6\beta_1}\ \langle\omega^{1,2,6} - \omega^{1,3,5} + \omega^{2,3,4}\rangle + 2e^{(-13\beta_1)/2}\ \langle\omega^{1,2,7} - \omega^{1,4,5} + \omega^{2,3,5}, \omega^{1,3,6} + \omega^{2,3,5}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,4,6} - \omega^{2,3,6}, \omega^{1,3,7} + 2\omega^{2,3,6} + \omega^{2,4,5}\rangle + 3e^{(-15\beta_1)/2}\ \langle\omega^{1,4,7} + \omega^{2,4,6}, \omega^{1,5,6} - \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,4,5}\rangle + 2e^{-8\beta_1}\ \langle\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}, \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{(-17\beta_1)/2}\ \langle\omega^{2,5,7} + \omega^{3,4,7} +$

$\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{(-23\beta_1)/2}\ \langle(-\omega^{1,5,6,7}+\omega^{2,4,6,7}-\omega^{3,4,5,7})/3\rangle + 2e^{-11\beta_1}\ \langle(\omega^{1,4,6,7}-2\omega^{2,3,6,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6})/5, (2\omega^{1,4,6,7}+\omega^{2,3,6,7}+3\omega^{2,4,5,7}-\omega^{3,4,5,6})/5\rangle + 2e^{(-21\beta_1)/2}\ \langle(-\omega^{1,3,6,7}+\omega^{1,4,5,7}-3\omega^{2,3,5,7}-\omega^{2,4,5,6})/4, (-\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6})/4\rangle + 3e^{-10\beta_1}\ \langle(\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}+3\omega^{2,3,5,6})/4, (4\omega^{1,2,6,7}-3\omega^{1,3,5,7}-\omega^{1,4,5,6}+5\omega^{2,3,4,7}+3\omega^{2,3,5,6})/12, (4\omega^{1,2,6,7}+3\omega^{1,3,5,7}+5\omega^{1,4,5,6}-\omega^{2,3,4,7}-3\omega^{2,3,5,6})/12\rangle + 2e^{(-19\beta_1)/2}\ \langle(-\omega^{1,2,5,7}-\omega^{1,3,4,7}+2\omega^{1,3,5,6}-3\omega^{2,3,4,6})/5, (-\omega^{1,2,5,7}-\omega^{1,3,4,7}-3\omega^{1,3,5,6}+2\omega^{2,3,4,6})/5\rangle + e^{-9\beta_1}\ \langle(\omega^{1,2,4,7}+\omega^{1,2,5,6}+\omega^{1,3,4,6})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-14\beta_1}\ \langle(-\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7})/2\rangle + e^{(-27\beta_1)/2}\ \langle(\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7})/2\rangle + 2e^{(-25\beta_1)/2}\ \langle(-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6})/2, -\omega^{1,2,4,6,7}\rangle + 2e^{-13\beta_1}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7}+2\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/3\rangle + e^{-12\beta_1}\ \langle(-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{(-33\beta_1)/2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-16\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{(-31\beta_1)/2}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{(-35\beta_1)/2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+5Y^2+6Y^3+5Y^4+2Y^5+Y^6)(1+Y)$.

5.3.20 Algebra $\mathcal{G}_{7,1.8}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (3\xi_1^1)/2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & (7\xi_1^1)/2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_1^5 & \xi_2^4 & 0 & 4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4+\xi_2^6 & -\xi_2^5 & \xi_2^4 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{(3\beta_1)/2}\ \langle x_3\rangle+e^{3\beta_1}\ \langle x_4\rangle+e^{(7\beta_1)/2}\ \langle x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{2\beta_1}\ \langle \delta_3^5,\delta_1^4,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7\rangle+e^{(5\beta_1)/2}\ \langle\delta_1^5\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+e^{(7\beta_1)/2}\ \langle\delta_3^7\rangle+e^{(3\beta_1)/2}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_2^5]=\delta_3^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^5]=\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_3^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^7+\delta_1^6)$.

ch($H^1_{\mathcal{G}}$)= $e^{(5\beta_1)/2}\ \langle\delta_1^5\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_1^4,\delta_2^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1}\ \langle x_3\omega^{2,5}\rangle+3e^{-\beta_1}\ \langle 2x_2\omega^{1,2}-x_3\omega^{1,3}-2x_4\omega^{1,4}+x_5\omega^{1,5},-x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}+x_6\omega^{3,5},x_4\omega^{1,4}+x_6\omega^{2,4}\rangle+e^{-3\beta_1}\ \langle x_1\omega^{1,4}+x_2(\omega^{1,6}-\omega^{2,4})-x_3(\omega^{1,5}-\omega^{3,4})+x_5(\omega^{3,7}+\omega^{4,5})\rangle+e^{-7/2\beta_1}\ \langle -x_1(\omega^{1,5}-\omega^{3,4})+2x_2\omega^{2,5}-x_3(\omega^{1,6}-3\omega^{2,4}+2\omega^{3,5})+x_4\omega^{4,5}+x_5\omega^{4,6}\rangle+4e^{-3/2\beta_1}\ \langle x_3\omega^{1,2}+x_6\omega^{2,5},x_1\omega^{1,3}+2x_2\omega^{2,3}+5x_3\omega^{1,2}-5x_4(\omega^{1,5}-2/5\omega^{3,4})+x_6\omega^{3,6},-x_1\omega^{1,3}-x_2\omega^{2,3}-3x_3\omega^{1,2}+3x_4(\omega^{1,5}-1/3\omega^{3,4})+x_5\omega^{1,6},-x_2\omega^{2,3}-x_3\omega^{1,2}+x_4\omega^{1,5}+x_5\omega^{2,4}\rangle+3e^{-2\beta_1}\ \langle -x_1\omega^{1,2}+2x_4\omega^{1,6}+x_6\omega^{2,6},x_1\omega^{1,2}-x_3\omega^{2,3}-x_4(\omega^{1,6}+\omega^{3,5})+x_6\omega^{1,7},x_1\omega^{1,2}-x_4(\omega^{1,6}-\omega^{2,4})\rangle+e^{-5/2\beta_1}\ \langle -x_1\omega^{2,3}-x_2(\omega^{1,5}-\omega^{3,4})-x_3\omega^{1,4}+x_4\omega^{3,6}+x_5(\omega^{1,7}+2\omega^{2,6})+x_6\omega^{3,7}\rangle+e^{-1/2\beta_1}\ \langle -x_2\omega^{1,3}+x_5\omega^{1,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1/2}+e^{-7\beta_1/2}+4e^{-9\beta_1/2}+5e^{-11\beta_1/2}+3e^{-13\beta_1/2}+e^{-15\beta_1/2}+e^{-4\beta_1}+5e^{-5\beta_1}+2e^{-6\beta_1}+2e^{-7\beta_1}+e^{-8\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-13\beta_1/2}+2e^{-15\beta_1/2}+5e^{-17\beta_1/2}+4e^{-19\beta_1/2}+3e^{-21\beta_1/2}+e^{-23\beta_1/2}+4e^{-8\beta_1}+3e^{-9\beta_1}+3e^{-10\beta_1}+e^{-11\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$3e^{-23\beta_1/2}+2e^{-25\beta_1/2}+4e^{-27\beta_1/2}+e^{-29\beta_1/2}+e^{-33\beta_1/2}+2e^{-11\beta_1}+2e^{-12\beta_1}+2e^{-13\beta_1}+2e^{-14\beta_1}+e^{-16\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-29\beta_1/2}+e^{-31\beta_1/2}+e^{-33\beta_1/2}+2e^{-35\beta_1/2}+2e^{-15\beta_1}+e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-37\beta_1/2}+e^{-18\beta_1}+e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,53,120,152,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,15,26,27,20,11,3)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,18,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,6,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,4}-\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}+\omega^{1,3,5}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{3,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{2,4,6}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}+\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle+e^{(-3\beta_1)/2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{(-5\beta_1)/2}\ \langle\omega^{1,3}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,6},\omega^{2,4}\rangle + e^{(-11\beta_1)/2}\ \langle\omega^{2,5}\rangle + e^{(-9\beta_1)/2}\ \langle\omega^{1,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{(-13\beta_1)/2}\ \langle\omega^{1,2,5}\rangle + e^{(-15\beta_1)/2}\ \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-8\beta_1}\ \langle\omega^{1,2,7}+\omega^{3,4,5}\rangle + 2e^{(-17\beta_1)/2}\ \langle\omega^{1,5,6}-\omega^{3,4,6},\omega^{2,3,7}+\omega^{3,4,6}\rangle + e^{-9\beta_1}\ \langle\omega^{1,4,7}-\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-11\beta_1}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6}\rangle + 2e^{(-23\beta_1)/2}\ \langle\omega^{1,3,6,7}-\omega^{2,3,4,7},\omega^{1,2,5,7}\rangle + e^{-12\beta_1}\ \langle\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{(-25\beta_1)/2}\ \langle\omega^{1,4,5,7}-\omega^{2,3,6,7}\rangle + e^{(-27\beta_1)/2}\ \langle\omega^{1,5,6,7}+\omega^{2,4,5,7}-\omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-15\beta_1}\ \langle\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7},\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7}\rangle + e^{(-29\beta_1)/2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{(-31\beta_1)/2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{(-35\beta_1)/2}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{(-37\beta_1)/2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-18\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,9,18,23,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,5,6,6,5,3,1)$

Total dimension of trivial cohomology: 30.

Length associated polynomial: 30.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{3,5}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,5}=-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}$; $d^\star\omega^{1,3,5}=\omega^{1,7}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=\omega^{2,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{4,7}$; $d^\star\omega^{3,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,4,7}+\omega^{3,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,4,7}-\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,2,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=$

$\omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7}+\omega^{1,3,5,6,7}-\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,6} + \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,5} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,5} + \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}+\omega^{2,4,7}+\omega^{3,5,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5}+\omega^{1,3,6}+2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{2,4,5} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + 2\omega^{2,4,7} + \omega^{3,5,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{2,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{2,4,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,5} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}-\omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}+\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,4,6} + 2\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + \omega^{1,3,6,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}-\omega^{2,4,5,7}+\omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} - \omega^{1,3,5,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,6,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7} + \omega^{2,4,5,7}$;

$\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{2,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{(-3\beta_1)/2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{(-5\beta_1)/2} \langle\omega^{1,3}\rangle + e^{(-11\beta_1)/2} \langle\omega^{2,5}\rangle + e^{(-9\beta_1)/2} \langle\omega^{1,5} - \omega^{3,4}\rangle + 2e^{-5\beta_1} \langle\omega^{1,6} - \omega^{3,5}, \omega^{2,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{(-13\beta_1)/2} \langle\omega^{1,2,5} - \omega^{1,3,6} + \omega^{2,3,4}\rangle + e^{(-15\beta_1)/2} \langle 2\omega^{1,3,7} + \omega^{1,4,5} + \omega^{2,3,6}\rangle + e^{-8\beta_1} \langle\omega^{1,2,7} - 2\omega^{1,4,6} + \omega^{3,4,5}\rangle + 2e^{(-17\beta_1)/2} \langle\omega^{1,5,6} + \omega^{2,4,5} - \omega^{3,4,6}, \omega^{2,3,7} + \omega^{2,4,5} + \omega^{3,4,6}\rangle + e^{-9\beta_1} \langle\omega^{1,4,7} + 2\omega^{2,4,6} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{(-27\beta_1)/2} \langle(-\omega^{1,5,6,7} - \omega^{2,4,5,7} + \omega^{3,4,6,7})/3\rangle + e^{(-25\beta_1)/2} \langle(-\omega^{1,4,5,7} + \omega^{2,3,6,7} - 2\omega^{2,4,5,6})/6\rangle + e^{-12\beta_1} \langle(\omega^{1,2,6,7} + 2\omega^{2,3,5,7} + \omega^{3,4,5,6})/6\rangle + 2e^{(-23\beta_1)/2} \langle(\omega^{1,2,5,7} - \omega^{1,3,6,7} + \omega^{2,3,4,7})/3, (-\omega^{1,2,5,7} - \omega^{1,3,6,7} + \omega^{1,4,5,6})/3\rangle + e^{-11\beta_1} \langle(-\omega^{1,2,4,7} + 2\omega^{1,3,5,7} + \omega^{2,3,5,6})/6\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{(-35\beta_1)/2} \langle-\omega^{2,4,5,6,7}\rangle + e^{(-29\beta_1)/2} \langle\omega^{1,3,4,6,7}\rangle + e^{(-31\beta_1)/2} \langle(-\omega^{1,2,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + 2e^{-15\beta_1} \langle(\omega^{1,2,4,6,7} + \omega^{1,3,5,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,4,6,7} - 2\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-19\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-18\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{(-37\beta_1)/2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-20\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.3.21 Algebra $\mathcal{G}_{7,1.9}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_6$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_7$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & (3\xi_1^1)/2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & (5\xi_1^1)/2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & (7\xi_1^1)/2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^3-\xi_1^4 & \xi_1^2 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5-\xi_1^3 & \xi_2^4 & 0 & (9\xi_1^1)/2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^6, ad\ x_2=-\delta_1^4+\delta_3^7, ad\ x_3=-(\delta_1^6+\delta_2^7), ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{(3\beta_1)/2}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{(5\beta_1)/2}\ \langle x_4\rangle+e^{(7\beta_1)/2}\ \langle x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{(9\beta_1)/2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{4\beta_1}\ \langle x_6\rangle+e^{(9\beta_1)/2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{(5\beta_1)/2}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{3\beta_1}\ \langle\delta_1^6,\delta_2^7\rangle+2e^{\beta_1}\ \langle\delta_3^6,\delta_2^4\rangle+3e^{(3\beta_1)/2}\ \langle\delta_2^3,\delta_1^4,\delta_3^7\rangle+e^{\beta_1/2}\ \langle\delta_1^2\rangle+e^{(7\beta_1)/2}\ \langle\delta_1^7\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_3^6,\delta_2^3]=\delta_2^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_2^3,\delta_1^4]=\delta_1^6$; $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_2^3,\delta_2^4]=\delta_2^6$; $[\delta_1^4,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_1^3]=-\delta_1^5$; $[\delta_1^2,\delta_2^5]=\delta_2^6-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^7-\delta_1^4$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{(5\beta_1)/2}\ \langle\delta_1^5\rangle+2e^{(3\beta_1)/2}\ \langle\delta_2^3,\delta_1^4\rangle+e^{\beta_1/2}\ \langle\delta_1^2\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_1^2]=\delta_1^5$; $[\delta_1^3,\delta_2^4]=\delta_1^6$; $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_2^3,\delta_1^4]=\delta_1^6$; $[\delta_2^3,\delta_2^4]=-\delta_1^5$; $[\delta_2^5,\delta_1^2]=2\delta_1^5$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=5e^{-3/2\beta_1}\ \langle -x_1\omega^{1,2}+2x_3(\omega^{1,5}-1/2\omega^{2,3})+x_5\omega^{2,5}, x_1\omega^{1,2}-x_3(\omega^{1,5}-\omega^{2,3})+x_5\omega^{1,6}, -x_3\omega^{2,3}+x_6\omega^{1,7}, x_3\omega^{2,3}+x_4\omega^{1,3}, x_1\omega^{1,2}-x_3(\omega^{1,5}-\omega^{2,3})+x_4\omega^{2,4}\rangle+4e^{-2\beta_1}\ \langle x_3(\omega^{1,6}+\omega^{2,5})+x_4(\omega^{1,5}+\omega^{2,3})+x_5\omega^{2,6}, -x_4\omega^{2,3}+x_5\omega^{1,7}, -x_2\omega^{1,4}-x_4\omega^{2,3}+x_5\omega^{3,4}, -x_2\omega^{1,4}-x_3(\omega^{1,6}+\omega^{2,5})-x_4\omega^{1,5}+x_7\omega^{4,6}\rangle+4e^{-5/2\beta_1}\ \langle -x_2(\omega^{1,3}-\omega^{2,4})+x_3(\omega^{1,7}-\omega^{3,4})+x_4\omega^{2,5}+x_5\omega^{2,7}, x_1\omega^{1,4}+x_2\omega^{1,3}+$

$x_4\omega^{2,5} + x_5\omega^{4,5}, x_2(\omega^{1,3} - \omega^{2,4}) + x_3(\omega^{2,6} + \omega^{3,4}) + x_4\omega^{1,6}, -x_1\omega^{1,4} - 6x_2(\omega^{1,3} - 1/6\omega^{2,4}) - x_3(\omega^{1,7} + 3\omega^{2,6} + 2\omega^{3,4}) - 3x_6\omega^{4,6} + x_7\omega^{4,7}\rangle + 3e^{-3\beta_1}\ \langle -2/3x_1(\omega^{1,3} - 5/2\omega^{2,4}) - 5/3x_2(\omega^{1,5} - 6/5\omega^{2,3}) - 5/3x_3(\omega^{2,7} - \omega^{4,5}) - 5/3x_4(\omega^{1,7} - 1/5\omega^{3,4}) + x_5\omega^{3,5}, x_1\omega^{2,4} + 2x_2\omega^{2,3} - x_3(\omega^{2,7} - \omega^{4,5}) - 2x_4\omega^{1,7} + x_6\omega^{4,7}, 1/3x_1(\omega^{1,3} - \omega^{2,4}) + 1/3x_2(\omega^{1,5} - 3\omega^{2,3}) + 1/3x_3(\omega^{2,7} - \omega^{4,5}) + 1/3x_4(\omega^{1,7} + \omega^{3,4}) + x_6\omega^{3,6}\rangle + e^{-4\beta_1}\ \langle x_2(\omega^{1,7} - \omega^{3,4}) - x_4\omega^{3,5} - x_5\omega^{3,7} + x_6\omega^{5,7}\rangle + 5e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_3\omega^{2,4} + x_5\omega^{2,3}, x_5\omega^{1,5}, -x_2\omega^{1,2} + x_3(\omega^{1,3} - \omega^{2,4}), -x_3\omega^{2,4} + x_6\omega^{1,6}, x_3\omega^{2,4} + x_4\omega^{1,4}\rangle + e^{-1/2\beta_1}\ \langle x_5\omega^{1,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{-7\beta_1/2} + 5e^{-9\beta_1/2} + 8e^{-11\beta_1/2} + 3e^{-13\beta_1/2} + 4e^{-4\beta_1} + 7e^{-5\beta_1} + 6e^{-6\beta_1} + e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = 3e^{-15\beta_1/2} + 7e^{-17\beta_1/2} + 6e^{-19\beta_1/2} + 2e^{-21\beta_1/2} + e^{-7\beta_1} + 5e^{-8\beta_1} + 7e^{-9\beta_1} + 3e^{-10\beta_1} + e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-21\beta_1/2} + 4e^{-23\beta_1/2} + 3e^{-25\beta_1/2} + 3e^{-27\beta_1/2} + e^{-29\beta_1/2} + 3e^{-11\beta_1} + 4e^{-12\beta_1} + 4e^{-13\beta_1} + 2e^{-14\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = 2e^{-29\beta_1/2} + 2e^{-31\beta_1/2} + e^{-35\beta_1/2} + 2e^{-14\beta_1} + e^{-15\beta_1} + 2e^{-16\beta_1} + e^{-17\beta_1} + e^{-18\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-37\beta_1/2} + e^{-17\beta_1} + e^{-19\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 58, 124, 156, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 23, 35, 35, 25, 12, 3)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 20, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 9, 9, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{5,6} = \omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} + \omega^{2,3,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} =$

$-\omega^{1,2,6,7}+\omega^{1,3,4,7}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d\omega^{5,6,7}=\omega^{1,3,5,7}-\omega^{1,4,6,7}-\omega^{2,3,5,6}+\omega^{2,4,5,7}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,5,6}-\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7}=\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,3,5,7}-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{(-3\beta_1)/2}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{(-9\beta_1)/2}\ \langle\omega^{1,5}\rangle+e^{-4\beta_1}\ \langle\omega^{1,3}\rangle+e^{-5\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}\rangle+2e^{(-11\beta_1)/2}\ \langle\omega^{1,7}-\omega^{3,4},\omega^{2,6}+\omega^{3,4}\rangle+e^{-6\beta_1}\ \langle\omega^{2,7}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{(-17\beta_1)/2}\ \langle\omega^{1,3,7},\omega^{1,5,6}-\omega^{2,4,7}\rangle+e^{-7\beta_1}\ \langle\omega^{1,2,7}\rangle+e^{(-15\beta_1)/2}\ \langle\omega^{1,4,6}\rangle+2e^{-8\beta_1}\ \langle\omega^{1,3,6}-\omega^{2,4,6},\omega^{1,4,7}+\omega^{2,4,6}\rangle+2e^{-9\beta_1}\ \langle\omega^{1,5,7}+\omega^{3,4,5},\omega^{2,3,7}+\omega^{3,4,5}\rangle+e^{(-19\beta_1)/2}\ \langle\omega^{1,6,7}+\omega^{2,5,7}+\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 2e^{-11\beta_1}\ \langle\omega^{1,2,6,7},\omega^{1,4,5,6}\rangle+2e^{-12\beta_1}\ \langle\omega^{1,3,5,7},\omega^{1,4,6,7}+\omega^{2,3,5,6}\rangle+2e^{(-23\beta_1)/2}\ \langle\omega^{1,4,5,7}+\omega^{2,4,5,6},\omega^{1,3,5,6}-\omega^{2,4,5,6}\rangle+e^{(-21\beta_1)/2}\ \langle\omega^{1,2,5,7}\rangle+e^{(-25\beta_1)/2}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7}\rangle+e^{-13\beta_1}\ \langle\omega^{2,3,6,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{(-29\beta_1)/2}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,5,7}\rangle+e^{-14\beta_1}\ \langle\omega^{1,2,3,6,7}\rangle+e^{-15\beta_1}\ \langle\omega^{1,3,4,6,7}\rangle+e^{(-31\beta_1)/2}\ \langle\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle+e^{-16\beta_1}\ \langle\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-17\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle+e^{(-37\beta_1)/2}\ \langle\omega^{1,3,4,5,6,7}\rangle+e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+4Y^2+5Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,10,20,24,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,6,9,9,6,3,1)$

Total dimension of trivial cohomology: 38.

Length associated polynomial: 38.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^6$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^7$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{1,2,3}=\omega^{1,7}-\omega^{2,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}+\omega^{4,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{6,7}$;

$d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,5} + \omega^{2,4,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7} + \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7}-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,7} - \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5} - \omega^{4,6}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{2,7}+\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,5}+2\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,6}+3\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}-\omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}+\omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,6} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}-\omega^{1,4,7}-\omega^{2,3,5}+\omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}-\omega^{1,5,6}-\omega^{2,3,6}+\omega^{2,4,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,5} + \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + 2\omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 2\omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7}+\omega^{1,5,6}+\omega^{2,3,6}-\omega^{2,4,7}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7}+\omega^{2,3,7}-\omega^{2,5,6}-\omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} - \omega^{2,3,6} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,3,7} + 2\omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} =$

$-\omega^{2,6,7} + 2\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,6,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,6} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,5,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{1,4,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,5,7} + 2\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,7} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + 2\omega^{2,3,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{1,3,5,6} - \omega^{1,4,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7} + 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,5,6} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{(-3\beta_1)/2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{(-9\beta_1)/2}\ \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-4\beta_1}\ \langle\omega^{1,3} - \omega^{2,4}\rangle + e^{-5\beta_1}\ \langle\omega^{1,6} + \omega^{2,5}\rangle + 2e^{(-11\beta_1)/2}\ \langle\omega^{1,7} - \omega^{3,4}, \omega^{2,6} + \omega^{3,4}\rangle + e^{-6\beta_1}\ \langle\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-7\beta_1}\ \langle\omega^{1,2,7} - \omega^{1,4,5}\rangle + 2e^{(-17\beta_1)/2}\ \langle\omega^{1,3,7} + \omega^{2,3,6}, \omega^{1,5,6} - 2\omega^{2,3,6} - \omega^{2,4,7}\rangle + e^{(-15\beta_1)/2}\ \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + 2e^{-8\beta_1}\ \langle\omega^{1,3,6} - \omega^{2,4,6}, \omega^{1,4,7} + \omega^{2,3,5} + 2\omega^{2,4,6}\rangle + 2e^{-9\beta_1}\ \langle\omega^{1,5,7} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{(-19\beta_1)/2}\ \langle\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-13\beta_1} \langle(-\omega^{2,3,6,7} + \omega^{3,4,5,6})/2\rangle + 2e^{(-23\beta_1)/2} \langle(\omega^{1,3,5,6} - \omega^{1,4,5,7} + \omega^{2,3,4,7} - 3\omega^{2,4,5,6})/4, (\omega^{1,3,5,6} + \omega^{1,4,5,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6})/4\rangle + e^{(-25\beta_1)/2} \langle(-\omega^{1,3,6,7} - \omega^{2,3,5,7})/2\rangle + 2e^{-12\beta_1} \langle(-\omega^{1,3,5,7} + \omega^{1,4,6,7} + \omega^{2,3,5,6} + 3\omega^{2,4,5,7})/4, (\omega^{1,3,5,7}+\omega^{1,4,6,7}+\omega^{2,3,5,6}+\omega^{2,4,5,7})/4\rangle+2e^{-11\beta_1} \langle(\omega^{1,2,6,7}-\omega^{1,4,5,6}-2\omega^{2,3,4,6})/3, (\omega^{1,2,6,7} + 2\omega^{1,4,5,6} + \omega^{2,3,4,6})/3\rangle + e^{(-21\beta_1)/2} \langle(-\omega^{1,2,5,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{(-31\beta_1)/2} \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-16\beta_1} \langle(\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-15\beta_1} \langle(\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + 2e^{(-29\beta_1)/2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}-2\omega^{2,3,4,5,6})/3, (\omega^{1,2,5,6,7}-2\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6})/3\rangle+e^{-14\beta_1} \langle(-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-19\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{(-37\beta_1)/2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-17\beta_1} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-20\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 5Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y)$.

5.3.22 Algebra $\mathcal{G}_{7,1.10}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_5]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & (3\xi_1^1)/2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & (5\xi_1^1)/2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & (7\xi_1^1)/2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 4\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & (9\xi_1^1)/2 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & \xi_2^4 & -\xi_1^3 & \xi_2^3 & (11\xi_1^1)/2\end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T{=}\langle t_1\rangle$.

ch$(\mathcal{G})$= $e^{\beta_1}\ \langle x_1\rangle+e^{(3\beta_1)/2}\ \langle x_2\rangle+e^{(5\beta_1)/2}\ \langle x_3\rangle+e^{(7\beta_1)/2}\ \langle x_4\rangle+e^{4\beta_1}\ \langle x_5\rangle+e^{(9\beta_1)/2}\ \langle x_6\rangle+e^{(11\beta_1)/2}\ \langle x_7\rangle$.

ch$(H^0_{\mathcal{G}})$= $e^{(11\beta_1)/2}\ \langle x_7\rangle$.

ch$(Z^1_{\mathcal{G}})$= $2e^{(5\beta_1)/2}\ \langle\delta_1^4,\delta_2^5\rangle+e^{(7\beta_1)/2}\ \langle\delta_1^6\rangle+e^{(9\beta_1)/2}\ \langle\delta_1^7\rangle+e^{4\beta_1}\ \langle\delta_2^7\rangle+2e^{3\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{(3\beta_1)/2}\ \langle\delta_1^3\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_2^3]=-\delta_1^6$; $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^4,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$.

ch$(H^1_{\mathcal{G}})$= $e^{(5\beta_1)/2}\ \langle\delta_1^4\rangle+2e^{3\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch$(H^2_{\mathcal{G}})$= $3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+2x_3\omega^{1,3}+x_5\omega^{1,5}, x_4\omega^{1,4}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_6\omega^{2,5}\rangle+e^{-3\beta_1}\ \langle -x_3\omega^{2,5}+x_4\omega^{1,7}\rangle+2e^{-2\beta_1}\ \langle x_4\omega^{1,6}, x_2\omega^{1,3}+x_4\omega^{2,5}+x_6\omega^{3,5}\rangle+3e^{-3/2\beta_1}\ \langle x_1\omega^{1,2}+3x_3\omega^{2,3}+2x_4\omega^{2,4}+x_6\omega^{2,6}, -x_3\omega^{2,3}+x_4\omega^{1,5}, -x_1\omega^{1,2}-2x_3\omega^{2,3}-x_4\omega^{2,4}+x_5\omega^{1,6}\rangle+2e^{-5/2\beta_1}\ \langle x_3(\omega^{1,5}+\omega^{2,4})+x_4\omega^{2,6}+x_5\omega^{1,7}+x_6\omega^{2,7}, -2x_1\omega^{1,3}-x_2\omega^{2,3}-x_3(\omega^{1,5}+4\omega^{2,4})-x_4(\omega^{2,6}+3\omega^{3,4})-x_5\omega^{1,7}-x_6\omega^{3,6}+x_7\omega^{3,7}\rangle+e^{-1/2\beta_1}\ \langle -x_5\omega^{1,4}+x_7\omega^{3,4}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{-7\beta_1/2}+e^{-9\beta_1/2}+4e^{-11\beta_1/2}+5e^{-13\beta_1/2}+e^{-15\beta_1/2}+2e^{-5\beta_1}+3e^{-6\beta_1}+2e^{-7\beta_1}$.

ch$(H^4_{\mathcal{G}})$=$e^{-15\beta_1/2}+2e^{-19\beta_1/2}+4e^{-21\beta_1/2}+2e^{-23\beta_1/2}+e^{-8\beta_1}+3e^{-9\beta_1}+3e^{-10\beta_1}+4e^{-11\beta_1}+e^{-12\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$e^{-23\beta_1/2} + e^{-25\beta_1/2} + e^{-27\beta_1/2} + 2e^{-29\beta_1/2} + e^{-31\beta_1/2} + e^{-33\beta_1/2} + e^{-12\beta_1} + 2e^{-13\beta_1} + e^{-14\beta_1} + 3e^{-15\beta_1} + 2e^{-16\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-33\beta_1/2} + 2e^{-35\beta_1/2} + e^{-39\beta_1/2} + e^{-41\beta_1/2} + e^{-16\beta_1} + e^{-17\beta_1} + e^{-20\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-43\beta_1/2} + e^{-21\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 11, 50, 116, 150, 111, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 5, 12, 19, 21, 16, 8, 2)$

S=$(45\beta_1)/2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 9, 19, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 4, 7, 7, 4, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = -\omega^{1,5,6} - \omega^{2,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{2,5,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{(-3\beta_1)/2}$ $\langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-5\beta_1}$ $\langle\omega^{1,5} + \omega^{2,4}\rangle + e^{(-11\beta_1)/2}$ $\langle\omega^{1,6}\rangle + e^{-6\beta_1}$ $\langle\omega^{2,6} - \omega^{3,4}\rangle + e^{(-13\beta_1)/2}$ $\langle\omega^{1,7} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{(-15\beta_1)/2}\ \langle\omega^{1,3,5}\rangle + e^{-8\beta_1}\ \langle\omega^{1,2,7}\rangle + e^{-9\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,4,5}\rangle + e^{(-19\beta_1)/2}\ \langle\omega^{1,5,6}+\omega^{2,4,6}\rangle + e^{-11\beta_1}\ \langle\omega^{1,6,7}-\omega^{2,5,7}\rangle + e^{-10\beta_1}\ \langle\omega^{1,4,7}+\omega^{3,4,5}\rangle + e^{(-21\beta_1)/2}\ \langle\omega^{1,5,7}+\omega^{2,4,7}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{(-23\beta_1)/2}\ \langle\omega^{1,3,4,6}\rangle + e^{-12\beta_1}\ \langle\omega^{1,2,5,7}\rangle + e^{-13\beta_1}\ \langle\omega^{1,3,5,7}\rangle + e^{(-25\beta_1)/2}\ \langle\omega^{1,2,6,7}\rangle + e^{(-27\beta_1)/2}\ \langle\omega^{2,3,5,7}-\omega^{2,4,5,6}\rangle + e^{-15\beta_1}\ \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}\rangle + e^{(-29\beta_1)/2}\ \langle\omega^{1,4,6,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{(-33\beta_1)/2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-16\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-17\beta_1}\ \langle\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle + e^{(-35\beta_1)/2}\ \langle\omega^{1,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-21\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{(-43\beta_1)/2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{(-45\beta_1)/2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+3Y^2+4Y^3+3Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,19,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,7,7,4,2,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}-\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=\omega^{2,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$;

$d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + 2\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 3\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = 3\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 2\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} =$

$2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{(-3\beta_1)/2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{(-11\beta_1)/2} \langle\omega^{1,6} - \omega^{2,5}\rangle + e^{-6\beta_1} \langle\omega^{2,6} - \omega^{3,4}\rangle + e^{(-13\beta_1)/2} \langle\omega^{1,7} + \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{(-15\beta_1)/2} \langle\omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-8\beta_1} \langle\omega^{1,2,7} - \omega^{2,3,5}\rangle + e^{-9\beta_1} \langle\omega^{1,3,7} - 2\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{(-19\beta_1)/2} \langle\omega^{1,5,6} - \omega^{2,3,7} + 2\omega^{2,4,6}\rangle + e^{-11\beta_1} \langle\omega^{1,6,7} - \omega^{2,5,7}\rangle + e^{-10\beta_1} \langle\omega^{1,4,7} + \omega^{2,5,6} + 2\omega^{3,4,5}\rangle + e^{(-21\beta_1)/2} \langle\omega^{1,5,7} + \omega^{2,4,7} - \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-15\beta_1} \langle(-\omega^{1,5,6,7} - \omega^{2,4,6,7})/2\rangle + e^{(-29\beta_1)/2} \langle(-\omega^{1,4,6,7} + \omega^{3,4,5,6})/2\rangle + e^{(-27\beta_1)/2} \langle(-\omega^{1,3,6,7} + 2\omega^{2,3,5,7} - \omega^{2,4,5,6})/6\rangle + e^{-13\beta_1} \langle(2\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle + e^{(-23\beta_1)/2} \langle(-\omega^{1,3,4,6} + \omega^{2,3,4,5})/2\rangle + e^{(-25\beta_1)/2} \langle(2\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6})/6\rangle + e^{-12\beta_1} \langle(\omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/3\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{(-35\beta_1)/2} \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-17\beta_1} \langle(-\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{(-33\beta_1)/2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-16\beta_1} \langle(-\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{(-43\beta_1)/2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-21\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{(-45\beta_1)/2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 3Y^2 + 4Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y).$

5.3.23 Algebra $\mathcal{G}_{7,1.11}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 4\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4 & -\xi_1^3 - \xi_1^4 + \xi_2^5 & \xi_2^4 & 5\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 + \xi_2^6 & -2\xi_1^3 - \xi_1^4 + \xi_2^5 & \xi_2^4 & 6\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_2^6, ad\ x_4 = -\delta_1^5 - \delta_2^6 + \delta_3^7, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + 2e^{3\beta_1}\ \langle x_3, x_4\rangle + e^{4\beta_1}\ \langle x_5\rangle + e^{5\beta_1}\ \langle x_6\rangle + e^{6\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{4\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + e^{5\beta_1}\ \langle \delta_1^7\rangle + 3e^{3\beta_1}\ \langle \delta_3^7, \delta_1^5, \delta_2^6\rangle + 3e^{2\beta_1}\ \langle \delta_1^3, \delta_1^4, \delta_2^5\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_1^5, \delta_1^3] = 2\delta_1^7$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -(\delta_2^7 + \delta_1^6)$; $[\delta_2^6, \delta_1^4] = \delta_1^7$; $[\delta_1^3, \delta_2^5] = -2\delta_2^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^4, \delta_2^5] = -(\delta_2^7 + \delta_1^6)$; $[\delta_1^4, \delta_2^4] = \delta_3^7 - \delta_2^6 - \delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle \delta_1^6\rangle + e^{3\beta_1}\ \langle \delta_1^5\rangle + 2e^{2\beta_1}\ \langle \delta_1^3, \delta_2^5\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_2^5] = 2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $2e^{-4\beta_1}\ \langle x_1\omega^{2,4} - x_3(\omega^{2,6} - \omega^{4,5}), -x_4(\omega^{1,7} + \omega^{2,6} - \omega^{3,5}) - x_5(\omega^{2,7} - \omega^{3,6}) + x_6\omega^{3,7}\rangle + 3e^{-3\beta_1}\ \langle -x_1\omega^{1,3} - 2x_2(\omega^{1,5} - \omega^{2,3} - \omega^{2,4}) - x_4(\omega^{1,6} - \omega^{2,5}) - x_5(\omega^{2,6} - \omega^{3,5} - 2\omega^{4,5}), x_1\omega^{1,3} + 2x_2(\omega^{1,5} - \omega^{2,3} - \omega^{2,4}) - 2x_4(\omega^{2,5} - 1/2\omega^{3,4}) - 2x_5\omega^{4,5} + x_6\omega^{3,6}, -1/3x_1\omega^{1,3} - 1/3x_2(\omega^{1,5} - \omega^{2,4}) - 1/3x_4(\omega^{1,6} - 2\omega^{3,4}) + x_5(\omega^{1,7} + 2/3\omega^{2,6} + 1/3\omega^{4,5}) + x_6\omega^{2,7}\rangle + 4e^{-\beta_1}\ \langle x_3(\omega^{1,3} + \omega^{1,4}) - x_4\omega^{1,4} + x_5\omega^{2,3}, x_3\omega^{1,4} + x_5\omega^{2,4}, -x_3(\omega^{1,3} + \omega^{1,4}) + x_4(\omega^{1,3} + \omega^{1,4}), x_3(\omega^{1,3} + 2\omega^{1,4}) - 2x_4\omega^{1,4} + x_6\omega^{3,4}\rangle + 5e^{-2\beta_1}\ \langle x_1\omega^{1,2} - 3x_3\omega^{2,3} + x_4(\omega^{2,3} + 3\omega^{2,4}) - x_5(\omega^{1,6} - \omega^{2,5}), 2x_1\omega^{1,2} + x_2\omega^{1,4} - x_3(\omega^{1,5} + 4\omega^{2,3}) + 2x_4(\omega^{2,3} + 2\omega^{2,4}) - 2x_5\omega^{1,6} + x_6\omega^{1,7}, x_3\omega^{2,4}, x_2(\omega^{1,3} + \omega^{1,4}) - x_3\omega^{1,5} + x_4\omega^{1,5}, 2x_1\omega^{1,2} + x_2(\omega^{1,3} + 2\omega^{1,4}) - 2x_3(\omega^{1,5} + 2\omega^{2,3}) + 2x_4(\omega^{2,3} + 2\omega^{2,4}) - 2x_5\omega^{1,6} + x_6\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-4\beta_1}+5e^{-5\beta_1}+5e^{-6\beta_1}+8e^{-7\beta_1}+3e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-8\beta_1}+e^{-9\beta_1}+6e^{-10\beta_1}+7e^{-11\beta_1}+5e^{-12\beta_1}+3e^{-13\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-13\beta_1}+5e^{-14\beta_1}+3e^{-15\beta_1}+5e^{-16\beta_1}+3e^{-17\beta_1}+e^{-18\beta_1}+e^{-19\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-17\beta_1}+e^{-18\beta_1}+3e^{-19\beta_1}+3e^{-20\beta_1}+e^{-21\beta_1}+e^{-22\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-21\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,52,117,151,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,14,22,23,19,11,3)$

S$=24\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,18,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,6,6,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}-\omega^{2,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,3}+\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}+\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}+\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$;

$d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,5},\omega^{2,3}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,7}-\omega^{3,5}+\omega^{4,5},\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-8\beta_1}\ \langle\omega^{1,2,6}\rangle + e^{-9\beta_1}\ \langle\omega^{1,2,7}+\omega^{2,3,5}\rangle + 3e^{-10\beta_1}\ \langle\omega^{1,3,7}-\omega^{3,4,5},\omega^{1,4,7}+\omega^{2,4,6},\omega^{2,3,6}+\omega^{3,4,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,5,7}+\omega^{2,3,7}+\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 3e^{-14\beta_1}\ \langle\omega^{1,2,6,7}-\omega^{2,3,5,6},\omega^{1,4,5,7},\omega^{1,3,5,7}\rangle + e^{-13\beta_1}\ \langle\omega^{1,3,4,7}+\omega^{2,3,4,6}\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7}+2\omega^{3,4,5,6}\rangle + e^{-16\beta_1}\ \langle\omega^{1,5,6,7}-2\omega^{2,3,6,7}-\omega^{2,4,6,7}-\omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-17\beta_1}\ \langle\omega^{1,2,3,6,7},\omega^{1,2,4,6,7}\rangle + 2e^{-19\beta_1}\ \langle\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle + e^{-20\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-21\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-22\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-24\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,9,18,23,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,3,5,6,6,5,3,1)$

Total dimension of trivial cohomology: 30.

Length associated polynomial: 30.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7}-\omega^{3,6}+\omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{1,5,6}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7}-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$;

$d^{\star}\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6} - \omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^{\star}\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^{\star}\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{1,2,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,5,6,7}-\omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 3\omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{1,6} = 3\omega^{1,6}$; $\Delta\omega^{1,7} = 2\omega^{1,7}-\omega^{2,6}+\omega^{3,5}-\omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,5}+\omega^{2,3}+\omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{1,5}+\omega^{2,3}+\omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 5\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}+\omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3}+2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5}-\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}+\omega^{1,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}+\omega^{1,3,6}-\omega^{2,3,5}+\omega^{2,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} - \omega^{1,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 3\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 3\omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 4\omega^{1,6,7} - \omega^{3,5,6} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,3,5} - \omega^{1,4,5} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + 3\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7} + \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{2,3,5} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + \omega^{2,3,7} + 2\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,4,7} - \omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} - \omega^{2,5,6} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 3\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{3,4,7} - \omega^{3,5,6} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,3,4,5}$;

$\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,3,7} + 2\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} - \omega^{1,2,5,6} + 3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 2\omega^{1,3,5,6} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,4,7} - \omega^{1,3,5,6} + 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 3\omega^{1,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,4,7} + 4\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7} - \omega^{1,4,5,7} + 3\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 3\omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,5,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} - \omega^{1,4,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{3,5,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 5\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 3\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 3\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-3\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1} \langle\omega^{1,3}\rangle + 2e^{-5\beta_1} \langle\omega^{1,5} - \omega^{2,4}, \omega^{2,3} - \omega^{2,4}\rangle + 2e^{-7\beta_1} \langle\omega^{1,7} - \omega^{3,5} + \omega^{4,5}, \omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-8\beta_1} \langle\omega^{1,2,6} + \omega^{1,3,5} - 2\omega^{1,4,5} - \omega^{2,3,4}\rangle + 3e^{-10\beta_1} \langle\omega^{1,3,7} + \omega^{2,3,6}, \omega^{1,4,7} - \omega^{2,3,6} + \omega^{2,4,6} - \omega^{3,4,5}, \omega^{1,5,6} - 2\omega^{2,3,6} - \omega^{2,4,6} - \omega^{3,4,5}\rangle + e^{-9\beta_1} \langle 5\omega^{1,2,7} - 3\omega^{1,3,6} - \omega^{1,4,6} + 3\omega^{2,3,5} - 4\omega^{2,4,5}\rangle + e^{-11\beta_1} \langle 2\omega^{1,5,7} + 2\omega^{2,3,7} - 3\omega^{2,4,7} + 3\omega^{2,5,6} + 2\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-16\beta_1} \langle(\omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7})/7\rangle + 3e^{-14\beta_1} \langle(-3\omega^{1,2,6,7} - \omega^{1,3,5,7} - 3\omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6} - 8\omega^{2,4,5,6})/11, (-8\omega^{1,2,6,7} + 12\omega^{1,3,5,7} + 3\omega^{1,4,5,7} - 2\omega^{2,3,4,7} + 10\omega^{2,3,5,6} -$

$3\omega^{2,4,5,6})/33, (-5\omega^{1,2,6,7} - 9\omega^{1,3,5,7} + 6\omega^{1,4,5,7} + 7\omega^{2,3,4,7} - 2\omega^{2,3,5,6} - 6\omega^{2,4,5,6})/33\rangle + e^{-15\beta_1} \langle(4\omega^{1,3,6,7} + 3\omega^{1,4,6,7} + \omega^{2,3,5,7} - 3\omega^{2,4,5,7} + 5\omega^{3,4,5,6})/60\rangle + e^{-13\beta_1} \langle(-2\omega^{1,2,5,7} - 3\omega^{1,3,4,7} + 3\omega^{1,3,5,6} + 2\omega^{1,4,5,6} - 2\omega^{2,3,4,6})/30\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-20\beta_1} \langle-\omega^{2,4,5,6,7}\rangle + 2e^{-19\beta_1} \langle(\omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} - 2\omega^{2,3,4,6,7})/3, (\omega^{1,3,5,6,7} + 2\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7})/3\rangle + 2e^{-17\beta_1} \langle(\omega^{1,2,3,6,7} + 2\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} - 2\omega^{2,3,4,5,6})/5, (-2\omega^{1,2,3,6,7} + \omega^{1,2,4,6,7} - 3\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/5\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-23\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-22\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-21\beta_1} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-24\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.3.24 Algebra $\mathcal{G}_{7,1.12}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_7$; $[x_2, x_4] = x_6$; $[x_2, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 4\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & 0 & -\xi_1^4+\xi_2^5 & \xi_2^4 & 5\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3-\xi_1^5+\xi_2^6 & -\xi_1^4+\xi_2^5 & \xi_2^4 & 6\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^7, ad\ x_3 = -\delta_2^7, ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : T=$\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + 2e^{4\beta_1}\ \langle x_3, x_5\rangle + e^{3\beta_1}\ \langle x_4\rangle + e^{5\beta_1}\ \langle x_6\rangle + e^{6\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{6\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $4e^{2\beta_1}\ \langle \delta_2^3, \delta_3^7, \delta_1^4, \delta_2^5\rangle + 2e^{4\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + e^{5\beta_1}\ \langle \delta_1^7\rangle + 3e^{3\beta_1}\ \langle \delta_1^3, \delta_1^5, \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^7$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -(\delta_2^7 + \delta_1^6)$; $[\delta_2^6, \delta_1^4] = \delta_1^7$; $[\delta_1^4, \delta_2^5] = -(\delta_2^7 + \delta_1^6)$; $[\delta_1^4, \delta_2^4] = -(\delta_2^6 + \delta_1^5)$.

ch($H^1_{\mathcal{G}}$)= $3e^{2\beta_1}\ \langle \delta_2^3, \delta_1^4, \delta_2^5\rangle + 2e^{3\beta_1}\ \langle \delta_1^3, \delta_1^5\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $4e^{-3\beta_1}\ \langle x_4(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{2,6}, -x_2\omega^{2,4} - x_3(\omega^{1,7} - \omega^{3,4} + \omega^{4,5}) - x_4(\omega^{2,3} + \omega^{2,5}) + x_5\omega^{1,7}, x_2(\omega^{1,3} + \omega^{1,5} - \omega^{2,4}) - x_4(\omega^{2,3} + \omega^{2,5}) + x_5(\omega^{3,4} - \omega^{4,5}), -2x_1\omega^{1,4} - 2x_2\omega^{2,4} - x_3(\omega^{1,7} + 3\omega^{2,6} - \omega^{3,4} - 2\omega^{4,5}) + x_4(\omega^{1,6} - 3\omega^{2,5}) - 3x_5\omega^{4,5} - x_6\omega^{4,6} + x_7\omega^{4,7}\rangle + 5e^{-2\beta_1}\ \langle x_4\omega^{1,3} + x_5\omega^{2,3}, -x_1\omega^{1,2} + x_3(\omega^{1,6} + \omega^{2,5}) + 2x_4\omega^{1,5} + x_5\omega^{2,5}, x_1\omega^{1,2} - 2x_3(\omega^{1,6} + \omega^{2,5}) - x_4\omega^{1,5} + x_5\omega^{1,6}, x_3\omega^{2,3}, x_1\omega^{1,2} - x_3(\omega^{1,6} + \omega^{2,5}) - x_4(\omega^{1,5} - \omega^{2,4})\rangle + 2e^{-4\beta_1}\ \langle x_1\omega^{1,3} + 2x_2\omega^{2,3} - 6x_4(\omega^{1,7} + \omega^{2,6} - 1/2\omega^{3,4}) - 6x_5(\omega^{2,7} - 1/3\omega^{3,5}) + x_6\omega^{3,6}, -x_1\omega^{1,3} - 2x_2\omega^{2,3} + 5x_4(\omega^{1,7} + \omega^{2,6} - 2/5\omega^{3,4}) + 5x_5(\omega^{2,7} - 1/5\omega^{3,5}) + x_7\omega^{3,7}\rangle + 5e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_5\omega^{1,3}, x_2\omega^{1,2} - x_3(\omega^{1,3} + \omega^{1,5}) + x_5\omega^{2,4}, x_3\omega^{2,4}, -x_2\omega^{1,2} + x_3(\omega^{1,3} + \omega^{1,5}) + x_4\omega^{1,4}, x_3\omega^{1,3} + x_7\omega^{3,4}\rangle$.

ch$(H^3_{\mathcal{G}})$=$5e^{-5\beta_1} + 6e^{-6\beta_1} + 9e^{-7\beta_1} + 4e^{-8\beta_1} + 2e^{-9\beta_1}$.

ch$(H^4_{\mathcal{G}})$=$3e^{-9\beta_1} + 6e^{-10\beta_1} + 7e^{-11\beta_1} + 7e^{-12\beta_1} + 4e^{-13\beta_1} + e^{-14\beta_1}$.

ch$(H^5_{\mathcal{G}})$=$e^{-13\beta_1} + 5e^{-14\beta_1} + 3e^{-15\beta_1} + 6e^{-16\beta_1} + 3e^{-17\beta_1} + 2e^{-18\beta_1} + e^{-19\beta_1}$.

ch$(H^6_{\mathcal{G}})$=$e^{-17\beta_1} + 2e^{-18\beta_1} + 2e^{-19\beta_1} + 3e^{-20\beta_1} + e^{-21\beta_1} + e^{-22\beta_1} + e^{-23\beta_1}$.

ch$(H^7_{\mathcal{G}})$=$e^{-21\beta_1} + e^{-23\beta_1} + e^{-24\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 12, 53, 120, 153, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 6, 16, 26, 28, 21, 11, 3)$

S=$25\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3} - \omega^{2,5}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3} + \omega^{1,2,5}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6} - \omega^{2,4,7} + \omega^{2,5,6}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4} + \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-4\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{T}})= 2e^{-5\beta_1}\ \langle\omega^{1,3},\omega^{1,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,7}-\omega^{3,4}+\omega^{4,5},\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{T}})= 2e^{-9\beta_1}\ \langle\omega^{1,2,7},\omega^{1,3,5}\rangle + e^{-10\beta_1}\ \langle\omega^{1,4,7}-\omega^{2,3,5}+\omega^{2,4,6}\rangle + 3e^{-11\beta_1}\ \langle\omega^{1,3,7}-\omega^{3,4,5},\omega^{1,5,7}+\omega^{3,4,5},\omega^{2,3,6}+\omega^{3,4,5}\rangle + e^{-12\beta_1}\ \langle\omega^{1,6,7}-\omega^{2,3,7}+\omega^{2,5,7}-2\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{T}})= e^{-13\beta_1}\ \langle\omega^{1,2,3,7}\rangle + 3e^{-14\beta_1}\ \langle\omega^{1,2,6,7},\omega^{1,3,4,7}+\omega^{2,3,4,6},\omega^{1,4,5,7}+\omega^{2,3,4,6}\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,5,7}\rangle + 2e^{-16\beta_1}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7}+2\omega^{3,4,5,6},\omega^{1,5,6,7}-\omega^{2,4,6,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{T}})= 2e^{-18\beta_1}\ \langle\omega^{1,2,3,6,7},\omega^{1,2,5,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}\rangle + 2e^{-20\beta_1}\ \langle\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7},\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{T}})= e^{-21\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-24\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{T}})= e^{-25\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{T}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathfrak{T}}$'s $0\le j\le 7$: $(1,3,5,7,7,5,3,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}+\omega^{4,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}+\omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{1,5,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7}+\omega^{3,5,6}-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7}-\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} =$

$\omega^{1,4,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7}-\omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}+\omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{3,4,6,7} + \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} - \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} + \omega^{3,4} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,6} + \omega^{2,3} + \omega^{2,5}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,3} + 2\omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,7}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}-\omega^{4,7}$; $\Delta\omega^{3,7} = 2\omega^{3,7}-\omega^{5,7}$; $\Delta\omega^{4,5} = -\omega^{1,7} + \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,6} + 4\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{3,7} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,5}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,3} + 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,4} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,7}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + 2\omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + 3\omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = -\omega^{1,3,7} + 2\omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} + \omega^{1,3,5} - \omega^{1,4,6} + 2\omega^{2,3,4} - \omega^{2,4,5}$; $\Delta\omega^{2,3,5} = -\omega^{1,5,6}+3\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7}+2\omega^{2,3,6}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} - \omega^{2,3,4} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} - \omega^{2,3,6} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,3,6} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} - \omega^{1,5,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 3\omega^{3,4,7} + \omega^{4,5,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{5,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + \omega^{3,4,7} - \omega^{3,5,6} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{3,6,7} + 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,7} + \omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,4} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,6} + 3\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,4,6} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{1,4,5,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6}$;

$\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,5,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} + 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 3\omega^{1,4,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,3,6,7} + 2\omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,5,7} + \omega^{1,4,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6} + 2\omega^{2,3,4,6} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + 3\omega^{2,3,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,5,6,7} + 2\omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,5,6} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7} + \omega^{4,5,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{3,4,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 4\omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,6} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,5,6} + 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} + \omega^{1,4,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,3,4,6,7} + \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} - \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,4,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{a}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-4\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}}) = 2e^{-5\beta_1}\ \langle\omega^{1,3}, \omega^{1,5} - \omega^{2,4}\rangle + e^{-6\beta_1}\ \langle\omega^{1,6} - 2\omega^{2,3} + \omega^{2,5}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,7} - \omega^{3,4} + \omega^{4,5}, \omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}}) = 2e^{-9\beta_1}\ \langle 2\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5}, 2\omega^{1,3,5} - \omega^{1,4,6} - 2\omega^{2,3,4} - \omega^{2,4,5}\rangle + e^{-10\beta_1}\ \langle\omega^{1,3,6} + 2\omega^{1,4,7} - 3\omega^{1,5,6} - \omega^{2,3,5} + 5\omega^{2,4,6}\rangle + 3e^{-11\beta_1}\ \langle\omega^{1,3,7} - \omega^{3,4,5}, 2\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,5}, 2\omega^{2,3,6} + \omega^{2,4,7} - \omega^{2,5,6} + 2\omega^{3,4,5}\rangle + e^{-12\beta_1}\ \langle\omega^{1,6,7} - \omega^{2,3,7} + \omega^{2,5,7} - 2\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}}) = 2e^{-16\beta_1}\ \langle(2\omega^{1,3,6,7} - \omega^{1,5,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,6,7} + 5\omega^{3,4,5,6})/14, (\omega^{1,3,6,7} + 3\omega^{1,5,6,7} + \omega^{2,3,5,7} - 3\omega^{2,4,6,7} - \omega^{3,4,5,6})/14\rangle + e^{-15\beta_1}\ \langle(5\omega^{1,3,5,7} - \omega^{1,4,6,7} - 3\omega^{2,3,4,7} + 2\omega^{2,3,5,6} + \omega^{2,4,5,7})/40\rangle + 3e^{-14\beta_1}\ \langle(-\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} - 3\omega^{2,4,5,6})/4, (\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6} - 2\omega^{2,3,4,6} - \omega^{2,4,5,6})/8, (\omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - 2\omega^{1,4,5,7} - \omega^{2,4,5,6})/8\rangle + e^{-13\beta_1}\ \langle(2\omega^{1,2,3,7} + \omega^{1,3,4,6} - \omega^{1,4,5,6} + \omega^{2,3,4,5})/7\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}}) = 2e^{-20\beta_1}\ \langle -\omega^{2,4,5,6,7}, (\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-19\beta_1}\ \langle(\omega^{1,3,4,6,7} - 2\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7})/6\rangle + 2e^{-18\beta_1}\ \langle(\omega^{1,2,3,6,7} - 2\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} - 2\omega^{2,3,4,5,6})/5, (-2\omega^{1,2,3,6,7} - \omega^{1,2,5,6,7} - 3\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/5\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-24\beta_1} \left\langle\omega^{2,3,4,5,6,7}\right\rangle + e^{-23\beta_1} \left\langle-\omega^{1,3,4,5,6,7}\right\rangle + e^{-21\beta_1} \left\langle\omega^{1,2,4,5,6,7}\right\rangle.$
$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-25\beta_1} \left\langle\omega^{1,2,3,4,5,6,7}\right\rangle.$
$P(Y) = (1+Y^2)^2(1+Y)^3.$

5.3.25 Algebra $\mathcal{G}_{7,1.13}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_5]=x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_1^2 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & 5\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^4+\xi_2^5+\xi_2^6 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 2\xi_1^2+\xi_2^3 & 6\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_1^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch$(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+2e^{5\beta_1}\ \langle x_5,x_6\rangle+e^{6\beta_1}\ \langle x_7\rangle$.

ch$(H^0_{\mathcal{G}})= e^{5\beta_1}\ \langle x_5\rangle+e^{6\beta_1}\ \langle x_7\rangle$.

ch$(Z^1_{\mathcal{G}})= 3e^{4\beta_1}\ \langle\delta_1^5,\delta_1^6,\delta_2^7\rangle+e^{5\beta_1}\ \langle\delta_1^7\rangle+3e^{3\beta_1}\ \langle\delta_1^4,\delta_2^5,\delta_2^6\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle+2e^{\beta_1}\ \langle\delta_1^2,\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^3]=-\delta_1^7$; $[\delta_1^6,\delta_1^2]=-2\delta_1^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_1^2]=-\delta_1^5$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^6)$; $[\delta_2^5,\delta_1^3]=\delta_1^7$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^2]=-2\delta_2^7+\delta_1^6$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$; $[\delta_2^4,\delta_1^2]=-\delta_2^5+\delta_1^4$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$.

ch$(H^1_{\mathcal{G}})= 2e^{4\beta_1}\ \langle\delta_1^5,\delta_1^6\rangle+2e^{3\beta_1}\ \langle\delta_1^4,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_1^2\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^2]=-\delta_1^5$; $[\delta_2^6,\delta_1^2]=3\delta_1^6$; $[\delta_2^4,\delta_1^2]=2\delta_1^4$.

ch$(H^2_{\mathcal{G}})= 7e^{-2\beta_1}\ \langle x_3\omega^{1,4}+x_5\omega^{1,7}, -x_3\omega^{2,3}+x_4\omega^{1,5}+x_6\omega^{2,5}, -x_1\omega^{1,2}-2x_3\omega^{2,3}+x_4(\omega^{1,5}+2\omega^{1,6})+x_6\omega^{2,6}, x_1\omega^{1,2}+3x_3\omega^{2,3}-x_4(\omega^{1,5}+\omega^{1,6}-\omega^{2,4}), x_1\omega^{1,2}+2x_3\omega^{2,3}-x_4(\omega^{1,5}+\omega^{1,6})+x_6\omega^{1,7}, -x_1\omega^{1,2}+x_2\omega^{1,3}-2x_3\omega^{2,3}+x_4(\omega^{1,5}+\omega^{1,6})+x_6\omega^{3,4}, x_3\omega^{2,3}-x_4\omega^{1,5}+x_7\omega^{3,5}\rangle+3e^{-3\beta_1}\ \langle x_4(\omega^{1,7}+\omega^{2,5}+\omega^{2,6})+x_6\omega^{2,7}, x_1\omega^{1,3}+2x_2\omega^{2,3}+3x_3\omega^{2,4}+x_4(\omega^{2,5}+\omega^{2,6}+2\omega^{3,4})+x_6(\omega^{3,5}+\omega^{3,6}), -x_1\omega^{1,3}+3x_2(\omega^{1,4}-5/3\omega^{2,3})-3x_3(\omega^{1,6}+2\omega^{2,4})-x_4(\omega^{1,7}+3\omega^{2,6}+4\omega^{3,4})+3x_5\omega^{3,6}-3x_6\omega^{3,6}+x_7\omega^{3,7}\rangle+4e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, x_3\omega^{1,3}+x_5\omega^{1,5}, x_5\omega^{1,6}, -x_4\omega^{2,3}+x_6\omega^{1,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-5\beta_1}+5e^{-6\beta_1}+10e^{-7\beta_1}+4e^{-8\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-9\beta_1}+3e^{-10\beta_1}+6e^{-11\beta_1}+8e^{-12\beta_1}+4e^{-13\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$3e^{-14\beta_1}+2e^{-15\beta_1}+3e^{-16\beta_1}+4e^{-17\beta_1}+3e^{-18\beta_1}+e^{-19\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$3e^{-19\beta_1}+2e^{-20\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-24\beta_1}+e^{-25\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,12,51,117,150,111,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,7,14,21,22,16,8,2)$

S=$26\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,19,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,7,7,4,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,5}-\omega^{1,6}-\omega^{2,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,5}-\omega^{1,2,6}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,5}-\omega^{1,3,6}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,5}-\omega^{1,4,6}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,5}-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,5}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}+\omega^{1,2,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,5}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,7}+\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,5}-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-6\beta_1}\ \langle\omega^{1,5}+\omega^{2,4}\rangle + 3e^{-7\beta_1}\ \langle\omega^{2,5},\omega^{1,7}+\omega^{3,4},\omega^{2,6}-\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-9\beta_1}\ \langle\omega^{1,2,7}\rangle + 2e^{-10\beta_1}\ \langle\omega^{2,3,5},\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-11\beta_1}\ \langle\omega^{1,4,7}\rangle + 3e^{-12\beta_1}\ \langle\omega^{2,5,6}+\omega^{3,4,5},\omega^{1,5,7}+\omega^{2,4,7}-2\omega^{3,4,5}-\omega^{3,4,6},\omega^{1,6,7}-\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= 3e^{-14\beta_1}\ \langle\omega^{1,2,5,7},\omega^{1,2,6,7},\omega^{1,3,4,7}\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6}\rangle + 2e^{-16\beta_1}\ \langle\omega^{1,4,6,7},\omega^{2,3,5,7}-\omega^{2,4,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}+2\omega^{3,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= 3e^{-19\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,5,7},\omega^{1,3,4,6,7}\rangle + e^{-20\beta_1}\ \langle\omega^{2,3,4,5,7}+\omega^{2,3,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-24\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-25\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-26\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+3Y^2+4Y^3+3Y^4+Y^5+Y^6)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,2,9,19,23,16,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,2,4,7,7,4,2,1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,5}=-\omega^7$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}+\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}+\omega^{2,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}+\omega^{3,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}+\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{2,3,5,7}+\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,5,6,7}-\omega^{2,4,5,7}+\omega^{2,4,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=$

$\omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = 2\omega^{1,5} + \omega^{1,6}$; $\Delta\omega^{1,6} = \omega^{1,5} + \omega^{1,6} + \omega^{2,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,6} + 2\omega^{2,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,5} + \omega^{3,6}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6}$; $\Delta\omega^{3,7} = 4\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{5,7} + 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,2,6}$; $\Delta\omega^{1,2,6} = \omega^{1,2,5} + 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,5} + \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,7} + 2\omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,5} + 2\omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5} + \omega^{1,4,6}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,5} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 3\omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} + \omega^{1,3,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 3\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,5,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + 3\omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{2,6,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = -\omega^{2,5,7} + 2\omega^{2,6,7} + \omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 3\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} - \omega^{2,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 3\omega^{3,5,7} - \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = -\omega^{3,5,7} + 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + \omega^{1,2,4,5} + 3\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,4,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} - \omega^{1,2,6,7} - \omega^{1,3,4,7} + 3\omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 3\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,6,7} + 3\omega^{2,3,4,7} - \omega^{2,3,5,6}$;

$\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} - \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = -\omega^{2,3,5,7} + 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{2,4,5,7} + 2\omega^{2,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,5,6,7} + \omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{3,4,5,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 4\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} - \omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{2,3,4,5,7} + 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-6\beta_1} \langle\omega^{1,5} - 2\omega^{1,6} + \omega^{2,4}\rangle + 3e^{-7\beta_1} \langle\omega^{2,5}, \omega^{1,7} + \omega^{3,4}, \omega^{2,6} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-9\beta_1} \langle 2\omega^{1,2,7} - \omega^{1,3,5} - \omega^{2,3,4}\rangle + 2e^{-10\beta_1} \langle\omega^{2,3,5}, 2\omega^{1,3,7} + \omega^{1,4,5} - 3\omega^{1,4,6} + \omega^{2,3,6}\rangle + e^{-11\beta_1} \langle 5\omega^{1,4,7} + 2\omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,5} + 3\omega^{2,4,6}\rangle + 3e^{-12\beta_1} \langle\omega^{2,5,6} + \omega^{3,4,5}, \omega^{1,5,7} + \omega^{2,4,7} - 2\omega^{3,4,5} - \omega^{3,4,6}, \omega^{1,6,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-17\beta_1} \langle(\omega^{1,5,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,6})/6\rangle + 2e^{-16\beta_1} \langle\omega^{1,4,6,7}, (-\omega^{1,4,5,7} + 3\omega^{2,3,5,7} + \omega^{2,3,6,7} - 2\omega^{2,4,5,6})/15\rangle + e^{-15\beta_1} \langle(3\omega^{1,3,5,7} - \omega^{1,3,6,7} - \omega^{1,4,5,6} + 2\omega^{2,3,4,7} + 5\omega^{2,3,5,6})/40\rangle + 3e^{-14\beta_1} \langle(2\omega^{1,2,5,7} + 5\omega^{1,2,6,7} - 13\omega^{1,3,4,7} - 4\omega^{1,3,5,6} - 2\omega^{2,3,4,5} - 4\omega^{2,3,4,6})/18, (\omega^{1,2,5,7} - 2\omega^{1,2,6,7} - 2\omega^{1,3,4,7} - 2\omega^{1,3,5,6} - \omega^{2,3,4,5} - 2\omega^{2,3,4,6})/9, (4\omega^{1,2,5,7} + \omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6} + 5\omega^{2,3,4,5} + \omega^{2,3,4,6})/9\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-20\beta_1} \langle(-\omega^{1,3,5,6,7} - 2\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7})/6\rangle + 3e^{-19\beta_1} \langle\omega^{1,3,4,6,7}, (\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} - 2\omega^{2,3,4,5,6})/3, (-\omega^{1,2,5,6,7} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-25\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-24\beta_1} \langle -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-26\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + 4Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

5.3.26 Algebra $\mathcal{G}_{7,1.14}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_6$; $[x_2,x_3]=x_5$; $[x_2,x_5]=-x_7$; $[x_2,x_6]=-x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 5\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & 5\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_1^5+\xi_1^6 & -\xi_1^4 & \xi_1^3-2\xi_2^4 & \xi_1^3 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_1^6, ad\ x_5=\delta_2^7, ad\ x_6=\delta_2^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{4\beta_1}\ \langle x_4\rangle+2e^{5\beta_1}\ \langle x_5,x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{5\beta_1}\ \langle x_5-x_6\rangle+e^{7\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=3e^{3\beta_1}\ \langle\delta_2^5,\delta_2^6,\delta_1^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+e^{5\beta_1}\ \langle\delta_2^7\rangle+2e^{4\beta_1}\ \langle\delta_1^5,\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_2^4,\delta_1^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_2^4]=2\delta_2^7$; $[\delta_2^5,\delta_1^3]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=-\delta_2^7$; $[\delta_2^3,\delta_1^5]=-\delta_2^7$; $[\delta_2^3,\delta_1^6]=-\delta_2^7$; $[\delta_2^3,\delta_1^4]=\delta_1^6$; $[\delta_2^3,\delta_1^3]=\delta_2^5+\delta_1^4$; $[\delta_1^5,\delta_2^4]=2\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_2^7$; $[\delta_2^4,\delta_1^3]=\delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})=\ 2e^{3\beta_1}\ \langle\delta_2^6,\delta_1^4\rangle+e^{6\beta_1}\ \langle\delta_1^7\rangle+e^{4\beta_1}\ \langle\delta_1^5\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^4]=2\delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})=\ 4e^{-2\beta_1}\ \langle x_1\omega^{1,2}+3x_2\omega^{1,3}-x_3\omega^{2,3}+3x_4(\omega^{1,5}+\omega^{1,6}+1/3\omega^{2,4}), x_6\omega^{2,5}, x_2\omega^{1,3}-x_3(\omega^{1,4}+\omega^{2,3})+x_4(\omega^{1,5}+\omega^{1,6})+x_5\omega^{3,4}, x_5\omega^{2,5}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+2x_3\omega^{1,3}-2x_4\omega^{1,4}+x_5(\omega^{1,5}+\omega^{1,6}), x_2\omega^{1,2}-x_3\omega^{1,3}+2x_4\omega^{1,4}+x_7\omega^{1,7}, -x_2\omega^{1,2}+x_3\omega^{1,3}-x_4(\omega^{1,4}+\omega^{2,3})+x_5\omega^{1,6}+x_6\omega^{1,5}\rangle+2e^{-4\beta_1}\ \langle -1/2x_1\omega^{1,4}+x_2(\omega^{1,5}+\omega^{1,6}+\omega^{2,4})+1/2x_3\omega^{3,4}-5/2x_5(\omega^{2,7}-2/5\omega^{4,5}-2/5\omega^{4,6}), x_1\omega^{2,3}+x_6\omega^{2,7}\rangle+e^{-3\beta_1}\ \langle -x_1\omega^{1,3}+2x_2\omega^{2,3}+x_3\omega^{2,4}+x_4(\omega^{2,5}+\omega^{2,6})+x_6(\omega^{3,5}+\omega^{3,6})\rangle+e^{\beta_1}\ \langle -x_4\omega^{1,2}+x_7\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{-5\beta_1}+3e^{-6\beta_1}+5e^{-7\beta_1}+3e^{-8\beta_1}+3e^{-9\beta_1}+e^{-10\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=3e^{-10\beta_1}+2e^{-11\beta_1}+6e^{-12\beta_1}+3e^{-13\beta_1}+2e^{-14\beta_1}+2e^{-15\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$e^{-15\beta_1} + 3e^{-16\beta_1} + 3e^{-17\beta_1} + 2e^{-18\beta_1} + 2e^{-19\beta_1} + 2e^{-20\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-20\beta_1} + 3e^{-21\beta_1} + e^{-23\beta_1} + e^{-24\beta_1} + e^{-25\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-25\beta_1} + e^{-26\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 49, 115, 148, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 6, 11, 17, 18, 13, 7, 2)$

S=$27\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = \omega^{2,5} + \omega^{2,6} - \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,7} = -\omega^{1,2,5} - \omega^{1,2,6} + \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} + \omega^{2,3,5} + \omega^{2,3,6}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6}$; $d\omega^{4,7} = -\omega^{1,3,7} + \omega^{2,4,5} + \omega^{2,4,6}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = -\omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = -\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,4,5} - \omega^{1,2,4,6}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,5} + \omega^{2,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} + \omega^{2,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{2,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = -\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-2\beta_1}$ $\langle\omega^2\rangle$.

ch($H^2_{\mathbb{C}}$)= $2e^{-6\beta_1}$ $\langle\omega^{1,6}, \omega^{1,5} + \omega^{2,4}\rangle + e^{-7\beta_1}$ $\langle\omega^{2,5}\rangle$.

ch($H^3_{\mathbb{C}}$)= $2e^{-10\beta_1}$ $\langle\omega^{1,4,6}, \omega^{1,2,7} - \omega^{2,3,6}\rangle + e^{-11\beta_1}$ $\langle\omega^{1,5,6} + \omega^{2,4,6}\rangle + e^{-12\beta_1}$ $\langle\omega^{1,4,7} - \omega^{3,4,5} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-15\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{1,4,5,6}\rangle + 2e^{-17\beta_1}\ \langle\omega^{1,4,6,7}+\omega^{3,4,5,6},\omega^{2,3,5,7}\rangle + e^{-16\beta_1}\ \langle\omega^{1,3,5,7}-\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}\rangle + 2e^{-21\beta_1}\ \langle\omega^{2,3,4,5,7},\omega^{1,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-25\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-26\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-27\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,5}=\omega^7$; $d^\star\omega^{2,6}=\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,5}=\omega^{3,7}$; $d^\star\omega^{2,3,6}=\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=\omega^{4,7}$; $d^\star\omega^{2,4,6}=\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{5,7}-\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,5,7}+\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}+\omega^{3,4,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,5,7}-\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,5,7}-\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}-\omega^{1,3,4,7}-\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,5,7}+\omega^{1,3,6,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,5,7}+\omega^{1,4,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=\omega^{2,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}+\omega^{3,4,5,7}-\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,5,7}+\omega^{1,3,4,6,7}$;

$d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,7} = 3\omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,5} + 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7}+\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7}-\omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{2,7}+2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{5,7} + 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,2,6} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,5} + 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 3\omega^{1,3,7} - \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 3\omega^{1,4,7} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{1,6,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = -\omega^{1,5,7} + 2\omega^{1,6,7} - \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + \omega^{2,3,5} + \omega^{2,3,6}$; $\Delta\omega^{2,3,6} = \omega^{2,3,5} + 3\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,4,5} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7} + 3\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6}+2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7}+3\omega^{3,4,7}+\omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7}+2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} - \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = -\omega^{3,5,7} + 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,5}+3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}-\omega^{1,2,5,6}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,5} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 3\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{1,3,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = -\omega^{1,3,5,7} + 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 3\omega^{1,4,5,7} - \omega^{1,4,6,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,4,5,7} + \omega^{1,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,7} + 2\omega^{2,3,4,5} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6}+\omega^{2,3,4,5}+3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} + 3\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,4,7} + 3\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,6,7} + \omega^{2,3,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,5,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6}$;

$\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,6,7} + 3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= 2e^{-6\beta_1} \langle\omega^{1,6}, \omega^{1,5} + \omega^{2,4}\rangle + e^{-7\beta_1} \langle 2\omega^{2,5} - \omega^{2,6} + \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{-10\beta_1} \langle\omega^{1,4,6}, 2\omega^{1,2,7} - \omega^{1,4,5} + 3\omega^{2,3,5} - \omega^{2,3,6}\rangle + e^{-11\beta_1} \langle\omega^{1,3,7} + 3\omega^{1,5,6} - \omega^{2,4,5} + 2\omega^{2,4,6}\rangle + e^{-12\beta_1} \langle\omega^{1,4,7} - 5\omega^{2,3,7} - 2\omega^{2,5,6} - 3\omega^{3,4,5} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 2e^{-17\beta_1} \langle -\omega^{2,3,5,7}, (\omega^{1,4,5,7} + 3\omega^{1,4,6,7} - \omega^{2,3,6,7} + 2\omega^{3,4,5,6})/15\rangle + e^{-16\beta_1} \langle (2\omega^{1,3,5,7} + \omega^{1,3,6,7} + 3\omega^{2,3,4,7} - \omega^{2,4,5,6})/15\rangle + e^{-15\beta_1} \langle (\omega^{1,2,5,7} - 3\omega^{1,2,6,7} + 2\omega^{1,3,4,7} - 5\omega^{1,4,5,6} + \omega^{2,3,5,6})/40\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-21\beta_1} \langle\omega^{2,3,4,5,7}, (-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-20\beta_1} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} + 2\omega^{1,3,4,6,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-26\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-25\beta_1} \langle -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-27\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \;=\; (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.3.27 Algebra $\mathcal{G}_{7,1.15}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_7$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 3\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 4\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 5\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^2 & \xi_2^5 & \xi_2^4 & 6\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3 - \xi_1^4 + \xi_2^6 & \xi_1^2 + \xi_2^5 & \xi_2^4 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^7, ad\ x_3 = -\delta_2^7, ad\ x_4 = -(\delta_1^5 + \delta_2^7), ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{3\beta_1}\ \langle x_2\rangle + 2e^{4\beta_1}\ \langle x_3, x_4\rangle + e^{5\beta_1}\ \langle x_5\rangle + e^{6\beta_1}\ \langle x_6\rangle + e^{7\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_1}\ \langle \delta_2^3, \delta_2^4\rangle + 2e^{4\beta_1}\ \langle \delta_1^5, \delta_2^7\rangle + e^{5\beta_1}\ \langle \delta_1^6\rangle + e^{6\beta_1}\ \langle \delta_1^7\rangle + 4e^{3\beta_1}\ \langle \delta_3^7, \delta_1^3, \delta_1^4, \delta_2^6\rangle + 2e^{2\beta_1}\ \langle \delta_1^2, \delta_2^5\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_1^2] = \delta_2^6 + \delta_1^3$; $[\delta_1^5, \delta_1^2] = -\delta_1^7$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -\delta_1^6$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^7$; $[\delta_1^3, \delta_1^2] = \delta_1^6$; $[\delta_1^3, \delta_2^4] = -\delta_2^7$; $[\delta_1^4, \delta_2^6] = -\delta_1^7$; $[\delta_1^4, \delta_2^5] = -\delta_1^6$; $[\delta_1^4, \delta_2^4] = -(\delta_2^7 + \delta_1^5)$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_1^2, \delta_2^5] = \delta_2^7 - \delta_1^5$; $[\delta_1^2, \delta_2^4] = \delta_3^7 - \delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1}\ \langle \delta_2^3\rangle + 3e^{3\beta_1}\ \langle \delta_1^3, \delta_1^4, \delta_2^6\rangle + 2e^{2\beta_1}\ \langle \delta_1^2, \delta_2^5\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_2^6 + \delta_1^3$.

ch($H^2_{\mathcal{G}}$)= $4e^{-2\beta_1}\ \langle x_2(\omega^{1,3} + \omega^{1,4}), -x_2\omega^{1,4} - x_5\omega^{2,3} + x_6\omega^{3,4}, -x_3\omega^{1,5} + x_4\omega^{1,5}, x_2\omega^{1,4} + x_7\omega^{3,5}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3(\omega^{1,3} + \omega^{1,4}), x_2\omega^{1,2} - x_3\omega^{1,4} + x_6\omega^{2,3}, x_3\omega^{1,4} + x_6\omega^{2,4}, -x_2\omega^{1,2} + x_3\omega^{1,4} + x_4\omega^{1,3}\rangle + 5e^{-3\beta_1}\ \langle x_1\omega^{1,2} - x_3\omega^{1,6} + 2x_4\omega^{2,4} + x_5\omega^{2,5}, -x_1\omega^{1,2} + 2x_3\omega^{1,6} - x_4\omega^{2,4} + x_6\omega^{2,6}, x_3\omega^{2,3}, x_3\omega^{2,4}, -x_3\omega^{1,6} + x_4\omega^{1,6}\rangle + 5e^{-4\beta_1}\ \langle x_1(\omega^{1,3} + \omega^{1,4}) + x_2(\omega^{1,6} + 2\omega^{2,3} + 5\omega^{2,4}) + 5x_3(\omega^{1,7} + \omega^{2,5} - \omega^{3,4}) - 5x_4(\omega^{1,7} - 1/5\omega^{2,5} - 2/5\omega^{3,4}) + x_5(\omega^{3,5} + \omega^{4,5}), x_2\omega^{2,4} + x_3(\omega^{1,7} + \omega^{2,5} - \omega^{3,4}) + x_4\omega^{2,5} + x_5\omega^{2,6} + x_6\omega^{2,7}, -x_1(\omega^{1,3} + 2\omega^{1,4}) - 2x_2(\omega^{1,6} + 1/2\omega^{2,3} + 3/2\omega^{2,4}) - 3x_3(\omega^{1,7} + \omega^{2,5} - \omega^{3,4}) + 3x_4(\omega^{1,7} - 2/3\omega^{2,5} - 1/3\omega^{3,4}) - 2x_5\omega^{4,5} + x_6\omega^{3,6}, x_1\omega^{1,4} +$

$2x_2(\omega^{2,3} + 2\omega^{2,4}) + 2x_3(\omega^{1,7} + \omega^{2,5} - \omega^{3,4}) - 2x_4(\omega^{1,7} - 3/2\omega^{2,5}) + x_5(\omega^{2,6} + 2\omega^{4,5}) + x_6\omega^{4,6}, x_1\omega^{1,4} + x_2(\omega^{1,6} - \omega^{2,3} - \omega^{2,4}) - x_3(\omega^{1,7} + \omega^{2,5} - \omega^{3,4}) + x_4(\omega^{1,7} + \omega^{2,5}) + x_5\omega^{4,5} + x_7\omega^{3,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-5\beta_1} + 2e^{-6\beta_1} + 7e^{-7\beta_1} + 9e^{-8\beta_1} + 5e^{-9\beta_1} + 3e^{-10\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-11\beta_1} + 8e^{-12\beta_1} + 6e^{-13\beta_1} + 7e^{-14\beta_1} + 5e^{-15\beta_1} + e^{-17\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$2e^{-16\beta_1} + 3e^{-17\beta_1} + 5e^{-18\beta_1} + 5e^{-19\beta_1} + 2e^{-20\beta_1} + 2e^{-21\beta_1} + e^{-22\beta_1} + e^{-24\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-21\beta_1} + 2e^{-22\beta_1} + 3e^{-23\beta_1} + e^{-24\beta_1} + 2e^{-25\beta_1} + e^{-26\beta_1} + e^{-28\beta_1}$.

ch($H^7_{\mathcal{G}}$)=$e^{-26\beta_1} + e^{-27\beta_1} + e^{-29\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 54, 120, 153, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 18, 27, 28, 21, 11, 3)$

S=$30\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3} - \omega^{2,4}$; $d\omega^{1,7} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} - \omega^{1,2,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-3\beta_1}\ \langle\omega^2\rangle + e^{-4\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-7\beta_1}\ \langle\omega^{1,6},\omega^{2,3}\rangle + e^{-8\beta_1}\ \langle\omega^{1,7}+\omega^{2,5}-\omega^{3,4}\rangle + e^{-9\beta_1}\ \langle\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 3e^{-12\beta_1}\ \langle\omega^{1,5,6},\omega^{1,3,7}+\omega^{2,3,5},\omega^{1,4,7}-\omega^{2,3,5}\rangle + e^{-11\beta_1}\ \langle\omega^{1,2,7}\rangle + 2e^{-14\beta_1}\ \langle\omega^{1,6,7}-\omega^{2,3,7}-\omega^{2,5,6},\omega^{2,4,7}-\omega^{2,5,6}\rangle + e^{-13\beta_1}\ \langle\omega^{1,5,7}-\omega^{2,4,6}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-16\beta_1}\ \langle\omega^{1,3,4,7},\omega^{1,3,5,6}\rangle + 3e^{-18\beta_1}\ \langle\omega^{1,3,6,7}-\omega^{2,3,5,6},\omega^{2,3,4,7}-\omega^{2,3,5,6},\omega^{1,4,6,7}+\omega^{2,3,5,6}\rangle + e^{-17\beta_1}\ \langle\omega^{1,2,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{1,5,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-21\beta_1}\ \langle\omega^{1,2,3,6,7}\rangle + 2e^{-23\beta_1}\ \langle\omega^{1,3,5,6,7}+\omega^{2,3,4,5,7},\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-22\beta_1}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-25\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-26\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-27\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-29\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-30\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y^2)^2(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,7,7,5,3,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^7$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,7}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}+\omega^{4,7}$; $d^\star\omega^{2,3,5}=\omega^{5,7}$; $d^\star\omega^{2,3,6}=\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{1,6,7}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}-\omega^{2,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,4,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,7}-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}-\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,3,5,7}+\omega^{1,4,5,7}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}+\omega^{1,4,6,7}-\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{2,3,5,7}$;

$d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}+\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,6} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{1,6} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,7} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,7}$; $\Delta\omega^{4,5} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,7} + 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7}+\omega^{5,6}$; $\Delta\omega^{5,7} = 4\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}+\omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,7} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + 2\omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,7} + 2\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,3,5} - \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}+\omega^{2,3,7}+\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,6}+2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,7} - \omega^{1,5,6} + 2\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,3,6} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} + \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 3\omega^{3,5,7} - \omega^{4,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} - \omega^{3,5,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} + \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,6} + 3\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 2\omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,3,5,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6} + \omega^{1,4,5,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} =$

$\omega^{1,2,6,7}+3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,6,7}+\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,3,6,7}+\omega^{2,3,4,7}+2\omega^{2,3,5,6}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,5,6,7}+2\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7}=\omega^{2,3,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6}=\omega^{1,4,6,7}+\omega^{2,3,5,6}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=-\omega^{1,5,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{1,5,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{2,3,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}-\omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7}=-\omega^{3,5,6,7}+2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=4\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,5,6}+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{a}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-3\beta_1}\ \langle\omega^2\rangle+e^{-4\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{-5\beta_1}\ \langle\omega^{1,3}\rangle+2e^{-7\beta_1}\ \langle\omega^{1,6}-\omega^{2,4},\omega^{2,3}-\omega^{2,4}\rangle+e^{-8\beta_1}\ \langle\omega^{1,7}+\omega^{2,5}-\omega^{3,4}\rangle+e^{-9\beta_1}\ \langle\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-11\beta_1}\ \langle\omega^{1,2,7}-\omega^{1,3,6}+\omega^{2,3,4}\rangle+3e^{-12\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,3,5}-\omega^{2,4,5},\omega^{1,4,7}-\omega^{2,3,5}+2\omega^{2,4,5},\omega^{1,5,6}+\omega^{2,4,5}\rangle+2e^{-14\beta_1}\ \langle\omega^{1,6,7}-\omega^{2,3,7}-\omega^{2,5,6},\omega^{2,4,7}-\omega^{2,5,6}\rangle+e^{-13\beta_1}\ \langle\omega^{1,5,7}+2\omega^{2,3,6}-\omega^{2,4,6}+3\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= e^{-19\beta_1}\ \langle(-\omega^{1,5,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6})/3\rangle+3e^{-18\beta_1}\ \langle(\omega^{1,4,6,7}+\omega^{2,3,5,6}-2\omega^{2,4,5,6})/3,(-3\omega^{1,3,6,7}-\omega^{1,4,6,7}-3\omega^{2,3,4,7}+5\omega^{2,3,5,6}-4\omega^{2,4,5,6})/12,(-\omega^{1,3,6,7}+\omega^{1,4,6,7}+3\omega^{2,3,4,7}-\omega^{2,3,5,6})/4\rangle+2e^{-16\beta_1}\ \langle(\omega^{1,3,4,7}+\omega^{1,3,5,6}-2\omega^{1,4,5,6}+2\omega^{2,3,4,5})/5,(2\omega^{1,3,4,7}-3\omega^{1,3,5,6}+\omega^{1,4,5,6}-\omega^{2,3,4,5})/5\rangle+e^{-17\beta_1}\ \langle(3\omega^{1,2,6,7}-\omega^{1,3,5,7}-2\omega^{1,4,5,7}-\omega^{2,3,4,6})/15\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= e^{-25\beta_1}\ \langle-\omega^{2,4,5,6,7}\rangle+2e^{-23\beta_1}\ \langle(\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}+2\omega^{2,3,4,5,7})/3,(\omega^{1,3,5,6,7}+2\omega^{1,4,5,6,7}-\omega^{2,3,4,5,7})/3\rangle+e^{-22\beta_1}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6})/3\rangle+e^{-21\beta_1}\ \langle(-\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-29\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-27\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-26\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-30\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y^2)^2(1+Y)^3$.

5.3.28 Algebra $\mathcal{G}_{7,1.16}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_5]=x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^3 & \xi_4^5 & 4\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_4^6 & 0 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^3+\xi_2^5+\xi_2^6 & \xi_4^7 & \xi_1^2+\xi_2^3+\xi_2^4 & \xi_4^5+\xi_4^6 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^3+\delta_4^6, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^5+\delta_2^7), ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_1^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+2e^{3\beta_1}\ \langle x_3,x_4\rangle+2e^{4\beta_1}\ \langle x_5,x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle x_5\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $5e^{2\beta_1}\ \langle\delta_1^4,\delta_1^3,\delta_2^5,\delta_2^6,\delta_4^7\rangle+3e^{3\beta_1}\ \langle\delta_1^5,\delta_1^6,\delta_2^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+5e^{\beta_1}\ \langle\delta_1^2,\delta_2^3,\delta_2^4,\delta_4^5,\delta_4^6\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_1^4,\delta_4^5]=-\delta_1^5$; $[\delta_1^4,\delta_4^6]=-\delta_1^6$; $[\delta_1^5,\delta_1^2]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^6,\delta_4^5]=-\delta_1^7$; $[\delta_1^6,\delta_4^6]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^3,\delta_2^5]=-\delta_1^7$; $[\delta_1^3,\delta_2^6]=-\delta_1^7$; $[\delta_1^3,\delta_2^3]=-(\delta_2^7+\delta_1^5)$; $[\delta_1^3,\delta_2^4]=-\delta_1^6$; $[\delta_2^5,\delta_1^2]=-\delta_2^7+\delta_1^5$; $[\delta_2^5,\delta_2^4]=-\delta_2^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_2^6,\delta_2^3]=\delta_2^7$; $[\delta_2^6,\delta_4^5]=-\delta_2^7$; $[\delta_2^6,\delta_4^6]=-\delta_2^7$; $[\delta_4^7,\delta_2^4]=\delta_2^7$; $[\delta_1^2,\delta_2^3]=-\delta_1^3$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^2,\delta_4^5]=\delta_4^7$; $[\delta_2^3,\delta_2^4]=-\delta_2^6$; $[\delta_2^3,\delta_4^5]=\delta_4^7$; $[\delta_2^4,\delta_4^5]=\delta_4^7-\delta_2^5$; $[\delta_2^4,\delta_4^6]=-\delta_2^6$; $[\delta_4^5,\delta_4^6]=\delta_4^7$.

ch($H^1_{\mathcal{G}}$)= $4e^{2\beta_1}\ \langle\delta_1^4,\delta_2^5,\delta_2^6,\delta_4^7\rangle+e^{3\beta_1}\ \langle\delta_1^5\rangle+4e^{\beta_1}\ \langle\delta_1^2,\delta_2^4,\delta_4^5,\delta_2^3\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^5]=-2\delta_1^5$; $[\delta_1^2,\delta_4^5]=\delta_4^7$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^4,\delta_4^5]=-\delta_1^5$; $[\delta_2^5,\delta_2^4]=\delta_1^5$; $[\delta_4^5,\delta_2^6]=-\delta_1^5$; $[\delta_4^5,\delta_2^4]=-\delta_4^7+\delta_2^5$; $[\delta_4^5,\delta_2^3]=-\delta_4^7$; $[\delta_2^6,\delta_2^3]=-\delta_1^5$; $[\delta_2^4,\delta_2^3]=\delta_2^6$; $[\delta_2^4,\delta_4^7]=\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $11e^{-2\beta_1}\ \langle x_1\omega^{1,2}-x_3(\omega^{1,5}+\omega^{1,6}-\omega^{2,3})+x_4\omega^{2,4}, x_3(\omega^{1,5}+\omega^{2,3})+x_5\omega^{2,5}, x_3(\omega^{1,6}+\omega^{2,4})+x_5\omega^{2,6}, x_4\omega^{1,5}, x_4\omega^{1,6}, x_3\omega^{1,5}+x_4\omega^{2,3}+x_6\omega^{2,5}, -x_1\omega^{1,2}+x_3(\omega^{1,5}+2\omega^{1,6}-\omega^{2,3})+x_6\omega^{2,6}, -x_3\omega^{2,3}+x_5\omega^{1,7}, x_2\omega^{1,3}, -x_2\omega^{1,4}+x_3\omega^{2,4}+x_5\omega^{3,4}, -x_2\omega^{1,4}+x_7\omega^{4,5}\rangle+6e^{-\beta_1}\ \langle x_3\omega^{1,4}+x_5\omega^{2,4}, x_4\omega^{1,3}, x_4\omega^{1,4}, x_2\omega^{1,2}+x_5\omega^{2,3}, x_5\omega^{1,5}, x_5\omega^{1,6}\rangle+8e^{-3\beta_1}\ \langle x_1\omega^{1,4}+2x_2\omega^{2,4}+5x_3(\omega^{1,7}+\omega^{2,5}+\omega^{2,6}-$

$2/5\omega^{3,4}) + 5x_5(\omega^{2,7} + 1/5\omega^{4,5} + 1/5\omega^{4,6}), -x_1\omega^{1,3} - x_2(\omega^{1,5} + \omega^{1,6}) + x_3\omega^{1,7} + x_4(\omega^{1,7}+\omega^{2,5}+\omega^{2,6}-\omega^{3,4})+x_6\omega^{2,7}, -x_2\omega^{1,5}+x_4(\omega^{1,7}+\omega^{2,5})+x_6\omega^{3,5}, -2x_1\omega^{1,3} - 2x_2(\omega^{1,5} + 3/2\omega^{1,6} - \omega^{2,3}) + x_4(\omega^{2,6} - \omega^{3,4}) + x_6\omega^{3,6}, -x_2(\omega^{1,6} + \omega^{2,4}) - 3x_3(\omega^{1,7} + \omega^{2,5} + 2/3\omega^{2,6} - 1/3\omega^{3,4}) - 3x_5(\omega^{2,7} - 1/3\omega^{3,6}), -x_1\omega^{1,4} - 2x_2\omega^{2,4} - 4x_3(\omega^{1,7} + \omega^{2,5} + \omega^{2,6} - 1/4\omega^{3,4}) - 4x_5\omega^{2,7} + x_7\omega^{4,7}, -x_2(\omega^{1,5} - \omega^{2,3}) + x_3\omega^{2,5} + x_5\omega^{3,5}, x_1(\omega^{1,3} + \omega^{1,4}) + x_2(\omega^{1,5} + \omega^{1,6} - \omega^{2,3} + \omega^{2,4}) + 3x_3(\omega^{1,7} + \omega^{2,5} + \omega^{2,6} - 1/3\omega^{3,4}) + 3x_5(\omega^{2,7} + 1/3\omega^{4,6}) + x_6\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-4\beta_1} + 13e^{-5\beta_1} + 15e^{-6\beta_1} + 10e^{-7\beta_1} + e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=4e^{-8\beta_1} + 14e^{-9\beta_1} + 15e^{-10\beta_1} + 7e^{-11\beta_1} + e^{-12\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=5e^{-12\beta_1} + 9e^{-13\beta_1} + 9e^{-14\beta_1} + 4e^{-15\beta_1} + e^{-16\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-15\beta_1} + 3e^{-16\beta_1} + 4e^{-17\beta_1} + 2e^{-18\beta_1} + e^{-19\beta_1} + e^{-20\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-19\beta_1} + e^{-20\beta_1} + e^{-21\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 59, 128, 158, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 25, 40, 41, 28, 13, 3)$

S=$22\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 20, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 10, 10, 7, 3, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{1,6} - \omega^{2,3}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5} - \omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,5} - \omega^{1,3,6}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,5} - \omega^{1,4,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,3,5}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,3,6}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,5} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6}$; $d\omega^{4,5,7} = \omega^{1,3,4,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,5,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,6}$;

$d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 3e^{-5\beta_1}\ \langle\omega^{1,5},\omega^{1,6},\omega^{2,4}\rangle + 2e^{-6\beta_1}\ \langle\omega^{1,7}+\omega^{2,5},\omega^{2,6}-\omega^{3,4}\rangle + 2e^{-7\beta_1}\ \langle\omega^{2,7}-\omega^{3,5}-\omega^{3,6}-\omega^{4,5},\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 3e^{-8\beta_1}\ \langle\omega^{1,2,7},\omega^{1,3,5},\omega^{1,4,6}\rangle + 4e^{-9\beta_1}\ \langle\omega^{1,3,7},\omega^{1,5,6},\omega^{1,4,7}+\omega^{2,4,5},\omega^{2,4,6}\rangle + 3e^{-10\beta_1}\ \langle\omega^{1,5,7}-\omega^{2,3,7}+\omega^{2,5,6}+\omega^{3,4,5},\omega^{1,6,7}-\omega^{2,5,6}-\omega^{3,4,5},\omega^{2,4,7}-\omega^{3,4,5}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-12\beta_1}\ \langle\omega^{1,2,6,7},\omega^{1,2,5,7},\omega^{1,4,5,6}\rangle + 4e^{-13\beta_1}\ \langle\omega^{1,3,5,7},\omega^{1,3,6,7},\omega^{1,4,6,7}-\omega^{2,4,5,6},\omega^{2,3,4,7}+\omega^{2,3,5,6}+\omega^{2,4,5,6}\rangle + 3e^{-14\beta_1}\ \langle\omega^{1,5,6,7}-\omega^{2,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6},\omega^{2,3,5,7}+\omega^{2,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6},\omega^{2,4,6,7}+\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-15\beta_1}\ \langle\omega^{1,2,3,5,7},\omega^{1,2,3,6,7}\rangle + 2e^{-16\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,6,7}\rangle + 3e^{-17\beta_1}\ \langle\omega^{1,3,5,6,7},\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7},\omega^{2,3,4,5,7}+\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-19\beta_1}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-20\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-21\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-22\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,20,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,10,10,7,3,1)$

Total dimension of trivial cohomology: 42.

Length associated polynomial: 42.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,7}-\omega^{2,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}+\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,7}-\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7}-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7}-\omega^{2,3,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}+\omega^{2,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} =$

$-\omega^{3,4,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} + \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7} + \omega^{4,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,5} + \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,5}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,7} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{3,5} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{3,7} - \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} - \omega^{6,7}$; $\Delta\omega^{6,7} = -\omega^{5,7} + 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,2,6}$; $\Delta\omega^{1,2,6} = \omega^{1,2,5} + 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,7} + \omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,5} + 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = \omega^{1,4,5} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} - \omega^{1,4,7} + 2\omega^{1,5,6} - \omega^{2,3,5} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,3,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,4,5} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{1,6,7} + 2\omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,7} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{2,4,7} + 2\omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{2,6,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = -\omega^{2,5,7} + 2\omega^{2,6,7} + \omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 3\omega^{3,4,7} - \omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} - \omega^{2,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} - \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = -\omega^{3,5,7} + 3\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7} - \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = -\omega^{4,5,7} + 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5} + \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,5} + 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 3\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,5} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} - \omega^{1,2,4,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,3,4,5} =$

$\omega^{1,2,4,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,4,5} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + 2\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} - \omega^{1,2,6,7} - \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,4,7} + 2\omega^{1,4,5,6} + \omega^{2,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + 2\omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7} - \omega^{1,4,6,7} + 2\omega^{2,3,4,7} - \omega^{2,3,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,6,7} - \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,5,6,7} + \omega^{2,3,5,7} - \omega^{2,3,6,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + 2\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{3,4,5,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5} + \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} - \omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= 3e^{-5\beta_1}\ \langle\omega^{1,5} - \omega^{2,3}, \omega^{1,6} - \omega^{2,3}, \omega^{2,4}\rangle + 2e^{-6\beta_1}\ \langle\omega^{1,7} + \omega^{2,5}, \omega^{2,6} - \omega^{3,4}\rangle + 2e^{-7\beta_1}\ \langle\omega^{2,7} - \omega^{3,5} - \omega^{3,6} - \omega^{4,5}, \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= 3e^{-8\beta_1}\ \langle\omega^{1,2,7} - \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,4}, \omega^{1,3,5} - \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,4}, \omega^{1,4,6} + \omega^{2,3,4}\rangle + 4e^{-9\beta_1}\ \langle\omega^{1,3,7} + \omega^{2,3,5}, \omega^{1,4,7} + \omega^{2,3,6} + 2\omega^{2,4,5}, \omega^{1,5,6} + \omega^{2,3,5} - \omega^{2,3,6} - \omega^{2,4,5}, \omega^{2,4,6}\rangle + 3e^{-10\beta_1}\ \langle\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}, \omega^{1,6,7} - \omega^{2,5,6} - \omega^{3,4,5}, \omega^{2,4,7} - \omega^{3,4,5} - \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= 3e^{-14\beta_1}\ \langle(\omega^{1,5,6,7} - \omega^{2,3,5,7} - 2\omega^{2,3,6,7} - 2\omega^{2,4,5,7} + 5\omega^{2,4,6,7} + 7\omega^{3,4,5,6})/12, (\omega^{1,5,6,7} - \omega^{2,3,5,7} - 2\omega^{2,3,6,7} - 2\omega^{2,4,5,7} - 7\omega^{2,4,6,7} - 5\omega^{3,4,5,6})/12, (-5\omega^{1,5,6,7} - 7\omega^{2,3,5,7} - 2\omega^{2,3,6,7} - 2\omega^{2,4,5,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6})/12\rangle + 4e^{-13\beta_1}\ \langle(-\omega^{1,4,5,7} + 3\omega^{1,4,6,7} - 2\omega^{2,3,4,7} - \omega^{2,3,5,6} - 5\omega^{2,4,5,6})/8, (-8\omega^{1,3,6,7} - \omega^{1,4,5,7} + 3\omega^{1,4,6,7} + 6\omega^{2,3,4,7} + 7\omega^{2,3,5,6} + 3\omega^{2,4,5,6})/24, (\omega^{1,4,5,7} + \omega^{1,4,6,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6} + \omega^{2,4,5,6})/4, \omega^{1,3,5,7}\rangle + 3e^{-12\beta_1}\ \langle(\omega^{1,2,5,7} + 2\omega^{1,2,6,7} - 3\omega^{1,3,4,7} - \omega^{1,3,5,6} - 8\omega^{1,4,5,6} + 5\omega^{2,3,4,5} - 8\omega^{2,3,4,6})/21, (-2\omega^{1,2,5,7} - 4\omega^{1,2,6,7} + 6\omega^{1,3,4,7} +$

$2\omega^{1,3,5,6} - 5\omega^{1,4,5,6} + 11\omega^{2,3,4,5} - 5\omega^{2,3,4,6})/21, (8\omega^{1,2,5,7} - 5\omega^{1,2,6,7} - 3\omega^{1,3,4,7} - 8\omega^{1,3,5,6} - \omega^{1,4,5,6} - 2\omega^{2,3,4,5} - \omega^{2,3,4,6})/21\rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}})= 3e^{-17\beta_1} \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7} - 2\omega^{2,3,4,6,7})/3, (-\omega^{1,4,5,6,7} + 2\omega^{2,3,4,5,7} + \omega^{2,3,4,6,7})/3, -\omega^{1,3,5,6,7}\rangle + 2e^{-16\beta_1} \langle(\omega^{1,3,4,6,7} - \omega^{2,3,4,5,6})/2, (-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle + 2e^{-15\beta_1} \langle(-\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,2,4,6,7} + \omega^{1,3,4,5,6})/4, -\omega^{1,2,3,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}})= e^{-21\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-20\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}})= e^{-22\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.3.29 Algebra $\mathcal{G}_{7,1.17}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_5] = x_6$; $[x_2, x_6] = x_7$; $[x_3, x_4] = -x_7$; $[x_3, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^4 - \xi_1^5 & \xi_2^3 & -\xi_1^3 & 4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^6 + \xi_2^6 & \xi_1^4 + \xi_2^4 - 2\xi_1^5 & 2\xi_2^4 - \xi_1^5 - \xi_2^5 & -\xi_1^3 + \xi_2^3 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_1^6, ad\ x_5 = -\delta_2^6, ad\ x_6 = -(\delta_1^7 + \delta_2^7), ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1 \rangle$.

ch($\mathcal{G}$)= $2e^{\beta_1}\ \langle x_1, x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + 2e^{3\beta_1}\ \langle x_4, x_5\rangle + e^{4\beta_1}\ \langle x_6\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{4\beta_1}\ \langle \delta_1^7, \delta_2^7\rangle + 2e^{3\beta_1}\ \langle \delta_1^6, \delta_2^6\rangle + 4e^{2\beta_1}\ \langle \delta_1^4, \delta_2^4, \delta_1^5, \delta_2^5\rangle + 2e^{\beta_1}\ \langle \delta_1^3, \delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_2^3] = -(\delta_2^7 + \delta_1^7)$; $[\delta_2^6, \delta_1^3] = \delta_2^7 + \delta_1^7$; $[\delta_1^4, \delta_2^4] = \delta_2^7 - \delta_1^7$; $[\delta_1^4, \delta_1^5] = 2\delta_1^7$; $[\delta_1^4, \delta_2^3] = -\delta_1^6$; $[\delta_2^4, \delta_1^5] = 2(\delta_2^7 + \delta_1^7)$; $[\delta_2^4, \delta_2^5] = 2\delta_2^7$; $[\delta_2^4, \delta_1^3] = \delta_1^6$; $[\delta_1^5, \delta_2^5] = -\delta_2^7 + \delta_1^7$; $[\delta_1^5, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_1^3] = \delta_2^6$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle \delta_1^7\rangle + 3e^{2\beta_1}\ \langle \delta_2^4, \delta_1^5, \delta_1^4\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^5] = 2\delta_1^7$; $[\delta_2^4, \delta_1^4] = 2\delta_1^7$; $[\delta_1^5, \delta_1^4] = -2\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $4e^{-\beta_1}\ \langle x_1\omega^{1,2} - x_2\omega^{1,2} + 2x_3(\omega^{1,3} - 1/2\omega^{2,3}) - 2x_4(\omega^{1,4} - 3/2\omega^{1,5} - 1/2\omega^{2,4}) + x_5(\omega^{1,5} + \omega^{2,5}), -x_1\omega^{1,2} + x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4(\omega^{1,4} - 1/2\omega^{1,5} + 1/2\omega^{2,5}) + x_5(\omega^{1,4} + \omega^{2,5}), -x_1\omega^{1,2} + 2x_2\omega^{1,2} - 2x_3(\omega^{1,3} - 1/2\omega^{2,3}) + 3x_4(\omega^{1,4} - \omega^{1,5} - 1/3\omega^{2,4} + 2/3\omega^{2,5}) + x_5\omega^{2,5} + x_7\omega^{1,7}, -3/2x_1\omega^{1,2} + 3/2x_2\omega^{1,2} - 3/2x_3(\omega^{1,3} + 1/3\omega^{2,3}) + 1/2x_4(\omega^{1,4} - 3\omega^{1,5} - \omega^{2,4} + 3\omega^{2,5}) + 2x_5\omega^{2,5} + x_6\omega^{3,4}\rangle + e^{-4\beta_1}\ \langle -x_1(\omega^{2,6} - \omega^{3,4}) + x_2(\omega^{1,6} + \omega^{3,5}) + x_3\omega^{4,5} - x_4\omega^{5,6} + x_5\omega^{4,6}\rangle + 2e^{\beta_1}\ \langle x_4\omega^{1,2} + x_5\omega^{1,2}, x_7\omega^{1,4}\rangle + 3e^{-2\beta_1}\ \langle x_2\omega^{1,3} + x_4(\omega^{1,6} + \omega^{3,5}), -x_3(\omega^{1,4} - \omega^{1,5} - \omega^{2,4} + \omega^{2,5}) + x_4(\omega^{1,6} + \omega^{2,6}) + x_5(\omega^{1,6} + \omega^{2,6}), x_1\omega^{1,3} + x_3(\omega^{2,4} - \omega^{2,5}) + x_4(\omega^{1,6} + \omega^{3,4})\rangle$.

ch($H^3_{\mathcal{G}}$)=$3e^{-4\beta_1} + 8e^{-5\beta_1} + e^{-6\beta_1} + 3e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-6\beta_1}+7e^{-8\beta_1}+3e^{-9\beta_1}+5e^{-10\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-11\beta_1}+3e^{-12\beta_1}+2e^{-13\beta_1}+4e^{-14\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-14\beta_1}+3e^{-15\beta_1}+3e^{-17\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=2e^{-18\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,48,114,147,110,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,10,15,16,12,7,2)$

S=$19\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}-\omega^{2,5}$; $d\omega^7=-\omega^{1,6}-\omega^{2,6}+\omega^{3,4}-\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,5}$; $d\omega^{1,7}=\omega^{1,2,6}-\omega^{1,3,4}+\omega^{1,3,5}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}+\omega^{2,3,5}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,4}-\omega^{2,3,5}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,6}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}-\omega^{2,4,5}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,5,6}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{2,5,7}-\omega^{3,4,6}+\omega^{3,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,6}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,6}=\omega^{1,2,4,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}+\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,5,7}+\omega^{1,3,4,6}-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}+\omega^{2,3,4,6}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}-\omega^{2,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,7}-\omega^{2,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,3,4,6}+\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,5,7}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7}=\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$;

$d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= 2e^{-\beta_1}\ \langle\omega^1,\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-4\beta_1}\ \langle\omega^{1,5}+\omega^{2,4},\omega^{1,4}\rangle + e^{-5\beta_1}\ \langle\omega^{1,6}+\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1}\ \langle\omega^{1,2,6}\rangle + 3e^{-8\beta_1}\ \langle\omega^{1,5,6}+\omega^{2,4,6},\omega^{2,3,7}-\omega^{2,4,6},\omega^{1,3,7}+\omega^{2,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-11\beta_1}\ \langle\omega^{1,2,6,7}-\omega^{2,3,4,7}+\omega^{2,4,5,6},\omega^{1,3,4,7}+\omega^{2,3,4,7},\omega^{1,3,5,7}+\omega^{2,4,5,6}\rangle + e^{-13\beta_1}\ \langle\omega^{1,4,6,7}-3\omega^{1,5,6,7}-3\omega^{2,4,6,7}+\omega^{2,5,6,7}+2\omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-14\beta_1}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}\rangle + 2e^{-15\beta_1}\ \langle\omega^{1,3,5,6,7},\omega^{1,3,4,6,7}-\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^{-18\beta_1}\ \langle\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-19\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = \omega^{1,7}-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5} = \omega^{1,7}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,5} = \omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{2,3,6} = -\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,6}$; $d^\star\omega^{2,4,6} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,5,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}-\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{1,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = \omega^{1,4,7}-\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,5,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = \omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}+\omega^{1,5,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} =$

$\omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7} + \omega^{2,5,7} - \omega^{3,4,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7} - \omega^{3,5,7}$; $d^\star\omega^{2,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7} + \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,2,5,7} + \omega^{1,3,4,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,4,5,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 4\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,5}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{1,7} = 3\omega^{1,7} - \omega^{2,7} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,4} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{1,6} + 2\omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = -\omega^{1,7} + 3\omega^{2,7} - \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} + 2\omega^{3,4} - \omega^{3,5}$; $\Delta\omega^{3,5} = \omega^{2,6} - \omega^{3,4} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,6} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} - \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7} + \omega^{5,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{4,7} + 4\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 3\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 3\omega^{1,2,7} - \omega^{1,4,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4} - \omega^{1,3,5} + \omega^{2,3,5}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} - \omega^{1,3,4} + 2\omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 3\omega^{1,3,6} + \omega^{2,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,6} + \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,4,6} + \omega^{2,5,6}$; $\Delta\omega^{1,4,7} = 3\omega^{1,4,7} + \omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,7}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + 2\omega^{1,5,6} - \omega^{2,4,6} + \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,4,7} + 3\omega^{1,5,7} - \omega^{2,4,7} - \omega^{2,5,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} - \omega^{1,3,5} + 2\omega^{2,3,4} - \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{1,3,4} - \omega^{2,3,4} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,3,6} + 3\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,7} + 2\omega^{2,3,7} + \omega^{2,4,6} + \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,4,6} - \omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,7} - \omega^{1,5,7} + 3\omega^{2,4,7} + \omega^{2,5,7}$; $\Delta\omega^{2,5,6} = \omega^{1,4,6} + \omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,4,7} - \omega^{1,5,7} + \omega^{2,4,7} + 3\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7} + \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6} + 3\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 3\omega^{3,4,6} - \omega^{3,5,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 3\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{3,4,6} + 3\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} =$

$3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{2,6,7} - \omega^{3,4,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 4\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7} + \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = \omega^{4,6,7} + 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} - \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = -\omega^{1,2,3,4} + 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 4\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} + \omega^{1,2,5,6} + \omega^{1,3,4,5} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 3\omega^{1,2,4,7} + \omega^{1,2,5,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 3\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,4,7} + 3\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 3\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7} + \omega^{1,2,4,6} + 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 3\omega^{1,3,4,6} - \omega^{1,3,5,6} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,6} + 3\omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{1,3,5,7} + 2\omega^{1,4,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,4,5,7} = 3\omega^{1,4,5,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{1,5,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,4,6,7} + 2\omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,4,6} - \omega^{1,3,5,6} + 3\omega^{2,3,4,6} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,3,5,7} + 2\omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,4,6} + \omega^{1,3,5,6} - \omega^{2,3,4,6} + 3\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,4,7} - \omega^{1,3,5,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,7} + 3\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + 2\omega^{2,4,6,7} + \omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{1,4,6,7} + \omega^{2,4,6,7} + 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,6,7} + 3\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + 3\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,6} + 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,3,5,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,4,6,7} + 2\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,6,7} + 3\omega^{1,3,4,5,6} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6} + 3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 4\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1}\ \langle \omega^1, \omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-4\beta_1}\ \langle \omega^{1,5} + \omega^{2,4}, \omega^{1,4} - \omega^{2,5}\rangle + e^{-5\beta_1}\ \langle \omega^{1,6} - \omega^{2,6} + \omega^{3,4} + \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1}\ \langle 2\omega^{1,2,6} - \omega^{1,3,4} - 3\omega^{1,3,5} - 3\omega^{2,3,4} - \omega^{2,3,5}\rangle + 3e^{-8\beta_1}\ \langle \omega^{1,4,6} + \omega^{2,3,7} - \omega^{2,4,6} - \omega^{2,5,6}, 3\omega^{1,3,7} + 4\omega^{2,3,7} - \omega^{2,4,6} - 2\omega^{2,5,6} - 2\omega^{3,4,5}, 3\omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{2,4,6} - 2\omega^{2,5,6} - 2\omega^{3,4,5}\rangle.$

ch$(H^4_{\mathbb{C}})$= $e^{-13\beta_1}$ $\langle(-\omega^{1,4,6,7}+3\omega^{1,5,6,7}+3\omega^{2,4,6,7}-\omega^{2,5,6,7}-2\omega^{3,4,5,7})/24\rangle$ + $3e^{-11\beta_1}$ $\langle(2\omega^{1,2,6,7}+4\omega^{1,3,4,7}-4\omega^{1,3,5,7}+\omega^{1,4,5,6}+\omega^{2,3,4,7}-6\omega^{2,3,5,7}+4\omega^{2,4,5,6})/15,(-12\omega^{1,2,6,7}-4\omega^{1,3,4,7}-\omega^{1,3,5,7}+19\omega^{1,4,5,6}-11\omega^{2,3,4,7}+16\omega^{2,3,5,7}-29\omega^{2,4,5,6})/180,(-12\omega^{1,2,6,7}+16\omega^{1,3,4,7}+19\omega^{1,3,5,7}-\omega^{1,4,5,6}+29\omega^{2,3,4,7}-4\omega^{2,3,5,7}+11\omega^{2,4,5,6})/180\rangle$.

ch$(H^5_{\mathbb{C}})$= $2e^{-15\beta_1}$ $\langle(-\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7})/2,(-\omega^{1,3,4,6,7}+\omega^{2,3,5,6,7})/2\rangle$ + $e^{-14\beta_1}$ $\langle(-\omega^{1,2,4,6,7}+\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}+\omega^{2,3,4,5,7})/4\rangle$.

ch$(H^6_{\mathbb{C}})$= $2e^{-18\beta_1}$ $\langle\omega^{2,3,4,5,6,7},-\omega^{1,3,4,5,6,7}\rangle$.

ch$(H^7_{\mathbb{C}})$= $e^{-19\beta_1}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y)$.

5.3.30 Algebra $\mathcal{G}_{7,1.18}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_6+x_7$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix}\xi_1^1&0&0&0&0&0&0\\0&2\xi_1^1&0&0&0&0&0\\\xi_1^3&\xi_2^3&3\xi_1^1&0&0&0&0\\\xi_1^4&\xi_2^4&0&3\xi_1^1&0&0&0\\\xi_1^5&\xi_2^5&0&\xi_2^4&4\xi_1^1&0&0\\\xi_1^6&\xi_2^6&\xi_3^6&-(\xi_1^3+\xi_1^4)&0&5\xi_1^1&0\\\xi_1^7&\xi_2^7&\xi_3^7&-\xi_1^3+\xi_2^5&\xi_2^4&0&5\xi_1^1\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^6+\delta_3^7, ad\ x_3=-(\delta_2^6+\delta_2^7), ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{2\beta_1}\ \langle x_2\rangle+2e^{3\beta_1}\ \langle x_3,x_4\rangle+e^{4\beta_1}\ \langle x_5\rangle+2e^{5\beta_1}\ \langle x_6,x_7\rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}})= 2e^{5\beta_1}\ \langle x_6,x_7\rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_1}\ \langle\delta_2^3,\delta_2^4\rangle+3e^{3\beta_1}\ \langle\delta_1^5,\delta_2^6,\delta_2^7\rangle+2e^{4\beta_1}\ \langle\delta_1^6,\delta_1^7\rangle+5e^{2\beta_1}\ \langle\delta_3^6,\delta_1^4,\delta_3^7,\delta_1^3,\delta_2^5\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^4,\delta_1^3]=\delta_1^7+\delta_1^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^4]=-(\delta_2^7+\delta_2^6)$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1}\ \langle\delta_2^3\rangle+e^{4\beta_1}\ \langle\delta_1^6\rangle+e^{3\beta_1}\ \langle\delta_1^5\rangle+4e^{2\beta_1}\ \langle\delta_1^4,\delta_3^6,\delta_1^3,\delta_2^5\rangle+e^0\ \langle t_1\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^6]=\delta_1^5$; $[\delta_1^4,\delta_1^3]=\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-4\beta_1}\ \langle x_3\omega^{2,6}\rangle+8e^{-2\beta_1}\ \langle -x_1\omega^{1,2}+2x_3\omega^{1,5}-2x_4\omega^{1,5}+x_5\omega^{2,5},-x_2\omega^{1,3}+x_6\omega^{3,5},x_3\omega^{2,4},x_1\omega^{1,2}-x_3\omega^{1,5}+x_4(\omega^{1,5}-\omega^{2,3})+x_5\omega^{1,6},x_3\omega^{2,3},-x_4\omega^{2,3}+x_5\omega^{1,7},-x_2(\omega^{1,3}+\omega^{1,4})-x_4\omega^{2,3}+x_5\omega^{3,4},x_1\omega^{1,2}-x_3\omega^{1,5}+x_4(\omega^{1,5}+\omega^{2,4})\rangle+6e^{-3\beta_1}\ \langle x_3(\omega^{1,6}-\omega^{1,7}+\omega^{2,5})-x_4(\omega^{1,6}-\omega^{1,7}+\omega^{2,5})+x_5(\omega^{2,6}-\omega^{2,7}),x_1\omega^{1,3}+2x_2(\omega^{2,3}+5/2\omega^{2,4})-5x_3(\omega^{3,4}-\omega^{1,7})+2x_4(\omega^{3,4}-5/2\omega^{1,7}+5/2\omega^{2,5})+x_5(\omega^{3,5}+5\omega^{2,7}),-x_2(\omega^{2,3}+\omega^{2,4})+x_3(\omega^{3,4}-\omega^{1,7})+x_4(\omega^{1,7}-\omega^{2,5})-x_5\omega^{2,7}+x_6\omega^{3,6},x_1\omega^{1,3}+x_2(\omega^{1,5}+2\omega^{2,3}+3\omega^{2,4})-3x_3(\omega^{3,4}-\omega^{1,7})+x_4(\omega^{3,4}-4\omega^{1,7}+3\omega^{2,5})+3x_5\omega^{2,7}+x_6(\omega^{3,7}+\omega^{4,7}),-2x_2(\omega^{2,3}+\omega^{2,4})+x_3(\omega^{3,4}-\omega^{1,6}-\omega^{2,5})+2x_4\omega^{1,6}-x_6\omega^{4,6}+x_7\omega^{4,6},-x_1\omega^{1,4}-4x_2(\omega^{2,3}+1/2\omega^{2,4})+x_3(\omega^{3,4}-\omega^{1,7})+3x_4(\omega^{1,6}+1/3\omega^{1,7})-3x_6\omega^{4,6}+x_7\omega^{4,7}\rangle+5e^{-\beta_1}\ \langle x_2\omega^{1,2}-$

$x_3\omega^{1,3} + x_4\omega^{1,3} + x_6\omega^{1,6}, x_4\omega^{1,3} + x_6\omega^{1,7}, x_3\omega^{1,3} - x_4\omega^{1,3} + x_5\omega^{2,3}, -x_2\omega^{1,2} + x_3(\omega^{1,3} + \omega^{1,4}) + x_5\omega^{2,4}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-4\beta_1} + 9e^{-5\beta_1} + 9e^{-6\beta_1} + 12e^{-7\beta_1} + e^{-8\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-8\beta_1} + 6e^{-9\beta_1} + 13e^{-10\beta_1} + 7e^{-11\beta_1} + 3e^{-12\beta_1} + 2e^{-13\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$6e^{-13\beta_1} + 8e^{-14\beta_1} + 5e^{-15\beta_1} + 4e^{-16\beta_1} + e^{-18\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$2e^{-16\beta_1} + 3e^{-17\beta_1} + 2e^{-18\beta_1} + 3e^{-19\beta_1} + e^{-20\beta_1} + e^{-21\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-20\beta_1} + e^{-21\beta_1} + e^{-22\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 13, 56, 123, 155, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 8, 20, 32, 33, 24, 12, 3)$

S=$23\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{2,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} + \omega^{2,3,5}$; $d\omega^{6,7} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,3,7} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,3,7} + \omega^{1,2,4,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7}$; $d\omega^{2,6,7} = \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,3,5,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6} + \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-2\beta_1}$ $\langle\omega^2\rangle + e^{-3\beta_1}$ $\langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{X}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle+e^{-5\beta_1}\ \langle\omega^{1,5}\rangle+2e^{-7\beta_1}\ \langle\omega^{2,6},\omega^{2,7}-\omega^{4,5}\rangle+2e^{-6\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4},\omega^{1,7}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{X}})= 3e^{-9\beta_1}\ \langle\omega^{1,3,7},\omega^{1,3,6}+\omega^{2,3,5},\omega^{1,4,6}-\omega^{2,3,5}\rangle+e^{-8\beta_1}\ \langle\omega^{1,2,7}\rangle+4e^{-10\beta_1}\ \langle\omega^{2,3,6},\omega^{2,4,6},\omega^{1,5,6}-\omega^{2,4,7}+\omega^{3,4,5},\omega^{1,5,7}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{X}})= 4e^{-13\beta_1}\ \langle\omega^{1,2,6,7},\omega^{1,3,5,7},\omega^{1,4,5,6}+\omega^{2,3,4,7},\omega^{1,4,5,7}+\omega^{2,3,4,7}\rangle+3e^{-14\beta_1}\ \langle\omega^{1,3,6,7}+\omega^{2,3,5,7},\omega^{2,4,5,6},\omega^{1,4,6,7}+\omega^{2,3,5,6}-\omega^{2,3,5,7}\rangle+e^{-15\beta_1}\ \langle\omega^{2,3,6,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{X}})= 2e^{-17\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,4,6,7}\rangle+2e^{-16\beta_1}\ \langle\omega^{1,2,3,6,7},\omega^{1,3,4,5,7}\rangle+e^{-18\beta_1}\ \langle\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle+e^{-19\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{X}})= e^{-20\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle+e^{-21\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle+e^{-22\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{X}})= e^{-23\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{X}}$'s $0\le j\le 7$: $(1,3,10,19,24,17,7,1)$

Dimensions of $H^j_{\mathfrak{X}}$'s $0\le j\le 7$: $(1,3,6,8,8,6,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^6-\omega^7$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{1,2,3}=\omega^{1,6}+\omega^{1,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,7}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,6}=\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,6}+\omega^{4,6}+\omega^{4,7}$; $d^\star\omega^{2,3,5}=\omega^{5,6}+\omega^{5,7}$; $d^\star\omega^{2,3,6}=\omega^{6,7}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,6}-\omega^{1,4,6}-\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,6}-\omega^{1,5,7}-\omega^{2,3,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{1,6,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,6}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{2,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,6}-\omega^{4,5,6}-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,3,5,6}+\omega^{1,4,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,4,6,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,6,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{3,4,6,7}$;

$d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6} + \omega^{7}$; $\Delta\omega^{7} = \omega^{6} + 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{1,7} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,6} + \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + 2\omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 2\omega^{4,6} + \omega^{4,7}$; $\Delta\omega^{4,7} = -\omega^{3,6} + \omega^{4,6} + 3\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6} + \omega^{5,7}$; $\Delta\omega^{5,7} = \omega^{5,6} + 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{1,4,7} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} + \omega^{1,4,7} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + \omega^{1,4,6} + 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6} + \omega^{1,5,7} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,6} + \omega^{1,5,7} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,4,6} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} - \omega^{1,2,7} - \omega^{1,4,5} + 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 3\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,3,7} - \omega^{2,4,7}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} - \omega^{2,3,6} + 2\omega^{2,3,7} + \omega^{2,4,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{2,3,6} + \omega^{2,3,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{1,5,7} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} - \omega^{3,5,6} + 2\omega^{4,5,6} + \omega^{4,5,7}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,6} + \omega^{4,5,6} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,3,7} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + \omega^{1,2,3,7} + 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{1,4,5,6} - \omega^{1,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,6} + 2\omega^{1,4,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,5,6} + \omega^{1,4,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 3\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,2,5,7} - \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6} - \omega^{2,3,4,7}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6} + 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6} - \omega^{2,3,5,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} - \omega^{2,3,5,6} + 2\omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} =$

$-\omega^{1,4,6,7}-\omega^{2,3,5,6}+\omega^{2,3,5,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}-\omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7}=-\omega^{3,5,6,7}+3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=2\omega^{1,2,3,4,6}-\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,4,7}=-\omega^{1,2,3,4,6}+3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6}=3\omega^{1,2,3,5,6}-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7}=-\omega^{1,2,3,5,6}+2\omega^{1,2,3,5,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,5,6}+\omega^{1,2,3,5,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{2,3,4,5,6}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,3,5,6,7}+2\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,6}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}-\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,5,7}=-\omega^{1,2,3,4,5,6}+2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\Phi})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-2\beta_1}\ \langle\omega^2\rangle+e^{-3\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\Phi})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle+e^{-5\beta_1}\ \langle\omega^{1,5}-\omega^{2,3}+\omega^{2,4}\rangle+2e^{-7\beta_1}\ \langle\omega^{2,6},\omega^{2,7}-\omega^{4,5}\rangle+2e^{-6\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4},\omega^{1,7}-\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\Phi})= 3e^{-9\beta_1}\ \langle\omega^{1,3,7},2\omega^{1,3,6}+\omega^{1,4,7}+\omega^{2,3,5}-\omega^{2,4,5},2\omega^{1,4,6}-\omega^{1,4,7}-\omega^{2,3,5}+3\omega^{2,4,5}\rangle+e^{-8\beta_1}\ \langle\omega^{1,2,7}-\omega^{1,4,5}\rangle+4e^{-10\beta_1}\ \langle\omega^{2,4,6},\omega^{1,5,6}+2\omega^{2,3,7}-\omega^{2,4,7}+3\omega^{3,4,5},\omega^{1,5,7}+\omega^{3,4,5},\omega^{2,3,6}+\omega^{2,3,7}+\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\Phi})= 3e^{-14\beta_1}\ \langle-\omega^{2,4,5,6},(\omega^{1,4,6,7}+\omega^{2,3,5,6}-\omega^{2,3,5,7}+3\omega^{2,4,5,7})/8,(-8\omega^{1,3,6,7}-\omega^{1,4,6,7}-\omega^{2,3,5,6}-7\omega^{2,3,5,7}+5\omega^{2,4,5,7})/40\rangle+e^{-15\beta_1}\ \langle(-\omega^{2,3,6,7}+\omega^{3,4,5,6})/2\rangle+4e^{-13\beta_1}\ \langle\omega^{1,3,5,7},(4\omega^{1,2,6,7}+5\omega^{1,3,5,6}+3\omega^{1,4,5,6}+7\omega^{1,4,5,7}+4\omega^{2,3,4,6}+5\omega^{2,3,4,7})/28,(2\omega^{1,2,6,7}-\omega^{1,3,5,6}-2\omega^{1,4,5,6}-5\omega^{2,3,4,6}-\omega^{2,3,4,7})/7,(-\omega^{1,3,5,6}+\omega^{1,4,5,6}-3\omega^{1,4,5,7}-\omega^{2,3,4,7})/4\rangle.$

$\mathrm{ch}(H^5_{\Phi})= e^{-19\beta_1}\ \langle-\omega^{2,4,5,6,7}\rangle+e^{-18\beta_1}\ \langle(-\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7})/3\rangle+2e^{-16\beta_1}\ \langle-\omega^{1,3,4,5,7},(-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6})/2\rangle+2e^{-17\beta_1}\ \langle(-\omega^{1,2,5,6,7}+2\omega^{1,3,4,6,7}+\omega^{2,3,4,5,6}+2\omega^{2,3,4,5,7})/5,(-2\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-3\omega^{2,3,4,5,6}-\omega^{2,3,4,5,7})/5\rangle.$

$\mathrm{ch}(H^6_{\Phi})= e^{-22\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-21\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-20\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\Phi})= e^{-23\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y^2)(1+Y).$

5.3.31 Algebra $\mathcal{G}_{7,1.19}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_7$; $[x_1,x_5]=x_6$; $[x_2,x_4]=x_6$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & 0 & \xi_3^5 & 0 & 2\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & \xi_3^5 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^4 & -\xi_1^5 & 0 & 3\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_2^6+\delta_1^7), ad\ x_5=-(\delta_1^6+\delta_3^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $3e^{\beta_1}\ \langle x_1,x_2,x_3\rangle+2e^{2\beta_1}\ \langle x_4,x_5\rangle+2e^{3\beta_1}\ \langle x_6,x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $2e^{3\beta_1}\ \langle x_6,x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $6e^{2\beta_1}\ \langle\delta_1^6,\delta_2^6,\delta_3^6,\delta_1^7,\delta_2^7,\delta_3^7\rangle+4e^{\beta_1}\ \langle\delta_1^4,\delta_3^5,\delta_2^4,\delta_1^5\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^4]=-(\delta_1^7+\delta_2^6)$; $[\delta_3^5,\delta_1^5]=\delta_3^7+\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $4e^{2\beta_1}\ \langle\delta_3^6,\delta_2^6,\delta_2^7,\delta_1^6\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $6e^{-2\beta_1}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})+3x_3(\omega^{1,4}-2/3\omega^{3,5})+x_4\omega^{4,5}, -2x_1\omega^{1,4}-3x_2(\omega^{1,5}-2/3\omega^{2,4})+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5}, x_2\omega^{1,4}, x_1\omega^{2,4}+x_3\omega^{3,5}+x_5\omega^{3,6}+x_6\omega^{5,6}, x_1\omega^{1,4}-x_2\omega^{2,4}+x_5\omega^{3,7}+x_6\omega^{5,7}, x_3\omega^{1,5}\rangle+11e^{-\beta_1}\ \langle -2x_1\omega^{1,2}-x_3\omega^{2,3}+3x_4(\omega^{1,5}-2/3\omega^{2,4})+x_5\omega^{2,5}, -x_1\omega^{1,3}-2x_2\omega^{2,3}-4x_3\omega^{1,2}-4x_4(\omega^{2,5}+1/4\omega^{3,4})+x_6\omega^{3,6}, -x_2\omega^{1,3}+x_6\omega^{3,7}, -x_4\omega^{1,5}+x_6\omega^{2,6}, -x_2\omega^{1,2}-x_3\omega^{1,3}-x_4\omega^{3,5}+x_6\omega^{2,7}, -x_3\omega^{1,2}+x_7\omega^{2,6}, x_1\omega^{1,2}+x_3\omega^{2,3}-x_4(\omega^{1,5}-2\omega^{2,4})+x_7\omega^{2,7}, -x_1\omega^{2,3}+x_2\omega^{1,2}-x_3\omega^{1,3}-x_4\omega^{3,5}+x_5\omega^{2,4}, x_2\omega^{1,2}+x_6\omega^{1,6}, x_1\omega^{1,3}+x_2\omega^{2,3}+3x_3\omega^{1,2}+3x_4(\omega^{2,5}+1/3\omega^{3,4})+x_6\omega^{1,7}, x_2\omega^{2,3}+2x_3\omega^{1,2}+2x_4(\omega^{2,5}+1/2\omega^{3,4})+x_5\omega^{1,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-2\beta_1}+15e^{-3\beta_1}+9e^{-4\beta_1}+3e^{-5\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$3e^{-5\beta_1}+22e^{-6\beta_1}+7e^{-7\beta_1}$.

ch($H^5_{\mathcal{G}}$)=$6e^{-7\beta_1}+11e^{-8\beta_1}+6e^{-9\beta_1}+2e^{-10\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-9\beta_1}+3e^{-10\beta_1}+7e^{-11\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$3e^{-12\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,11,55,120,157,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,6,17,28,32,25,11,3)$

S=$13\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{1,4}-\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,4}+\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,3,7}+\omega^{1,4,5}$; $d\omega^{6,7}=\omega^{1,4,6}-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}+\omega^{1,2,4,5}$; $d\omega^{2,6,7}=\omega^{1,2,4,6}-\omega^{1,2,5,7}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}$; $d\omega^{3,5,6}=\omega^{2,3,4,5}$; $d\omega^{3,5,7}=\omega^{1,3,4,5}$; $d\omega^{3,6,7}=\omega^{1,3,4,6}-\omega^{1,3,5,7}-\omega^{2,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,6}+\omega^{2,4,5,7}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=\omega^{1,2,3,4,6}-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7}=\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $3e^{-\beta_1}$ $\langle\omega^1,\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{-2\beta_1}$ $\langle\omega^{2,3}\rangle+3e^{-3\beta_1}$ $\langle\omega^{1,5},\omega^{2,5}+\omega^{3,4},\omega^{1,4}\rangle+e^{-4\beta_1}$ $\langle\omega^{2,6}-\omega^{3,7}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})$= $3e^{-5\beta_1}$ $\langle\omega^{1,2,6},\omega^{2,3,7}+\omega^{2,4,5},\omega^{2,3,6}+\omega^{3,4,5}\rangle$ + $6e^{-6\beta_1}$ $\langle\omega^{1,4,6},\omega^{1,5,7},\omega^{1,5,6}+\omega^{2,5,7}+\omega^{3,4,7},\omega^{2,4,6}+\omega^{2,5,7}+\omega^{3,4,7},\omega^{1,4,7}-\omega^{3,5,7},\omega^{2,5,6}+\omega^{3,4,6}+\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=\ 6e^{-7\beta_1}\ \langle\omega^{1,2,4,6},\omega^{1,2,5,6},\omega^{1,2,5,7},\omega^{1,2,4,7}+\omega^{2,3,5,7},\omega^{1,3,5,7},\omega^{2,3,4,6}+\omega^{2,3,5,7}\rangle+3e^{-8\beta_1}\ \langle\omega^{1,3,6,7}-\omega^{2,4,5,7},\omega^{1,2,6,7}-\omega^{3,4,5,6},\omega^{2,3,6,7}-\omega^{2,4,5,6}+\omega^{3,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})=\ e^{-9\beta_1}\ \langle\omega^{1,2,3,6,7}\rangle+3e^{-10\beta_1}\ \langle\omega^{1,2,5,6,7},\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7},\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7}\rangle+e^{-11\beta_1}\ \langle\omega^{1,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})=\ 3e^{-12\beta_1}\ \langle\omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7},\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})=\ e^{-13\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^5$; $d^\star\omega^{1,4}=-\omega^7$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{3,5}=-\omega^7$; $d^\star\omega^{1,2,3}=-\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,7}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{1,3,5}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,6}=-\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{5,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}+\omega^{5,7}$; $d^\star\omega^{1,4,6}=\omega^{6,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,6}$; $d^\star\omega^{2,3,5}=\omega^{2,7}$; $d^\star\omega^{2,4,5}=\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{3,4,5}=-\omega^{4,7}$; $d^\star\omega^{3,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,2,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,6}-\omega^{2,4,6}+\omega^{2,5,7}$; $d^\star\omega^{1,2,4,6}=\omega^{2,6,7}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,4,7}-\omega^{3,4,6}+\omega^{3,5,7}$; $d^\star\omega^{1,3,4,6}=\omega^{3,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{1,6,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}+\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6}=\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=-\omega^{1,4,6,7}-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=-\omega^{2,4,6,7}$; $d^\star\omega^{2,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,2,4,6,7}-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{3,5}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,7}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = -\omega^{1,6} + 2\omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = -\omega^{1,7} + 2\omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = \omega^{2,6} - \omega^{3,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,6} + 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} - \omega^{2,3,5}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} - \omega^{1,3,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{3,4,6} + \omega^{3,5,7}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,4,6} - \omega^{2,5,7}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + \omega^{1,5,7} + \omega^{2,4,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,6} + \omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,7} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6} + \omega^{2,4,6} - \omega^{2,5,7}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{1,5,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} - \omega^{2,4,6} + 2\omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6} - \omega^{3,5,7}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,4,6} - \omega^{1,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,4,7} - \omega^{3,4,6} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,2,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,6} + 2\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + 2\omega^{1,3,4,6} - \omega^{1,3,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} + \omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,3,4,6} + \omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,5,6} + \omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,4,6} + \omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6} + \omega^{1,2,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{2,4,5,6} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{1,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = -\omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,7}$;

$\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,2,4,5,6} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,5,6} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= 3e^{-\beta_1}\ \langle\omega^1,\omega^2,\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1}\ \langle\omega^{2,3}\rangle + 3e^{-3\beta_1}\ \langle\omega^{1,5}-\omega^{2,4},\omega^{2,5}+\omega^{3,4},\omega^{1,4}-\omega^{3,5}\rangle + e^{-4\beta_1}\ \langle\omega^{2,6}-\omega^{3,7}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 3e^{-5\beta_1}\ \langle\omega^{1,2,6}-\omega^{1,3,7}-\omega^{1,4,5},\omega^{1,3,6}+2\omega^{2,3,7}+\omega^{2,4,5},\omega^{1,2,7}-2\omega^{2,3,6}-\omega^{3,4,5}\rangle + 6e^{-6\beta_1}\ \langle\omega^{1,5,6}+\omega^{2,5,7}+\omega^{3,4,7},\omega^{2,4,6}+\omega^{2,5,7}+\omega^{3,4,7},2\omega^{1,4,6}+\omega^{2,4,7}-\omega^{3,5,6},2\omega^{1,5,7}-\omega^{2,4,7}+\omega^{3,5,6},\omega^{1,4,7}-\omega^{3,5,7},\omega^{2,5,6}+\omega^{3,4,6}+\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= 3e^{-8\beta_1}\ \langle(-\omega^{2,3,6,7}+\omega^{2,4,5,6}-\omega^{3,4,5,7})/3,(-\omega^{1,3,6,7}+2\omega^{1,4,5,6}+\omega^{2,4,5,7})/6,(-\omega^{1,2,6,7}+2\omega^{1,4,5,7}+\omega^{3,4,5,6})/6\rangle + 6e^{-7\beta_1}\ \langle(\omega^{1,2,5,6}+\omega^{1,3,4,6}-2\omega^{1,3,5,7}+3\omega^{2,3,4,7})/5,(\omega^{1,2,5,6}+\omega^{1,3,4,6}+3\omega^{1,3,5,7}-2\omega^{2,3,4,7})/5,(-\omega^{1,2,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}-3\omega^{2,3,5,7})/8,(\omega^{1,2,4,7}+\omega^{1,3,5,6}-3\omega^{2,3,4,6}-\omega^{2,3,5,7})/8,(2\omega^{1,2,4,6}-\omega^{1,2,5,7}-\omega^{1,3,4,7}+3\omega^{2,3,5,6})/5,(-\omega^{1,2,4,6}-2\omega^{1,2,5,7}-2\omega^{1,3,4,7}+\omega^{2,3,5,6})/5\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-11\beta_1}\ \langle\omega^{1,4,5,6,7}\rangle + 3e^{-10\beta_1}\ \langle(\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7})/2,(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2,(\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7})/2\rangle + e^{-9\beta_1}\ \langle(-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}-\omega^{1,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= 3e^{-12\beta_1}\ \langle\omega^{2,3,4,5,6,7},-\omega^{1,3,4,5,6,7},\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-13\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

5.3.32 Algebra $\mathcal{G}_{7,1.20}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_4$; $[x_2, x_4] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^3 & 5\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & 2\xi_1^4 & 0 & 0 & 6\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_1^4 & -\xi_1^3 & \xi_2^3 & 7\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & -\xi_2^4 & \xi_5^7 & \xi_2^3 & 8\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3 + \delta_5^7, ad\ x_3 = -\delta_2^4, ad\ x_4 = -\delta_2^6, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{3\beta_1}\ \langle x_3\rangle + e^{5\beta_1}\ \langle x_4\rangle + e^{6\beta_1}\ \langle x_5\rangle + e^{7\beta_1}\ \langle x_6\rangle + e^{8\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{8\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{6\beta_1}\ \langle\delta_1^6, \delta_2^7\rangle + e^{4\beta_1}\ \langle\delta_1^4\rangle + 2e^{2\beta_1}\ \langle\delta_1^3, \delta_5^7\rangle + e^{7\beta_1}\ \langle\delta_1^7\rangle + 2e^{5\beta_1}\ \langle\delta_1^5, \delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_1^4, \delta_2^4] = \delta_1^7$; $[\delta_1^4, \delta_5^7] = -2\delta_2^7$; $[\delta_1^4, \delta_2^3] = -\delta_2^6$; $[\delta_1^3, \delta_1^5] = \delta_1^7$; $[\delta_1^3, \delta_2^6] = -\delta_1^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_2^3] = -\delta_2^4$; $[\delta_1^5, \delta_5^7] = -\delta_1^7$; $[\delta_1^5, \delta_2^3] = -(\delta_2^7 + \delta_1^6)$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle\delta_1^4\rangle + e^{6\beta_1}\ \langle\delta_1^6\rangle + e^{5\beta_1}\ \langle\delta_1^5\rangle + e^{2\beta_1}\ \langle\delta_1^3\rangle + e^0\ \langle t_1\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^3] = 2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $2e^{-6\beta_1}\ \langle 3x_1\omega^{2,4} + 2x_2\omega^{2,5} + 3x_3(\omega^{2,6} - 1/3\omega^{3,5}) + x_4\omega^{4,5}, x_1(\omega^{1,5} - \omega^{2,4}) + x_2(\omega^{1,6} + \omega^{3,4}) - x_3(\omega^{1,7} + \omega^{2,6} - \omega^{3,5}) + x_4\omega^{3,7} + x_6\omega^{4,7}\rangle + e^{-8\beta_1}\ \langle -x_1(\omega^{2,6} - \omega^{3,5}) - x_3\omega^{4,5} + x_4\omega^{5,6}\rangle + e^{2\beta_1}\ \langle x_4\omega^{1,2} + x_5\omega^{1,3}\rangle + e^{-3\beta_1}\ \langle x_1\omega^{1,3} - x_3\omega^{1,4} - x_4(\omega^{1,6} - \omega^{2,5}) - x_5(\omega^{2,6} - \omega^{3,5})\rangle + e^{3\beta_1}\ \langle x_5\omega^{1,2}\rangle + 2e^{-\beta_1}\ \langle 2/3x_2\omega^{1,2} - 1/3x_3\omega^{1,3} + 1/3x_4\omega^{1,4} - 2/3x_5(\omega^{1,5} - 3/2\omega^{2,4}), -x_2\omega^{1,2} + x_3\omega^{1,3} + x_5\omega^{1,5} + x_6\omega^{3,4}\rangle + 3e^{-2\beta_1}\ \langle -x_1\omega^{1,2} + x_2\omega^{1,3} + x_3\omega^{2,3} - x_4\omega^{1,5} + x_5(\omega^{1,6} - 2\omega^{2,5} + \omega^{3,4}), x_2\omega^{1,3} - x_4\omega^{1,5} - 2x_5\omega^{2,5} + x_6\omega^{1,7}, x_4\omega^{1,5} + 2x_5\omega^{2,5} + x_6\omega^{3,5}\rangle + e^{-4\beta_1}\ \langle x_3\omega^{2,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-4\beta_1} + e^{-5\beta_1} + e^{-6\beta_1} + 2e^{-7\beta_1} + 3e^{-8\beta_1} + 3e^{-9\beta_1} + e^{-10\beta_1} + e^{-11\beta_1} + e^{-12\beta_1} + e^{-13\beta_1} + e^{-14\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-10\beta_1}+e^{-11\beta_1}+2e^{-12\beta_1}+e^{-13\beta_1}+3e^{-14\beta_1}+3e^{-15\beta_1}+e^{-16\beta_1}+2e^{-17\beta_1}+e^{-18\beta_1}+2e^{-19\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-17\beta_1}+2e^{-18\beta_1}+e^{-19\beta_1}+2e^{-20\beta_1}+e^{-21\beta_1}+3e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}+e^{-25\beta_1}+e^{-27\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-20\beta_1}+e^{-23\beta_1}+2e^{-24\beta_1}+2e^{-25\beta_1}+2e^{-28\beta_1}+e^{-29\beta_1}+e^{-30\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-26\beta_1}+e^{-30\beta_1}+e^{-31\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,50,114,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,12,17,17,15,10,3)$

S=$32\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,4}=\omega^{1,2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{2,3,5}$; $d\omega^{4,6}=-\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{4,7}=-\omega^{1,4,6}-\omega^{2,3,7}-\omega^{2,4,5}$; $d\omega^{5,6}=\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,5,6}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=\omega^{1,2,3,5}$; $d\omega^{1,4,6}=\omega^{1,2,3,6}$; $d\omega^{1,4,7}=\omega^{1,2,3,7}+\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{2,3,5,6}$; $d\omega^{4,5,7}=-\omega^{1,4,5,6}-\omega^{2,3,5,7}$; $d\omega^{4,6,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,7}=\omega^{1,2,3,5,7}$; $d\omega^{1,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{2,3,5,6,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,5,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-6\beta_1}\ \langle\omega^5\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-7\beta_1}\ \langle\omega^{1,5}\rangle + e^{-8\beta_1}\ \langle\omega^{1,6}+\omega^{3,4}\rangle + e^{-9\beta_1}\ \langle\omega^{2,6}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-10\beta_1}\ \langle\omega^{1,2,6}\rangle + e^{-12\beta_1}\ \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-14\beta_1}\ \langle\omega^{2,4,6}-\omega^{3,4,5}\rangle + e^{-15\beta_1}\ \langle\omega^{1,5,7}+\omega^{2,4,7}-\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-17\beta_1}\ \langle\omega^{1,3,4,7}+\omega^{2,3,4,6}\rangle + e^{-18\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7}\rangle + e^{-20\beta_1}\ \langle\omega^{1,4,5,7}+\omega^{2,3,6,7}\rangle + e^{-22\beta_1}\ \langle\omega^{1,5,6,7}-2\omega^{2,4,6,7}+\omega^{3,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-23\beta_1}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-24\beta_1}\ \langle\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-25\beta_1}\ \langle\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7}\rangle + e^{-28\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-26\beta_1}\ \langle\omega^{1,2,3,4,6,7}\rangle + e^{-30\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-31\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-32\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^4$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}-\omega^{4,5}$; $d^\star\omega^{2,3,6}=-\omega^{4,6}$; $d^\star\omega^{2,3,7}=-\omega^{4,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}+\omega^{1,3,6}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}+\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,6}=\omega^{1,4,6}-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,7}=\omega^{1,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,3,5,6}-$

$\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{4,5} = \omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,5} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,3,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,3,7} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + 2\omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} + \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 3\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 4\omega^{3,6,7} - \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = -\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,7} + 4\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 3\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,7} + 2\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,6,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,4,6,7}$;

$\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{1,3,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7} + 2\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + 2\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,6,7} + 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-6\beta_1} \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-4\beta_1} \langle\omega^{1,3}\rangle + e^{-7\beta_1} \langle\omega^{1,5} - \omega^{2,4}\rangle + e^{-8\beta_1} \langle\omega^{1,6} - 2\omega^{2,5} + \omega^{3,4}\rangle + e^{-9\beta_1} \langle\omega^{2,6} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-10\beta_1} \langle\omega^{1,2,6} - 2\omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-12\beta_1} \langle 2\omega^{1,3,7} - \omega^{1,4,5} + \omega^{2,3,6}\rangle + e^{-14\beta_1} \langle\omega^{1,5,6} - 2\omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-15\beta_1} \langle\omega^{1,5,7} + \omega^{2,4,7} + 3\omega^{2,5,6} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-22\beta_1} \langle(-\omega^{1,5,6,7} + 2\omega^{2,4,6,7} - \omega^{3,4,5,7})/6\rangle + e^{-20\beta_1} \langle(-\omega^{1,4,5,7} - \omega^{2,3,6,7} - 2\omega^{2,4,5,6})/6\rangle + e^{-18\beta_1} \langle(\omega^{1,2,6,7} - 2\omega^{1,3,5,7} + \omega^{2,3,4,7})/6\rangle + e^{-17\beta_1} \langle(\omega^{1,2,5,7} - 3\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-28\beta_1} \langle-\omega^{2,4,5,6,7}\rangle + e^{-25\beta_1} \langle(\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-24\beta_1} \langle(\omega^{1,2,5,6,7} - 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/6\rangle + e^{-23\beta_1} \langle(\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-31\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-30\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-26\beta_1} \langle\omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-32\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.3.33 Algebra $\mathcal{G}_{7,1.21}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_6$; $[x_2, x_6] = x_7$; $[x_4, x_5] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_1^3/2 & 0 & \xi_2^4 & 4\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4 & (-\xi_1^3 - 2\xi_1^4)/2 & \xi_2^4 & 5\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^6 & \xi_1^5 & -\xi_1^4 & 7\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_2^6, ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_2^7, ad\ x_7 = 0.$

Maximal torus : T=$\langle t_1 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{2\beta_1}\ \langle x_2 \rangle + 2e^{3\beta_1}\ \langle x_3, x_4 \rangle + e^{4\beta_1}\ \langle x_5 \rangle + e^{5\beta_1}\ \langle x_6 \rangle + e^{7\beta_1}\ \langle x_7 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{7\beta_1}\ \langle x_7 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_1}\ \langle \delta_2^3, \delta_2^4 \rangle + 2e^{3\beta_1}\ \langle \delta_2^6, \delta_1^5 \rangle + 2e^{2\beta_1}\ \langle \delta_1^3, \delta_1^4 \rangle + e^{6\beta_1}\ \langle \delta_1^7 \rangle + e^{5\beta_1}\ \langle \delta_2^7 \rangle + 2e^{4\beta_1}\ \langle \delta_3^7, \delta_1^6 \rangle + e^0\ \langle t_1 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_1^4] = \delta_2^6$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^5] = -1/2\delta_2^7$; $[\delta_1^3, \delta_1^4] = 1/2\delta_1^6$; $[\delta_2^4, \delta_1^6] = \delta_2^7$; $[\delta_2^4, \delta_1^5] = \delta_1^6$; $[\delta_2^4, \delta_1^4] = \delta_2^6 + \delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1}\ \langle \delta_2^3 \rangle + e^{2\beta_1}\ \langle \delta_1^3 \rangle + e^{6\beta_1}\ \langle \delta_1^7 \rangle + e^{4\beta_1}\ \langle \delta_3^7 \rangle + e^0\ \langle t_1 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^7] = -\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $4e^{-2\beta_1}\ \langle -2x_1\omega^{1,2} + 3x_3\omega^{1,5} - x_4\omega^{2,4} + x_5\omega^{2,5}, x_3\omega^{2,3}, x_3\omega^{2,4}, -x_3\omega^{1,5} + x_4\omega^{1,5} \rangle + 2e^{-4\beta_1}\ \langle 3/2x_1(\omega^{1,5} - \omega^{2,4}) - x_2(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) + x_4(\omega^{2,6} + \omega^{3,5} - 1/2\omega^{4,5}) + x_5(\omega^{3,6} + \omega^{4,6}), x_1\omega^{2,4} - x_3(\omega^{2,6} - \omega^{4,5}) \rangle + 3e^{-3\beta_1}\ \langle -3x_1(\omega^{1,3} + 7/6\omega^{1,4}) - 7/2x_2(\omega^{1,5} - 2/7\omega^{2,3}) - 7/2x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) + 7/2x_4(\omega^{1,6} - 3/7\omega^{3,4}) + 3/2x_5\omega^{3,5} + x_6\omega^{3,6}, 2x_1(\omega^{1,3} + 5/4\omega^{1,4}) + 5/2x_2(\omega^{1,5} - 2/5\omega^{2,3}) + 5/2x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) - 5/2x_4(\omega^{1,6} - 3/5\omega^{3,4}) - 1/2x_5\omega^{3,5} + x_7\omega^{3,7}, x_2\omega^{2,4} + x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) + x_4\omega^{2,5} + x_5\omega^{2,6} \rangle + e^{3\beta_1}\ \langle x_7\omega^{1,3} \rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3(\omega^{1,3} + 2\omega^{1,4}) + x_4\omega^{1,4} + x_7\omega^{1,7}, 2x_3\omega^{1,3} + x_5\omega^{2,3} + x_6\omega^{3,4} \rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-4\beta_1} + 4e^{-5\beta_1} + 3e^{-6\beta_1} + 5e^{-7\beta_1} + 2e^{-8\beta_1} + 2e^{-9\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-8\beta_1}+e^{-9\beta_1}+3e^{-10\beta_1}+2e^{-11\beta_1}+4e^{-12\beta_1}+3e^{-13\beta_1}+2e^{-14\beta_1}+e^{-15\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-13\beta_1}+e^{-15\beta_1}+3e^{-16\beta_1}+4e^{-17\beta_1}+4e^{-18\beta_1}+e^{-19\beta_1}+e^{-20\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-19\beta_1}+3e^{-20\beta_1}+3e^{-21\beta_1}+e^{-22\beta_1}+e^{-23\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-22\beta_1}+e^{-23\beta_1}+e^{-24\beta_1}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,11,50,114,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,5,12,17,17,15,10,3)$

S=$25\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}-\omega^{2,4}$; $d\omega^7=-\omega^{2,6}+\omega^{4,5}$; $d\omega^{1,6}=\omega^{1,2,3}+\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{2,4,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{2,4,6}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{2,5,6}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}-\omega^{2,4,7}-\omega^{4,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,4,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,6}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,5,6}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}+\omega^{1,2,4,7}+\omega^{1,4,5,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=\omega^{2,3,4,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,4,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{2,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{3,4,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{2,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}+\omega^{2,3,4,7}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}+\omega^{2,4,5,7}$; $d\omega^{1,2,3,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,3,4,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7}=\omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,3,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{a}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + 2e^{-5\beta_1}\ \langle\omega^{1,5},\omega^{2,3}\rangle + e^{-6\beta_1}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}})= e^{-8\beta_1}\ \langle\omega^{1,2,6}\rangle + e^{-9\beta_1}\ \langle\omega^{1,3,6}+\omega^{2,3,5}\rangle + e^{-10\beta_1}\ \langle\omega^{1,5,6}-\omega^{2,4,6}+\omega^{3,4,5}\rangle + e^{-12\beta_1}\ \langle\omega^{1,5,7}+\omega^{2,3,7}+\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}})= e^{-13\beta_1}\ \langle\omega^{1,3,5,6}-\omega^{2,3,4,6}\rangle + e^{-16\beta_1}\ \langle\omega^{1,4,6,7}-\omega^{2,3,5,7}\rangle + e^{-17\beta_1}\ \langle(2\omega^{1,5,6,7}-\omega^{2,3,6,7}-2\omega^{2,4,6,7}+\omega^{3,4,5,7})/2\rangle + e^{-15\beta_1}\ \langle\omega^{1,2,6,7}+\omega^{1,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}})= 2e^{-20\beta_1}\ \langle\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7},\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}\rangle + e^{-19\beta_1}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-21\beta_1}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{a}})= e^{-22\beta_1}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-23\beta_1}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-24\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-25\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{a}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathfrak{a}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^5$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^6$; $d^\star\omega^{2,6}=-\omega^7$; $d^\star\omega^{4,5}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,6}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=\omega^{1,6}-\omega^{2,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,5}$; $d^\star\omega^{1,3,5}=-\omega^{3,6}$; $d^\star\omega^{1,4,5}=-\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,4,6}=-\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,6}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{5,6}$; $d^\star\omega^{2,3,6}=-\omega^{3,7}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=-\omega^{2,7}+\omega^{5,6}$; $d^\star\omega^{2,4,6}=-\omega^{4,7}$; $d^\star\omega^{2,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,5,6}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=-\omega^{3,7}$; $d^\star\omega^{4,5,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,6}-\omega^{1,4,6}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,6}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,2,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6}=\omega^{1,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,3,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{1,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,3,7}+\omega^{3,5,6}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}+\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,3,7}-\omega^{1,3,5,6}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,3,4,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}-\omega^{1,4,6,7}-\omega^{2,3,5,7}$;

$d^{\star}\omega^{1,2,3,5,6} = \omega^{1,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,3,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,3,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 3\omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} - \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{2,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,5,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,2,7} + 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,5} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7} + \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,3,5} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + \omega^{2,3,7} + \omega^{2,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{2,3,7} + 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,4,7} - \omega^{3,5,6} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} + \omega^{1,2,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,3,7} + 2\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,3,7} + \omega^{1,3,5,6} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,4,7} - \omega^{1,3,5,6} + 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 2\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,5,6} - \omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{1,4,5,7} + 2\omega^{2,3,4,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} =$

$2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,4,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{3,5,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 3\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-3\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-4\beta_1} \langle\omega^{1,3}\rangle + 2e^{-5\beta_1} \langle\omega^{1,5} - \omega^{2,4}, \omega^{2,3} - \omega^{2,4}\rangle + e^{-6\beta_1} \langle\omega^{1,6} + \omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-8\beta_1} \langle\omega^{1,2,6} - 2\omega^{1,3,5} + \omega^{1,4,5} + 2\omega^{2,3,4}\rangle + e^{-9\beta_1} \langle 2\omega^{1,3,6} + \omega^{1,4,6} + \omega^{2,3,5}\rangle + e^{-10\beta_1} \langle\omega^{1,2,7} + 2\omega^{1,5,6} - \omega^{2,3,6} - \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-12\beta_1} \langle\omega^{1,5,7} + \omega^{2,3,7} - 3\omega^{2,4,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-17\beta_1} \langle(-2\omega^{1,5,6,7} + \omega^{2,3,6,7} + 2\omega^{2,4,6,7} - \omega^{3,4,5,7})/10\rangle + e^{-16\beta_1} \langle(\omega^{1,4,6,7} - \omega^{2,3,5,7} + 2\omega^{2,4,5,7})/6\rangle + e^{-15\beta_1} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{3,4,5,6})/8\rangle + e^{-13\beta_1} \langle(-\omega^{1,2,3,7} + 3\omega^{1,3,5,6} + \omega^{1,4,5,6} - \omega^{2,3,4,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-21\beta_1} \langle-\omega^{2,4,5,6,7}\rangle + 2e^{-20\beta_1} \langle(\omega^{1,3,5,6,7} - \omega^{1,4,5,6,7} - 2\omega^{2,3,4,6,7})/3, (\omega^{1,3,5,6,7} + 2\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7})/3\rangle + e^{-19\beta_1} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-24\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-23\beta_1} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-22\beta_1} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-25\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.4 rank 2 Lie Algebras

5.4.1 Algebra $\mathcal{G}_{7,2.1(i_\lambda)}$

Adjoint cohomology

$[x_1,x_2] = x_4$; $[x_1,x_3] = x_5$; $[x_1,x_4] = x_6$; $[x_1,x_6] = x_7$; $[x_2,x_3] = x_6$; $[x_2,x_5] = \lambda x_7$; $[x_3,x_4] = x_7(\lambda-1)$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & a & -\xi_1^2(\lambda+1) & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & b & -\xi_1^2\lambda & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5\lambda+\xi_2^6 & \xi_1^4(-\lambda+1)+\xi_3^6 & c & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

with $a = (\xi_1^3(-\lambda+2))/\lambda + \xi_3^5$, $b = (2\xi_1^3(-\lambda+1))/\lambda + \xi_3^5$, $c = (\xi_1^3(\lambda^2-3\lambda+2))/\lambda + \xi_3^5$.

$ad\ x_1 = \delta_3^5, ad\ x_2 = -\delta_1^4+\delta_3^6, ad\ x_3 = -(\delta_1^5+\delta_2^6), ad\ x_4 = -\lambda\delta_3^7+\delta_3^7-\delta_1^6, ad\ x_5 = -\lambda\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0$.

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + 2e^0\ \langle t_2,t_1\rangle + 2e^{\beta_1}\ \langle\delta_1^3,\delta_3^5\rangle + 2e^{\beta_2}\ \langle\delta_1^4,\delta_3^6\rangle + 2e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle + e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle + 2e^{\beta_1+\beta_2}\ \langle\delta_1^6,\delta_3^7\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{3\beta_1}\ \langle\delta_2^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3] = \lambda^{-1}(2\delta_3^6 - 2\delta_1^4 - 2\lambda^2\delta_3^6 - \lambda^2\delta_1^4)$; $[\delta_1^2,\delta_1^5] = -\lambda(\delta_1^6+\delta_3^7+\lambda\delta_3^7)$; $[\delta_1^2,\delta_2^5] = -(\delta_1^5+\lambda\delta_2^6)$; $[\delta_1^2,\delta_3^5] = \delta_3^6-\delta_1^4$; $[\delta_1^2,\delta_2^6] = -\delta_1^6+\delta_3^7+\lambda\delta_3^7$; $[\delta_1^2,\delta_2^7] = -\delta_1^7$; $[\delta_1^3,\delta_1^4] = 2\delta_1^6(-1+\lambda^{-1})$; $[\delta_1^3,\delta_1^5] = \delta_2^7(2-\lambda)$; $[\delta_1^3,\delta_3^5] = -(\delta_2^6+\delta_1^5)$; $[\delta_1^3,\delta_1^6] = \delta_1^7(-3+2\lambda^{-1}+\lambda)$; $[\delta_1^3,\delta_2^6] = \delta_2^7(-2+\lambda)$; $[\delta_1^3,\delta_3^6] = -\delta_1^6 - 3\delta_3^7 + 2\lambda^{-1}\delta_3^7 + \lambda\delta_3^7$; $[\delta_1^3,\delta_3^7] = -\delta_1^7$; $[\delta_1^4,\delta_1^5] = \delta_1^7$; $[\delta_1^4,\delta_2^5] = \delta_2^7(1-\lambda)$; $[\delta_1^4,\delta_3^5] = -\delta_1^6+\delta_3^7-\lambda\delta_3^7$; $[\delta_1^4,\delta_2^6] = -\delta_1^7$; $[\delta_1^5,\delta_3^5] = -\lambda\delta_2^7$; $[\delta_1^5,\delta_3^6] = -\delta_1^7$; $[\delta_2^5,\delta_3^6] = -\delta_2^7$; $[\delta_3^5,\delta_1^6] = \delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + 2e^0\ \langle t_2,t_1\rangle + e^{\beta_1}\ \langle\delta_1^3\rangle + e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_1^5\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_1+\beta_2}\ \langle\delta_3^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3] = -3\lambda\delta_1^4$; $[\delta_1^2,\delta_2^5] = (\lambda-1)\delta_1^5$; $[\delta_1^2,\delta_1^5] = -2\lambda\delta_3^7$; $[\delta_1^3,\delta_1^4] = 2\lambda^{-1}(1-\lambda)^2\delta_3^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3} + x_6\lambda\omega^{3,5}\rangle + 4e^{-2\beta_1}\ \langle -1/2x_1\lambda\omega^{1,3} + 1/4x_2(\lambda^2\omega^{1,4}-2\lambda\omega^{1,4}+2\lambda\omega^{2,3}-4\omega^{2,3})+1/4x_3\lambda^2\omega^{1,5}+x_4\omega^{1,6}-1/4x_6\lambda^2\omega^{4,5}-1/2x_7\omega^{3,7}(\lambda-2), x_1\omega^{1,3}(\lambda-1/2)+3/4x_2(\lambda\omega^{1,4}-2/3\omega^{1,4}+2/3\omega^{2,3})+3/4x_3\lambda\omega^{1,5}+x_4\lambda\omega^{2,5}+1/4x_6\lambda\omega^{4,5}-x_7\omega^{3,7}(\lambda-1/2), -1/2x_1\omega^{1,3}+3/4x_2(\lambda\omega^{1,4}-2\omega^{1,4}-2/3\omega^{2,3})+3/4x_3\lambda\omega^{1,5}+x_4\omega^{3,4}-3/4x_6\lambda\omega^{4,5}+1/2x_7\omega^{3,7}, -1/2x_1\omega^{1,3}+1/4x_2(\lambda\omega^{1,4}-2\omega^{1,4}+2\omega^{2,3})+1/4x_3\lambda\omega^{1,5}+x_5\omega^{3,5}-1/4x_6\lambda\omega^{4,5}+1/2x_7\omega^{3,7}\rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2}(\lambda-1)+x_3\lambda\omega^{1,3}+x_4\omega^{2,3}+x_7\lambda\omega^{4,5}(\lambda-1), -x_2\lambda\omega^{1,2}+x_3\lambda\omega^{1,3}+x_5\omega^{1,5}+x_7\lambda\omega^{4,5}(\lambda-1), -x_2\omega^{1,2}(\lambda-1)+x_3\omega^{1,3}(\lambda-1)+x_4\omega^{1,4}+x_7\lambda\omega^{4,5}(\lambda-1)\rangle + e^{-4\beta_1+\beta_2}\ \langle -x_2\omega^{1,5}+x_4\omega^{3,5}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle -x_1\omega^{1,2}(\lambda-1)+x_3(\lambda\omega^{1,4}-\omega^{1,4}+\omega^{2,3})+x_5\lambda\omega^{2,5}+x_7\omega^{4,6}(\lambda-1), -x_1\omega^{1,2}(\lambda-1)+2x_3(\lambda\omega^{1,4}-1/2\lambda\omega^{2,3}-1/2\omega^{1,4}+1/2\omega^{2,3})+x_4\lambda\omega^{2,4}+x_7\omega^{4,6}(\lambda-1), x_1\omega^{1,2}(\lambda-2)-x_3(\lambda\omega^{1,4}-\lambda\omega^{2,3}-2\omega^{1,4}+2\omega^{2,3})+x_7(\lambda\omega^{2,7}-2\lambda\omega^{4,6}+2\omega^{4,6})\rangle + e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}+x_5\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1(\lambda^2\omega^{1,4}-3/2\lambda^2\omega^{2,3}-3/2\lambda\omega^{1,4}+5/2\lambda\omega^{2,3}+1/2\omega^{1,4}-1/2\omega^{2,3})-x_2\omega^{2,4}(\lambda^2-5/2\lambda+1)-1/2x_3(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}-2\lambda\omega^{1,6}-2\lambda\omega^{2,5}+\omega^{1,6}-\omega^{3,4})+x_4\omega^{2,6}(\lambda-1/2)-1/2x_5(\lambda^2\omega^{4,5}+\lambda\omega^{3,6}-2\lambda\omega^{4,5}-2\omega^{3,6}), -x_1\lambda(\lambda\omega^{1,4}-5/2\lambda\omega^{2,3}-1/2\omega^{1,4}+1/2\omega^{2,3})+x_2\lambda\omega^{2,4}(\lambda-1/2)+1/2x_3(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}-2\lambda\omega^{1,6}+\lambda\omega^{2,5}+3\lambda\omega^{3,4}+\omega^{1,6}-\omega^{3,4})+1/2x_5(\lambda^2\omega^{4,5}+5\lambda\omega^{3,6}+\lambda\omega^{4,5}-\omega^{3,6})+x_6\omega^{2,7}(\lambda-1/2)+x_7\omega^{4,7}(\lambda-1/2)\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + e^{0}\ \langle -x_6\omega^{2,3}+x_7\omega^{3,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_7\omega^{3,5}\rangle + e^{-3\beta_1}\ \langle -1/3x_1\lambda^2\omega^{1,5}+1/3x_2(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}-2\lambda\omega^{1,6}+\lambda\omega^{2,5}+3\lambda\omega^{3,4}+4\omega^{1,6}-4\omega^{3,4})+1/3x_3\lambda^2\omega^{3,5}+1/3x_4(\lambda^2\omega^{3,6}+4\lambda\omega^{3,6}+\lambda\omega^{4,5}-4\omega^{3,6})+1/3x_6\lambda(\lambda\omega^{5,6}+3\omega^{3,7}-\omega^{5,6})+x_7\lambda\omega^{5,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=3e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+2e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+4e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+5e^{-4\beta_1-\beta_2}+2e^{-4\beta_1}+e^{-5\beta_1-\beta_2}+4e^{-5\beta_1}+e^{-6\beta_1+\beta_2}+2e^{-6\beta_1}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=3e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+e^{-5\beta_1-\beta_2}+5e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+3e^{-6\beta_1-\beta_2}+3e^{-6\beta_1-2\beta_2}+e^{-6\beta_1}+4e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+2e^{-8\beta_1-\beta_2}+e^{-8\beta_1}+e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-3\beta_2}+2e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-2\beta_2}+4e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2}+4e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1-3\beta_2}+e^{-9\beta_1-\beta_2}+2e^{-9\beta_1-2\beta_2}+e^{-10\beta_1-\beta_2}+e^{-10\beta_1-2\beta_2}+e^{-11\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-3\beta_2}+2e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-3\beta_2}+e^{-9\beta_1-4\beta_2}+2e^{-10\beta_1-3\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-2\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-4\beta_2}+e^{-12\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,14,55,123,155,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,8,20,31,33,24,12,3)$

The preceding results are valid for:

$$-(\lambda^2-\lambda+1)(\lambda+1/3)(\lambda-1/2)^2(\lambda+1)(\lambda-1)^3(\lambda-2)^4\lambda \neq 0.$$

S=$4(3\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,19,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,8,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}-\omega^{2,3}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}\lambda+\omega^{3,4}(-\lambda+1)$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,5}\lambda+\omega^{1,3,4}(\lambda-1)$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}(\lambda-1)$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}\lambda$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{2,4,5}\lambda$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,3,7}-\omega^{1,5,6}+\omega^{3,4,5}(\lambda-1)$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,5,6}\lambda+\omega^{3,4,6}(\lambda-1)$; $d\omega^{1,2,7}=\omega^{1,2,3,4}(-\lambda+1)$; $d\omega^{1,3,7}=\omega^{1,2,3,5}\lambda$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}\lambda$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=\omega^{1,3,4,5}(-\lambda+1)$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,5,6}\lambda+\omega^{1,3,4,6}(-\lambda+1)$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5}(-\lambda+1)$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}+\omega^{2,3,4,6}(-\lambda+1)$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}\lambda$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}+\omega^{2,3,5,6}\lambda$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}\lambda$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}+\omega^{3,4,5,6}(-\lambda+1)$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}(\lambda-1)$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}(\lambda-1)$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}\lambda$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}\lambda$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6}\lambda$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}(\lambda-1)$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}(\lambda-1)$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}+\omega^{2,3,4,5,6}\lambda$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7}=\omega^{1,2,3,4,5,6}(-\lambda+1)$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}\lambda$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-4\beta_1}\ \langle\omega^{1,5}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7}+\omega^{3,6}(2\lambda-1)+\omega^{4,5}\lambda\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{2,5}+\omega^{3,4}\rangle+e^{-5\beta_1}\ \langle\omega^{3,5}\rangle$.

ch$(H^3_{\mathcal{G}})$= $e^{-6\beta_1}$ $\langle\omega^{1,3,5}\rangle + 2e^{-4\beta_1-2\beta_2}$ $\langle\omega^{1,2,7}+\omega^{2,4,5}(\lambda-1),\omega^{2,3,6}+\omega^{2,4,5}\rangle +$ $e^{-6\beta_1-\beta_2}$ $\langle\omega^{1,3,7}-\omega^{3,4,5}\lambda\rangle + e^{-7\beta_1-\beta_2}$ $\langle\omega^{1,5,7}+\omega^{3,5,6}(\lambda-1)\rangle + e^{-5\beta_1-\beta_2}$ $\langle\omega^{2,3,5}\rangle +$ $e^{-5\beta_1-2\beta_2}$ $\langle\omega^{1,4,7}-\omega^{2,5,6}\lambda-\omega^{3,4,6}\lambda\rangle + e^{-3\beta_1-3\beta_2}$ $\langle\omega^{2,4,6}\rangle$.

ch$(H^4_{\mathcal{G}})$= $e^{-9\beta_1-\beta_2}$ $\langle\omega^{1,3,5,7}\rangle + 2e^{-8\beta_1-2\beta_2}$ $\langle\omega^{1,3,6,7}+\omega^{3,4,5,6}\lambda,\omega^{1,4,5,7}\rangle +$ $e^{-7\beta_1-2\beta_2}$ $\langle\omega^{1,2,5,7}+\omega^{2,3,5,6}(-\lambda+1)\rangle + e^{-7\beta_1-3\beta_2}$ $\langle\omega^{1,4,6,7}-\omega^{2,3,6,7}(\lambda+1)-$ $\omega^{2,4,5,7}\lambda\rangle + e^{-5\beta_1-3\beta_2}$ $\langle\omega^{2,3,4,6}\rangle + e^{-6\beta_1-3\beta_2}$ $\langle\omega^{1,2,6,7}+\omega^{2,4,5,6}(-\lambda+1)\rangle +$ $e^{-6\beta_1-4\beta_2}$ $\langle\omega^{2,4,6,7}\rangle$.

ch$(H^5_{\mathcal{G}})$= $e^{-7\beta_1-4\beta_2}$ $\langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-3\beta_2}$ $\langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}(-\lambda+$ $1)\rangle + e^{-11\beta_1-2\beta_2}$ $\langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2}$ $\langle\omega^{2,3,4,5,6}\rangle + e^{-8\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,6,7}\rangle +$ $e^{-10\beta_1-3\beta_2}$ $\langle\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}\rangle$.

ch$(H^6_{\mathcal{G}})$= $e^{-10\beta_1-4\beta_2}$ $\langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2}$ $\langle\omega^{1,3,4,5,6,7}\rangle +$ $e^{-11\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$.

ch$(H^7_{\mathcal{G}})$= $e^{-12\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

The preceding results are valid for:

$$(\lambda^2-\lambda+1)(\lambda-2)\lambda \neq 0.$$

Comments: singular values.

$\mathcal{G}_{7,2.1(i_\lambda)}$ is isomorphic to $\mathcal{G}_{7,2.1(i_{\lambda'})}$ if and only if $\lambda = \lambda'$. The values excluded by the computer are not necessarily singular, i.e. give a different cohomology than the generic one: some of them appear from the order used by the computer in solving systems of equations . Computing again for each of these values shows that only $\lambda^2-\lambda+1=0$, $\lambda=2$ are singular values for the trivial cohomology and that only $\lambda^2-\lambda+1=0$, $\lambda=2$, $\lambda=1/2$, $\lambda=0$ are singular values for the adjoint cohomology.

changes in adjoint cohomology:

case $\lambda = 0$:

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & \xi_1^4+\xi_3^6 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

Change $ad\ x_1 = \delta_2^4 + \delta_3^5$;

Add in $\mathrm{ch}(H^0_{\mathcal{G}})$ $e^{3\beta_1}$ $\langle x_5\rangle$; change in $\mathrm{ch}(Z^1_{\mathcal{G}})$ $2e^{\beta_1}$ $\langle \delta_1^3, \delta_3^5\rangle$ to $2e^{\beta_1}$ $\langle \delta_2^4, \delta_3^5\rangle$;

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = \delta_3^7 - \delta_1^6$; $[\delta_1^2, \delta_2^4] = \delta_3^6 - \delta_1^4$; $[\delta_1^5, \delta_1^4] = -\delta_1^7$; $[\delta_1^5, \delta_3^6] = -\delta_1^7$; $[\delta_2^5, \delta_1^4] = -\delta_2^7$; $[\delta_2^5, \delta_3^6] = -\delta_2^7$; $[\delta_3^5, \delta_1^4] = -\delta_3^7$; $[\delta_3^5, \delta_3^6] = -\delta_3^7$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_2^6, \delta_1^4] = \delta_1^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_3^6, \delta_2^4] = -\delta_3^7$.

change in $\mathrm{ch}(H^1_{\mathcal{G}})$ e^{β_1} $\langle \delta_1^3\rangle$ to e^{β_1} $\langle \delta_2^4\rangle$ and add $e^{3\beta_1}$ $\langle \delta_2^4\rangle$;

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_2^5, \delta_1^4] = -\delta_2^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$.

Add in $\mathrm{ch}(H^2_{\mathcal{G}})$ $e^{-4\beta_1}$ $\langle x_2(-\omega^{1,7} + \omega^{3,6}) + x_4\omega^{3,7}\rangle$;

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(\underline{2}, 14, \underline{56}, \underline{124}, 155, 114, 45, 7)$ [5]

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(\underline{2}, \underline{9}, \underline{21}, \underline{33}, \underline{34}, 24, 12, 3)$;

Add $e^{-7\beta_1}$ in $\mathrm{ch}(H^3_{\mathcal{G}})$ and $\mathrm{ch}(H^4_{\mathcal{G}})$; the multiplicity of $e^{-4\beta_1}$ in $\mathrm{ch}(H^3_{\mathcal{G}})$ is 3.

case $\lambda = 1/2$: The only change is that δ_2^3 enters: $\dim(Z^1_{\mathcal{G}})$=15 which implies $\dim(H^1_{\mathcal{G}})$=9 and $\dim(H^2_{\mathcal{G}})$=21 ;

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 3\xi_1^3 + \xi_3^5 & (-3\xi_1^2)/2 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 2\xi_1^3 + \xi_3^5 & (-\xi_1^2)/2 & 2\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & (-\xi_1^5 + 2\xi_2^6)/2 & (\xi_1^4 + 2\xi_3^6)/2 & 3/2\xi_1^3 + \xi_3^5 & 3\xi_1^1 + \xi_2^2 \end{pmatrix}$$

Add in $\mathrm{ch}(Z^1_{\mathcal{G}})$ and in $\mathrm{ch}(H^1_{\mathcal{G}})$ $e^{2\beta_1 - \beta_2}$ $\langle \delta_2^3\rangle$

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: add $[\delta_2^3, \delta_1^2] = -3/2\delta_3^5 + \delta_1^3$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_1^3] = 3\delta_2^5$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: add $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_1^3] = 3\delta_2^5$.

Add in $\mathrm{ch}(H^2_{\mathcal{G}})$ $e^{2\beta_1 - \beta_2}$ $\langle x_5\omega^{1,2}\rangle$

case $\lambda = 2$: The only change are $\dim(Z^3_{\mathcal{G}})$=125, $\dim(Z^4_{\mathcal{G}})$=157, $\dim(Z^5_{\mathcal{G}})$=115, which implies $\dim(H^3_{\mathcal{G}})$=33, $\dim(H^4_{\mathcal{G}})$=37, $\dim(H^5_{\mathcal{G}})$=27, $\dim(H^6_{\mathcal{G}})$=13. In $\mathrm{ch}(H^3_{\mathcal{G}})$ $e^{-3\beta_1 - 2\beta_2}$ has multiplicity 3 and $e^{-5\beta_1 - \beta_2}$ has multiplicity 2; in $\mathrm{ch}(H^4_{\mathcal{G}})$ $e^{-7\beta_1 - 2\beta_2}$ and $e^{-5\beta_1 - \beta_2}$ have multiplicity 2 and there

[5] underlined dimensions are those differing from the generic ones

are new terms $e^{3\beta_1-2\beta_2}$ and $e^{-9\beta_1-\beta_2}$; in $\mathrm{ch}(H^5_{\mathcal{G}})$ $e^{-7\beta_1-2\beta_2}$ and $e^{-9\beta_1-\beta_2}$ have multiplicity 2 and $e^{-9\beta_1-2\beta_2}$ has multiplicity 3 ; in $\mathrm{ch}(H^6_{\mathcal{G}})$ there is a new term $e^{-9\beta_1-2\beta_2}$.

case $\lambda^2-\lambda+1=0$**:** The only change is $\dim(Z^4_{\mathcal{G}})$=156 which implies $\dim(H^4_{\mathcal{G}})$=34, $\dim(H^5_{\mathcal{G}})$=25. $e^{-6\beta_1-2\beta_2}$ has multiplicity 4 in $\mathrm{ch}(H^4_{\mathcal{G}})$ and appears as a new term in $\mathrm{ch}(H^5_{\mathcal{G}})$.

changes in trivial cohomology:

case $\lambda=2$**:**

Add in $\mathrm{ch}(H^2_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2}$ $\langle\omega^{2,7}-\omega^{4,6}\rangle$ and correspondingly in $\mathrm{ch}(H^5_{\mathbb{C}})$ $e^{-9\beta_1-2\beta_2}$ $\langle\omega^{1,3,4,5,6}\rangle$; in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2}$ $\langle\omega^{1,2,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-9\beta_1-2\beta_2}$ $\langle\omega^{1,5,6,7}+\omega^{3,4,5,7}\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,\underline{11},19,\underline{25},17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,\underline{7},\underline{9},\underline{9},\underline{7},3,1)$

$P(Y)\ =\ (1+2Y+4Y^2+2Y^3+Y^4)(1+Y^2)(1+Y)$.

case $\lambda^2-\lambda+1=0$**:**

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-6\beta_1-2\beta_2}$ $\langle\omega^{1,6,7}+(1-\lambda)\omega^{2,5,7}-\lambda\omega^{3,4,7}-\omega^{4,5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-6\beta_1-2\beta_2}$ $\langle\omega^{2,3,4,5}\rangle$

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,\underline{20},24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,\underline{9},\underline{9},6,3,1)$

$P(Y)\ =\ (1+2Y+4Y^2+5Y^3+4Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,19,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,8,8,6,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^5$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,5}=-\omega^7\bar\lambda$; $d^\star\omega^{3,4}=\omega^7(-\bar\lambda+1)$; $d^\star\omega^{1,2,3}=\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}$; $d^\star\omega^{1,2,5}=\omega^{1,7}\bar\lambda-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}(\bar\lambda-1)-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{5,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}(\bar\lambda-1)+\omega^{4,6}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}\bar\lambda+\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}\bar\lambda$; $d^\star\omega^{2,5,6}=\omega^{6,7}\bar\lambda$; $d^\star\omega^{3,4,5}=\omega^{5,7}(\bar\lambda-1)$;

$d^\star\omega^{3,4,6} = \omega^{6,7}(\bar\lambda-1)$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7}(-\bar\lambda+1) - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}\bar\lambda - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}\bar\lambda + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}\bar\lambda - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7}(-\bar\lambda+1) + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7}(-\bar\lambda+1) - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7}(-\bar\lambda+1) - \omega^{3,4,7}\bar\lambda - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7}(-\bar\lambda+1)$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}\bar\lambda$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}\bar\lambda$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}(-\bar\lambda+1)$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7}(\bar\lambda-1) + \omega^{1,3,4,7}\bar\lambda + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7}(\bar\lambda-1) - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}\bar\lambda - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}\bar\lambda - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}(\bar\lambda-1) - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7}(\bar\lambda-1) + \omega^{3,4,6,7}\bar\lambda$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7}(-\bar\lambda+1) - \omega^{1,3,4,6,7}\bar\lambda - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6}+\omega^{2,5}(\lambda-1)+\omega^{3,4}\lambda$; $\Delta\omega^{1,7} = \omega^{1,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+1) + \omega^{3,6}(-\lambda+1) - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{1,6}(\bar\lambda-1) + \omega^{2,5}(\lambda\bar\lambda+1) + \omega^{3,4}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2) + \omega^{4,6}\lambda$; $\Delta\omega^{3,4} = \omega^{1,6}\bar\lambda + \omega^{2,5}(\lambda\bar\lambda-\lambda-1) + \omega^{3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,6} = \omega^{1,7}(-\bar\lambda+1) + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7}(\lambda\bar\lambda+1)+\omega^{5,6}(-\lambda+1)$; $\Delta\omega^{4,5} = -\omega^{1,7}-\omega^{3,6}+2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7}\bar\lambda+2\omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}(\lambda\bar\lambda+2)$; $\Delta\omega^{5,6} = \omega^{3,7}(-\bar\lambda+1) + 3\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{6,7} = \omega^{6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}(\lambda\bar\lambda+1) + \omega^{1,3,4}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}\lambda$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2) + \omega^{1,4,6}\lambda + \omega^{2,3,6}(\lambda-1) + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}-\omega^{1,4,5}+\omega^{2,3,5}(\lambda-1)$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}(\lambda\bar\lambda+1) + \omega^{1,5,6}(-\lambda+1) + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7}\bar\lambda + 2\omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5}(\lambda-1)$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}(\lambda\bar\lambda+1) + \omega^{2,3,7} + \omega^{3,4,6}(-\lambda+1)$; $\Delta\omega^{1,5,6} = \omega^{1,3,7}(-\bar\lambda+1) + 2\omega^{1,5,6} - \omega^{3,4,5}\lambda$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+1)+\omega^{3,5,6}(-\lambda+1)$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{2,5,7}(\lambda-1)+\omega^{3,4,7}\lambda+\omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6}\bar\lambda+\omega^{2,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,5} = \omega^{1,3,6}(\bar\lambda-1) + \omega^{1,4,5} + \omega^{2,3,5}(\lambda\bar\lambda+1)$; $\Delta\omega^{2,3,6} = \omega^{1,2,7}(\bar\lambda-1) + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} + \omega^{2,5,6}(-\lambda+1) - \omega^{3,4,6}\lambda$;

$\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,6}(\bar\lambda - 1) - \omega^{2,3,6} + \omega^{2,4,5}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7}(-\bar\lambda + 1) + \omega^{2,5,6}(\lambda\bar\lambda + 2) + \omega^{3,4,6}(\lambda\bar\lambda - \bar\lambda - 1)$; $\Delta\omega^{2,5,7} = \omega^{1,6,7}(\bar\lambda - 1) + \omega^{2,5,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{3,4,7}(\lambda\bar\lambda - \bar\lambda - 1) + \omega^{4,5,6}\lambda$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} - \omega^{1,5,6}\bar\lambda + \omega^{3,4,5}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,6} = \omega^{1,4,7}(-\bar\lambda + 1) - \omega^{2,3,7}\bar\lambda + \omega^{2,5,6}(\lambda\bar\lambda - \lambda - 1) + \omega^{3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,7} = \omega^{1,6,7}\bar\lambda + \omega^{2,5,7}(\lambda\bar\lambda - \lambda - 1) + \omega^{3,4,7}(\lambda\bar\lambda + 2) + \omega^{4,5,6}(\lambda - 1)$; $\Delta\omega^{3,5,6} = \omega^{1,5,7}(-\bar\lambda + 1) + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}(\lambda\bar\lambda + 1) - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{2,5,7}\bar\lambda + \omega^{3,4,7}(\bar\lambda - 1) + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}(\lambda\bar\lambda + 2)$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 4)$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}(\lambda\bar\lambda - \lambda - \bar\lambda + 4)$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}(\lambda\bar\lambda + 2)$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} + \omega^{1,2,5,6}(-\lambda + 1) - \omega^{1,3,4,6}\lambda - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}(\lambda\bar\lambda + 1)$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}(\lambda - 1)$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7}(-\bar\lambda + 1) + \omega^{1,2,5,6}(\lambda\bar\lambda + 2) + \omega^{1,3,4,6}(\lambda\bar\lambda - \bar\lambda - 1) + \omega^{2,3,4,5}\lambda$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{1,3,4,7}(\lambda\bar\lambda - \bar\lambda - 1) + \omega^{1,4,5,6}\lambda + \omega^{2,3,5,6}(\lambda - 1)$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) - \omega^{2,3,4,7}\lambda - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7}\bar\lambda + \omega^{1,2,5,6}(\lambda\bar\lambda - \lambda - 1) + \omega^{1,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{2,3,4,5}(\lambda - 1)$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7}(\lambda\bar\lambda - \lambda - 1) + \omega^{1,3,4,7}(\lambda\bar\lambda + 2) + \omega^{1,4,5,6}(\lambda - 1)$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}(\lambda\bar\lambda + 1) - \omega^{1,4,5,7} + \omega^{2,3,5,7}(\lambda - 1) - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7}\bar\lambda + \omega^{1,3,4,7}(\bar\lambda - 1) + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}(-\lambda + 1)$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}(\lambda\bar\lambda + 1) + \omega^{2,3,6,7} + \omega^{2,4,5,7}(\lambda - 1)$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) - \omega^{3,4,5,7}\lambda$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}\bar\lambda + \omega^{1,3,4,6}(\bar\lambda - 1) + \omega^{2,3,4,5}(2\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7}(\bar\lambda - 1) + \omega^{2,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 1)$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7}\bar\lambda + 2\omega^{2,3,4,7} + \omega^{2,4,5,6}(\lambda - 1)$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7}(\bar\lambda - 1) + \omega^{1,4,5,6} + \omega^{2,3,5,6}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7}(\bar\lambda - 1) + \omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{3,4,5,6}\lambda$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{2,3,4,7}(\bar\lambda - 1) + \omega^{2,4,5,6}(\lambda\bar\lambda + 1)$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7}(\bar\lambda - 1) - \omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 3) + \omega^{3,4,6,7}(\lambda\bar\lambda - \bar\lambda - 1)$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}(-\bar\lambda + 1) - \omega^{2,3,5,7}\bar\lambda + \omega^{3,4,5,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7}\bar\lambda + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7}(\lambda\bar\lambda - \lambda - 1) + \omega^{3,4,6,7}(\lambda\bar\lambda + 1)$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}(2\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 3)$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}(\lambda - 1)$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}(\lambda\bar\lambda + 2)$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}\lambda$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7}(\bar\lambda - 1) + \omega^{1,2,4,5,6}(\lambda\bar\lambda + 1)$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}(\lambda - 1)$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}(\lambda\bar\lambda - \lambda - \bar\lambda + 2) + \omega^{1,3,4,6,7}(\lambda\bar\lambda - \bar\lambda - 1) + \omega^{2,3,4,5,7}\lambda$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7}\bar\lambda + \omega^{1,3,4,5,6}(\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7}(\lambda\bar\lambda - \lambda - 1) + \omega^{1,3,4,6,7}(\lambda\bar\lambda + 1) + \omega^{2,3,4,5,7}(\lambda - 1)$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}(\bar\lambda - 1) + \omega^{2,3,4,5,6}(2\lambda\bar\lambda - \lambda - \bar\lambda + 1)$; $\Delta\omega^{2,3,4,5,7} =$

$\omega^{1,2,5,6,7}\bar\lambda + \omega^{1,3,4,6,7}(\bar\lambda - 1) + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}(2\lambda\bar\lambda - \lambda - \bar\lambda + 2)$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1}\ \langle\omega^{1,5}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} + \omega^{3,6}(2\lambda - 1) + \omega^{4,5}\lambda\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}(2\bar\lambda - 1) + \omega^{2,5}(\bar\lambda - 2) - \omega^{3,4}(\bar\lambda + 1)\rangle + e^{-5\beta_1}\ \langle\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}\bar\lambda+\omega^{2,3,6}(-\lambda\bar\lambda+\bar\lambda-1)-\omega^{2,4,5}, \omega^{1,4,6}\bar\lambda-\omega^{2,3,6}(\bar\lambda+1)-\omega^{2,4,5}\rangle+e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle+e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,7}(\bar\lambda-2)+\omega^{1,5,6}(\lambda\bar\lambda-\lambda+1)+\omega^{3,4,5}(\lambda+1)\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7}(\bar\lambda-2)+\omega^{2,3,7}(2\lambda\bar\lambda-\lambda+1)+\omega^{2,5,6}\lambda(-\bar\lambda+2)+\omega^{3,4,6}(\lambda\bar\lambda+\lambda+1)\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,7} + \omega^{3,5,6}(\lambda-1)\rangle + e^{-5\beta_1-\beta_2}\ \langle-\omega^{1,3,6}\bar\lambda - \omega^{1,4,5}(\bar\lambda+1) + \omega^{2,3,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-8\beta_1-2\beta_2}\ \langle(\omega^{1,3,6,7}(-\bar\lambda^2+2\bar\lambda+2)+\omega^{1,4,5,7}(\lambda\bar\lambda^2-\lambda\bar\lambda+2\bar\lambda-2)+\omega^{2,3,5,7}(\lambda\bar\lambda^2-\lambda+\bar\lambda+2)+\omega^{3,4,5,6}(2\lambda\bar\lambda+\lambda+\bar\lambda+2))/((\lambda\bar\lambda-\lambda-\bar\lambda+4)(\lambda\bar\lambda+2)\bar\lambda), (\omega^{1,3,6,7}(\lambda\bar\lambda^2-2\lambda\bar\lambda-\bar\lambda^2+4\bar\lambda-2)+\omega^{1,4,5,7}(\lambda\bar\lambda+2)+\omega^{2,3,5,7}(-\lambda^2\bar\lambda^2+\lambda^2\bar\lambda+\lambda\bar\lambda^2-4\lambda\bar\lambda+\lambda+\bar\lambda-2)+\omega^{3,4,5,6}(-\lambda^2\bar\lambda-\lambda+\bar\lambda-2))/((\lambda\bar\lambda-\lambda-\bar\lambda+4)(\lambda\bar\lambda+2)\bar\lambda)\rangle+e^{-6\beta_1-4\beta_2}\ \langle-\omega^{2,4,6,7}\rangle+e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,2,6,7}(\bar\lambda+1)+\omega^{2,3,4,7}(\lambda\bar\lambda-\bar\lambda+1)+\omega^{2,4,5,6}(-\lambda+2))/((\lambda\bar\lambda-\lambda-\bar\lambda+3)(\lambda\bar\lambda+2))\rangle+e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,7}(\lambda\bar\lambda+\bar\lambda+1)+\omega^{1,3,4,7}\bar\lambda(\lambda-2)+\omega^{1,4,5,6}(2\lambda\bar\lambda-\bar\lambda+1)+\omega^{2,3,5,6}(\lambda-2))/((3\lambda\bar\lambda+2)(2\lambda\bar\lambda-\lambda-\bar\lambda+3))\rangle+e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,7}(\bar\lambda-1)-\omega^{2,3,4,6})/(\lambda\bar\lambda-\lambda-\bar\lambda+2)\rangle+e^{-7\beta_1-3\beta_2}\ \langle(\omega^{1,4,6,7}-\omega^{2,3,6,7}(\lambda+1)-\omega^{2,4,5,7}\lambda)/(2\lambda\bar\lambda+\lambda+\bar\lambda+2)\rangle + e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-8\beta_1-4\beta_2}\ \langle-\omega^{2,3,4,6,7}\rangle+e^{-8\beta_1-3\beta_2}\ \langle(\omega^{1,2,3,6,7}\bar\lambda+\omega^{1,2,4,5,7}(2\bar\lambda-1)-\omega^{2,3,4,5,6})/(5\lambda\bar\lambda-2\lambda-2\bar\lambda+2)\rangle + e^{-10\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7})/2\rangle + e^{-11\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6,7}(\lambda+1)+\omega^{1,3,4,6,7}(\lambda-2)+\omega^{2,3,4,5,7}(2\lambda-1))/(3(2\lambda\bar\lambda-\lambda-\bar\lambda+2))\rangle + e^{-7\beta_1-4\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-11\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y^2)(1+Y).$

The preceding results are valid for: $(\bar\lambda^2-\bar\lambda+1)(\lambda-3\bar\lambda-1)(\lambda^2-\lambda+1)(-(\lambda^2\bar\lambda^2-2\lambda^2\bar\lambda+\lambda^2-\lambda\bar\lambda-\lambda+2\bar\lambda-1))(\lambda-3\bar\lambda-1)(\bar\lambda-2)(\lambda\bar\lambda+\bar\lambda+1)(-(\bar\lambda-2))(\bar\lambda+1)(-\bar\lambda)(-(\bar\lambda-2))(\lambda-2)(-(\bar\lambda-2))(\lambda+1)\neq 0.$

Comments:singular values.

Among the values excluded by the computer, only $\lambda = 2$ and $\lambda^2 - \lambda + 1 = 0$ are singular values, i.e. give different cohomology than the generic one. Note that $\lambda = 0$ is not a singular value; however, for $\lambda = 0$, the eigenvectors computed in $\mathrm{ch}(H^3_{\mathbb{C}})$ for $2e^{-4\beta_1 - 2\beta_2}$ are proportional to $\varphi = \omega^{2,3,6} + \omega^{2,4,5}$. Change in $\mathrm{ch}(H^3_{\mathbb{C}})$ $2e^{-4\beta_1-2\beta_2}$ $\langle ... \rangle$ to $2e^{-4\beta_1-2\beta_2}$ $\langle \varphi, \psi \rangle$ with $\psi = \omega^{1,2,7} - \omega^{1,4,6} - 2\omega^{2,4,5}$. (Note that the generic nonharmonic cocycles computed by the first program for $\lambda = 0$ are φ, and $\omega^{1,2,7} - \omega^{2,4,7}$ which is cohomologous to $\psi/2$). Change also in $\mathrm{ch}(H^4_{\mathbb{C}})$ $2e^{-8\beta_1-2\beta_2}$ $\langle .. \rangle$ to $2e^{-8\beta_1-2\beta_2}$ $\langle (\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6})/4, (-\omega^{1,3,6,7} - 3\omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6})/4 \rangle$.

Case $\lambda = 2$ **:**

Add in $\mathrm{ch}(H^2_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2}$ $\langle \omega^{2,7} - \omega^{4,6} \rangle$ and correspondingly in $\mathrm{ch}(H^5_{\mathbb{C}})$ $e^{-9\beta_1-2\beta_2}$ $\langle (\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6})/2 \rangle$; in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2}$ $\langle \omega^{1,2,6} + \omega^{2,3,4} \rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-9\beta_1-2\beta_2}$ $\langle (-(\omega^{1,5,6,7} + \omega^{3,4,5,7}))/2 \rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{11}, 19, \underline{25}, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{7}, \underline{9}, \underline{9}, \underline{7}, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 36.

Riemann Hyp.: no

$P(Y) \ = \ (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y).$

Case $\lambda^2 - \lambda + 1 = 0$ **:**

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-6\beta_1-2\beta_2}$ $\langle \omega^{1,6,7} + \omega^{2,5,7}(-\lambda + 1) - \omega^{3,4,7}\lambda - \omega^{4,5,6} \rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-6\beta_1-2\beta_2}$ $\langle (\lambda\omega^{1,2,5,6} + \lambda\omega^{1,3,4,6} + \omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5})/4 \rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, \underline{20}, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, \underline{9}, \underline{9}, 6, 3, 1)$

Total dimension of trivial cohomology: 38.

Length associated polynomial: 36.

Riemann Hyp.: no

$P(Y) \ = \ (1 + 2Y + 4Y^2 + 5Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y).$

5.4.2 Algebra $\mathcal{G}_{7,2.1(ii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & -\xi_1^2 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5+\xi_2^6 & -\xi_1^4+\xi_3^6 & \xi_1^3+\xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_1^6+\delta_3^7), ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{3\beta_1}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=\ e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^6,\delta_3^7\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+2e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{\beta_2}\ \langle\delta_1^4,\delta_3^6\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=\delta_2^6+\delta_1^5$; $[\delta_2^5,\delta_1^4]=\delta_2^7$; $[\delta_2^5,\delta_3^6]=-\delta_2^7$; $[\delta_1^6,\delta_1^3]=-\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_1^5]=-(\delta_3^7+\delta_1^6)$; $[\delta_1^2,\delta_2^6]=\delta_3^7-\delta_1^6$; $[\delta_1^2,\delta_1^3]=-\delta_1^4$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_2^7$; $[\delta_2^6,\delta_1^4]=\delta_1^7$; $[\delta_2^6,\delta_1^3]=-\delta_2^7$; $[\delta_1^4,\delta_2^4]=-(\delta_3^7+\delta_1^6)$; $[\delta_3^6,\delta_1^3]=-\delta_3^7+\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{\beta_2}\ \langle\delta_3^6\rangle+e^{\beta_1}\ \langle\delta_1^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=\delta_2^6$; $[\delta_1^2,\delta_2^6]=-2\delta_1^6$; $[\delta_3^6,\delta_1^3]=2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=\ e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,5}\rangle+4e^{-2\beta_1}\ \langle x_2\omega^{1,4}+x_3\omega^{1,5},-1/2x_1\omega^{1,3}+3/2x_4(\omega^{1,6}+1/3\omega^{3,4})+x_6\omega^{3,6},-1/2x_1\omega^{1,3}+x_2\omega^{2,3}+1/2x_4(\omega^{1,6}-\omega^{3,4})+x_5\omega^{3,5},-1/2x_1\omega^{1,3}+x_2\omega^{2,3}+1/2x_4(\omega^{1,6}-2\omega^{2,5}-3\omega^{3,4})+x_6\omega^{1,7}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle+e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}+x_5\omega^{2,6}\rangle+2e^{-\beta_1-\beta_2}\ \langle-2x_1\omega^{1,4}+2x_2\omega^{2,4}-2x_3(\omega^{1,6}-1/2\omega^{2,5}-1/2\omega^{3,4})+x_5\omega^{4,5},2x_1(\omega^{1,4}+1/2\omega^{2,3})-x_2\omega^{2,4}+2x_3(\omega^{1,6}-1/2\omega^{2,5}-1/2\omega^{3,4})+x_4\omega^{2,6}+x_5(\omega^{1,7}+2\omega^{3,6})+x_6\omega^{2,7}\rangle+e^{-2\beta_2}\ \langle x_1\omega^{2,4}-$

$x_5(\omega^{2,7}-\omega^{4,6})\rangle + e^0\ \langle -x_4\omega^{1,2}+x_6\omega^{2,3}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + 3e^{-\beta_2}\ \langle -1/2x_1\omega^{1,2} - 1/2x_3\omega^{2,3} + 1/2x_4\omega^{2,4} + 3/2x_5\omega^{1,6} + x_6\omega^{2,6}, -1/2x_1\omega^{1,2} + 1/2x_3\omega^{2,3} - 1/2x_4\omega^{2,4} + 1/2x_5(\omega^{1,6}+2\omega^{2,5}), x_3\omega^{1,4}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}+x_6\omega^{3,5}\rangle + e^{-3\beta_1}\ \langle -x_1\omega^{1,5} - 2x_2(\omega^{1,6}-1/2\omega^{2,5}-1/2\omega^{3,4}) + x_3\omega^{3,5} + x_4\omega^{3,6} + x_6\omega^{5,6}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_4\omega^{2,3} + x_6\omega^{2,5}, x_3\omega^{1,3} - x_4\omega^{2,3} + x_6\omega^{3,4}, -x_3\omega^{1,3} + x_4\omega^{1,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}\rangle$.

ch$(H^3_{\mathcal{G}})$=$3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 2e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + 4e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 5e^{-4\beta_1-\beta_2} + 2e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + 4e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 2e^{-6\beta_1} + e^{-2\beta_2} + e^{-3\beta_2}$.

ch$(H^4_{\mathcal{G}})$=$3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + 3e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + 5e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + 3e^{-6\beta_1-\beta_2} + 3e^{-6\beta_1-2\beta_2} + e^{-6\beta_1} + 4e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2} + 2e^{-8\beta_1-\beta_2} + e^{-8\beta_1} + e^{-9\beta_1}$.

ch$(H^5_{\mathcal{G}})$=$e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-3\beta_2} + 2e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2} + 4e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + 4e^{-8\beta_1-2\beta_2} + 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + e^{-10\beta_1-\beta_2} + e^{-10\beta_1-2\beta_2} + e^{-11\beta_1-\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + 2e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-2\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-2\beta_2}$.

ch$(H^7_{\mathcal{G}})$=$e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-4\beta_2} + e^{-12\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 14, 56, 123, 155, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 8, 21, 32, 33, 24, 12, 3)$

S=$4(3\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$;

$d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-5\beta_1} \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7} + 2\omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}+\omega^{2,4,5}, \omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}-\omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{2,3,7}+\omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_2} \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-9\beta_1-\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,2,5,7} - \omega^{2,3,5,6}\rangle + 2e^{-8\beta_1-2\beta_2} \langle\omega^{1,3,6,7} + \omega^{3,4,5,6}, \omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{2,3,4,7} + \omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-10\beta_1-4\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-12\beta_1-4\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} =$

$\omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,5} = \omega^{1,6} + 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = 3\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 3\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 3\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} =$

$-\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 3\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 3\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} - \omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 3\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle 2\omega^{1,6} - \omega^{2,5} - \omega^{3,4}\rangle + e^{-5\beta_1}\ \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} + 2\omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,5}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} - 2\omega^{2,3,6} - \omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,7} - 2\omega^{1,5,6} - \omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7} + 3\omega^{2,3,7} - \omega^{2,5,6} + 2\omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2}\ \langle -\omega^{2,4,6,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7})/3\rangle + 2e^{-8\beta_1-2\beta_2}\ \langle(\omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6})/3, (\omega^{1,3,6,7} - 2\omega^{2,3,5,7} - \omega^{3,4,5,6})/3\rangle + e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle(-\omega^{1,2,6,7} - 2\omega^{2,3,4,7} - \omega^{2,4,5,6})/6\rangle + e^{-7\beta_1-2\beta_2}\ \langle(-2\omega^{1,2,5,7} + \omega^{1,3,4,7} + 3\omega^{1,4,5,6} + \omega^{2,3,5,6})/15\rangle + e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,7} - \omega^{2,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-8\beta_1-4\beta_2}\ \langle -\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-11\beta_1-2\beta_2}\ \langle -\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/6\rangle + e^{-7\beta_1-4\beta_2}\ \langle -\omega^{1,2,4,6,7}\rangle + e^{-8\beta_1-3\beta_2}\ \langle(\omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/6\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-11\beta_1-4\beta_2} \left\langle\omega^{2,3,4,5,6,7}\right\rangle + e^{-12\beta_1-3\beta_2} \left\langle-\omega^{1,3,4,5,6,7}\right\rangle + e^{-10\beta_1-4\beta_2} \left\langle\omega^{1,2,4,5,6,7}\right\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-12\beta_1-4\beta_2} \left\langle\omega^{1,2,3,4,5,6,7}\right\rangle$.

$P(Y) = (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

5.4.3 Algebra $\mathcal{G}_{7,2.1(iii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -2\xi_1^3 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^4 & -\xi_1^3+\xi_2^4 & -\xi_1^2 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5+\xi_2^6 & \xi_5^7 & -2\xi_1^3+\xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_5^7, ad\ x_3=-\delta_2^6, ad\ x_4=-\delta_1^6+\delta_3^7, ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\langle x_1\rangle+e^{\beta_2}\langle x_2\rangle+e^{2\beta_1}\langle x_3\rangle+e^{\beta_1+\beta_2}\langle x_4\rangle+e^{3\beta_1}\langle x_5\rangle+e^{2\beta_1+\beta_2}\langle x_6\rangle+e^{3\beta_1+\beta_2}\langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{3\beta_1+\beta_2}\langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{3\beta_1-\beta_2}\langle\delta_2^5\rangle+2e^{\beta_1+\beta_2}\langle\delta_1^6,\delta_3^7\rangle+2e^{\beta_2}\langle\delta_1^4,\delta_5^7\rangle+e^{-\beta_1+\beta_2}\langle\delta_1^2\rangle+e^{2\beta_1+\beta_2}\langle\delta_1^7\rangle+e^{3\beta_1}\langle\delta_2^7\rangle+2e^{2\beta_1}\langle\delta_1^5,\delta_2^6\rangle+2e^{\beta_1}\langle\delta_1^3,\delta_2^4\rangle+2e^0\langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=\delta_2^6+\delta_1^5$; $[\delta_2^5,\delta_5^7]=-\delta_2^7$; $[\delta_1^6,\delta_1^3]=2\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_1^5]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_1^3]=-2\delta_3^7$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_1^6$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_1^5]=-(\delta_3^7+\delta_1^6)$; $[\delta_1^2,\delta_2^6]=\delta_3^7-\delta_1^6$; $[\delta_1^2,\delta_1^3]=-2\delta_5^7-\delta_1^4$; $[\delta_1^2,\delta_2^4]=\delta_5^7-\delta_1^4$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^5,\delta_5^7]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=2\delta_2^7$; $[\delta_5^7,\delta_1^3]=-2\delta_3^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{3\beta_1-\beta_2}\langle\delta_2^5\rangle+e^{-\beta_1+\beta_2}\langle\delta_1^2\rangle+e^{\beta_1+\beta_2}\langle\delta_1^6\rangle+e^{2\beta_1}\langle\delta_1^5\rangle+e^{\beta_2}\langle\delta_1^4\rangle+e^{\beta_1}\langle\delta_1^3\rangle+2e^0\langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_1^2,\delta_1^5]=-2\delta_1^6$; $[\delta_1^4,\delta_1^3]=-2\delta_1^6$; $[\delta_1^2,\delta_1^3]=-3\delta_1^4$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{\beta_1-2\beta_2}\langle x_3\omega^{2,4}\rangle+e^{-4\beta_1+\beta_2}\langle -x_2\omega^{1,5}+x_4\omega^{3,5}\rangle+3e^{-3\beta_1}\langle -x_1\omega^{1,5}-x_2(\omega^{1,6}-2\omega^{2,5}-\omega^{3,4})-2x_3\omega^{3,5}+x_4(\omega^{3,6}+\omega^{4,5}),-x_1\omega^{1,5}-2x_2(\omega^{1,6}-1/2\omega^{2,5}-\omega^{3,4})-2x_3\omega^{3,5}+2x_4\omega^{3,6}+x_6\omega^{5,6},x_2\omega^{2,5}-x_4(\omega^{1,7}-\omega^{3,6})+x_6\omega^{3,7}\rangle+e^0\langle -x_4\omega^{1,2}+x_5\omega^{1,3}\rangle+3e^{-\beta_1}\langle -x_2\omega^{1,2}+x_3\omega^{1,3}-x_4\omega^{1,4}+x_5\omega^{1,5},x_3\omega^{1,3}-x_4(\omega^{1,4}-\omega^{2,3}),x_3\omega^{1,3}-2x_4\omega^{1,4}+x_6\omega^{3,4}\rangle+e^{\beta_1-\beta_2}\langle x_5\omega^{2,3}\rangle+e^{2\beta_1-2\beta_2}\langle x_5\omega^{2,4}\rangle+4e^{-2\beta_1}\langle -x_1\omega^{1,3}-x_2(\omega^{1,4}-2\omega^{2,3})+x_3\omega^{1,5}-x_4(\omega^{1,6}-\omega^{2,5})+x_5\omega^{3,5},-2x_1\omega^{1,3}-$

$3x_2(\omega^{1,4}-\omega^{2,3})+3x_3\omega^{1,5}-2x_4(\omega^{1,6}-3/2\omega^{2,5}-1/2\omega^{3,4}), 2x_1\omega^{1,3}+3x_2(\omega^{1,4}-4/3\omega^{2,3})-3x_3\omega^{1,5}+2x_4(\omega^{1,6}-2\omega^{2,5})+x_6\omega^{1,7}, -2x_1\omega^{1,3}-2x_2(\omega^{1,4}-2\omega^{2,3})+3x_3\omega^{1,5}-2x_4(\omega^{1,6}-2\omega^{2,5})+x_6\omega^{4,5}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}+x_4\omega^{1,5}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})+2x_2\omega^{2,4}+x_3(\omega^{1,6}-2\omega^{2,5}-\omega^{3,4})+x_4\omega^{2,6}, x_1(\omega^{1,4}-\omega^{2,3})-x_2\omega^{2,4}-x_3(\omega^{1,6}-2\omega^{2,5}-\omega^{3,4})+x_5(\omega^{1,7}-\omega^{3,6}+\omega^{4,5})+x_6\omega^{2,7}\rangle + e^{-4\beta_1}\ \langle -x_2(\omega^{1,7}-\omega^{3,6}+\omega^{4,5})+x_4(\omega^{3,7}+\omega^{5,6})+x_6\omega^{5,7}\rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2}-x_3(\omega^{1,4}+\omega^{2,3})-x_5(\omega^{1,6}-\omega^{3,4}), x_5\omega^{2,5}, x_3(\omega^{1,4}-\omega^{2,3})+x_4\omega^{2,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$3e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+2e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+4e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+2e^{-3\beta_1}+5e^{-4\beta_1-\beta_2}+3e^{-4\beta_1}+e^{-5\beta_1-\beta_2}+4e^{-5\beta_1}+e^{-6\beta_1+\beta_2}+2e^{-6\beta_1}+e^{-7\beta_1}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2}+3e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-\beta_2}+5e^{-5\beta_1-2\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+3e^{-6\beta_1-\beta_2}+3e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-6\beta_1}+4e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-7\beta_1}+2e^{-8\beta_1-\beta_2}+e^{-8\beta_1}+e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-4\beta_1-4\beta_2}+2e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2}+3e^{-6\beta_1-3\beta_2}+3e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-2\beta_2}+4e^{-7\beta_1-3\beta_2}+2e^{-7\beta_1-4\beta_2}+4e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-\beta_2}+2e^{-9\beta_1-2\beta_2}+e^{-9\beta_1-3\beta_2}+e^{-10\beta_1-\beta_2}+e^{-10\beta_1-2\beta_2}+e^{-11\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-6\beta_1-4\beta_2}+2e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-3\beta_2}+3e^{-8\beta_1-4\beta_2}+2e^{-9\beta_1-3\beta_2}+2e^{-9\beta_1-4\beta_2}+2e^{-10\beta_1-3\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-2\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-9\beta_1-4\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-4\beta_2}+e^{-12\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,14,58,125,159,118,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,8,23,36,39,32,17,4)$

S=$4(3\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,10,19,24,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,7,8,8,7,4,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^6=-\omega^{1,4}-\omega^{2,3}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}+\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,5}-\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{2,3,7}+\omega^{2,5,6}-\omega^{3,4,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}+\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$;

$d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle + e^{-3\beta_1}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + e^{-4\beta_1}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}\rangle + e^{-5\beta_1}\ \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}+\omega^{2,3,6}, \omega^{1,4,6}-\omega^{2,3,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,7}-\omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7}-\omega^{2,5,6}\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,7} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle + 2e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,3,6,7} + \omega^{3,4,5,6}, \omega^{1,4,5,7}\rangle + e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,5,7}+\omega^{2,3,5,6}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,2,4,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,2,6,7}-\omega^{2,3,4,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-11\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-8\beta_1-4\beta_2}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 10, 19, 24, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 7, 8, 8, 7, 4, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 36.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + 2\omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 2\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 3\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$;

$\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 3\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,4,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,5,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle + e^{-3\beta_1}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + e^{-4\beta_1}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6} - 2\omega^{2,5} - \omega^{3,4}\rangle + e^{-5\beta_1}\ \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6} + \omega^{1,4,5} - \omega^{2,3,5}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} + \omega^{2,3,6} - \omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle 2\omega^{1,3,7} + \omega^{1,5,6} - \omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7} - \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,7} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1-4\beta_2}\ \langle -\omega^{2,4,6,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{2,4,5,7})/3\rangle + 2e^{-8\beta_1-2\beta_2}\ \langle(\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{3,4,5,6})/3, (\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,5,7})/3\rangle + e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle(-\omega^{1,2,6,7} + \omega^{2,3,4,7} - 2\omega^{2,4,5,6})/6\rangle + e^{-7\beta_1-2\beta_2}\ \langle(\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6} + \omega^{2,3,5,6})/4\rangle + e^{-5\beta_1-3\beta_2}\ \langle(-\omega^{1,2,4,7} - \omega^{2,3,4,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-9\beta_1-4\beta_2}\ \langle -\omega^{2,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2}\ \langle -\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-11\beta_1-2\beta_2}\ \langle -\omega^{1,3,5,6,7}\rangle +$

$e^{-9\beta_1-3\beta_2}$ $\langle(-\omega^{1,2,5,6,7} - 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/6\rangle$ + $e^{-7\beta_1-4\beta_2}$ $\langle -\omega^{1,2,4,6,7}\rangle$ + $e^{-8\beta_1-3\beta_2}$ $\langle(\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{A}})=$ $e^{-11\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-12\beta_1-3\beta_2}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-10\beta_1-4\beta_2}$ $\langle\omega^{1,2,4,5,6,7}\rangle$ + $e^{-9\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{A}})=$ $e^{-12\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y)^3$.

5.4.4 Algebra $\mathcal{G}_{7,2.1(iv)}$

Adjoint cohomology

$[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_1^3 & -2\xi_1^2 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_1^5 & \xi_3^6 & \xi_3^5 - 2\xi_1^3 & -\xi_1^2 & 2\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^4 + \xi_3^6 & \xi_3^5 - \xi_1^3 & 3\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_3^5, ad\ x_2 = \delta_3^6, ad\ x_3 = -\delta_1^5 + \delta_4^7, ad\ x_4 = -(\delta_1^6 + \delta_3^7), ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{2\beta_1} \langle x_3 \rangle + e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{3\beta_1} \langle x_5 \rangle + e^{2\beta_1+\beta_2} \langle x_6 \rangle + e^{3\beta_1+\beta_2} \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{3\beta_1+\beta_2} \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^5 \rangle + 2e^{\beta_1+\beta_2} \langle \delta_1^6, \delta_3^7 \rangle + 2e^{2\beta_1} \langle \delta_1^5, \delta_4^7 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + e^{2\beta_1+\beta_2} \langle \delta_1^7 \rangle + e^{3\beta_1} \langle \delta_2^7 \rangle + 2e^{\beta_2} \langle \delta_1^4, \delta_3^6 \rangle + 2e^{\beta_1} \langle \delta_1^3, \delta_3^5 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^5, \delta_1^4] = \delta_2^7$; $[\delta_2^5, \delta_3^6] = -\delta_2^7$; $[\delta_1^6, \delta_1^3] = \delta_1^7$; $[\delta_1^6, \delta_3^5] = -\delta_1^7$; $[\delta_1^5, \delta_1^2] = 2\delta_1^6$; $[\delta_1^5, \delta_1^4] = \delta_1^7$; $[\delta_1^5, \delta_3^6] = -\delta_1^7$; $[\delta_1^5, \delta_1^3] = \delta_2^7$; $[\delta_1^5, \delta_3^5] = -\delta_2^7$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_4^7] = 2\delta_3^7$; $[\delta_1^2, \delta_1^3] = -4\delta_3^6 - 3\delta_1^4$; $[\delta_1^2, \delta_3^5] = \delta_3^6$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_4^7, \delta_1^4] = \delta_1^7$; $[\delta_4^7, \delta_1^3] = \delta_2^7$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_1^4, \delta_3^5] = -(\delta_3^7 + \delta_1^6)$; $[\delta_3^6, \delta_1^3] = \delta_3^7 + \delta_1^6$; $[\delta_1^3, \delta_3^5] = \delta_4^7 - \delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^5 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + e^{\beta_1+\beta_2} \langle \delta_1^6 \rangle + e^{2\beta_1} \langle \delta_1^5 \rangle + e^{\beta_2} \langle \delta_1^4 \rangle + e^{\beta_1} \langle \delta_1^3 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_1^2, \delta_1^5] = -2\delta_1^6$; $[\delta_1^2, \delta_1^3] = -3\delta_1^4$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-5\beta_1+\beta_2} \langle x_2\omega^{3,5} \rangle + e^{-4\beta_1} \langle x_1\omega^{3,5} + x_2(\omega^{3,6} + \omega^{4,5}) \rangle + e^{-3\beta_1+\beta_2} \langle -x_2\omega^{1,3} + x_4\omega^{1,5} \rangle + e^{-4\beta_1+\beta_2} \langle -x_2\omega^{1,5} + x_4\omega^{3,5} \rangle + e^{-3\beta_1} \langle -x_1\omega^{1,5} - x_2(\omega^{1,6} + \omega^{2,5} - 2\omega^{3,4}) + x_3\omega^{3,5} + x_4(\omega^{3,6} + \omega^{4,5}) \rangle + e^{2\beta_1-2\beta_2} \langle x_5\omega^{2,4} \rangle + e^{\beta_1-2\beta_2} \langle x_3\omega^{2,4} + x_5\omega^{2,6} \rangle + 3e^{-\beta_1-\beta_2} \langle x_2\omega^{2,4} - x_3\omega^{3,4} + x_5\omega^{4,5}, x_1\omega^{2,3} + x_3\omega^{3,4} + x_5\omega^{3,6}, -x_1\omega^{1,4} - x_2\omega^{2,4} - x_3(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{1,7} + x_6\omega^{2,7} \rangle + e^{-2\beta_1+\beta_2} \langle x_4\omega^{1,3} \rangle + 3e^{-\beta_1} \langle x_3\omega^{1,3} - x_4(\omega^{1,4} - \omega^{2,3}), -x_2\omega^{1,2} + x_3\omega^{1,3} - x_4\omega^{1,4} + x_5\omega^{1,5}, x_4\omega^{1,4} +$

$x_6\omega^{3,4}\rangle + e^0\ \langle x_4\omega^{1,2} + x_5\omega^{1,3}\rangle + e^{\beta_1-\beta_2}\ \langle -x_3\omega^{1,2} + x_5\omega^{1,4}\rangle + 3e^{-\beta_2}\ \langle -x_3(\omega^{1,4} - \omega^{2,3}) - x_4\omega^{2,4} + x_5\omega^{2,5}, -x_1\omega^{1,2} + 3x_3(\omega^{1,4} - 2/3\omega^{2,3}) + 2x_4\omega^{2,4} + x_5\omega^{1,6}, x_3\omega^{1,4} + x_4\omega^{2,4} + x_5\omega^{3,4}\rangle + 4e^{-2\beta_1}\ \langle 1/2x_1\omega^{1,3} - 1/2x_2(\omega^{1,4} - \omega^{2,3}) - 1/2x_3\omega^{1,5} - 1/2x_4(\omega^{1,6} + \omega^{2,5} - 2\omega^{3,4}), x_1\omega^{1,3} + x_2\omega^{2,3} - x_4(\omega^{1,6} + \omega^{2,5}) + x_6\omega^{1,7}, x_2\omega^{2,3} + x_5\omega^{3,5}, x_1\omega^{1,3} + x_2\omega^{2,3} - x_3\omega^{1,5} - x_4(\omega^{1,6} + \omega^{2,5}) + x_6\omega^{4,5}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{-\beta_1-\beta_2} + 3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 2e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + 4e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 5e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1} + e^{-5\beta_1+\beta_2} + e^{-5\beta_1-\beta_2} + 4e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 2e^{-6\beta_1} + e^{-3\beta_2}$.

ch$(H^4_{\mathcal{G}})$=$3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + 4e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + 5e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + 3e^{-6\beta_1-\beta_2} + 3e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-6\beta_1} + 4e^{-7\beta_1-\beta_2} + 2e^{-7\beta_1-2\beta_2} + 2e^{-8\beta_1-\beta_2} + e^{-8\beta_1} + e^{-9\beta_1}$.

ch$(H^5_{\mathcal{G}})$=$e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 3e^{-6\beta_1-3\beta_2} + 2e^{-6\beta_1-4\beta_2} + 2e^{-7\beta_1-2\beta_2} + 4e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + 4e^{-8\beta_1-2\beta_2} + 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-10\beta_1-\beta_2} + 2e^{-10\beta_1-2\beta_2} + e^{-10\beta_1-3\beta_2} + e^{-11\beta_1-\beta_2} + e^{-11\beta_1-2\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + 2e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + e^{-10\beta_1-2\beta_2} + 3e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-4\beta_2} + 2e^{-11\beta_1-2\beta_2} + 2e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-2\beta_2}$.

ch$(H^7_{\mathcal{G}})$=$e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-11\beta_1-4\beta_2} + e^{-12\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 14, 58, 124, 157, 118, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 8, 23, 35, 36, 30, 17, 4)$

S=$4(3\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 10, 19, 24, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 7, 8, 8, 7, 4, 1)$

$d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,7} = -\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$;

$d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-\beta_1-\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-4\beta_1} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-5\beta_1} \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7} + \omega^{1,5,6}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,5,6}\rangle + e^{-7\beta_1-\beta_2} \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-7\beta_1-2\beta_2} \langle\omega^{1,2,5,7} - \omega^{2,3,5,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,6,7} - \omega^{2,4,5,6}\rangle + e^{-9\beta_1-\beta_2} \langle\omega^{1,3,5,7}\rangle + 2e^{-8\beta_1-2\beta_2} \langle\omega^{1,4,5,7}, \omega^{1,3,6,7} - \omega^{2,3,5,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7} - 2\omega^{2,3,6,7} - \omega^{2,4,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle + e^{-11\beta_1-3\beta_2} \langle\omega^{3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-3\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-12\beta_1-4\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 10, 19, 24, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 7, 8, 8, 7, 4, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 36.

Riemann Hyp.: no

$d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$;

$d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = 2\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{1,7} - \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + 3\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + 2\omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 3\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7} + \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} =$

$\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + 3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 3\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} + \omega^{1,2,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 3\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{g}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-\beta_1-\beta_2} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{g}}) = e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-4\beta_1} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} + \omega^{2,5} - 2\omega^{3,4}\rangle + e^{-5\beta_1} \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{3,6} + \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{g}}) = e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6} + 2\omega^{1,4,5} - \omega^{2,3,5}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{2,3,6}, \omega^{1,4,6} - 2\omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7} + \omega^{1,5,6} + 2\omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,5,6} + 3\omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_2} \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{g}}) = e^{-6\beta_1-4\beta_2} \langle-\omega^{2,4,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle(-\omega^{1,4,6,7} + 2\omega^{2,3,6,7} + \omega^{2,4,5,7})/6\rangle + 2e^{-8\beta_1-2\beta_2} \langle(-\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} + 3\omega^{3,4,5,6})/4, (\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,5,7} - \omega^{3,4,5,6})/4\rangle + e^{-9\beta_1-\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2} \langle(2\omega^{1,2,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6})/6\rangle + e^{-7\beta_1-2\beta_2} \langle(-3\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6})/12\rangle + e^{-5\beta_1-3\beta_2} \langle(\omega^{1,2,4,7} - \omega^{2,3,4,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{g}}) = e^{-11\beta_1-3\beta_2} \langle\omega^{3,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle-\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-11\beta_1-2\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle(-2\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/6\rangle + e^{-7\beta_1-4\beta_2} \langle-\omega^{1,2,4,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle(\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-11\beta_1-4\beta_2} \left\langle\omega^{2,3,4,5,6,7}\right\rangle + e^{-12\beta_1-3\beta_2} \left\langle-\omega^{1,3,4,5,6,7}\right\rangle + e^{-10\beta_1-4\beta_2} \left\langle\omega^{1,2,4,5,6,7}\right\rangle + e^{-11\beta_1-3\beta_2} \left\langle-\omega^{1,2,3,5,6,7}\right\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-12\beta_1-4\beta_2} \left\langle\omega^{1,2,3,4,5,6,7}\right\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y)^3$.

5.4.5 Algebra $\mathcal{G}_{7,2.1(v)}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_1^3+\xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^4 & \xi_3^4 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 & -\xi_1^4 & \xi_1^3 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^6, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = -(\delta_1^6 + \delta_3^7), ad\ x_5 = -\delta_2^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1\rangle + e^{\beta_2} \langle x_2\rangle + e^{2\beta_1} \langle x_3\rangle + e^{\beta_1+\beta_2} \langle x_4\rangle + e^{3\beta_1} \langle x_5\rangle + e^{2\beta_1+\beta_2} \langle x_6\rangle + e^{3\beta_1+\beta_2} \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{2\beta_1+\beta_2} \langle x_6\rangle + e^{3\beta_1+\beta_2} \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^5\rangle + 2e^{\beta_1+\beta_2} \langle \delta_1^6, \delta_3^7\rangle + 2e^{2\beta_1} \langle \delta_2^6, \delta_1^5\rangle + 2e^{\beta_2} \langle \delta_3^6, \delta_1^4\rangle + 2e^{\beta_1} \langle \delta_2^4, \delta_1^3\rangle + e^{-\beta_1+\beta_2} \langle \delta_3^4\rangle + e^{2\beta_1+\beta_2} \langle \delta_1^7\rangle + e^{3\beta_1} \langle \delta_2^7\rangle + 2e^0 \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_3^4] = -\delta_2^6$; $[\delta_2^5, \delta_1^4] = \delta_2^7$; $[\delta_1^6, \delta_1^3] = -\delta_1^7$; $[\delta_2^6, \delta_1^3] = -\delta_2^7$; $[\delta_3^6, \delta_1^3] = -\delta_3^7 + \delta_1^6$; $[\delta_2^4, \delta_1^5] = \delta_2^7$; $[\delta_2^4, \delta_1^4] = \delta_3^7 + \delta_1^6$; $[\delta_2^4, \delta_1^3] = \delta_2^6 + \delta_1^5$; $[\delta_3^4, \delta_1^5] = \delta_3^7 + \delta_1^6$; $[\delta_3^4, \delta_1^3] = 2\delta_3^6 + \delta_1^4$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_1^4, \delta_1^3] = -\delta_3^7 + \delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^5\rangle + e^{-\beta_1+\beta_2} \langle \delta_3^4\rangle + e^{2\beta_1+\beta_2} \langle \delta_1^7\rangle + e^{\beta_1+\beta_2} \langle \delta_1^6\rangle + e^{2\beta_1} \langle \delta_1^5\rangle + e^{\beta_2} \langle \delta_1^4\rangle + e^{\beta_1} \langle \delta_1^3\rangle + 2e^0 \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5, \delta_3^4] = \delta_1^5$; $[\delta_3^4, \delta_1^3] = 3\delta_1^4$; $[\delta_1^6, \delta_1^3] = -\delta_1^7$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-4\beta_1+\beta_2} \langle -x_2\omega^{1,5} + x_4\omega^{3,5}\rangle + e^{2\beta_1-2\beta_2} \langle x_5\omega^{2,4}\rangle + e^{\beta_1-2\beta_2} \langle x_3\omega^{2,4} + x_5\omega^{2,6}\rangle + e^{-2\beta_2} \langle -x_1\omega^{2,4} + x_5\omega^{2,7}\rangle + 2e^{-\beta_1-\beta_2} \langle -2x_1(\omega^{1,4} - 3/2\omega^{2,3}) + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5}, x_1\omega^{2,3} + x_6\omega^{2,7}\rangle + e^{-3\beta_1+\beta_2} \langle -x_2\omega^{1,3} + x_4\omega^{1,5}\rangle + 4e^{-2\beta_1} \langle x_1\omega^{1,3} + x_2\omega^{1,4} + x_3\omega^{1,5} + x_4\omega^{2,5} + x_5\omega^{3,5}, 3x_1\omega^{1,3} + 2x_2(\omega^{1,4} - \omega^{2,3}) + 2x_3\omega^{1,5} + 2x_4(\omega^{2,5} + 1/2\omega^{3,4}), 4x_1\omega^{1,3} + 3x_2(\omega^{1,4} - 2/3\omega^{2,3}) + 3x_3\omega^{1,5} + 2x_4\omega^{2,5} + x_6\omega^{1,7}, -2x_1\omega^{1,3} - x_2(\omega^{1,4} - \omega^{2,3}) - x_3\omega^{1,5} - x_4\omega^{2,5} + x_6\omega^{3,6}\rangle + e^{-3\beta_1} \langle -x_1\omega^{1,5} + x_2(\omega^{2,5} + \omega^{3,4}) + x_3\omega^{3,5} + x_4\omega^{3,6} + 3x_6(\omega^{3,7} + 1/3\omega^{5,6})\rangle +$

$e^{-4\beta_1}$ $\langle -x_1\omega^{3,5} + x_4\omega^{3,7} + x_6\omega^{5,7}\rangle + e^0$ $\langle -x_4\omega^{1,2} + x_7\omega^{1,6}\rangle + 3e^{-\beta_1}$ $\langle x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_7\omega^{1,7}, x_3\omega^{1,3} - x_4\omega^{1,4} + x_6\omega^{1,6}, x_3\omega^{1,3} - x_4(\omega^{1,4} - \omega^{2,3})\rangle + e^{\beta_1-\beta_2}$ $\langle x_5\omega^{2,3}\rangle + 3e^{-\beta_2}$ $\langle -x_1\omega^{1,2} + 2x_3(\omega^{1,4} - 1/2\omega^{2,3}) + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_3\omega^{1,4} + x_5\omega^{2,5}, -x_3\omega^{1,4} + x_6\omega^{2,6}\rangle + e^{-2\beta_1+\beta_2}$ $\langle x_4\omega^{1,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 2e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 4e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 5e^{-4\beta_1-\beta_2} + 3e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + 4e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 2e^{-6\beta_1} + e^{-2\beta_2} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + 3e^{-4\beta_1-2\beta_2} + 4e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + 5e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + 3e^{-6\beta_1-\beta_2} + 3e^{-6\beta_1-2\beta_2} + e^{-6\beta_1} + 4e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2} + 3e^{-8\beta_1-\beta_2} + e^{-8\beta_1} + e^{-9\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-3\beta_2} + 2e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2} + 4e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-\beta_2} + 4e^{-8\beta_1-2\beta_2} + 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + e^{-10\beta_1-\beta_2} + e^{-10\beta_1-2\beta_2} + e^{-11\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + 2e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-2\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-2\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-4\beta_2} + e^{-12\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 57, 124, 157, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 22, 34, 36, 26, 12, 3)$

S=$4(3\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$;

$d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{3,4}\rangle + e^{-5\beta_1} \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7} + 2\omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1} \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}+\omega^{2,4,5}, \omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{2,4,7}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,5,6} + \omega^{3,4,5}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_2} \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle + e^{-8\beta_1-\beta_2} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-8\beta_1-\beta_2} \langle\omega^{1,3,5,6}\rangle + e^{-9\beta_1-\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle + 2e^{-8\beta_1-2\beta_2} \langle\omega^{1,4,5,7}, \omega^{1,3,6,7} + \omega^{3,4,5,6}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,6,7} - \omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{2,3,6,7} + \omega^{2,4,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-10\beta_1-4\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-4\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 36.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$;

$d^\star\omega^{2,3,4} = \omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,6} + 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = 3\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7} + 3\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} =$

$2\omega^{1,2,6,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5}=2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,3,7}+2\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7}=3\omega^{1,3,4,7}+\omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7}=2\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,5,7}+\omega^{1,3,4,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7}=2\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,5,6}-\omega^{1,3,4,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,2,4,7}+\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,2,6,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,2,5,7}+\omega^{1,4,5,6}+2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7}=\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6}=\omega^{2,3,4,7}+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7}=3\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,4,5,7}-\omega^{2,3,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7}=2\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=2\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{-4\beta_1}\ \langle\omega^{1,5}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}-\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{-3\beta_1-\beta_2}\ \langle 2\omega^{1,6}+\omega^{2,5}-\omega^{3,4}\rangle+e^{-5\beta_1}\ \langle\omega^{3,5}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7}+2\omega^{3,6}+\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-6\beta_1}\ \langle\omega^{1,3,5}\rangle+e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6}+\omega^{1,4,5}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6}-\omega^{2,3,6},\omega^{1,2,7}-2\omega^{2,3,6}-\omega^{2,4,5}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{2,4,7}\rangle+e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,7}+2\omega^{1,5,6}+\omega^{3,4,5}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7}-3\omega^{2,3,7}-\omega^{2,5,6}-\omega^{3,4,6}\rangle+e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,7}+\omega^{3,5,6}\rangle+e^{-8\beta_1-\beta_2}\ \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-6\beta_1-4\beta_2}\ \langle-\omega^{2,4,6,7}\rangle+e^{-7\beta_1-3\beta_2}\ \langle(\omega^{2,3,6,7}+\omega^{2,4,5,7})/2\rangle+2e^{-8\beta_1-2\beta_2}\ \langle(-\omega^{1,3,6,7}+\omega^{1,4,5,7}-3\omega^{2,3,5,7}-\omega^{3,4,5,6})/4,(\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}+\omega^{3,4,5,6})/4\rangle+e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle+e^{-8\beta_1-\beta_2}\ \langle-\omega^{1,3,5,6}\rangle+e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,2,6,7}+2\omega^{2,3,4,7}-\omega^{2,4,5,6})/6\rangle+e^{-7\beta_1-2\beta_2}\ \langle(\omega^{1,2,5,7}+\omega^{1,3,4,7}-3\omega^{1,4,5,6}+\omega^{2,3,5,6})/12\rangle+e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,7}-\omega^{2,3,4,6})/2\rangle+e^{-4\beta_1-3\beta_2}\ \langle-\omega^{1,2,4,6}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-8\beta_1-4\beta_2}\ \langle-\omega^{2,3,4,6,7}\rangle+e^{-10\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7})/2\rangle+e^{-11\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle+e^{-9\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7})/6\rangle+e^{-7\beta_1-4\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle+e^{-8\beta_1-3\beta_2}\ \langle(\omega^{1,2,3,6,7}+2\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6})/6\rangle.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-11\beta_1-4\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-12\beta_1-3\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{A}})= e^{-12\beta_1-4\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+3Y^2+Y^4)(1+Y)^3.$

5.4.6 Algebra $\mathcal{G}_{7,2.2}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_4] = 2x_7$; $[x_2, x_3] = x_4$; $[x_2, x_6] = x_7$; $[x_3, x_5] = -x_7$; $[x_3, x_6] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & 2\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1 + \xi_2^2 & \xi_3^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & \xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 - \xi_2^6 + \xi_3^6 & 2\xi_2^4 - \xi_1^6 & 2\xi_3^4 + \xi_1^5 - \xi_1^6 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = \delta_3^4 - \delta_1^5, ad\ x_3 = -(\delta_2^4 + \delta_1^6), ad\ x_4 = -2\delta_1^7, ad\ x_5 = \delta_3^7, ad\ x_6 = -(\delta_2^7 + \delta_3^7), ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + 2e^{\beta_3}\ \langle x_2, x_3 \rangle + e^{2\beta_3}\ \langle x_4 \rangle + 2e^{\beta_1+\beta_3}\ \langle x_5, x_6 \rangle + e^{\beta_1+2\beta_3}\ \langle x_7 \rangle.$

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+2\beta_2}\ \langle x_7 \rangle.$

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+2\beta_3}\ \langle \delta_1^4 \rangle + 4e^{\beta_1}\ \langle \delta_3^5, \delta_2^5, \delta_2^6, \delta_3^6 \rangle + 3e^0\ \langle \delta_3^2, t_1, t_2 \rangle + e^{2\beta_3}\ \langle \delta_1^7 \rangle + 2e^{\beta_1+\beta_3}\ \langle \delta_2^7, \delta_3^7 \rangle + 4e^{\beta_3}\ \langle \delta_2^4, \delta_3^4, \delta_1^5, \delta_1^6 \rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^6] = \delta_1^7$; $[\delta_1^4, \delta_3^6] = -\delta_1^7$; $[\delta_3^5, \delta_2^4] = -2\delta_3^7$; $[\delta_3^5, \delta_1^6] = \delta_3^7$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_3^2, \delta_2^5] = -\delta_3^5$; $[\delta_3^2, \delta_2^6] = -\delta_3^6 + \delta_2^5$; $[\delta_3^2, \delta_3^6] = \delta_3^5$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_3^2, \delta_1^6] = \delta_1^5$; $[\delta_2^5, \delta_2^4] = -\delta_2^7$; $[\delta_2^5, \delta_3^4] = \delta_3^7$; $[\delta_2^5, \delta_1^6] = \delta_2^7$; $[\delta_2^6, \delta_2^4] = -\delta_2^7$; $[\delta_2^6, \delta_3^4] = -\delta_3^7 - 2\delta_2^7$; $[\delta_2^6, \delta_1^5] = -\delta_2^7$; $[\delta_2^6, \delta_1^6] = \delta_2^7$; $[\delta_3^6, \delta_2^4] = \delta_2^7$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_3^6, \delta_1^5] = -\delta_3^7$; $[\delta_3^6, \delta_1^6] = \delta_3^7$; $[\delta_2^4, \delta_1^5] = 2\delta_1^7$; $[\delta_3^4, \delta_1^6] = 2\delta_1^7$; $[\delta_1^5, \delta_1^6] = 2\delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+2\beta_3}\ \langle \delta_1^4 \rangle + 3e^{\beta_1}\ \langle \delta_3^5, \delta_2^6, \delta_2^5 \rangle + 3e^0\ \langle \delta_3^2, t_1, t_2 \rangle + 2e^{\beta_3}\ \langle \delta_3^4, \delta_2^4 \rangle.$

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^2, \delta_2^6] = 2\delta_2^5$; $[\delta_3^2, \delta_2^5] = -\delta_3^5$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 2e^{-2\beta_1+\beta_3}\ \langle x_4\omega^{1,5}, x_4\omega^{1,6} \rangle + e^{-2\beta_1}\ \langle x_2(\omega^{1,5} - 2\omega^{1,6}) + x_3\omega^{1,6} + 2x_4(\omega^{1,7} + 1/2\omega^{5,6}) \rangle + 3e^{\beta_1-2\beta_3}\ \langle x_5\omega^{3,4}, -1/2x_1\omega^{2,3} + x_6\omega^{2,4}, 1/2x_1\omega^{2,3} + x_5\omega^{2,4} \rangle + 3e^{-2\beta_3}\ \langle -1/2x_1\omega^{3,6} - 2x_2\omega^{3,4} - 1/2x_5(\omega^{3,7} - 2\omega^{4,6}), -1/3x_1(\omega^{1,4} - 3/2\omega^{2,5} + \omega^{2,6} + 1/2\omega^{3,6}) + 4/3x_2(\omega^{2,4} + 3/4\omega^{3,4}) - 2x_3(\omega^{2,4} + 2/3\omega^{3,4}) + 1/3x_5(\omega^{2,7} + 3/2\omega^{3,7} - \omega^{4,5}) - 1/2x_6(\omega^{2,7} + 5/3\omega^{3,7} - 2\omega^{4,5}), 1/2x_1(\omega^{2,6} + \omega^{3,5} - \omega^{3,6}) + 2x_2(\omega^{2,4} + \omega^{3,4}) - 2x_3\omega^{3,4} + 1/2x_5(\omega^{2,7} + 2\omega^{3,7} - 2\omega^{4,5}) - 1/2x_6(\omega^{3,7} -$

$2\omega^{4,6})\rangle + 2e^{-\beta_1-\beta_3}\ \langle 2/3x_1\omega^{1,6} - 1/3x_2(\omega^{2,6} + \omega^{3,5} + 2\omega^{3,6}) - 2/3x_3\omega^{3,6} - 1/3x_4(\omega^{3,7} + 2\omega^{4,6}) + 2/3x_5(\omega^{1,7} + 3/2\omega^{5,6}), -2/3x_1(\omega^{1,5} + 10/9\omega^{1,6}) - 4/9x_2(\omega^{1,4} - 3/2\omega^{2,5} - 1/3\omega^{2,6} - 4/3\omega^{3,5} + 4/3\omega^{3,6}) + 1/3x_3(\omega^{2,6} + \omega^{3,5} + 2/9\omega^{3,6}) + 1/3x_4(\omega^{2,7} + 1/9\omega^{3,7} + 2\omega^{4,5} + 14/9\omega^{4,6}) + 16/27x_5\omega^{1,7} + 2/3x_6(\omega^{1,7} + 3/2\omega^{5,6})\rangle + 5e^{-\beta_1}\ \langle 2/3x_2\omega^{1,2} - 2x_3\omega^{1,2} + x_4(\omega^{2,5} - 2/3\omega^{2,6}), 2/3x_2\omega^{1,2} + x_4(\omega^{1,4} + 1/3\omega^{2,6}), 2x_2\omega^{1,3} + x_4\omega^{3,6}, 2x_2(\omega^{1,2} + \omega^{1,3}) - 2x_3\omega^{1,3} + x_4(\omega^{2,6} + \omega^{3,5}), 2/3x_2(\omega^{1,2} + 3/2\omega^{1,3}) - x_3\omega^{1,3} + 1/3x_4\omega^{2,6} + x_5\omega^{1,5}\rangle + 8e^{-\beta_3}\ \langle x_1(\omega^{1,2} + \omega^{1,3}) - x_2\omega^{2,3} + x_3\omega^{2,3} + x_4(\omega^{2,4} + \omega^{3,4}) + x_6\omega^{2,5}, -x_1(\omega^{1,2} + \omega^{1,3}) + x_2\omega^{2,3} - x_3\omega^{2,3} - x_4(\omega^{2,4} + \omega^{3,4}) + x_5\omega^{2,5}, -x_1(\omega^{1,2} - \omega^{1,3}) - x_2\omega^{2,3} - x_4(\omega^{2,4} - \omega^{3,4}) + x_6\omega^{2,6}, -x_1\omega^{1,3} - x_4\omega^{3,4} + x_5\omega^{3,5}, x_2\omega^{2,3} + x_5\omega^{2,6}, x_5\omega^{3,6}, x_1\omega^{1,2} + 3/2x_4(\omega^{2,4} + 1/3\omega^{3,4}) + x_6\omega^{1,4}, -x_1\omega^{1,3} - 3/2x_4\omega^{3,4} + x_5\omega^{1,4}\rangle + e^{0}\ \langle -1/2x_4\omega^{2,3} + x_6\omega^{1,2}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-4\beta_3} + 9e^{-\beta_1-2\beta_3} + 6e^{-\beta_1-3\beta_3} + 8e^{-2\beta_1-\beta_3} + 4e^{-2\beta_1-2\beta_3} + 2e^{-3\beta_1-\beta_3} + e^{-3\beta_1} + 8e^{-3\beta_3} + 3e^{-4\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=7e^{-\beta_1-4\beta_3} + 6e^{-\beta_1-5\beta_3} + 2e^{-2\beta_1-2\beta_3} + 8e^{-2\beta_1-3\beta_3} + 6e^{-2\beta_1-4\beta_3} + 4e^{-3\beta_1-2\beta_3} + 6e^{-3\beta_1-3\beta_3} + e^{-4\beta_1-2\beta_3} + e^{-4\beta_3} + 2e^{-5\beta_3} + e^{-6\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-\beta_1-6\beta_3} + 2e^{-\beta_1-7\beta_3} + 4e^{-2\beta_1-5\beta_3} + 6e^{-2\beta_1-6\beta_3} + 6e^{-3\beta_1-4\beta_3} + 6e^{-3\beta_1-5\beta_3} + 4e^{-4\beta_1-4\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-2\beta_1-7\beta_3} + e^{-2\beta_1-8\beta_3} + 4e^{-3\beta_1-6\beta_3} + 2e^{-3\beta_1-7\beta_3} + 2e^{-4\beta_1-5\beta_3} + 3e^{-4\beta_1-6\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-3\beta_1-8\beta_3} + 2e^{-4\beta_1-7\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 15, 59, 130, 159, 116, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 9, 25, 42, 44, 30, 14, 3)$

S=$4(\beta_1 + 2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 19, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 9, 9, 7, 3, 1)$

$d\omega^4 = -\omega^{2,3}$; $d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -2\omega^{1,4} - \omega^{2,6} + \omega^{3,5} - \omega^{3,6}$; $d\omega^{1,4} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,6} - \omega^{1,3,5} + \omega^{1,3,6}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -2\omega^{1,2,4} - \omega^{2,3,5} + \omega^{2,3,6}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -2\omega^{1,3,4} - \omega^{2,3,6}$; $d\omega^{4,5} = \omega^{1,2,4} - \omega^{2,3,5}$; $d\omega^{4,6} = \omega^{1,3,4} - \omega^{2,3,6}$; $d\omega^{4,7} = -\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5} - \omega^{3,4,6}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + 2\omega^{1,4,5} - \omega^{2,5,6} - \omega^{3,5,6}$; $d\omega^{6,7} = -\omega^{1,3,7} + 2\omega^{1,4,6} - \omega^{3,5,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,5} - \omega^{1,2,3,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,6}$; $d\omega^{1,4,5} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = \omega^{1,2,3,6}$; $d\omega^{1,4,7} = \omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5} + \omega^{1,3,4,6}$; $d\omega^{1,5,7} = \omega^{1,2,5,6} + \omega^{1,3,5,6}$; $d\omega^{1,6,7} = \omega^{1,3,5,6}$; $d\omega^{2,3,7} = -2\omega^{1,2,3,4}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{2,3,4,5} + \omega^{2,3,4,6}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$;

$d\omega^{2,5,7} = 2\omega^{1,2,4,5} + \omega^{2,3,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,6} + \omega^{2,3,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{2,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} + 2\omega^{1,3,4,5} - \omega^{2,3,5,6}$; $d\omega^{3,6,7} = 2\omega^{1,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5} - \omega^{2,3,5,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{2,3,5,7} - \omega^{2,4,5,6} - \omega^{3,4,5,6}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} - \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - 2\omega^{1,4,5,6}$; $d\omega^{1,2,4,7} = \omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} + \omega^{1,3,4,5,6}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,5,7} = 2\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = 2\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,7} = \omega^{2,3,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} - 2\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - 2\omega^{1,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} - \omega^{2,3,5,6,7}$; $d\omega^{1,2,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,5,6,7}$; $d\omega^{2,3,5,6,7} = -2\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-\beta_2}\ \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}, \omega^{1,6}\rangle + 2e^{-3\beta_2}\ \langle\omega^{2,4}, \omega^{3,4}\rangle + 3e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}, \omega^{1,4} - \omega^{3,5}, \omega^{2,6} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 2e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}, \omega^{1,2,6}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,5,6}\rangle + e^{-4\beta_2}\ \langle\omega^{2,3,4}\rangle + 3e^{-\beta_1-4\beta_2}\ \langle\omega^{2,4,5}, \omega^{2,3,7} - 2\omega^{3,4,5}, \omega^{2,4,6} + \omega^{3,4,5}\rangle + 2e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7} - \omega^{2,5,6} + \omega^{3,5,6}, \omega^{1,3,7} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 3e^{-3\beta_1-4\beta_2}\ \langle\omega^{1,2,5,7} + \omega^{1,4,5,6}, \omega^{1,2,6,7} + \omega^{1,4,5,6}, \omega^{1,3,6,7}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,5,6,7}\rangle + e^{-\beta_1-6\beta_2}\ \langle\omega^{2,3,4,7}\rangle + 2e^{-2\beta_1-5\beta_2}\ \langle\omega^{1,2,4,7} + \omega^{2,4,5,6} - \omega^{3,4,5,6}, \omega^{1,3,4,7} + \omega^{3,4,5,6}\rangle + 2e^{-2\beta_1-6\beta_2}\ \langle\omega^{2,4,6,7} + \omega^{3,4,5,7}, \omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 3e^{-3\beta_1-6\beta_2}\ \langle(2\omega^{1,2,4,5,7} - \omega^{2,3,5,6,7})/2, (2\omega^{1,2,4,6,7} - \omega^{2,3,5,6,7})/2, \omega^{1,3,4,6,7}\rangle + 2e^{-4\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7}, \omega^{1,3,5,6,7}\rangle + 2e^{-2\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}, \omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = 2e^{-4\beta_1-7\beta_2}\ \langle\omega^{1,2,4,5,6,7}, \omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-4\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 19, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 9, 9, 7, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,4} = -2\omega^7$; $d^\star\omega^{2,3} = -\omega^4$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{3,5} = \omega^7$; $d^\star\omega^{3,6} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,4} - \omega^{2,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = -2\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{1,2,6} = \omega^{1,7}-\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = -2\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{1,3,5} = -\omega^{1,7} + \omega^{5,6}$; $d^\star\omega^{1,3,6} = \omega^{1,7}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,5} = 2\omega^{5,7}$; $d^\star\omega^{1,4,6} = 2\omega^{6,7}$; $d^\star\omega^{2,3,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{2,3,6} = \omega^{2,7} - \omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{2,3,7} = -\omega^{4,7}$; $d^\star\omega^{2,4,6} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = \omega^{4,7}$; $d^\star\omega^{3,4,6} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = -\omega^{5,7}-\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -2\omega^{2,3,7}+\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,2,7} + \omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,4,7} - \omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = 2\omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{1,4,7} + 2\omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,4,7} + 2\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,4,7} + 2\omega^{3,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,5,7} + \omega^{1,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -2\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,4,7}$; $d^\star\omega^{2,3,4,6} = \omega^{2,4,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,5,7} - \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,4,7} + 2\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,4,7} + \omega^{1,3,4,7} + 2\omega^{2,3,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,5,7} - \omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,4,5,7}+\omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,4,6,7}+\omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,4,5,7}-2\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,5,7} + \omega^{1,4,6,7} - 2\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{2,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,5,7} - \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} - 2\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{1,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 7\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = 5\omega^{1,4}+\omega^{2,6}-\omega^{3,5}+2\omega^{3,6}$; $\Delta\omega^{1,7} = 3\omega^{1,7}-2\omega^{5,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{1,4} + 2\omega^{2,6} - 2\omega^{3,5} + \omega^{3,6}$; $\Delta\omega^{2,7} = 6\omega^{2,7} - \omega^{3,7} - \omega^{4,5} - \omega^{4,6}$; $\Delta\omega^{3,5} = -\omega^{1,4}-2\omega^{2,6}+2\omega^{3,5}-\omega^{3,6}$; $\Delta\omega^{3,6} = 2\omega^{1,4}+\omega^{2,6}-\omega^{3,5}+\omega^{3,6}$; $\Delta\omega^{3,7} = -\omega^{2,7} + 5\omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{2,7} - \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 4\omega^{4,7}$; $\Delta\omega^{5,6} = -2\omega^{1,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 7\omega^{5,7} + \omega^{6,7}$; $\Delta\omega^{6,7} = \omega^{5,7} + 6\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 5\omega^{1,2,4} + \omega^{2,3,5} - 2\omega^{2,3,6}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - 2\omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,2,7} = 3\omega^{1,2,7} - \omega^{1,3,7} - \omega^{1,4,5} - \omega^{1,4,6} + 2\omega^{2,5,6}$; $\Delta\omega^{1,3,4} = 5\omega^{1,3,4} + \omega^{2,3,6}$; $\Delta\omega^{1,3,5} = -2\omega^{1,2,6} + 2\omega^{1,3,5} - \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = \omega^{1,2,6} - \omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = -\omega^{1,2,7} + 2\omega^{1,3,7} - \omega^{1,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + 5\omega^{1,4,5} - \omega^{2,5,6} - 2\omega^{3,5,6}$; $\Delta\omega^{1,4,6} = -\omega^{1,2,7} - \omega^{1,3,7} + 5\omega^{1,4,6} - \omega^{3,5,6}$; $\Delta\omega^{1,4,7} = 4\omega^{1,4,7} + \omega^{2,6,7} - \omega^{3,5,7} + 2\omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{1,6,7}$; $\Delta\omega^{1,6,7} = \omega^{1,5,7} + \omega^{1,6,7}$; $\Delta\omega^{2,3,5} = \omega^{1,2,4} + 2\omega^{2,3,5} - \omega^{2,3,6}$; $\Delta\omega^{2,3,6} = -2\omega^{1,2,4} + \omega^{1,3,4} - \omega^{2,3,5} + 3\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 5\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5} + \omega^{3,4,6}$;

$\Delta\omega^{2,4,6} = -\omega^{2,3,7} + 2\omega^{2,4,6} - 2\omega^{3,4,5} + \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} - \omega^{3,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{1,2,7} - \omega^{1,4,5} + 2\omega^{2,5,6} + \omega^{3,5,6}$; $\Delta\omega^{2,5,7} = 5\omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{1,4,7} + \omega^{2,5,7} + 6\omega^{2,6,7} - 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{2,3,7} - 2\omega^{2,4,6} + 2\omega^{3,4,5} - \omega^{3,4,6}$; $\Delta\omega^{3,4,6} = \omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,4,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{1,3,7} - 2\omega^{1,4,5} - \omega^{1,4,6} + \omega^{2,5,6} + 3\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{1,4,7} - \omega^{2,5,7} - 2\omega^{2,6,7} + 6\omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{1,4,7} + 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{1,4,7} - \omega^{2,5,7} + \omega^{2,6,7} - \omega^{3,5,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 4\omega^{4,5,7} + \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = \omega^{4,5,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 6\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 6\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = -\omega^{1,2,3,5} + 4\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{1,3,4,5} + \omega^{1,3,4,6} - 2\omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5} + 2\omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + 6\omega^{1,2,4,6} - 2\omega^{1,3,4,5} + \omega^{1,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7} = 3\omega^{1,2,4,7} - \omega^{1,3,4,7} + \omega^{2,3,5,7} - 2\omega^{2,3,6,7} - 2\omega^{2,4,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,5,7} + 2\omega^{1,2,6,7} - 2\omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7} - 2\omega^{1,2,4,6} + 6\omega^{1,3,4,5} - \omega^{1,3,4,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5} + 5\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,4,7} + 2\omega^{1,3,4,7} + \omega^{2,3,6,7} - 2\omega^{3,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,6} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,5,7} - 2\omega^{1,2,6,7} + 2\omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,5,7} + \omega^{1,2,6,7} - \omega^{1,3,5,7} + 5\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 3\omega^{1,4,5,7} + \omega^{1,4,6,7} - \omega^{2,5,6,7} - 2\omega^{3,5,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,5,7} + 2\omega^{1,4,6,7} - \omega^{3,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,6} = -\omega^{2,3,4,5} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = -2\omega^{1,2,3,7} + 2\omega^{1,2,4,5} + \omega^{1,2,4,6} - \omega^{1,3,4,5} + 4\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,2,4,7} + 5\omega^{2,3,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = -2\omega^{1,2,4,7} + \omega^{1,3,4,7} + 5\omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -2\omega^{1,2,4,7} - \omega^{2,3,5,7} + 2\omega^{2,4,5,6} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,5,7} + 2\omega^{2,4,6,7} - 2\omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = -\omega^{1,4,5,7} + 5\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -2\omega^{1,3,4,7} + \omega^{2,3,5,7} - \omega^{2,3,6,7} + \omega^{2,4,5,6} + 3\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,5,7} - 2\omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = -2\omega^{1,4,5,7} - \omega^{1,4,6,7} + 5\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 6\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = -\omega^{1,2,3,4,5} + 7\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 4\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + 5\omega^{1,2,4,5,6} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,5,7} + 2\omega^{1,2,4,6,7} - 2\omega^{1,3,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,5,7} - \omega^{1,2,3,6,7} + \omega^{1,2,4,5,6} + 6\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,5,7} - 2\omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{1,2,3,4,7} + 3\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,5,6,7} = 2\omega^{1,2,4,5,7} + \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 5\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 7\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + 2e^{-\beta_2}\ \langle\omega^2, \omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= 2e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}, \omega^{1,6}\rangle + 2e^{-3\beta_2}\ \langle\omega^{2,4}, \omega^{3,4}\rangle + 3e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}, \omega^{2,6} +$

$\omega^{3,5}, \omega^{1,4} - \omega^{3,5} - 3\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{O}})= 2e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}, \omega^{1,2,6} + \omega^{1,3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,5,6}\rangle + e^{-4\beta_2} \langle\omega^{2,3,4}\rangle + 3e^{-\beta_1-4\beta_2} \langle\omega^{2,4,5}, \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} - 2\omega^{3,4,5} - 3\omega^{3,4,6}\rangle + 2e^{-2\beta_1-3\beta_2} \langle 9\omega^{1,2,7} + 2\omega^{1,4,5} + 3\omega^{1,4,6} - 11\omega^{2,5,6} + 6\omega^{3,5,6}, 3\omega^{1,3,7} - \omega^{1,4,5} + \omega^{2,5,6} - 3\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{O}})= 2e^{-2\beta_1-6\beta_2} \langle\omega^{3,4,6,7}, (-\omega^{2,4,6,7} - \omega^{3,4,5,7})/2\rangle + e^{-\beta_1-6\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-4\beta_1-4\beta_2} \langle-\omega^{1,5,6,7}\rangle + 3e^{-3\beta_1-4\beta_2} \langle-\omega^{1,3,6,7}, (3\omega^{1,2,5,7} + 5\omega^{1,2,6,7} + 7\omega^{1,3,5,7} + \omega^{1,4,5,6})/12, (3\omega^{1,2,5,7} - \omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{1,4,5,6})/12\rangle + 2e^{-2\beta_1-5\beta_2} \langle(9\omega^{1,2,4,7} + 63\omega^{1,3,4,7} - 20\omega^{2,3,5,7} + 3\omega^{2,3,6,7} - 31\omega^{2,4,5,6} + 60\omega^{3,4,5,6})/1353, (-189\omega^{1,2,4,7} + 30\omega^{1,3,4,7} - 31\omega^{2,3,5,7} - 63\omega^{2,3,6,7} - 251\omega^{2,4,5,6} + 93\omega^{3,4,5,6})/1353\rangle$.

$\mathrm{ch}(H^5_{\mathcal{O}})= 2e^{-2\beta_1-7\beta_2} \langle-\omega^{2,3,4,6,7}, \omega^{2,3,4,5,7}\rangle + 2e^{-4\beta_1-5\beta_2} \langle-\omega^{1,3,5,6,7}, \omega^{1,2,5,6,7}\rangle + 3e^{-3\beta_1-6\beta_2} \langle\omega^{1,3,4,6,7}, (-3\omega^{1,2,4,5,7} - 10\omega^{1,2,4,6,7} - 11\omega^{1,3,4,5,7} + \omega^{2,3,5,6,7})/21, (-6\omega^{1,2,4,5,7} + \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,5,6,7})/21\rangle$.

$\mathrm{ch}(H^6_{\mathcal{O}})= e^{-3\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + 2e^{-4\beta_1-7\beta_2} \langle-\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{O}})= e^{-4\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.4.7 Algebra $\mathcal{G}_{7,2.3}$ (7-dim generic filiform Lie algebra) †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 3\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 4\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_2^6 & \xi_2^5 & \xi_2^4 & \xi_2^3 & 5\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1+\beta_2}\ \langle x_5\rangle + e^{4\beta_1+\beta_2}\ \langle x_6\rangle + e^{5\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{5\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{4\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{5\beta_1}\ \langle\delta_2^7\rangle + e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_2^6] = -\delta_1^7$; $[\delta_1^3, \delta_2^5] = -\delta_1^6$; $[\delta_1^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^3, \delta_2^3] = -\delta_1^4$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^5, \delta_2^4] = -\delta_1^7$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{5\beta_1}\ \langle\delta_2^7\rangle + e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-6\beta_1}\ \langle x_2\omega^{1,7}\rangle + e^{-5\beta_1}\ \langle x_2\omega^{1,6}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3}\rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} + 4x_3\omega^{2,3} + 3x_4\omega^{2,4} + 2x_5\omega^{2,5} + x_6\omega^{2,6}, -x_1\omega^{1,2} - 3x_3\omega^{2,3} - 2x_4\omega^{2,4} - x_5\omega^{2,5} + x_7\omega^{2,7}, 2x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_5(\omega^{2,5} + \omega^{3,4}) + x_6\omega^{3,5} + x_7\omega^{4,5}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} + 5x_2\omega^{2,3} + 6x_3\omega^{2,4} + 3x_4(\omega^{2,5} + \omega^{3,4}) + x_5(\omega^{2,6} + 2\omega^{3,5}) + x_6\omega^{3,6}, -x_1\omega^{1,3} - x_2\omega^{2,3} - 2x_3\omega^{2,4} - 2x_4\omega^{3,4} + x_5(\omega^{2,6} - \omega^{3,5}) + x_6\omega^{2,7} + x_7\omega^{3,7}\rangle + e^{-\beta_1}\ \langle x_2\omega^{1,2}\rangle + e^{4\beta_1-\beta_2}\ \langle x_7\omega^{2,3}\rangle + e^{3\beta_1-\beta_2}\ \langle x_6\omega^{2,3} + x_7\omega^{2,4}\rangle + 2e^{2\beta_1-\beta_2}\ \langle x_5\omega^{2,3} + x_6\omega^{2,4} + x_7\omega^{2,5}, x_5\omega^{2,3} + x_6\omega^{2,4} + x_7\omega^{3,4}\rangle + e^{\beta_1-\beta_2}\ \langle 2x_4\omega^{2,3} +$

$2x_5\omega^{2,4} + x_6(\omega^{2,5} + \omega^{3,4}) + x_7\omega^{3,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle -9/5x_1\omega^{1,4} - 6/5x_3(\omega^{2,5} + 1/2\omega^{3,4}) + 6/5x_4(\omega^{2,6} - 2\omega^{3,5}) + x_5(\omega^{2,7} + 1/5\omega^{3,6} - 13/5\omega^{4,5}) + x_6(\omega^{3,7} - 4/5\omega^{4,6}) + x_7\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{2\beta_1-2\beta_2}+e^{\beta_1-2\beta_2}+3e^{-\beta_1-2\beta_2}+2e^{-2\beta_1-2\beta_2}+3e^{-3\beta_1-2\beta_2}+e^{-4\beta_1-\beta_2}+e^{-4\beta_1-2\beta_2}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2}+3e^{-6\beta_1-\beta_2}+2e^{-7\beta_1-\beta_2}+2e^{-8\beta_1-\beta_2}+e^{-9\beta_1-\beta_2}+e^{-10\beta_1-\beta_2}+e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-3\beta_2}+2e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-3\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-7\beta_1-3\beta_2}+3e^{-8\beta_1-2\beta_2}+2e^{-9\beta_1-2\beta_2}+3e^{-10\beta_1-2\beta_2}+2e^{-11\beta_1-2\beta_2}+e^{-12\beta_1-2\beta_2}+e^{-13\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-6\beta_1-4\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-3\beta_2}+2e^{-10\beta_1-4\beta_2}+3e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-3\beta_2}+2e^{-13\beta_1-3\beta_2}+e^{-14\beta_1-3\beta_2}+e^{-15\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-10\beta_1-5\beta_2}+e^{-11\beta_1-4\beta_2}+e^{-13\beta_1-4\beta_2}+e^{-14\beta_1-6\beta_2}+e^{-15\beta_1-4\beta_2}+e^{-15\beta_1-5\beta_2}+e^{-16\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-15\beta_1-6\beta_2}+e^{-16\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 53, 119, 149, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 17, 25, 23, 14, 7, 2)$

S$=2(8\beta_1 + 3\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 9, 18, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 4, 6, 6, 4, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6}$; $d\omega^{5,6} = -\omega^{1,4,6}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6}$; $d\omega^{6,7} = -\omega^{1,5,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} -$

$\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-6\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,5} - \omega^{3,4}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{3,6} + \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,4,7}\rangle + e^{-10\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-10\beta_1-3\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-11\beta_1-3\beta_2} \langle\omega^{1,3,6,7}\rangle + e^{-13\beta_1-3\beta_2} \langle\omega^{1,5,6,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,3,4,5}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,4,7} - \omega^{2,3,5,6}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-11\beta_1-4\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-13\beta_1-4\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-15\beta_1-4\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-16\beta_1-5\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-15\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-16\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 9, 18, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 4, 6, 6, 4, 2, 1)$

Total dimension of trivial cohomology: 26.

Length associated polynomial: 26.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} - \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6} - \omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$;

$d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = \omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + 2\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{1,5,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + \omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 3\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 3\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 2\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} =$

$\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-6\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,5} - \omega^{3,4}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{3,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{1,3,6} + \omega^{1,4,5}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{1,5,6}\rangle + e^{-10\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-10\beta_1-4\beta_2} \langle(\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6})/3\rangle + e^{-8\beta_1-4\beta_2} \langle(-\omega^{2,3,4,7} + \omega^{2,3,5,6})/2\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,3,4,5}\rangle + e^{-13\beta_1-3\beta_2} \langle-\omega^{1,5,6,7}\rangle + e^{-11\beta_1-3\beta_2} \langle(\omega^{1,3,6,7} - \omega^{1,4,5,7})/2\rangle + e^{-10\beta_1-3\beta_2} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-10\beta_1-5\beta_2} \langle-\omega^{2,3,4,5,6}\rangle + e^{-15\beta_1-4\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-13\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-11\beta_1-4\beta_2} \langle(\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-15\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-16\beta_1-5\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-16\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + 3Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

5.4.8 Algebra $\mathcal{G}_{7,2.4}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_5] = -x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_2^2+\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_2^3 & \xi_2^2+2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^3 & \xi_2^2+3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^5 & 0 & \xi_2^3 & \xi_2^2+4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_1^5 & -\xi_1^4 & \xi_1^3 & -\xi_1^2 & 2\xi_2^2+3\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -\delta_1^4, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6 + \delta_2^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1+\beta_2}\ \langle x_5\rangle + e^{4\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{3\beta_1+\beta_2}\ \langle \delta_1^6, \delta_2^7\rangle + e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^7\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^6, \delta_1^2] = \delta_1^7$; $[\delta_2^6, \delta_1^2] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^3] = -\delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^3, \delta_1^5] = -\delta_2^7 + \delta_1^6$; $[\delta_2^3, \delta_1^4] = \delta_1^5$; $[\delta_2^3, \delta_1^3] = \delta_1^4$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^7, \delta_1^2] = \delta_1^7$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1}\ \langle\delta_2^6\rangle + e^{3\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^7\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6, \delta_1^2] = 2\delta_1^6$; $[\delta_1^6, \delta_1^2] = \delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} + 2x_2\omega^{2,3} + 3x_3\omega^{2,4} + x_4(\omega^{2,5} + 2\omega^{3,4}) + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4(\omega^{2,5} - \omega^{3,4}) + x_5\omega^{2,6} + x_6\omega^{3,6}\rangle + e^{3\beta_1-\beta_2}\ \langle x_6\omega^{2,3}\rangle + e^{2\beta_1-\beta_2}\ \langle x_5\omega^{2,3} + x_6\omega^{2,4}\rangle + 2e^{-\beta_2}\ \langle -2/5x_1\omega^{1,2} - 1/5x_3\omega^{2,3} + 1/5x_4\omega^{2,4} + 3/5x_5\omega^{2,5} + x_6\omega^{2,6}, 1/5x_1\omega^{1,2} + 3/5x_3\omega^{2,3} + 2/5x_4\omega^{2,4} + 6/5x_5(\omega^{2,5} - 5/6\omega^{3,4}) + x_6\omega^{1,7}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,3} + x_5\omega^{2,7} + x_6\omega^{3,7}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_7\omega^{1,6}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_3\omega^{1,3} + x_7\omega^{1,7}, x_6\omega^{1,6}\rangle + e^{-2\beta_1}\ \langle x_3\omega^{1,4}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,5}\rangle$.

ch($H^3_{\mathcal{G}}$)=$2e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2} + 2e^{-5\beta_1-\beta_2} + 2e^{-5\beta_1-2\beta_2} + 2e^{-6\beta_1-\beta_2} + e^{-8\beta_1-\beta_2} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-2\beta_1-4\beta_2} + 2e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-3\beta_2} + 2e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-2\beta_2}+2e^{-9\beta_1-3\beta_2}+e^{-10\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-6\beta_1-5\beta_2} + e^{-8\beta_1-5\beta_2} + e^{-8\beta_1-6\beta_2} + 2e^{-9\beta_1-4\beta_2} + 2e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-3\beta_2} + 2e^{-10\beta_1-4\beta_2} + e^{-12\beta_1-4\beta_2} + e^{-13\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-9\beta_1-6\beta_2} + 2e^{-10\beta_1-5\beta_2} + e^{-12\beta_1-7\beta_2} + e^{-13\beta_1-5\beta_2} + e^{-13\beta_1-6\beta_2} + e^{-14\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-13\beta_1-7\beta_2} + e^{-14\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 49, 115, 147, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 7, 12, 17, 17, 12, 7, 2)$

S=$7(2\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,2,7} + \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} + \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6}$; $d\omega^{5,7} = -\omega^{1,4,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = -\omega^{1,2,3,5}$; $d\omega^{1,4,7} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,4,7}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,7}-2\omega^{2,6}+\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,5,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,3,7}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,3,6}-\omega^{2,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,3,4,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,3,6,7}+\omega^{2,4,5,6}\rangle + e^{-8\beta_1-5\beta_2}\ \langle\omega^{2,3,6,7}-\omega^{2,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-13\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-14\beta_1-6\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-13\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-14\beta_1-7\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,5} = \omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{1,7}-\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = \omega^{3,7}$; $d^\star\omega^{2,4,5} = \omega^{4,7}$; $d^\star\omega^{2,5,6} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,3,7}-\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,4,7}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7}-\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,3,6,7}-\omega^{2,4,5,6}$;

$d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,4,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{1,7} + \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,3,7} + \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{1,5,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$;

$\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,7} - 2\omega^{2,6} + \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-6\beta_1-2\beta_2} \langle\omega^{1,3,6} - \omega^{1,4,5}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,5,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,3,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,3,7} - 2\omega^{2,3,6} + \omega^{2,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-8\beta_1-5\beta_2} \langle(-\omega^{2,3,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-9\beta_1-4\beta_2} \langle(-\omega^{1,3,6,7} + 2\omega^{1,4,5,7} - \omega^{2,4,5,6})/6\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-9\beta_1-6\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-13\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2} \langle(-\omega^{1,2,4,6,7} + 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/6\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-13\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-14\beta_1-6\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-14\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.4.9 Algebra $\mathcal{G}_{7,2.5}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_4] = x_6$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & \xi_1^1+2\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 2(\xi_1^1+\xi_2^2) & 0 \\ \xi_1^7 & \xi_2^7 & \xi_2^6 & -2\xi_1^4+\xi_2^5 & -\xi_2^4 & \xi_2^3 & 3\xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_2^6, ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : T=$\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+2\beta_2}\ \langle x_5\rangle + e^{2\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1+2\beta_2}\ \langle\delta_1^6\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^7\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_2^7\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_2^6\rangle + 2e^{\beta_1+\beta_2}\ \langle\delta_1^4, \delta_2^5\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5, \delta_2^4] = \delta_1^7$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_1^3, \delta_2^6] = -\delta_1^7$; $[\delta_1^3, \delta_2^5] = -\delta_1^6$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -2\delta_2^7$; $[\delta_1^4, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_2^4] = 2\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle\delta_1^5\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_2^7\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4, \delta_2^4] = -2\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle + e^{-3\beta_1}\ \langle -x_2\omega^{1,4} + x_5\omega^{1,7}\rangle + e^{-2\beta_1-2\beta_2}\ \langle -x_1(\omega^{3,4} + \omega^{1,6}) - x_2(\omega^{3,5} - \omega^{2,6}) - x_3\omega^{4,5} - x_4\omega^{4,6} + x_5\omega^{5,6}\rangle + e^{-2\beta_1}\ \langle x_3\omega^{1,4} + x_5\omega^{1,6}\rangle + e^{\beta_2}\ \langle x_5\omega^{1,2}\rangle + e^{-\beta_1}\ \langle 1/2x_2\omega^{1,2} + 1/2x_3\omega^{1,3} - 1/2x_4\omega^{1,4} + x_5\omega^{1,5}\rangle + e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,5}\rangle + 2e^{-\beta_2}\ \langle -x_1\omega^{1,2} + x_4\omega^{2,4} - x_5\omega^{2,5} + x_7\omega^{2,7}, -1/2x_3\omega^{2,3} + x_4(\omega^{1,5} + 1/2\omega^{2,4}) + 2x_5\omega^{2,5} + x_6\omega^{3,5}\rangle + e^{\beta_1-\beta_2}\ \langle x_4\omega^{2,3}\rangle + e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,3} + x_3\omega^{2,4} + x_4\omega^{3,4}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,3} + x_4(\omega^{3,5} - \omega^{2,6})\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2} + 4e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-4\beta_2} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-3\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-2\beta_1-4\beta_2} + 2e^{-3\beta_1-5\beta_2} + e^{-3\beta_1-6\beta_2} + 2e^{-4\beta_1-4\beta_2} + e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-6\beta_2} + 3e^{-5\beta_1-3\beta_2} + 2e^{-5\beta_1-5\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2} + 2e^{-7\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_1-6\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-6\beta_2} + e^{-5\beta_1-7\beta_2} + e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2}+e^{-6\beta_1-8\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-7\beta_1-7\beta_2}+2e^{-8\beta_1-5\beta_2}+e^{-8\beta_1-6\beta_2}+e^{-9\beta_1-5\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-8\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-8\beta_1-7\beta_2} + e^{-8\beta_1-9\beta_2} + e^{-9\beta_1-6\beta_2} + e^{-9\beta_1-8\beta_2} + e^{-10\beta_1-7\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-9\beta_1-9\beta_2} + e^{-10\beta_1-8\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 12, 49, 114, 149, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 6, 12, 16, 18, 14, 7, 2)$

S=$10\beta_1 + 9\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 19, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 6, 6, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{1,4,6}$; $d\omega^{5,6} = -\omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,7}$; $d\omega^{5,6,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{I}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{I}})= e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,6}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{I}})= e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,3,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-3\beta_1-5\beta_2}\ \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,5,6}+\omega^{2,4,6}-2\omega^{3,4,5}\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{2,6,7}-\omega^{3,5,7}+\omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{I}})= e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,2,3,7}\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,3,4,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7}\rangle + e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-6\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-5\beta_1-6\beta_2}\ \langle\omega^{2,3,5,7}-\omega^{2,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{I}})= e^{-8\beta_1-6\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-6\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-8\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{I}})= e^{-10\beta_1-8\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-9\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{I}})= e^{-10\beta_1-9\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1-Y+3Y^2-Y^3+Y^4)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{I}}$'s $0\le j\le 7$: $(1,2,8,19,22,16,7,1)$

Dimensions of $H^j_{\mathcal{I}}$'s $0\le j\le 7$: $(1,2,3,6,6,3,2,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}-\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}+\omega^{1,3,6}-\omega^{1,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}-\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$;

$d^{\star}\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = 3\omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,6} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 3\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = 3\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,3,7} + 2\omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = 3\omega^{2,4,7}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,3,7} + \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,6,7} + 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 2\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = 3\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 3\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} =$

$2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 3\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,6} - \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-5\beta_1-3\beta_2} \langle\omega^{1,3,7} - \omega^{1,4,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,5}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{2,6,7} - \omega^{3,5,7} + \omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-5\beta_1-6\beta_2} \langle(\omega^{2,3,5,7} - \omega^{2,4,5,6})/2\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{2,3,5,6}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle-\omega^{1,3,4,7}\rangle + e^{-6\beta_1-5\beta_2} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle + e^{-5\beta_1-4\beta_2} \langle(-\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5})/3\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-8\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-9\beta_1-6\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-6\beta_2} \langle(\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-9\beta_1-9\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-8\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-9\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) \ = \ (1 - Y + 3Y^2 - Y^3 + Y^4)(1 + Y)^3.$

5.4.10 Algebra $\mathcal{G}_{7,2.6}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_6$; $[x_2, x_3] = x_5$; $[x_2, x_6] = x_7$; $[x_3, x_4] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_2^2+\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & \xi_2^2+2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & 2\xi_2^2+\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & \xi_2^2+3\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^6 & \xi_1^4 & 2\xi_2^4 & -\xi_1^3 & 2\xi_2^2+3\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_1^6, ad\ x_5 = 0, ad\ x_6 = -\delta_2^7, ad\ x_7 = 0.$

Maximal torus : T=$\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+2\beta_2}\ \langle x_5\rangle + e^{3\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+2\beta_2}\ \langle x_5\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle \delta_1^5\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_2^5, \delta_1^4\rangle + e^{3\beta_1}\ \langle \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_2^3\rangle + e^{2\beta_1+2\beta_2}\ \langle \delta_1^7\rangle + e^{3\beta_1+\beta_2}\ \langle \delta_2^7\rangle + e^{2\beta_1+\beta_2}\ \langle \delta_1^6\rangle + e^{2\beta_1}\ \langle \delta_2^4\rangle + e^{\beta_2}\ \langle \delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5, \delta_2^4] = -2\delta_1^7$; $[\delta_2^5, \delta_2^4] = -2\delta_2^7$; $[\delta_2^6, \delta_1^3] = \delta_2^7$; $[\delta_2^3, \delta_1^6] = \delta_2^7$; $[\delta_2^3, \delta_1^4] = \delta_1^6$; $[\delta_2^3, \delta_1^3] = \delta_2^5 + \delta_1^4$; $[\delta_1^4, \delta_2^4] = \delta_2^7$; $[\delta_2^4, \delta_1^3] = \delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle \delta_1^5\rangle + e^{3\beta_1}\ \langle \delta_2^6\rangle + e^{2\beta_1+2\beta_2}\ \langle \delta_1^7\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^4\rangle + e^{2\beta_1}\ \langle \delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^5, \delta_2^4] = -2\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $2e^{-\beta_1}\ \langle x_3\omega^{1,3} - x_4\omega^{1,4} + x_5\omega^{1,5}, x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_7\omega^{1,7}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_5\omega^{1,6}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,6} + x_5\omega^{4,6}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_6\omega^{2,5}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,3} + x_6\omega^{2,7}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3} + 2x_2\omega^{2,3} + x_3\omega^{2,4} + x_4\omega^{2,6} + x_6\omega^{3,6}\rangle + e^{\beta_1}\ \langle x_4\omega^{1,2} + x_7\omega^{1,5}\rangle + e^{2\beta_1-\beta_2}\ \langle x_6\omega^{2,3}\rangle + 2e^{-\beta_2}\ \langle -2/3x_1\omega^{1,2} - 1/3x_3\omega^{2,3} + 1/3x_4\omega^{2,4} + x_6\omega^{2,6}, x_5\omega^{2,5}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,3} + x_4\omega^{1,6}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{\beta_1-3\beta_2} + 2e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-2\beta_2} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-\beta_2} + 2e^{-5\beta_1-2\beta_2} + e^{-5\beta_1} + e^{-6\beta_1-\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-2\beta_1-4\beta_2} + e^{-2\beta_1-5\beta_2} + e^{-3\beta_1-5\beta_2} + 2e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-5\beta_1-5\beta_2} + 2e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-2\beta_2} + e^{-8\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-5\beta_1-6\beta_2} + 2e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-6\beta_1-7\beta_2} + e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-8\beta_1-4\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-9\beta_1-4\beta_2} + 2e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-7\beta_1-7\beta_2} + 2e^{-9\beta_1-6\beta_2} + e^{-9\beta_1-8\beta_2} + e^{-10\beta_1-5\beta_2} + e^{-10\beta_1-7\beta_2} + e^{-11\beta_1-6\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-10\beta_1-8\beta_2} + e^{-11\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 49, 115, 148, 110, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 7, 12, 17, 18, 13, 7, 2)$

S=$11\beta_1 + 8\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 8, 17, 22, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 3, 4, 4, 3, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{2,6} + \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{2,3,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{2,3,6}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{2,4,6}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = -\omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,6}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,6}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{2,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,5}+\omega^{2,4}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,5,6}+\omega^{2,4,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,3,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,6,7}-\omega^{1,4,5,6}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,4,6,7}\rangle + e^{-5\beta_1-6\beta_2}\ \langle\omega^{2,3,5,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,3,5,7}+\omega^{2,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-9\beta_1-6\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-11\beta_1-7\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-11\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,8,17,22,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,3,4,4,3,2,1)$

Total dimension of trivial cohomology: 20.

Length associated polynomial: 20.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,6}=-\omega^7$; $d^\star\omega^{3,4}=\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=-\omega^{3,5}$; $d^\star\omega^{1,2,6}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,6}=-\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,6}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=-\omega^{5,7}$; $d^\star\omega^{3,4,6}=-\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=\omega^{1,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=\omega^{1,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,5,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=\omega^{1,6,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}-\omega^{3,4,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,5,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,2,6,7}+\omega^{1,3,4,7}-\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,4,5,7}-\omega^{3,4,5,6}$;

$d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} - \omega^{4,5}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 3\omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = 3\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{2,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,6,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 3\omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + 3\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} =$

$\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,3,7} - 2\omega^{1,5,6} - \omega^{2,4,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle 2\omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-5\beta_1-6\beta_2}\ \langle -\omega^{2,3,5,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle(-\omega^{1,3,5,7} - 2\omega^{2,3,4,7} - \omega^{2,4,5,6})/6\rangle + e^{-7\beta_1-4\beta_2}\ \langle(-\omega^{1,2,6,7} + \omega^{1,3,4,7} + 2\omega^{1,4,5,6})/6\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-7\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-9\beta_1-6\beta_2}\ \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-10\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-11\beta_1-7\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-11\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

5.4.11 Algebra $\mathcal{G}_{7,2.7}$ †

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_4$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_1^2 & \xi_1^1+2\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_2^6 & \xi_2^4 & 0 & \xi_2^3 & 4\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3, ad\ x_2 = -\delta_1^3, ad\ x_3 = -(\delta_1^4 + \delta_2^5), ad\ x_4 = -\delta_1^6, ad\ x_5 = 0, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+2\beta_2}\ \langle x_5\rangle + e^{3\beta_1+\beta_2}\ \langle x_6\rangle + e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+2\beta_2}\ \langle x_5\rangle + e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = 2e^{\beta_1+\beta_2}\ \langle \delta_1^4, \delta_2^5\rangle + e^{2\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{4\beta_1}\ \langle\delta_2^7\rangle + e^{3\beta_1}\ \langle\delta_2^6\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^2] = -\delta_1^5$; $[\delta_1^4, \delta_2^4] = -\delta_1^7$; $[\delta_1^4, \delta_2^3] = -\delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_1^3, \delta_2^6] = -\delta_1^7$; $[\delta_1^3, \delta_2^4] = -\delta_1^6$; $[\delta_1^3, \delta_2^3] = -(\delta_2^5 + \delta_1^4)$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = \delta_2^5 - \delta_1^4$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{2\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{4\beta_1}\ \langle\delta_2^7\rangle + e^{3\beta_1}\ \langle\delta_2^6\rangle + e^{2\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^2] = -\delta_1^5$; $[\delta_1^2, \delta_2^4] = -2\delta_1^4$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_5\omega^{1,5}, x_4\omega^{1,4}\rangle + e^{-4\beta_1+\beta_2}\ \langle x_5\omega^{1,7}\rangle + 3e^{-\beta_2}\ \langle -x_3\omega^{2,3} + x_4\omega^{1,5}, x_1\omega^{1,2} + 3x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} - 2x_3\omega^{2,3} - x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + e^{-3\beta_1}\ \langle x_4\omega^{1,7}\rangle + 2e^{-2\beta_1}\ \langle x_2\omega^{1,3}, x_4\omega^{1,6}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,6} + x_5\omega^{4,6}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} + 2x_2\omega^{2,3} + 3x_3\omega^{2,4} + x_4(\omega^{2,6} + 2\omega^{3,4}) + x_6\omega^{3,6}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4(\omega^{2,6} - \omega^{3,4}) + x_6\omega^{2,7} + x_7\omega^{3,7}\rangle + e^{2\beta_1-\beta_2}\ \langle x_6\omega^{2,3} + x_7\omega^{2,4}\rangle + e^{3\beta_1-2\beta_2}\ \langle x_7\omega^{2,5}\rangle + e^{\beta_1-\beta_2}\ \langle x_4\omega^{2,3} + x_6\omega^{2,4} + x_7\omega^{3,4}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_6\omega^{2,5} + x_7\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{2\beta_1-3\beta_2}+e^{\beta_1-3\beta_2}+e^{-\beta_1-2\beta_2}+2e^{-\beta_1-3\beta_2}+2e^{-2\beta_1-2\beta_2}+2e^{-2\beta_1-3\beta_2}+4e^{-3\beta_1-2\beta_2}+2e^{-4\beta_1-\beta_2}+e^{-4\beta_1-2\beta_2}+3e^{-5\beta_1-\beta_2}+e^{-5\beta_1-2\beta_2}+2e^{-6\beta_1-\beta_2}+e^{-7\beta_1}+e^{-8\beta_1-\beta_2}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-2\beta_1-4\beta_2}+2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+2e^{-4\beta_1-4\beta_2}+3e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-2\beta_2}+3e^{-6\beta_1-3\beta_2}+e^{-7\beta_1-2\beta_2}+2e^{-7\beta_1-3\beta_2}+3e^{-8\beta_1-2\beta_2}+e^{-9\beta_1-\beta_2}+e^{-9\beta_1-2\beta_2}+e^{-4\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-4\beta_1-5\beta_2}+e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-6\beta_2}+2e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+2e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-3\beta_2}+2e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-7\beta_1-6\beta_2}+e^{-9\beta_1-5\beta_2}+2e^{-10\beta_1-5\beta_2}+e^{-10\beta_1-7\beta_2}+e^{-11\beta_1-4\beta_2}+e^{-11\beta_1-6\beta_2}+e^{-12\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-11\beta_1-7\beta_2}+e^{-12\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,13,52,120,150,111,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,8,16,25,25,16,8,2)$

S=$12\beta_1+7\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,9,19,23,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,4,7,7,4,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}$; $d\omega^{6,7}=-\omega^{1,4,7}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,4,5,6,7}=$

$-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1-2\beta_2} \langle\omega^{1,3,7}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,5,6} + \omega^{2,4,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,5,6} + \omega^{3,4,5}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{2,3,4,5}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{1,2,5,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{2,3,5,7} - \omega^{2,4,5,6}\rangle + e^{-9\beta_1-4\beta_2} \langle\omega^{1,5,6,7} + \omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-9\beta_1-5\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{1,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-12\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + 4Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 9, 19, 23, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 2, 4, 7, 7, 4, 2, 1)$

Total dimension of trivial cohomology: 28.

Length associated polynomial: 28.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{1,2,3} = \omega^{1,5} - \omega^{2,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,3,5} = -\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} =$

$-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = \omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + 2\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = -\omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 2\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$;

$\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-5\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-6\beta_1-2\beta_2} \langle\omega^{1,3,7} - \omega^{1,4,6}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,5,6} - \omega^{2,3,7} + 2\omega^{2,4,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,5,6} + \omega^{3,4,5}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7} - \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-6\beta_1-5\beta_2} \langle(\omega^{2,3,5,7} - \omega^{2,4,5,6})/2\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{2,3,4,5}\rangle + e^{-9\beta_1-4\beta_2} \langle(-\omega^{1,5,6,7} - \omega^{2,4,6,7})/2\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle(2\omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle + e^{-8\beta_1-3\beta_2} \langle(\omega^{1,2,6,7} - \omega^{1,3,4,7})/2\rangle + e^{-6\beta_1-4\beta_2} \langle(\omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/3\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-7\beta_1-6\beta_2} \langle-\omega^{2,3,4,5,6}\rangle + e^{-10\beta_1-5\beta_2} \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-9\beta_1-5\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-11\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-12\beta_1-6\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-12\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 3Y^2 + 4Y^3 + 3Y^4 + Y^5 + Y^6)(1 + Y).$

5.4.12 Algebra $\mathcal{G}_{7,2.8}$ †

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & \xi_1^2 & \xi_1^1+2\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & -\xi_1^4+\xi_2^5 & -\xi_1^3 & \xi_2^3 & 2\xi_1^2 & 2(\xi_1^1+\xi_2^2) \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{2\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+2\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle\delta_1^5\rangle+2e^{2\beta_1+\beta_2}\ \langle\delta_1^6,\delta_2^7\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{\beta_1+2\beta_2}\ \langle\delta_1^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^4,\delta_2^5\rangle+e^{\beta_2}\ \langle\delta_1^3\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^3]=-\delta_1^7$; $[\delta_1^6,\delta_1^2]=-2\delta_1^7$; $[\delta_2^6,\delta_1^2]=-2\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^3]=\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^2]=-\delta_2^5+\delta_1^4$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_1^4,\delta_2^3]=-(\delta_2^7+\delta_1^6)$; $[\delta_1^4,\delta_1^2]=-\delta_1^5$; $[\delta_2^5,\delta_1^3]=\delta_1^7$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_1^3,\delta_2^3]=-(\delta_2^5+\delta_1^4)$; $[\delta_2^3,\delta_1^2]=\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle\delta_1^5\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6,\delta_1^2]=3\delta_1^6$; $[\delta_2^4,\delta_1^2]=2\delta_1^4$; $[\delta_1^4,\delta_1^2]=-\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1+\beta_2}\ \langle x_5\omega^{1,6}\rangle+2e^{-2\beta_1}\ \langle x_3\omega^{1,4}+x_5\omega^{1,7},-x_2\omega^{1,3}-x_4\omega^{1,6}+x_7\omega^{3,6}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_6\omega^{2,5}\rangle+4e^{-\beta_2}\ \langle -x_1\omega^{1,2}-2x_3\omega^{2,3}+x_4\omega^{1,5}+x_6\omega^{2,6},x_1\omega^{1,2}+3x_3\omega^{2,3}-x_4(\omega^{1,5}-\omega^{2,4}),x_1\omega^{1,2}+2x_3\omega^{2,3}-x_4\omega^{1,5}+x_6\omega^{1,7},-x_1\omega^{1,2}-2x_3\omega^{2,3}+x_4\omega^{1,5}+x_6\omega^{3,4}\rangle+2e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}+x_6\omega^{2,7},x_4\omega^{2,5}+x_6\omega^{3,5}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle+3e^{-\beta_1}\ \langle x_3\omega^{1,3}+x_5\omega^{1,5},-x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4},x_6\omega^{1,6}\rangle+e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,3}-5x_2\omega^{2,3}-6x_3\omega^{2,4}-x_4(\omega^{1,7}+3\omega^{2,6}+4\omega^{3,4})-3x_6\omega^{3,6}+x_7\omega^{3,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-3\beta_2}+3e^{-\beta_1-3\beta_2}+e^{-\beta_1-4\beta_2}+e^{-2\beta_1-2\beta_2}+4e^{-2\beta_1-3\beta_2}+6e^{-3\beta_1-2\beta_2}+4e^{-4\beta_1-\beta_2}+e^{-5\beta_1-\beta_2}+e^{-5\beta_1}+e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-\beta_1-5\beta_2}+2e^{-2\beta_1-4\beta_2}+2e^{-3\beta_1-3\beta_2}+3e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-2\beta_2}+4e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+3e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-2\beta_2}+4e^{-6\beta_1-3\beta_2}+2e^{-7\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2}+2e^{-5\beta_1-5\beta_2}+e^{-5\beta_1-7\beta_2}+3e^{-6\beta_1-4\beta_2}+3e^{-6\beta_1-6\beta_2}+e^{-7\beta_1-3\beta_2}+3e^{-7\beta_1-5\beta_2}+2e^{-8\beta_1-4\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-9\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-6\beta_1-7\beta_2}+2e^{-7\beta_1-6\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-8\beta_1-6\beta_2}+e^{-8\beta_1-8\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-9\beta_1-7\beta_2}+e^{-10\beta_1-6\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-9\beta_1-8\beta_2}+e^{-10\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,13,51,119,152,112,44,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,8,15,23,26,19,9,2)$

S=$2(5\beta_1+4\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,10,20,24,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,5,9,9,5,2,1)$

$d\omega^3=-\omega^{1,2}$; $d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,5}-\omega^{2,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,5}$; $d\omega^{3,4}=-\omega^{1,2,4}$; $d\omega^{3,5}=-\omega^{1,2,5}$; $d\omega^{3,6}=-\omega^{1,2,6}-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,2,7}-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{2,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}+\omega^{1,2,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,4,5}$; $d\omega^{3,4,6}=-\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{1,2,4,7}-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,7}+\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{2,4,5,6}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}+\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7}=-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$;

$d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,5}\rangle + 2e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,7}+\omega^{3,4}, \omega^{2,6}-\omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,7}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,2,7}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-5\beta_2}\ \langle\omega^{2,5,7}\rangle + 2e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,5,7}+\omega^{2,4,7}-2\omega^{3,4,5}, \omega^{2,5,6}+\omega^{3,4,5}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,6,7}-\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,2,6,7}, \omega^{1,3,4,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,3,4,6}\rangle + e^{-8\beta_1-4\beta_2}\ \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-5\beta_2}\ \langle\omega^{1,2,5,7}\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6}\rangle + e^{-4\beta_1-6\beta_2}\ \langle\omega^{2,3,5,7}\rangle + e^{-7\beta_1-5\beta_2}\ \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}+2\omega^{3,4,5,6}\rangle + e^{-6\beta_1-6\beta_2}\ \langle\omega^{2,5,6,7}+\omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-7\beta_1-6\beta_2}\ \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,5,7}\rangle + e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-9\beta_1-5\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-10\beta_1-7\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+4Y^2+5Y^3+4Y^4+Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,10,20,24,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,2,5,9,9,5,2,1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}-\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{4,6}$; $d^\star\omega^{1,3,7} = -\omega^{4,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}+\omega^{3,5,6}$;

$d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = 2\omega^{2,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,7} + \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{3,7} = 3\omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,7} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 2\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = \omega^{1,2,7} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,4,7} + \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,7} + \omega^{2,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 2\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{2,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 4\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 4\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,7} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} =$

$-\omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = 2\omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,5}\rangle + 2e^{-3\beta_1-2\beta_2} \langle\omega^{1,7} + \omega^{3,4}, \omega^{2,6} - \omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,7} - \omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,6} - \omega^{1,3,4}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-3\beta_2} \langle 2\omega^{1,2,7} - \omega^{1,3,5} - \omega^{2,3,4}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2} \langle 2\omega^{1,4,7} + \omega^{1,5,6} + \omega^{2,4,6}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{2,5,7}\rangle + 2e^{-4\beta_1-4\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7} - 2\omega^{3,4,5}, \omega^{2,5,6} + \omega^{3,4,5}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,6,7} - \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-6\beta_2} \langle(-\omega^{2,5,6,7} - \omega^{3,4,5,7})/2\rangle + e^{-4\beta_1-6\beta_2} \langle-\omega^{2,3,5,7}\rangle + e^{-7\beta_1-5\beta_2} \langle(\omega^{1,5,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,6})/6\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle(\omega^{1,3,5,7} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6})/6\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,4,6}\rangle + 2e^{-6\beta_1-4\beta_2} \langle(-\omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/4, (\omega^{1,2,6,7} - 3\omega^{1,3,4,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6})/4\rangle + e^{-4\beta_1-5\beta_2} \langle(\omega^{1,2,5,7} + \omega^{2,3,4,5})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-7\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-9\beta_1-5\beta_2} \langle\omega^{1,3,4,6,7}\rangle + 2e^{-7\beta_1-6\beta_2} \langle(\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7} - 2\omega^{2,3,4,5,6})/3, (-\omega^{1,2,5,6,7} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/3\rangle + e^{-8\beta_1-5\beta_2} \langle(\omega^{1,2,4,6,7} + \omega^{1,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-7\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 4Y^2 + 5Y^3 + 4Y^4 + Y^5 + Y^6)(1 + Y).$

5.4.13 Algebra $\mathcal{G}_{7,2.9}$ †

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_4$; $[x_1,x_4]=x_6$; $[x_2,x_3]=x_5$; $[x_2,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_2^3 & 2\xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^3 & 0 & \xi_1^1+2\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^4 & \xi_2^3 & 0 & 3\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5 & 0 & -\xi_1^3 & 0 & \xi_1^1+3\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^4+\delta_2^5), ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_2^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{2\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+2\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+3\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+3\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_1+\beta_2}\ \langle\delta_1^4,\delta_2^5\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+e^{3\beta_2}\ \langle\delta_1^7\rangle+e^{\beta_1+2\beta_2}\ \langle\delta_2^7\rangle+e^{2\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^3]=-\delta_1^6$; $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_2^4,\delta_1^3]=\delta_1^6$; $[\delta_2^3,\delta_1^5]=\delta_2^7$; $[\delta_2^3,\delta_1^3]=\delta_2^5+\delta_1^4$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1+\beta_2}\ \langle\delta_1^4\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^4\rangle+e^{3\beta_2}\ \langle\delta_1^7\rangle+e^{2\beta_2}\ \langle\delta_1^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= 3e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_3\omega^{2,3}+x_4\omega^{2,4}, -x_1\omega^{1,2}-x_3\omega^{2,3}+x_6\omega^{2,6}, -x_3\omega^{2,3}+x_4\omega^{1,5}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+2x_3\omega^{1,3}+x_5\omega^{1,5}, x_4\omega^{1,4}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_7\omega^{1,7}\rangle+e^{-2\beta_1}\ \langle x_4\omega^{1,6}\rangle+e^{-2\beta_2}\ \langle -x_3\omega^{2,5}+x_4\omega^{1,7}\rangle+e^{2\beta_1-3\beta_2}\ \langle x_6\omega^{2,7}\rangle+e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}+x_6\omega^{3,5}\rangle+e^{-3\beta_1+2\beta_2}\ \langle x_7\omega^{1,6}\rangle+e^{-2\beta_1+\beta_2}\ \langle -x_5\omega^{1,4}+x_7\omega^{3,4}\rangle+e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3}-x_2\omega^{2,3}-x_3(\omega^{1,5}+\omega^{2,4})-x_4\omega^{2,6}-x_5\omega^{1,7}-x_6\omega^{3,6}+x_7\omega^{3,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-5\beta_2}+e^{-\beta_1-3\beta_2}+3e^{-\beta_1-4\beta_2}+e^{-2\beta_1-2\beta_2}+4e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2}+3e^{-4\beta_1-\beta_2}+e^{-5\beta_1+\beta_2}+e^{-5\beta_1}+e^{-5\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-6\beta_2}+e^{-3\beta_1-3\beta_2}+2e^{-3\beta_1-5\beta_2}+2e^{-3\beta_1-6\beta_2}+e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-4\beta_2}+2e^{-4\beta_1-5\beta_2}+2e^{-5\beta_1-3\beta_2}+2e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2}+e^{-6\beta_1}+e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-3\beta_1-7\beta_2} + 2e^{-4\beta_1-6\beta_2} + e^{-4\beta_1-8\beta_2} + e^{-5\beta_1-5\beta_2} + 2e^{-5\beta_1-7\beta_2} + 2e^{-6\beta_1-4\beta_2} + 3e^{-6\beta_1-6\beta_2} + e^{-6\beta_1-7\beta_2} + e^{-7\beta_1-3\beta_2} + 2e^{-7\beta_1-5\beta_2} + e^{-7\beta_1-6\beta_2} + e^{-8\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-5\beta_1-8\beta_2} + e^{-6\beta_1-7\beta_2} + e^{-7\beta_1-6\beta_2} + 2e^{-7\beta_1-7\beta_2} + e^{-7\beta_1-9\beta_2} + e^{-8\beta_1-5\beta_2} + e^{-8\beta_1-8\beta_2} + e^{-9\beta_1-7\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-8\beta_1-9\beta_2} + e^{-9\beta_1-8\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 50, 118, 151, 112, 44, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 7, 13, 21, 24, 18, 9, 2)$

S=$9(\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 10, 19, 24, 16, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 2, 5, 8, 8, 5, 2, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^4 = -\omega^{1,3}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{2,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5}$; $d\omega^{2,4} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,3,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,3,6}$; $d\omega^{4,7} = -\omega^{1,3,7} - \omega^{2,4,5}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = -\omega^{2,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{2,5,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,3,7}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,3,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,5,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{-4\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{2,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,7} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{1,2,7}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,5,6}+\omega^{2,4,6}\rangle + e^{-3\beta_1-5\beta_2}\ \langle\omega^{1,5,7}+\omega^{2,4,7}\rangle + e^{-2\beta_1-6\beta_2}\ \langle\omega^{2,5,7}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,4,7}+\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,3,4,6}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6}\rangle + e^{-4\beta_1-6\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,6,7}\rangle + e^{-3\beta_1-7\beta_2}\ \langle\omega^{2,3,5,7}\rangle + e^{-6\beta_1-6\beta_2}\ \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}\rangle + e^{-7\beta_1-5\beta_2}\ \langle\omega^{1,4,6,7}-\omega^{3,4,5,6}\rangle + e^{-5\beta_1-7\beta_2}\ \langle\omega^{2,5,6,7}+\omega^{3,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,3,4,5,6}\rangle + e^{-6\beta_1-7\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-7\beta_1-6\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-7\beta_1-7\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-9\beta_1-8\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-9\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-9\beta_1-9\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y+4Y^2+4Y^3+4Y^4+Y^5+Y^6)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,10,19,24,16,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,2,5,8,8,5,2,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^3$; $d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}-\omega^{3,4}$; $d^\star\omega^{1,2,5}=\omega^{1,7}-\omega^{3,5}$; $d^\star\omega^{1,2,6}=-\omega^{3,6}$; $d^\star\omega^{1,2,7}=-\omega^{3,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,4}=\omega^{4,5}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}+\omega^{2,5,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,4,6}=-\omega^{3,4,6}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7}=\omega^{2,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,3,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=-\omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=\omega^{2,3,5,6,7}$;

$d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{2,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{3,4}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,7} + \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,6} - \omega^{1,3,4}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{1,2,7} - \omega^{2,3,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,5,6} + \omega^{2,4,6}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{2,5,7}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{1,4,7} + \omega^{2,5,6} + 2\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-7\beta_2} \langle(-\omega^{2,5,6,7} - \omega^{3,4,5,7})/2\rangle + e^{-3\beta_1-7\beta_2} \langle-\omega^{2,3,5,7}\rangle + e^{-6\beta_1-6\beta_2} \langle(-\omega^{1,5,6,7} - \omega^{2,4,6,7})/2\rangle + e^{-7\beta_1-5\beta_2} \langle(-\omega^{1,4,6,7} + \omega^{3,4,5,6})/2\rangle + e^{-4\beta_1-6\beta_2} \langle(\omega^{1,3,5,7} + \omega^{2,3,4,7})/2\rangle + e^{-6\beta_1-4\beta_2} \langle(-\omega^{1,3,5,6} - \omega^{2,3,4,6})/2\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,4,6}\rangle + e^{-5\beta_1-5\beta_2} \langle(2\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6})/6\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-8\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-7\beta_1-7\beta_2} \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,3,4,5,6}\rangle + e^{-6\beta_1-7\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-7\beta_1-6\beta_2} \langle(-\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-9\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-9\beta_1-8\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-9\beta_1-9\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1 + Y + 4Y^2 + 4Y^3 + 4Y^4 + Y^5 + Y^6)(1 + Y)$.

5.4.14 Algebra $\mathcal{G}_{7,2.10}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_7$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_6] = x_7$; $[x_4, x_5] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2(\xi_1^1+\xi_2^2) & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & 0 & 0 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & 0 & 0 & \xi_2^4 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^3-\xi_1^6 & \xi_1^5 & -\xi_1^4 & 3\xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^7, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^7, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_2^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1+2\beta_2}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{3\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{3\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{\beta_1+2\beta_2}\ \langle\delta_1^3\rangle + e^{3\beta_1}\ \langle\delta_2^6\rangle + 2e^{\beta_1}\ \langle\delta_2^4, \delta_3^7\rangle + e^{2\beta_1+2\beta_2}\ \langle\delta_1^7\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_2^7\rangle + 2e^{2\beta_1+\beta_2}\ \langle\delta_2^3, \delta_1^6\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_2^4, \delta_2^3] = -\delta_2^7$; $[\delta_2^4, \delta_1^6] = \delta_2^7$; $[\delta_2^4, \delta_1^5] = \delta_1^6$; $[\delta_2^4, \delta_1^4] = \delta_1^5$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_2^3, \delta_1^4] = \delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{\beta_1+2\beta_2}\ \langle\delta_1^3\rangle + e^{3\beta_1}\ \langle\delta_2^6\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-4\beta_1}\ \langle x_2\omega^{1,6}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,6}\rangle + e^{\beta_1-2\beta_2}\ \langle x_6\omega^{2,3}\rangle + e^{2\beta_1-\beta_2}\ \langle x_6\omega^{2,4}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,4} + x_6(\omega^{2,7}+\omega^{3,4})\rangle + 2e^{-2\beta_1-2\beta_2}\ \langle -2/3x_1\omega^{1,3} + x_2\omega^{2,3} - 1/3x_4\omega^{3,4} + 1/3x_5\omega^{3,5} + x_6\omega^{3,6}, 2/3x_1(\omega^{1,3} + 3/2\omega^{2,6} - 3/2\omega^{4,5}) + x_2\omega^{2,3} + x_4(\omega^{2,7} - 2/3\omega^{3,4}) - 1/3x_5(\omega^{3,5} - 3\omega^{4,7}) + x_6\omega^{5,7}\rangle + e^{-\beta_1-\beta_2}\ \langle 2x_2\omega^{2,4} + 2x_4\omega^{2,5} + x_5(\omega^{2,6}+\omega^{4,5}) + x_6\omega^{4,6}\rangle + 2e^{-\beta_1}\ \langle x_3\omega^{1,3}, x_2\omega^{1,2} + x_3(\omega^{2,6} - \omega^{4,5}) + x_4\omega^{1,4}\rangle + e^{\beta_1}\ \langle x_3\omega^{2,4} + x_5\omega^{1,2}\rangle + 2e^{-\beta_2}\ \langle -2/3x_1\omega^{1,2} - 2/3x_3\omega^{2,3} - 1/3x_4\omega^{2,4} + 1/3x_5\omega^{2,5} + x_6\omega^{2,6}, x_3\omega^{2,3} + x_6\omega^{1,3}\rangle + e^{-2\beta_1}\ \langle x_4\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 2e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-3\beta_2}$.

ch$(H^4_{\mathcal{G}})$=$e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-3\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+2e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2}+2e^{-8\beta_1-3\beta_2}+e^{-9\beta_1-3\beta_2}$.

ch$(H^5_{\mathcal{G}})$=$2e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-6\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-6\beta_2}+e^{-7\beta_1-7\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-8\beta_1-6\beta_2}+2e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-10\beta_1-5\beta_2}+e^{-11\beta_1-5\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-8\beta_1-4\beta_2}+e^{-8\beta_1-7\beta_2}+2e^{-9\beta_1-6\beta_2}+2e^{-10\beta_1-5\beta_2}+e^{-10\beta_1-8\beta_2}+e^{-11\beta_1-6\beta_2}+e^{-11\beta_1-7\beta_2}+e^{-12\beta_1-6\beta_2}$.

ch$(H^7_{\mathcal{G}})$=$e^{-10\beta_1-6\beta_2}+e^{-11\beta_1-8\beta_2}+e^{-12\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,51,115,149,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,14,19,19,16,10,3)$

S=$4(3\beta_1+2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}$; $d\omega^7=-\omega^{1,3}-\omega^{2,6}+\omega^{4,5}$; $d\omega^{1,7}=\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,3}-\omega^{2,4,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,2,7}+\omega^{1,3,4}-\omega^{2,4,6}$; $d\omega^{5,6}=-\omega^{1,4,6}$; $d\omega^{5,7}=\omega^{1,3,5}-\omega^{1,4,7}-\omega^{2,5,6}$; $d\omega^{6,7}=\omega^{1,3,6}-\omega^{1,5,7}-\omega^{4,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,4,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,6}$; $d\omega^{1,5,7}=\omega^{1,2,5,6}$; $d\omega^{1,6,7}=\omega^{1,4,5,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=\omega^{2,3,4,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=\omega^{1,2,3,5}-\omega^{1,2,4,7}$; $d\omega^{2,6,7}=\omega^{1,2,3,6}-\omega^{1,2,5,7}+\omega^{2,4,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{2,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}+\omega^{3,4,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,3,4,5}-\omega^{2,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,4,6}-\omega^{1,4,5,7}$; $d\omega^{5,6,7}=-\omega^{1,3,5,6}-\omega^{1,4,6,7}$; $d\omega^{1,2,3,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,4,5,7}=\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,4,6}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,5,6}-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{1,2,3,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,3}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-4\beta_2}\ \langle\omega^{2,3,4}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,3,5}+\omega^{2,4,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,3,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-8\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,5,6,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,2,3,7} - \omega^{2,4,5,7}\rangle + e^{-6\beta_1-6\beta_2}\ \langle\omega^{2,3,4,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-9\beta_1-6\beta_2}\ \langle\omega^{1,2,3,6,7} - \omega^{2,4,5,6,7}\rangle + e^{-11\beta_1-6\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-10\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-7\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^7$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{4,5} = \omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6} = \omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}+\omega^{4,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}+\omega^{5,7}$; $d^\star\omega^{1,3,6} = \omega^{6,7}$; $d^\star\omega^{1,4,5} = -\omega^{1,7}-\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,6} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{2,7}$; $d^\star\omega^{2,4,6} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = -\omega^{3,7}$; $d^\star\omega^{4,5,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,5}+\omega^{2,4,7}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6}+\omega^{2,5,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,3,7}+\omega^{2,6,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,2,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{1,4,7}-\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,3,7}-\omega^{3,4,6}-\omega^{4,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{1,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,3,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{2,4,5,6} = \omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{3,4,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,3,7}-\omega^{2,3,4,6}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,3,4,7}-\omega^{2,3,5,6}-\omega^{2,4,6,7}$;

$d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,3,5,7} - \omega^{2,5,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,3,6,7}+\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,3,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} + \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}+\omega^{2,6}-\omega^{4,5}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{1,3} + 2\omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} - \omega^{3,4}$; $\Delta\omega^{3,4} = -\omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5} - \omega^{4,7}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{5,7}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{1,3} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,5}+3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{3,6}+3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}+\omega^{2,4,5}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}-\omega^{1,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,7}+2\omega^{1,3,4}-\omega^{2,4,6}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{1,4,7} - \omega^{2,5,6}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,5,7} - \omega^{4,5,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}+\omega^{2,6,7}-\omega^{4,5,7}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,5}+2\omega^{1,4,7}$; $\Delta\omega^{1,5,7} = -\omega^{1,3,6}+2\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}-\omega^{2,4,7}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}-\omega^{2,5,7}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}$; $\Delta\omega^{2,4,5} = \omega^{1,2,3}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,3,4} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{2,3,5} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,3,5} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{2,3,6} + 2\omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = \omega^{1,3,7} + 3\omega^{2,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = \omega^{2,5,7}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,6,7}+2\omega^{3,4,6}+\omega^{4,5,7}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6} + \omega^{4,6,7}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,3,6}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{1,3,7}+\omega^{3,4,6}+3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{3,5,6}+3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,5} + \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,5,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,5,7} + 3\omega^{1,3,4,5} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,6,7} + 2\omega^{1,3,4,6} + \omega^{1,4,5,7}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{2,4,6,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} + \omega^{1,4,6,7}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{2,5,6,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,4,6}+2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,3,5,6}+\omega^{1,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}+\omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}+\omega^{2,4,6,7}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,3,6}+\omega^{1,3,4,5}+2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,2,3,7}+\omega^{2,3,4,6}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,3,4,7}+\omega^{2,3,5,6}+2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = -\omega^{1,3,5,7} + 2\omega^{2,5,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{1,3,6,7}+2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,5,7} =$

$2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,4,6} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,3,5,6} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,5,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-2\beta_2} \langle 2\omega^{1,3} - \omega^{2,6} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{2,3,4}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,3,5} + \omega^{2,4,7}\rangle + e^{-6\beta_1-3\beta_2} \langle 2\omega^{1,3,6} + \omega^{1,5,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1-6\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-9\beta_1-4\beta_2} \langle -\omega^{1,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle(-\omega^{1,3,5,6} + \omega^{1,4,6,7})/2\rangle + e^{-6\beta_1-5\beta_2} \langle(-\omega^{1,2,3,7} - \omega^{2,3,4,6} + 2\omega^{2,4,5,7})/6\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-8\beta_1-7\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-11\beta_1-6\beta_2} \langle -\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-6\beta_2} \langle(\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7} - 2\omega^{2,4,5,6,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-12\beta_1-7\beta_2} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-12\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.4.15 Algebra $\mathcal{G}_{7,2.11}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -(\xi_1^3+\xi_1^4) & 0 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_2^4 & 0 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = \delta_3^6 - \delta_1^4, ad\ x_3 = -\delta_2^6, ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + 2e^{\beta_1+\beta_2}\ \langle x_3, x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_1}\ \langle \delta_2^3, \delta_2^4\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_1^5, \delta_2^6\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + 3e^{\beta_2}\ \langle \delta_3^6, \delta_1^3, \delta_1^4\rangle + e^{2\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + 2e^{2\beta_1}\ \langle \delta_3^7, \delta_2^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_1^5, \delta_2^4] = -\delta_1^7$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -(\delta_2^6 + \delta_1^5)$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1}\ \langle \delta_2^3\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + 2e^{\beta_2}\ \langle \delta_1^3, \delta_1^4\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + 2e^{2\beta_1}\ \langle \delta_3^7, \delta_2^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 3e^{-2\beta_1}\ \langle x_2(\omega^{1,3} + \omega^{1,4}), -x_3\omega^{1,5} + x_4\omega^{1,5}, x_2\omega^{1,4} + x_6\omega^{3,5}\rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3(\omega^{1,3} + \omega^{1,4}), x_2\omega^{1,2} + x_6\omega^{1,6}, -x_2\omega^{1,2} + x_3\omega^{1,4} + x_4\omega^{1,3}\rangle + e^{-2\beta_2}\ \langle x_3\omega^{2,6}\rangle + 4e^{-\beta_2}\ \langle x_1\omega^{1,2} + 2x_4\omega^{2,4} + x_5\omega^{2,5}, x_3\omega^{2,3}, x_3\omega^{2,4}, -x_1\omega^{1,2} - x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + 4e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} + 2x_2(\omega^{2,3} + 5/2\omega^{2,4}) + 5x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) - 5x_4(\omega^{1,6} - 2/5\omega^{3,4}) + x_5\omega^{3,5}, -x_2(\omega^{2,3} + \omega^{2,4}) - x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) + x_4\omega^{1,6} + x_6\omega^{3,6}, -x_1\omega^{1,3} - x_2(\omega^{2,3} + 3\omega^{2,4}) - 3x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) + 3x_4(\omega^{1,6} - 1/3\omega^{3,4}) + x_7\omega^{3,7}, -x_1\omega^{1,4} + 2x_2(\omega^{2,3} + 2\omega^{2,4}) + 2x_3(\omega^{1,6} + \omega^{2,5} - \omega^{3,4}) - 2x_4(\omega^{1,6} - 1/2\omega^{2,5}) + x_5\omega^{2,7} + x_7\omega^{4,7}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_6\omega^{1,7}\rangle + e^{-3\beta_1}\ \langle -x_3\omega^{1,7} + x_4\omega^{1,7}\rangle + e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,3}\rangle +$

$e^{2\beta_1-2\beta_2}\ \langle x_7\omega^{2,6}\rangle + e^{\beta_1-\beta_2}\ \langle -x_5\omega^{2,3}+x_7\omega^{3,4}\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}+x_7(\omega^{3,6}+\omega^{4,6})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{\beta_1-3\beta_2} + 3e^{-\beta_1-2\beta_2} + 4e^{-\beta_1-3\beta_2} + e^{-2\beta_1-\beta_2} + 4e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-\beta_2} + 5e^{-3\beta_1-2\beta_2} + 4e^{-4\beta_1-\beta_2} + 3e^{-5\beta_1-\beta_2} + e^{-5\beta_1} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=4e^{-2\beta_1-4\beta_2} + 3e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2} + e^{-4\beta_1-2\beta_2} + 5e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2} + 3e^{-5\beta_1-2\beta_2} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-\beta_2} + 4e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-4\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-3\beta_1-5\beta_2} + 2e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-6\beta_2} + 2e^{-5\beta_1-4\beta_2} + 2e^{-5\beta_1-5\beta_2} + 5e^{-6\beta_1-4\beta_2} + 2e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + 3e^{-7\beta_1-3\beta_2} + 3e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-3\beta_2} + e^{-8\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-5\beta_1-6\beta_2} + e^{-6\beta_1-5\beta_2} + 2e^{-7\beta_1-5\beta_2} + 2e^{-7\beta_1-6\beta_2} + e^{-7\beta_1-7\beta_2} + e^{-8\beta_1-4\beta_2} + 2e^{-8\beta_1-5\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-9\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-8\beta_1-6\beta_2} + e^{-8\beta_1-7\beta_2} + e^{-9\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 56, 124, 155, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 21, 33, 34, 24, 12, 3)$

$\mathrm{S}=9\beta_1 + 7\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{2,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,3} + \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7}$; $d\omega^{6,7} = \omega^{1,5,6} - \omega^{2,3,7} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} - \omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} + \omega^{1,2,4,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7}$; $d\omega^{2,6,7} = \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} =$

$-\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_1-\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-5\beta_1-2\beta_2} \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,7}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,5}, \omega^{1,4,6} - \omega^{2,3,5}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6}, \omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,5,6} - \omega^{2,4,7} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,5,7}, \omega^{1,4,5,7}\rangle + 2e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,6,7} + \omega^{2,3,5,7}, \omega^{1,4,6,7} - \omega^{2,3,5,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{2,3,4,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-6\beta_1-5\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{1,3,4,5,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-9\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-9\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,6,7}$;

$d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6}+\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7} - \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,4} = \omega^{2,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,6} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,6} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,3,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} - \omega^{3,5,6} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,6,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,3,5} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,3,7} + 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,6} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,6,7} + 2\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{3,5,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,6,7} =$

$\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,3,5,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}-\omega^{1,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_1-\beta_2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}-\omega^{2,4}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}-\omega^{3,4}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,7}\rangle+e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}-\omega^{1,4,5}\rangle+e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,7}\rangle+2e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6},\omega^{2,4,6}\rangle+2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}+\omega^{2,3,5}-\omega^{2,4,5},\omega^{1,4,6}-\omega^{2,3,5}+2\omega^{2,4,5}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,5,6}+2\omega^{2,3,7}-\omega^{2,4,7}+3\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-5\beta_2}\ \langle-\omega^{2,4,5,6}\rangle+e^{-5\beta_1-5\beta_2}\ \langle(-\omega^{2,3,6,7}+\omega^{3,4,5,6})/2\rangle+e^{-3\beta_1-5\beta_2}\ \langle-\omega^{2,3,4,6}\rangle+2e^{-7\beta_1-3\beta_2}\ \langle-\omega^{1,4,5,7},\omega^{1,3,5,7}\rangle+2e^{-6\beta_1-4\beta_2}\ \langle(\omega^{1,4,6,7}-\omega^{2,3,5,7}+2\omega^{2,4,5,7})/3,(-\omega^{1,3,6,7}-\omega^{2,3,5,7}+\omega^{2,4,5,7})/3\rangle+e^{-5\beta_1-4\beta_2}\ \langle(3\omega^{1,2,6,7}+\omega^{1,3,5,6}+2\omega^{1,4,5,6}+\omega^{2,3,4,7})/15\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_1-6\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle+e^{-5\beta_1-6\beta_2}\ \langle-\omega^{2,3,4,5,6}\rangle+e^{-8\beta_1-5\beta_2}\ \langle(\omega^{1,3,5,6,7}+\omega^{1,4,5,6,7})/2\rangle+e^{-8\beta_1-4\beta_2}\ \langle-\omega^{1,3,4,5,7}\rangle+e^{-7\beta_1-5\beta_2}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/3\rangle+e^{-6\beta_1-5\beta_2}\ \langle(-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-8\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-9\beta_1-6\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-8\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-9\beta_1-7\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)^2(1+Y^2)(1+Y)$.

5.4.16 Algebra $\mathcal{G}_{7,2.12}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_2,x_4]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1&0&0&0&0&0&0\\ \xi_1^2&\xi_2^2&0&0&0&0&0\\ 0&0&\xi_2^2&0&0&0&0\\ \xi_1^4&\xi_2^4&\xi_3^4&\xi_1^1+\xi_2^2&0&0&0\\ \xi_1^5&\xi_3^4&\xi_3^5&0&\xi_1^1+\xi_2^2&0&0\\ \xi_1^6&\xi_2^6&\xi_3^6&\xi_2^4&\xi_3^4&2\xi_1^1+\xi_2^2&0\\ \xi_1^7&\xi_2^7&\xi_3^7&-\xi_1^4&-\xi_1^5&\xi_1^2&\xi_1^1+2\xi_2^2\end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_1^6+\delta_2^7), ad\ x_5=-\delta_3^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : T=$\langle t_1,t_2,t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle$+$2e^{\beta_3}\ \langle x_2,x_3\rangle$+$2e^{\beta_1+\beta_3}\ \langle x_4,x_5\rangle$+$e^{2\beta_1+\beta_3}\ \langle x_6\rangle$+$e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{\beta_1}\ \langle\delta_3^5,\delta_2^4,\delta_3^4\rangle+3e^{\beta_1+\beta_3}\ \langle\delta_1^6,\delta_2^7,\delta_3^7\rangle+2e^{2\beta_1}\ \langle\delta_2^6,\delta_3^6\rangle+e^{2\beta_3}\ \langle\delta_1^7\rangle+2e^{\beta_3}\ \langle\delta_1^4,\delta_1^5\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_3^4]=-\delta_3^6$; $[\delta_3^5,\delta_1^5]=\delta_3^7$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+\delta_1^6$; $[\delta_3^6,\delta_1^2]=-\delta_3^7$; $[\delta_2^4,\delta_3^4]=\delta_3^6$; $[\delta_2^4,\delta_1^4]=\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_3^4,\delta_1^4]=\delta_3^7$; $[\delta_3^4,\delta_1^5]=\delta_2^7+\delta_1^6$; $[\delta_3^4,\delta_1^2]=\delta_1^5$; $[\delta_2^7,\delta_1^2]=\delta_1^7$.

ch($H^1_{\mathcal{G}}$)= $2e^{2\beta_1}\ \langle\delta_2^6,\delta_3^6\rangle$+$2e^{\beta_1}\ \langle\delta_2^4,\delta_3^4\rangle$+$e^{2\beta_3}\ \langle\delta_1^7\rangle$+$e^{\beta_1+\beta_3}\ \langle\delta_1^6\rangle$+$e^{-\beta_1+\beta_3}\ \langle\delta_1^2\rangle$+$2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6,\delta_1^2]=2\delta_1^6$; $[\delta_2^4,\delta_3^4]=\delta_3^6$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $3e^{-2\beta_1}\ \langle x_3\omega^{1,5},x_2\omega^{1,4},x_2\omega^{1,5}+x_3\omega^{1,4}\rangle+6e^{-\beta_3}\ \langle -2x_1\omega^{1,2}-x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5},x_4\omega^{2,4}+x_6\omega^{2,6},-2/3x_1\omega^{1,3}+1/3x_2\omega^{2,3}+1/3x_4\omega^{3,4}+x_6\omega^{3,6},-x_1\omega^{1,2}-x_3\omega^{2,3}-2x_4(\omega^{2,4}+1/2\omega^{3,5})+x_6\omega^{1,7},1/3x_1\omega^{1,3}-2/3x_2\omega^{2,3}+1/3x_4\omega^{3,4}+x_5\omega^{2,4},x_5\omega^{3,5}\rangle+3e^{-\beta_1-\beta_3}\ \langle -2x_1\omega^{1,4}+2x_2\omega^{2,4}+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5},x_1\omega^{1,4}-x_2\omega^{2,4}+x_5\omega^{3,6},-x_1\omega^{1,5}+x_2(\omega^{2,5}+\omega^{3,4})+x_3(\omega^{2,4}+\omega^{3,5})+x_4\omega^{3,6}+x_5\omega^{2,6}+x_6\omega^{5,6}\rangle+e^{2\beta_1-\beta_3}\ \langle x_6\omega^{2,3}\rangle+2e^{\beta_1-\beta_3}\ \langle x_4\omega^{2,3}+x_6\omega^{2,5},x_5\omega^{2,3}\rangle+e^{\beta_1-2\beta_3}\ \langle -x_1\omega^{2,3}+x_6\omega^{3,7}\rangle+2e^{-2\beta_3}\ \langle x_1\omega^{2,4}+x_5\omega^{3,7},-x_1(\omega^{2,5}+\omega^{3,4})+x_4\omega^{3,7}+x_5\omega^{2,7}+x_6\omega^{5,7}\rangle+e^{-2\beta_1+\beta_3}\ \langle x_7\omega^{1,6}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_3\omega^{1,3}+x_7\omega^{1,7},x_6\omega^{1,6},-x_3\omega^{1,2}+x_4\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-3\beta_3} + 6e^{-\beta_1-2\beta_3} + 3e^{-\beta_1-3\beta_3} + 5e^{-2\beta_1-\beta_3} + 3e^{-2\beta_1-2\beta_3} + e^{-2\beta_1-3\beta_3} + 4e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + 3e^{-4\beta_1-\beta_3} + 2e^{-2\beta_3} + 4e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$3e^{-\beta_1-4\beta_3} + e^{-\beta_1-5\beta_3} + e^{-2\beta_1-3\beta_3} + 7e^{-2\beta_1-4\beta_3} + e^{-3\beta_1-2\beta_3} + 8e^{-3\beta_1-3\beta_3} + 2e^{-3\beta_1-4\beta_3} + 6e^{-4\beta_1-2\beta_3} + 3e^{-4\beta_1-3\beta_3} + 2e^{-5\beta_1-2\beta_3} + e^{-4\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-2\beta_1-5\beta_3} + e^{-2\beta_1-6\beta_3} + 2e^{-3\beta_1-4\beta_3} + 4e^{-3\beta_1-5\beta_3} + e^{-3\beta_1-6\beta_3} + 2e^{-4\beta_1-3\beta_3} + 5e^{-4\beta_1-4\beta_3} + e^{-4\beta_1-5\beta_3} + e^{-5\beta_1-3\beta_3} + 4e^{-5\beta_1-4\beta_3} + 2e^{-6\beta_1-4\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-3\beta_1-6\beta_3} + e^{-4\beta_1-6\beta_3} + e^{-4\beta_1-7\beta_3} + 2e^{-5\beta_1-5\beta_3} + 2e^{-5\beta_1-6\beta_3} + 4e^{-6\beta_1-5\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-5\beta_1-7\beta_3} + 2e^{-6\beta_1-6\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 57, 123, 157, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 22, 33, 35, 25, 11, 3)$

S=$6\beta_1 + 7\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 21, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 9, 9, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{2,4} - \omega^{3,5}$; $d\omega^{1,7} = \omega^{1,2,4}+\omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{2,4,6} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,4,6} - \omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}$ $\langle\omega^1\rangle + 2e^{-\beta_2}$ $\langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-2\beta_2}\ \langle\omega^{2,3}\rangle + 2e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}+\omega^{3,4},\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6},\omega^{1,5,6}\rangle + e^{-\beta_1-4\beta_2}\ \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,3,6}+\omega^{2,4,5}\rangle + 2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,5,7}+\omega^{2,5,6}+\omega^{3,4,6},\omega^{1,4,7}-\omega^{3,5,6}\rangle + 2e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,5,7}+\omega^{3,4,7},\omega^{2,4,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,5,6},\omega^{1,2,4,6}\rangle + 2e^{-3\beta_1-4\beta_2}\ \langle\omega^{1,3,5,7}+\omega^{2,3,5,6},\omega^{1,2,5,7}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle + 2e^{-2\beta_1-5\beta_2}\ \langle\omega^{2,3,4,7},\omega^{2,3,5,7}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,3,6,7}+\omega^{2,4,5,6}\rangle + e^{-3\beta_1-5\beta_2}\ \langle\omega^{2,3,6,7}+\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7},\omega^{1,2,4,6,7}-\omega^{1,3,5,6,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1-6\beta_2}\ \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= 2e^{-6\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7},\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-7\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7}-\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}+\omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7}-\omega^{1,3,5,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}$;

$d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,4} = \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,6} + \omega^{3,5,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{3,5,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{2,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{2,4,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,5,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,2,4,7} + \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{2,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} =$

$\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1} \langle\omega^1\rangle + 2e^{-\beta_2} \langle\omega^2, \omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-2\beta_2} \langle\omega^{2,3}\rangle + 2e^{-\beta_1-2\beta_2} \langle\omega^{2,5} + \omega^{3,4}, \omega^{2,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})= e^{-3\beta_1-2\beta_2} \langle\omega^{1,3,6} + \omega^{1,4,5}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}, \omega^{1,5,6}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,3,7} + 2\omega^{2,3,6} + \omega^{2,4,5}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}, \omega^{1,4,7} + 2\omega^{2,4,6} - \omega^{3,5,6}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{2,5,7} + \omega^{3,4,7}, \omega^{2,4,7} - \omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}})= e^{-3\beta_1-5\beta_2} \langle(\omega^{2,3,6,7} + \omega^{2,4,5,7})/2\rangle + 2e^{-2\beta_1-5\beta_2} \langle-\omega^{2,3,5,7}, \omega^{2,3,4,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-4\beta_2} \langle(-\omega^{1,3,6,7} - 2\omega^{1,4,5,7} - \omega^{2,4,5,6})/6\rangle + 2e^{-3\beta_1-4\beta_2} \langle(-\omega^{1,2,5,7} - \omega^{1,3,4,7} - \omega^{2,3,4,6})/3, (-\omega^{1,2,4,7} + 2\omega^{1,3,5,7} + \omega^{2,3,5,6})/6\rangle + 2e^{-4\beta_1-3\beta_2} \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6})/2, (\omega^{1,2,4,6} - \omega^{1,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}})= e^{-4\beta_1-6\beta_2} \langle-\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + 2e^{-5\beta_1-5\beta_2} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2, (\omega^{1,2,4,6,7} - \omega^{1,3,5,6,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}})= e^{-5\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + 2e^{-6\beta_1-6\beta_2} \langle-\omega^{1,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}})= e^{-6\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 3Y^2 + 6Y^3 + 3Y^4 + 2Y^5 + Y^6)(1 + Y)$.

5.4.17 Algebra $\mathcal{G}_{7,2.13}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_6] = x_7$; $[x_4, x_5] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_1^3/2 & 0 & \xi_2^4 & 2\xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^4 & (-\xi_1^3)/2 & \xi_2^4 & 3\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^6 & \xi_1^5 & -\xi_1^4 & 3\xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_2^6, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_2^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{3\beta_1} \langle x_3 \rangle + e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{2\beta_1+\beta_2} \langle x_5 \rangle + e^{3\beta_1+\beta_2} \langle x_6 \rangle + e^{3\beta_1+2\beta_2} \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{3\beta_1+2\beta_2} \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^3 \rangle + e^{3\beta_1} \langle \delta_2^6 \rangle + e^{2\beta_1} \langle \delta_1^3 \rangle + e^{\beta_1} \langle \delta_2^4 \rangle + e^{2\beta_1+2\beta_2} \langle \delta_1^7 \rangle + e^{3\beta_1+\beta_2} \langle \delta_2^7 \rangle + e^{2\beta_2} \langle \delta_3^7 \rangle + e^{2\beta_1+\beta_2} \langle \delta_1^6 \rangle + e^{\beta_1+\beta_2} \langle \delta_1^5 \rangle + e^{\beta_2} \langle \delta_1^4 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_1^4] = \delta_2^6$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^5] = -1/2\delta_2^7$; $[\delta_1^3, \delta_1^4] = 1/2\delta_1^6$; $[\delta_2^4, \delta_1^6] = \delta_2^7$; $[\delta_2^4, \delta_1^5] = \delta_1^6$; $[\delta_2^4, \delta_1^4] = \delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{3\beta_1-\beta_2} \langle \delta_2^3 \rangle + e^{2\beta_1} \langle \delta_1^3 \rangle + e^{2\beta_1+2\beta_2} \langle \delta_1^7 \rangle + e^{2\beta_2} \langle \delta_3^7 \rangle + 2e^0 \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^7] = -\delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{2\beta_1-2\beta_2} \langle x_3\omega^{2,4} \rangle + 2e^{-\beta_2} \langle -2x_1\omega^{1,2} + 3x_3\omega^{1,5} - x_4\omega^{2,4} + x_5\omega^{2,5}, x_3\omega^{2,3} \rangle + 2e^{-3\beta_1} \langle -2x_1\omega^{1,3} - 3x_2\omega^{1,5} + 3x_4(\omega^{1,6} - 1/3\omega^{3,4}) + x_5\omega^{3,5}, x_1\omega^{1,3} + x_2(\omega^{1,5} - \omega^{2,3}) - x_4(\omega^{1,6} - \omega^{3,4}) + x_7\omega^{3,7} \rangle + e^{-4\beta_1} \langle -x_2(\omega^{1,6} - \omega^{3,4}) + x_4\omega^{3,5} + x_5\omega^{3,6} \rangle + e^{2\beta_1-\beta_2} \langle x_3\omega^{1,2} \rangle + e^{-2\beta_2} \langle x_1\omega^{2,4} - x_3(\omega^{2,6} - \omega^{4,5}) \rangle + e^{-\beta_1+2\beta_2} \langle x_7\omega^{1,3} \rangle + 2e^{-\beta_1} \langle x_2\omega^{1,2} - x_3\omega^{1,3} + x_4\omega^{1,4} + x_7\omega^{1,7}, 2x_3\omega^{1,3} + x_5\omega^{2,3} + x_6\omega^{3,4} \rangle + e^{-\beta_1-\beta_2} \langle x_2\omega^{2,4} + x_3(\omega^{1,6} - \omega^{3,4}) + x_4\omega^{2,5} + x_5\omega^{2,6} \rangle + e^{-2\beta_1} \langle x_4\omega^{1,5} \rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 2e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1} + e^{-6\beta_1-\beta_2} + e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+2e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-\beta_2}+e^{-7\beta_1-3\beta_2}+e^{-8\beta_1-\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-9\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-6\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-8\beta_1-6\beta_2}+e^{-9\beta_1-2\beta_2}+2e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-6\beta_2}+e^{-9\beta_1-5\beta_2}+2e^{-9\beta_1-6\beta_2}+2e^{-10\beta_1-5\beta_2}+e^{-11\beta_1-6\beta_2}+e^{-12\beta_1-4\beta_2}+e^{-12\beta_1-5\beta_2}+e^{-13\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-10\beta_1-6\beta_2}+e^{-12\beta_1-6\beta_2}+e^{-13\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,50,114,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,13,17,17,15,10,3)$

S$=13\beta_1+6\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}$; $d\omega^7=-\omega^{2,6}+\omega^{4,5}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{2,4,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{4,5}=-\omega^{1,2,5}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{2,4,6}$; $d\omega^{5,6}=-\omega^{1,4,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{2,5,6}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}-\omega^{4,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,4,5}$; $d\omega^{1,3,7}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,6}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=\omega^{1,2,5,6}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}+\omega^{1,4,5,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=\omega^{2,3,4,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,4,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{2,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}+\omega^{3,4,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{2,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}+\omega^{2,3,4,7}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}+\omega^{2,3,5,7}$; $d\omega^{1,2,3,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7}=\omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{2,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,3,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,4,7}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,6} + \omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6}\rangle + e^{-9\beta_1-4\beta_2}\ \langle(2\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7})/2\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,6,7} + \omega^{1,4,5,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-12\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-9\beta_1-6\beta_2}\ \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-10\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-13\beta_1-5\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-12\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-13\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \leq j \leq 7$: $(1, 3, 8, 17, 22, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \leq j \leq 7$: $(1, 3, 4, 4, 4, 4, 3, 1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{4,5} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = \omega^{1,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{1,7} - \omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,6} = -\omega^{3,7}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{2,7}$; $d^\star\omega^{2,4,6} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = -\omega^{3,7}$; $d^\star\omega^{4,5,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,2,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{1,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,3,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{1,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,3,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{3,4,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$;

$d^{\star}\omega^{1,2,3,4,6} = \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{1,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,3,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,3,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 2\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$;

$\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6} + \omega^{1,4,5} + 2\omega^{2,3,4}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,4,7}\rangle + e^{-6\beta_1-2\beta_2}\ \langle 2\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,4,5,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle(-2\omega^{1,5,6,7}+\omega^{2,3,6,7}-\omega^{3,4,5,7})/6\rangle + e^{-9\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + e^{-7\beta_1-4\beta_2}\ \langle(\omega^{1,2,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,4,7})/6\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_1-6\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2}\ \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-12\beta_1-4\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-5\beta_2}\ \langle(-\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-12\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-13\beta_1-5\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-13\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.4.18 Algebra $\mathcal{G}_{7,2.14}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_3,x_4]=-x_6$; $[x_3,x_5]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^4 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^5 & \xi_2^5 & \xi_2^4 & 3\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & \xi_2^5 & \xi_2^4 & 4\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_2^5, ad\ x_4=-\delta_1^5, ad\ x_5=-\delta_1^6+\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1+\beta_2}\ \langle\delta_1^6,\delta_3^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{3\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{4\beta_1}\ \langle\delta_2^7\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=\delta_3^7-\delta_1^6$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=\delta_3^7-\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^5$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^5]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=\delta_3^7-\delta_1^6$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{4\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{-4\beta_1}\ \langle -x_2(\omega^{1,6}-\omega^{3,5})+x_4\omega^{3,6}\rangle+e^{-5\beta_1}\ \langle -x_2(\omega^{1,7}-\omega^{3,6})+x_4\omega^{3,7}\rangle+2e^{-\beta_2}\ \langle x_1\omega^{1,2}-2x_3\omega^{2,3}+3x_4\omega^{2,4}+2x_5\omega^{2,5}+x_6\omega^{2,6},-x_1\omega^{1,2}+2x_3\omega^{2,3}-2x_4\omega^{2,4}-x_5\omega^{2,5}+x_7\omega^{2,7}\rangle+3e^{-2\beta_1}\ \langle -x_2(\omega^{1,4}+\omega^{2,3})+x_4\omega^{3,4},x_1\omega^{1,3}+2x_2(\omega^{1,4}-\omega^{2,3})+x_4\omega^{1,5}+x_5\omega^{1,6},-x_2\omega^{2,3}+x_4\omega^{1,5}+x_6\omega^{1,7}\rangle+e^{-3\beta_1}\ \langle -x_2(\omega^{1,5}-\omega^{3,4})+x_4\omega^{3,5}\rangle+e^{2\beta_1}\ \langle x_6\omega^{1,2}\rangle+e^{3\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle+2e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_4\omega^{2,3},2x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}+x_5\omega^{1,5}\rangle+e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})+3x_2\omega^{2,4}+x_3(\omega^{1,5}-\omega^{3,4})+2x_4\omega^{2,5}+x_5\omega^{2,6}+x_6\omega^{2,7}+x_7\omega^{4,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+2e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+3e^{-5\beta_1-\beta_2}+e^{-5\beta_1}+e^{-6\beta_1-\beta_2}+e^{-6\beta_1}+e^{-7\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2}+e^{-7\beta_1-\beta_2}+2e^{-7\beta_1-2\beta_2}+e^{-8\beta_1-\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-9\beta_1-\beta_2}+e^{-10\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2}+2e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-2\beta_2}+e^{-10\beta_1-3\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+2e^{-10\beta_1-5\beta_2}+2e^{-11\beta_1-4\beta_2}+e^{-11\beta_1-5\beta_2}+e^{-12\beta_1-3\beta_2}+e^{-12\beta_1-4\beta_2}+e^{-13\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-11\beta_1-5\beta_2}+e^{-12\beta_1-5\beta_2}+e^{-13\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,51,113,148,112,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,14,17,16,15,10,3)$

S=$13\beta_1+5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,4,4,4,4,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,4}-\omega^{2,3}$; $d\omega^6=-\omega^{1,5}+\omega^{3,4}$; $d\omega^7=-\omega^{1,6}+\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,6}=-\omega^{1,3,4}$; $d\omega^{1,7}=-\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,4}$; $d\omega^{2,6}=-\omega^{1,2,5}-\omega^{2,3,4}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}+\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,4,6}-\omega^{2,3,6}-\omega^{3,4,5}$; $d\omega^{5,7}=-\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,3,7}$; $d\omega^{6,7}=-\omega^{1,5,7}+\omega^{3,4,7}-\omega^{3,5,6}$; $d\omega^{1,2,6}=\omega^{1,2,3,4}$; $d\omega^{1,2,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=-\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}+\omega^{1,3,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}$; $d\omega^{1,6,7}=-\omega^{1,3,4,7}+\omega^{1,3,5,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}-\omega^{3,4,5,7}$; $d\omega^{1,2,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7}=$

$-\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-7\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{3,4,5}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-10\beta_1-2\beta_2} \langle\omega^{1,3,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,6,7} + \omega^{2,3,5,7} + 2\omega^{3,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,2,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-12\beta_1-3\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{1,2,4,5,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-13\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-12\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-13\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1-Y+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \leq j \leq 7$: $(1,3,8,17,22,17,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \leq j \leq 7$: $(1,3,4,4,4,4,3,1)$

Total dimension of trivial cohomology: 24.

Length associated polynomial: 24.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{3,4} = \omega^6$; $d^\star\omega^{3,5} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,6} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,6} + \omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{2,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = \omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{3,4,7} = \omega^{6,7}$; $d^\star\omega^{3,5,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,2,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,4,7} + \omega^{1,5,6} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,6,7} - \omega^{3,5,7}$;

$d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,4,7} + \omega^{2,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{2,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{1,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 3\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + 2\omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,7} + \omega^{1,4,6} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 3\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 3\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = 3\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 3\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,5,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,4,5,7} + 2\omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7}$;

$\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 3\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-7\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-6\beta_1-4\beta_2} \langle-\omega^{2,4,5,6}\rangle + e^{-8\beta_1-3\beta_2} \langle(-\omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6})/3\rangle + e^{-10\beta_1-2\beta_2} \langle-\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle(\omega^{1,2,5,7} - \omega^{1,4,5,6} + \omega^{2,3,5,6})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-10\beta_1-5\beta_2} \langle-\omega^{2,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-12\beta_1-3\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle(-\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-12\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-13\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-13\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 - Y + Y^2)(1 + Y)^3$.

5.4.19 Algebra $\mathcal{G}_{7,2.15}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix}
\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\
\xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\
\xi_1^3 & 0 & 3\xi_1^1 & 0 & 0 & 0 & 0\\
\xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\
\xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0\\
\xi_1^6 & \xi_2^6 & -\xi_1^4 & \xi_2^5-\xi_1^3 & \xi_2^4 & 3\xi_1^1+\xi_2^2 & 0\\
\xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & \xi_2^5-2\xi_1^3 & \xi_2^4 & 4\xi_1^1+\xi_2^2
\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_2^6, ad\ x_4=-\delta_1^5+\delta_3^7, ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_6\rangle+e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_3^7\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{3\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{4\beta_1}\ \langle\delta_2^7\rangle+e^{3\beta_1}\ \langle\delta_2^6\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_1^3]=2\delta_1^7$; $[\delta_1^5,\delta_2^5]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^6$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_1^3]=-2\delta_3^7-\delta_1^5$; $[\delta_1^2,\delta_2^5]=\delta_3^7-\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^6$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_1^5$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^5]=-2\delta_2^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{4\beta_1}\ \langle\delta_2^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_1^3]=-3\delta_1^5$; $[\delta_1^3,\delta_2^5]=-2\delta_2^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+e^{-2\beta_2}\ \langle x_1\omega^{2,4}-x_3(\omega^{2,6}-\omega^{4,5})\rangle+3e^{-3\beta_1}\ \langle -x_1\omega^{1,3}-2x_2(\omega^{1,5}-\omega^{2,3})-x_4\omega^{1,6}+x_5\omega^{3,5}, x_1\omega^{1,3}+x_2(\omega^{1,5}-3\omega^{2,3})+x_4(\omega^{1,6}+\omega^{3,4}), -x_1\omega^{1,3}-x_2(\omega^{1,5}-2\omega^{2,3})-x_4\omega^{1,6}+x_5\omega^{1,7}\rangle+e^{-4\beta_1}\ \langle -x_2(\omega^{1,6}-\omega^{3,4})+x_4\omega^{3,5}+x_5\omega^{3,6}\rangle+e^{-5\beta_1}\ \langle -x_2(\omega^{1,7}-\omega^{3,5})+x_4\omega^{3,6}+x_5\omega^{3,7}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_5\omega^{2,3}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}, x_5\omega^{1,5}\rangle+e^{\beta_1}\ \langle x_5\omega^{1,2}\rangle+e^{2\beta_1-\beta_2}\ \langle x_3\omega^{1,2}+x_7\omega^{2,5}\rangle+e^{\beta_1-\beta_2}\ \langle x_3\omega^{1,4}+x_5\omega^{2,4}\rangle+2e^{-\beta_2}\ \langle -x_1\omega^{1,2}+3x_3\omega^{2,3}-$

$2x_4\omega^{2,4} - x_5\omega^{2,5} + x_7\omega^{2,7}, x_3(\omega^{1,5} - 2\omega^{2,3}) + 2x_4\omega^{2,4} + 2x_5\omega^{2,5} + x_6\omega^{4,5}\rangle + e^{-2\beta_1}\langle -x_2\omega^{1,4} - x_4\omega^{2,3} + x_5\omega^{3,4}\rangle + e^{-\beta_1-\beta_2}\langle x_1\omega^{1,4} - x_2\omega^{2,4} - x_3(\omega^{1,6} - \omega^{3,4}) - x_5(\omega^{2,6} - \omega^{4,5})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2} + 3e^{-\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-\beta_2} + 4e^{-3\beta_1-2\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + 3e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1} + 2e^{-6\beta_1-\beta_2} + e^{-6\beta_1} + e^{-7\beta_1} + e^{-8\beta_1} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + 2e^{-4\beta_1-2\beta_2} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 5e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-\beta_2} + 4e^{-8\beta_1-2\beta_2} + 2e^{-9\beta_1-\beta_2} + e^{-10\beta_1-\beta_2} + e^{-11\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-6\beta_1-3\beta_2} + 2e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + 3e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + e^{-9\beta_1-2\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-5\beta_2} + 2e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-4\beta_2} + 2e^{-11\beta_1-2\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-2\beta_2} + e^{-13\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-8\beta_1-5\beta_2} + e^{-9\beta_1-4\beta_2} + 2e^{-10\beta_1-5\beta_2} + e^{-11\beta_1-3\beta_2} + 2e^{-11\beta_1-4\beta_2} + e^{-12\beta_1-5\beta_2} + e^{-13\beta_1-3\beta_2} + e^{-13\beta_1-4\beta_2} + e^{-14\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-11\beta_1-5\beta_2} + e^{-13\beta_1-5\beta_2} + e^{-14\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 54, 119, 152, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 18, 26, 26, 20, 11, 3)$

S=$14\beta_1 + 5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} + \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = -\omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} - \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,3,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} =$

$-\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle \omega^1 \rangle + e^{-\beta_2} \langle \omega^2 \rangle + e^{-3\beta_1} \langle \omega^3 \rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1} \langle \omega^{1,3} \rangle + e^{-3\beta_1-\beta_2} \langle \omega^{1,5} \rangle + e^{-\beta_1-2\beta_2} \langle \omega^{2,4} \rangle + e^{-5\beta_1-\beta_2} \langle \omega^{1,7} - \omega^{3,5} \rangle + e^{-3\beta_1-2\beta_2} \langle \omega^{2,6} - \omega^{4,5} \rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-8\beta_1-\beta_2} \langle \omega^{1,3,7} \rangle + e^{-4\beta_1-2\beta_2} \langle \omega^{1,2,6} \rangle + e^{-3\beta_1-3\beta_2} \langle \omega^{2,4,5} \rangle + e^{-5\beta_1-3\beta_2} \langle \omega^{2,4,7} - \omega^{2,5,6} \rangle + 2e^{-6\beta_1-2\beta_2} \langle \omega^{1,4,7}, \omega^{2,3,6} + \omega^{3,4,5} \rangle + e^{-8\beta_1-2\beta_2} \langle \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6} \rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1-3\beta_2} \langle \omega^{1,2,4,7} \rangle + 2e^{-8\beta_1-3\beta_2} \langle \omega^{1,2,6,7} - \omega^{2,3,5,6}, \omega^{1,4,5,7} \rangle + e^{-9\beta_1-2\beta_2} \langle \omega^{1,3,4,7} \rangle + e^{-11\beta_1-2\beta_2} \langle \omega^{1,3,6,7} \rangle + e^{-6\beta_1-4\beta_2} \langle \omega^{2,4,5,6} \rangle + e^{-10\beta_1-3\beta_2} \langle \omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{3,4,5,7} \rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-11\beta_1-3\beta_2} \langle \omega^{1,2,3,6,7} \rangle + e^{-13\beta_1-3\beta_2} \langle \omega^{1,3,5,6,7} \rangle + e^{-9\beta_1-4\beta_2} \langle \omega^{1,2,4,6,7} \rangle + e^{-11\beta_1-4\beta_2} \langle \omega^{1,4,5,6,7} + \omega^{2,3,4,6,7} \rangle + e^{-10\beta_1-5\beta_2} \langle \omega^{2,4,5,6,7} \rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-5\beta_2} \langle \omega^{1,2,4,5,6,7} \rangle + e^{-14\beta_1-4\beta_2} \langle \omega^{1,3,4,5,6,7} \rangle + e^{-13\beta_1-5\beta_2} \langle \omega^{2,3,4,5,6,7} \rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-14\beta_1-5\beta_2} \langle \omega^{1,2,3,4,5,6,7} \rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$;

$d^\star\omega^{1,2,3,4} = \omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}+\omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7}+2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}+\omega^{1,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}-\omega^{2,3,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = 3\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 2\omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 3\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7} - \omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 3\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7}+\omega^{3,4,7}+\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 3\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 3\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}+\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7}+\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} +$

$\omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,7} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 3\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,6,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-8\beta_1-\beta_2}\ \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,6} - 2\omega^{1,4,5} - \omega^{2,3,4}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,4,7} - \omega^{2,5,6}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,4,7} - \omega^{2,3,6} - \omega^{3,4,5}, \omega^{1,5,6} - 2\omega^{2,3,6} - \omega^{3,4,5}\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-6\beta_1-4\beta_2}\ \langle -\omega^{2,4,5,6}\rangle + e^{-10\beta_1-3\beta_2}\ \langle(\omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{3,4,5,7})/6\rangle + e^{-11\beta_1-2\beta_2}\ \langle -\omega^{1,3,6,7}\rangle + e^{-9\beta_1-2\beta_2}\ \langle(\omega^{1,3,4,7} - \omega^{1,3,5,6})/2\rangle + 2e^{-8\beta_1-3\beta_2}\ \langle(-\omega^{1,2,6,7} - \omega^{2,3,4,7} + 2\omega^{2,3,5,6})/3, (\omega^{1,4,5,7} + \omega^{2,3,4,7} - \omega^{2,3,5,6})/3\rangle + e^{-6\beta_1-3\beta_2}\ \langle(2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5})/6\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-10\beta_1-5\beta_2}\ \langle -\omega^{2,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2}\ \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-13\beta_1-3\beta_2}\ \langle -\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle(\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-11\beta_1-3\beta_2}\ \langle(-\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-13\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-14\beta_1-4\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-5\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-14\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y^2)^2(1+Y)^3.$

5.4.20 Algebra $\mathcal{G}_{7,2.16}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 4\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 2\xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -\xi_1^2 & \xi_2^5 & \xi_2^4 & 3\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3 + \xi_2^6 & \xi_2^5 & \xi_2^4 & 4\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^7, ad\ x_3 = -\delta_2^7, ad\ x_4 = -\delta_1^5, ad\ x_5 = -\delta_1^6, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{4\beta_1}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{3\beta_1+\beta_2}\ \langle x_6\rangle + e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{4\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{4\beta_1-\beta_2}\ \langle\delta_2^3\rangle + 2e^{\beta_2}\ \langle\delta_1^4, \delta_3^7\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{3\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{4\beta_1}\ \langle\delta_2^7\rangle + 2e^{3\beta_1}\ \langle\delta_1^3, \delta_2^6\rangle + e^{2\beta_1}\ \langle\delta_2^5\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_2^6 + \delta_1^3$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_1^4, \delta_1^3] = \delta_1^7$; $[\delta_1^4, \delta_2^6] = -\delta_1^7$; $[\delta_1^4, \delta_2^5] = -\delta_1^6$; $[\delta_1^4, \delta_2^4] = -\delta_1^5$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -\delta_1^6$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^3] = -\delta_1^6$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^4] = \delta_3^7 - \delta_1^4$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_1^3, \delta_2^4] = -\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $e^{4\beta_1-\beta_2}\ \langle\delta_2^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + 2e^{3\beta_1}\ \langle\delta_1^3, \delta_2^6\rangle + e^{2\beta_1}\ \langle\delta_2^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_2^6 + \delta_1^3$.

ch($H^2_{\mathcal{G}}$)= $e^{-5\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3\omega^{1,3}, x_2\omega^{1,2} + x_6\omega^{2,3}, x_4\omega^{1,4}\rangle + e^{3\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} - x_3\omega^{1,6} + 2x_4\omega^{2,4} + x_5\omega^{2,5}, -x_1\omega^{1,2} + 2x_3\omega^{1,6} - x_4\omega^{2,4} + x_6\omega^{2,6}, x_3\omega^{2,3}\rangle + 3e^{-4\beta_1}\ \langle x_1\omega^{1,3} + x_2(\omega^{1,6} - 3\omega^{2,3}) + 2x_4\omega^{3,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} - 2x_2(\omega^{1,6} - \omega^{2,3}) - x_4\omega^{3,4} + x_6\omega^{3,6}, -x_2\omega^{2,3} + x_4\omega^{1,7}\rangle + e^{2\beta_1-\beta_2}\ \langle x_3\omega^{1,4} + x_6\omega^{2,4}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,4} + x_3(\omega^{1,7} - \omega^{3,4}) + x_4\omega^{2,5} + x_5\omega^{2,6} + x_6\omega^{2,7}, -2x_1\omega^{1,4} - 2x_2\omega^{2,4} - x_3(\omega^{1,7} - \omega^{3,4}) - 4x_4\omega^{2,5} - x_5(\omega^{2,6} + 3\omega^{4,5}) - x_6\omega^{4,6} + x_7\omega^{4,7}\rangle + 2e^{-2\beta_1}\ \langle -x_2\omega^{1,4} -$

$x_5\omega^{2,3} + x_6\omega^{3,4}, x_4\omega^{1,5}\rangle + e^{-5\beta_1}\ \langle -x_2(\omega^{1,7} - \omega^{3,4}) + x_4\omega^{3,5} + x_5\omega^{3,6} + x_6\omega^{3,7}\rangle + e^{3\beta_1-\beta_2}\ \langle x_3\omega^{1,2}\rangle + e^{-3\beta_1}\ \langle x_4\omega^{1,6}\rangle + e^{\beta_1-\beta_2}\ \langle -x_3\omega^{1,5} + x_7\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-3\beta_2} + 3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2} + e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2} + 5e^{-5\beta_1-\beta_2} + e^{-5\beta_1} + 2e^{-6\beta_1-\beta_2} + e^{-6\beta_1} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1} + e^{-8\beta_1} + e^{-9\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-2\beta_1-4\beta_2} + 2e^{-3\beta_1-3\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-2\beta_2} + 2e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 4e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + 3e^{-7\beta_1-2\beta_2} + 3e^{-8\beta_1-2\beta_2} + 2e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + 2e^{-10\beta_1-\beta_2} + e^{-11\beta_1-\beta_2} + e^{-12\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-6\beta_1-4\beta_2} + e^{-6\beta_1-5\beta_2} + e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-2\beta_2} + 3e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-4\beta_2} + e^{-11\beta_1-2\beta_2} + e^{-11\beta_1-3\beta_2} + 2e^{-12\beta_1-2\beta_2} + e^{-13\beta_1-2\beta_2} + e^{-14\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2} + 2e^{-10\beta_1-5\beta_2} + 2e^{-11\beta_1-4\beta_2} + e^{-12\beta_1-3\beta_2} + e^{-13\beta_1-5\beta_2} + e^{-14\beta_1-3\beta_2} + e^{-14\beta_1-4\beta_2} + e^{-15\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-11\beta_1-5\beta_2} + e^{-14\beta_1-5\beta_2} + e^{-15\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 14, 55, 120, 153, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 8, 20, 28, 28, 21, 11, 3)$

S=$5(3\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 19, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 7, 7, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,3}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,3,6}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$;

$d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-4\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-5\beta_1} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-9\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{2,3,5}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,6,7} - \omega^{2,3,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,7} - \omega^{2,5,6}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-10\beta_1-2\beta_2} \langle\omega^{1,3,4,7}\rangle + e^{-12\beta_1-2\beta_2} \langle\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-8\beta_1-3\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{2,3,4,7} - \omega^{2,3,5,6}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,5,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-12\beta_1-3\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-14\beta_1-3\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-15\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-15\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,5,7,7,5,3,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,7}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}-\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,7}-\omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$;

$d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,7}+\omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7}+2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}+\omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7}+\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7}+2\omega^{1,4,6}-\omega^{2,3,4}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}+\omega^{1,5,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7}+\omega^{1,5,6}-\omega^{2,3,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,5,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,6,7} + \omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}+\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7}+2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,7}+\omega^{1,4,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7}+2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,4,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,5,6,7} + 2\omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$;

$\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7} + 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{A}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-4\beta_1} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{A}})= e^{-5\beta_1} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{A}})= e^{-9\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,2,7} + \omega^{2,3,4}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,7} - 2\omega^{1,5,6} - \omega^{2,3,5}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,6,7} - \omega^{2,3,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,7} - \omega^{2,5,6}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{2,3,6} + 2\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{A}})= e^{-6\beta_1-4\beta_2} \langle-\omega^{2,4,5,6}\rangle + e^{-10\beta_1-3\beta_2} \langle(-\omega^{1,5,6,7} + \omega^{3,4,5,6})/2\rangle + e^{-9\beta_1-3\beta_2} \langle(-\omega^{1,4,6,7} - 2\omega^{2,3,4,7} + \omega^{2,3,5,6})/6\rangle + e^{-7\beta_1-3\beta_2} \langle(-\omega^{1,4,5,6} + \omega^{2,3,4,5})/2\rangle + e^{-12\beta_1-2\beta_2} \langle-\omega^{1,3,6,7}\rangle + e^{-10\beta_1-2\beta_2} \langle(\omega^{1,3,4,7} - \omega^{1,3,5,6})/2\rangle + e^{-8\beta_1-3\beta_2} \langle(2\omega^{1,2,6,7} - \omega^{1,4,5,7} - \omega^{2,3,4,6})/6\rangle.$

$\mathrm{ch}(H^5_{\mathcal{A}})= e^{-10\beta_1-5\beta_2} \langle-\omega^{2,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-14\beta_1-3\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-12\beta_1-3\beta_2} \langle(-\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{A}})= e^{-14\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-15\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{A}})= e^{-15\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y^2)^2(1+Y)^3.$

5.4.21 Algebra $\mathcal{G}_{7,2.17}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^4 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_2^4 & -\xi_2^3 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_2^5, ad\ x_4=-\delta_1^5-\delta_2^6+\delta_3^7, ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0$.

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6,\delta_3^7\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^6,\delta_2^3]=\delta_1^7$; $[\delta_2^6,\delta_2^3]=\delta_2^7$; $[\delta_3^6,\delta_2^3]=\delta_3^7+\delta_2^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=\delta_3^7-\delta_2^6-\delta_1^5$; $[\delta_1^4,\delta_2^3]=-\delta_2^5$; $[\delta_3^7,\delta_2^3]=\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1-\beta_2}\ \langle\delta_2^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^6,\delta_2^3]=2\delta_2^6+\delta_1^5$; $[\delta_2^6,\delta_2^3]=\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,4}-\omega^{2,3})+2x_2\omega^{2,4}+x_3(\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,5}\rangle+e^{-2\beta_1+2\beta_2}\ \langle x_6\omega^{1,3}\rangle+3e^{-\beta_1}\ \langle 1/2x_2\omega^{1,2}-1/2x_3\omega^{1,3}+1/2x_4\omega^{1,4}+x_6\omega^{1,6}, 1/2x_2\omega^{1,2}+1/2x_3\omega^{1,3}-1/2x_4\omega^{1,4}+x_5\omega^{1,5}, 1/2x_2\omega^{1,2}+1/2x_3\omega^{1,3}-1/2x_4(\omega^{1,4}-2\omega^{2,3})\rangle+2e^{-3\beta_1+\beta_2}\ \langle -x_2\omega^{1,3}+x_6\omega^{1,7}, -x_2\omega^{1,3}+x_6\omega^{3,5}\rangle+4e^{-2\beta_1}\ \langle -1/2x_2(\omega^{1,4}+\omega^{2,3})+1/2x_4\omega^{3,4}+x_6\omega^{3,6}, x_1\omega^{1,3}+5/2x_2(\omega^{1,4}-1/5\omega^{2,3})-1/2x_4\omega^{3,4}+x_5\omega^{3,5}, x_2\omega^{1,4}+x_5\omega^{1,7}, -x_1\omega^{1,3}-5/2x_2(\omega^{1,4}-1/5\omega^{2,3})-x_4(\omega^{1,5}-$

$3/2\omega^{3,4}) + x_6\omega^{2,7}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_7\omega^{2,6}\rangle + 4e^{-\beta_2}\ \langle -x_1\omega^{1,2} + x_3(\omega^{1,4} + \omega^{2,3}) + x_7\omega^{2,7}, x_3\omega^{1,4} + x_5\omega^{1,6}, x_6\omega^{2,6}, x_3(\omega^{1,4} - \omega^{2,3}) + x_4\omega^{2,4}\rangle + e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$3e^{-\beta_1-3\beta_2} + 2e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + e^{-3\beta_1-\beta_2} + 5e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1} + 4e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + e^{-5\beta_1+\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1} + e^{-6\beta_1} + e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-2\beta_1-4\beta_2} + e^{-2\beta_1-5\beta_2} + 4e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2} + 2e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2} + 3e^{-4\beta_1-4\beta_2} + 4e^{-5\beta_1-2\beta_2} + 2e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 3e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-7\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-4\beta_1-5\beta_2} + 2e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2} + 5e^{-6\beta_1-4\beta_2} + e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + 4e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-8\beta_1-2\beta_2} + 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-7\beta_1-4\beta_2} + 2e^{-7\beta_1-6\beta_2} + e^{-8\beta_1-4\beta_2} + 2e^{-8\beta_1-5\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-8\beta_1-6\beta_2} + e^{-9\beta_1-6\beta_2} + e^{-10\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 13, 53, 122, 154, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 8, 17, 28, 31, 23, 12, 3)$

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^6 = -\omega^{2,4}$; $d\omega^7 = -\omega^{1,5} + \omega^{3,4}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = -\omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5} - \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}$; $d\omega^{6,7} = \omega^{1,5,6} - \omega^{2,4,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = \omega^{1,2,3,6} - \omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,2,3,7} + \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} + \omega^{1,3,4,6}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7} + \omega^{2,3,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$;

$d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - 2\omega^{3,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}, \omega^{2,3,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,5}, \omega^{1,2,7} + \omega^{2,3,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-8\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + 2e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,6,7} + \omega^{2,3,5,7}, \omega^{1,4,5,7} - \omega^{2,3,5,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,6,7} - \omega^{2,3,5,6}\rangle + 2e^{-6\beta_1-4\beta_2} \langle\omega^{1,4,6,7}, \omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{2,4,5,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,5,7}\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 2Y + 4Y^2 + 3Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} - \omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$;

$d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,5,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{1,5} = 2\omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}-\omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7}+\omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4}+\omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6}+\omega^{2,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7}+\omega^{3,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,7}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7}+\omega^{3,6}-\omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 3\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}-\omega^{2,5,7}-\omega^{3,5,6}$; $\Delta\omega^{2,3,4} = 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7}-\omega^{1,3,6}+\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7}+2\omega^{2,5,7}+\omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,7}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7}+2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,6,7}+\omega^{2,5,7}+2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + 2\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$;

$\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 3\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + 2\omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,6,7} + \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-3\beta_1}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7} - 2\omega^{3,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-6\beta_1-\beta_2}\ \langle\omega^{1,3,7}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} - \omega^{1,4,5} + 2\omega^{2,3,5}, \omega^{1,3,6} + \omega^{2,3,5}\rangle + 2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,6} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-5\beta_1-2\beta_2}\ \langle 2\omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-5\beta_2}\ \langle -\omega^{2,4,5,6}\rangle + 2e^{-6\beta_1-4\beta_2}\ \langle(\omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6})/4, (\omega^{1,4,6,7} + \omega^{2,3,6,7} + 3\omega^{2,4,5,7} - \omega^{3,4,5,6})/4\rangle + 2e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,3,6,7} + \omega^{1,4,5,7} - 2\omega^{2,3,5,7})/3, (-\omega^{1,3,6,7} - 2\omega^{1,4,5,7} + \omega^{2,3,5,7})/3\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-4\beta_2}\ \langle(-\omega^{1,2,6,7} - \omega^{1,4,5,6} + 2\omega^{2,3,5,6})/6\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_1-6\beta_2}\ \langle -\omega^{2,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-8\beta_1-4\beta_2}\ \langle(\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-9\beta_1-3\beta_2}\ \langle -\omega^{1,3,4,5,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle(\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-7\beta_1-4\beta_2}\ \langle(-\omega^{1,2,3,6,7} - 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/6\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + 2Y + 4Y^2 + 3Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y).$

5.4.22 Algebra $\mathcal{G}_{7,2.18}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_7$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & \xi_1^2 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3+\xi_2^5 & \xi_2^4 & 0 & 3\xi_1^1+\xi_2^2\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^7, ad\ x_3=-\delta_2^7, ad\ x_4=-(\delta_1^5+\delta_2^6), ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{-2\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+2e^{\beta_2}\ \langle\delta_1^4,\delta_3^7\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^4,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-(\delta_2^6+\delta_1^5)$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_1^3]=-\delta_1^5$; $[\delta_1^2,\delta_2^5]=\delta_2^6-\delta_1^5$; $[\delta_1^2,\delta_2^4]=\delta_3^7-\delta_1^4$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{-2\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_1^2]=\delta_2^5+\delta_1^3$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^2,\delta_1^3]=-\delta_1^5$; $[\delta_1^2,\delta_2^5]=-2\delta_1^5$; $[\delta_2^3,\delta_3^6]=\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{2\beta_1-3\beta_2}\ \langle x_3\omega^{2,6}\rangle+e^{-2\beta_2}\ \langle x_1\omega^{2,4}-x_3(\omega^{2,7}-\omega^{4,5})\rangle+4e^{-\beta_2}\ \langle -x_1\omega^{1,2}+2x_3\omega^{1,5}+x_5\omega^{2,5},x_1\omega^{1,2}-x_3\omega^{1,5}+x_5\omega^{1,6},x_3\omega^{2,3},x_1\omega^{1,2}-x_3\omega^{1,5}+x_4\omega^{2,4}\rangle+2e^{\beta_1-2\beta_2}\ \langle x_3(\omega^{1,6}+\omega^{2,5})+x_5\omega^{2,6},-x_3(\omega^{1,6}+\omega^{2,5})+x_7\omega^{4,6}\rangle+2e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,4}+x_3(\omega^{1,7}-\omega^{3,4})+x_4\omega^{2,5}+x_5\omega^{2,7},-x_1\omega^{1,4}-2x_2\omega^{2,4}-x_3(\omega^{1,7}-\omega^{3,4})+3x_4\omega^{1,6}-3x_6\omega^{4,6}+x_7\omega^{4,7}\rangle+e^{-3\beta_1+2\beta_2}\ \langle x_6\omega^{1,3}\rangle+4e^{-\beta_1}\ \langle x_2\omega^{1,2}-x_3\omega^{1,3}+x_6\omega^{1,6},x_3\omega^{1,3}+x_5\omega^{2,3},x_5\omega^{1,5},-x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}\rangle+e^{-3\beta_1+\beta_2}\ \langle x_4\omega^{1,3}+x_6\omega^{1,7}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,3}\rangle+2e^{-2\beta_1}\ \langle -x_4\omega^{2,3}+x_5\omega^{1,7},-x_2\omega^{1,4}-x_4\omega^{2,3}+$

$x_5\omega^{3,4}\rangle + e^{-4\beta_1+\beta_2}\ \langle -x_2\omega^{1,3} + x_6\omega^{3,5}\rangle + 2e^{-3\beta_1}\ \langle 1/5x_1\omega^{1,3} - 3/5x_2\omega^{2,3} + 2/5x_4\omega^{3,4} + 1/5x_5\omega^{3,5} + x_6\omega^{3,6}, 2/5x_1\omega^{1,3} + x_2(\omega^{1,5} + 4/5\omega^{2,3}) - x_4(\omega^{1,7} + 1/5\omega^{3,4}) - 3/5x_5\omega^{3,5} + x_6\omega^{4,7}\rangle + e^{-4\beta_1}\ \langle x_2(\omega^{1,7} - \omega^{3,4}) - x_4\omega^{3,5} - x_5\omega^{3,7} + x_6\omega^{5,7}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-3\beta_2} + e^{\beta_1-4\beta_2} + 2e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2} + 3e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-\beta_2} + 6e^{-3\beta_1-2\beta_2} + 5e^{-4\beta_1-\beta_2} + 2e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + 3e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + e^{-7\beta_1} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-\beta_1-5\beta_2} + 3e^{-2\beta_1-4\beta_2} + 3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2} + 2e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1-3\beta_2} + 3e^{-4\beta_1-4\beta_2} + e^{-5\beta_1-2\beta_2} + 5e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 6e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + 3e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-8\beta_1-\beta_2} + e^{-8\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-6\beta_2} + 3e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-3\beta_2} + 3e^{-6\beta_1-4\beta_2} + e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-7\beta_1-2\beta_2} + 2e^{-7\beta_1-3\beta_2} + 2e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + 2e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + e^{-9\beta_1-2\beta_2} + 2e^{-9\beta_1-3\beta_2} + e^{-10\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-5\beta_1-6\beta_2} + e^{-7\beta_1-5\beta_2} + 2e^{-7\beta_1-6\beta_2} + e^{-8\beta_1-4\beta_2} + 2e^{-8\beta_1-5\beta_2} + 2e^{-9\beta_1-4\beta_2} + e^{-9\beta_1-6\beta_2} + e^{-10\beta_1-3\beta_2} + e^{-10\beta_1-5\beta_2} + e^{-11\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-8\beta_1-6\beta_2} + e^{-10\beta_1-6\beta_2} + e^{-11\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 58, 125, 157, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 24, 36, 37, 27, 13, 3)$

S=$11\beta_1 + 6\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{2,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,4,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7} + \omega^{2,3,5}$; $d\omega^{6,7} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7}$; $d\omega^{2,6,7} = \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$;

$d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,6} + \omega^{2,3,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,5,6} - \omega^{2,4,7}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,7} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,6}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{2,6,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,2,6,7}, \omega^{1,4,5,6}\rangle + e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,3,7}\rangle + e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,4,5,7} + \omega^{2,3,4,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,2,5,7}\rangle + e^{-8\beta_1-3\beta_2}\ \langle\omega^{1,3,6,7} + \omega^{2,3,5,7}\rangle + e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,4,5,6}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,4,6,7} + \omega^{2,3,5,6}\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-8\beta_1-4\beta_2}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-7\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-7\beta_1-6\beta_2}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-11\beta_1-5\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-11\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,7} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$;

$d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7} - \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,6} + 3\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,4,7}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{2,3,6} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,6} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{1,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$;

$\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,7} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,5,6} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-3\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-4\beta_1} \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-7\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{1,4,5}\rangle + e^{-5\beta_1-2\beta_2} \langle 2\omega^{1,3,6} + \omega^{1,4,7} + \omega^{2,3,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,5,6} - 2\omega^{2,3,6} - \omega^{2,4,7}\rangle + 2e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,6}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,6,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-4\beta_1-5\beta_2} \langle -\omega^{2,4,5,6}\rangle + e^{-7\beta_1-4\beta_2} \langle(-\omega^{2,3,6,7} + \omega^{3,4,5,6})/2\rangle + e^{-6\beta_1-4\beta_2} \langle(\omega^{1,4,6,7} + \omega^{2,3,5,6} + 2\omega^{2,4,5,7})/6\rangle + e^{-8\beta_1-3\beta_2} \langle(-\omega^{1,3,6,7} - \omega^{2,3,5,7})/2\rangle + e^{-9\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-7\beta_1-3\beta_2} \langle(\omega^{1,3,5,6} + 2\omega^{1,4,5,7} + \omega^{2,3,4,7})/6\rangle + 2e^{-5\beta_1-4\beta_2} \langle(\omega^{1,2,6,7} - \omega^{1,4,5,6} - 2\omega^{2,3,4,6})/3, (\omega^{1,2,6,7} + 2\omega^{1,4,5,6} + \omega^{2,3,4,6})/3\rangle + e^{-6\beta_1-3\beta_2} \langle(-\omega^{1,2,5,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5})/3\rangle + e^{-7\beta_1-2\beta_2} \langle(-\omega^{1,2,3,7} - \omega^{1,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-6\beta_2} \langle -\omega^{2,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-9\beta_1-4\beta_2} \langle(\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-10\beta_1-3\beta_2} \langle -\omega^{1,3,4,5,7}\rangle + e^{-7\beta_1-5\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-8\beta_1-4\beta_2} \langle(-\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-10\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-11\beta_1-5\beta_2} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-11\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

5.4.23 Algebra $\mathcal{G}_{7,2.19}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_6$; $[x_1, x_4] = x_5$; $[x_1, x_5] = x_7$; $[x_2, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & 2\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & \xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^3-\xi_1^4 & \xi_1^2 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_2^4 & 0 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^6, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^6, ad\ x_4 = -(\delta_1^5 + \delta_2^6), ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_2}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+2\beta_2}\ \langle \delta_1^3\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_1^5, \delta_2^6\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + 2e^{\beta_1}\ \langle \delta_3^6, \delta_2^4\rangle + 2e^{\beta_2}\ \langle \delta_2^3, \delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{2\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_3^7\rangle + e^{2\beta_1}\ \langle \delta_2^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^5, \delta_2^4] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_3^6, \delta_2^3] = \delta_2^6$; $[\delta_2^3, \delta_1^4] = \delta_1^6$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_2^4] = \delta_2^6$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -(\delta_2^6 + \delta_1^5)$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^5] = \delta_2^6 - \delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+2\beta_2}\ \langle \delta_1^3\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5\rangle + e^{\beta_2}\ \langle \delta_2^3\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_3^7\rangle + e^{2\beta_1}\ \langle \delta_2^5\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^2, \delta_2^5] = -2\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-4\beta_1+\beta_2}\ \langle x_3\omega^{1,7}\rangle + 4e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_5\omega^{2,5}, -x_4\omega^{2,4} + x_5\omega^{1,6}, -x_1\omega^{1,2} - x_3\omega^{2,3} - x_4\omega^{2,4} + x_7\omega^{2,7}, x_3\omega^{2,3} + x_4\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle x_3\omega^{1,3}, x_2\omega^{1,2} + x_3\omega^{2,4}, x_5\omega^{1,5}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_3\omega^{1,5}\rangle + 3e^{-2\beta_1}\ \langle x_5\omega^{1,7}, x_2\omega^{1,4}, x_3(\omega^{1,6} + \omega^{2,5}) + x_4\omega^{1,5}\rangle + 3e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4} + x_2\omega^{1,3} + x_4\omega^{2,5} + x_5\omega^{4,5}, x_2(\omega^{1,3} - \omega^{2,4}) + x_3(\omega^{2,6} + \omega^{3,4}) + x_4\omega^{1,6}, -x_1\omega^{1,4} - x_2(\omega^{1,3} - 2\omega^{2,4}) + x_4\omega^{2,5} + x_5\omega^{2,7} + x_7\omega^{4,7}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,5} + x_3(\omega^{2,7} - \omega^{4,5}) + x_4\omega^{1,7}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,7} + x_6\omega^{5,7}\rangle + e^{3\beta_1-2\beta_2}\ \langle x_7\omega^{2,3}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{1,3}\rangle + 2e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,3} +$

$x_7\omega^{2,6}, -x_5\omega^{2,3} + x_7\omega^{3,4}\rangle + 2e^{-2\beta_2}\ \langle -2/5x_1\omega^{1,3} + 1/5x_2\omega^{2,3} + 1/5x_4\omega^{3,4} + 3/5x_5\omega^{3,5} + x_7\omega^{3,7}, 1/5x_1\omega^{1,3} - 3/5x_2\omega^{2,3} + 2/5x_4\omega^{3,4} + 1/5x_5\omega^{3,5} + x_6\omega^{3,6}\rangle + e^{\beta_1-2\beta_2}\ \langle 2x_4\omega^{2,3} + x_5(\omega^{2,6} - \omega^{3,4}) + x_7\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{2\beta_1-4\beta_2} + e^{\beta_1-3\beta_2} + e^{\beta_1-4\beta_2} + 2e^{-\beta_1-2\beta_2} + 4e^{-\beta_1-3\beta_2} + 5e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-\beta_2} + 5e^{-3\beta_1-2\beta_2} + 4e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2} + e^{-4\beta_1} + 2e^{-5\beta_1-\beta_2} + e^{-5\beta_1} + e^{-6\beta_1} + 3e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{\beta_1-5\beta_2} + 2e^{-\beta_1-5\beta_2} + 3e^{-2\beta_1-4\beta_2} + 2e^{-2\beta_1-5\beta_2} + 3e^{-3\beta_1-3\beta_2} + 4e^{-3\beta_1-4\beta_2} + e^{-4\beta_1-2\beta_2} + 4e^{-4\beta_1-3\beta_2} + 2e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-2\beta_2} + 4e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 3e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-2\beta_1-6\beta_2} + e^{-3\beta_1-5\beta_2} + e^{-3\beta_1-7\beta_2} + 2e^{-4\beta_1-5\beta_2} + 2e^{-4\beta_1-6\beta_2} + 2e^{-5\beta_1-4\beta_2} + 2e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2} + 4e^{-6\beta_1-4\beta_2} + 2e^{-6\beta_1-5\beta_2} + 2e^{-7\beta_1-3\beta_2} + 2e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-8\beta_1-3\beta_2} + e^{-8\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-4\beta_1-7\beta_2} + e^{-5\beta_1-6\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-6\beta_1-8\beta_2} + e^{-7\beta_1-5\beta_2} + 2e^{-7\beta_1-6\beta_2} + e^{-7\beta_1-7\beta_2} + e^{-8\beta_1-4\beta_2} + 2e^{-8\beta_1-5\beta_2} + e^{-8\beta_1-6\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-7\beta_1-8\beta_2} + e^{-8\beta_1-6\beta_2} + e^{-8\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 58, 125, 156, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 24, 36, 36, 25, 12, 3)$

S=$8(\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 20, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 9, 9, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,4}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,5} = -\omega^{1,3,4}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = \omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,4,7}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7}$; $d\omega^{2,3,5} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5} - \omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,4,7}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,6,7}$;

$d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-3\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,3}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6} + \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{1,3,6}-\omega^{2,4,6}, \omega^{2,3,5}+\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,6,7} + \omega^{2,5,7}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{2,3,7} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,5,7}, \omega^{1,4,6,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,3,4,6}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{2,3,4,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,3,5,6} - \omega^{2,4,5,6}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,3,6,7} + \omega^{2,3,5,7}\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-6\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 5Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 20, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 9, 9, 6, 3, 1)$

Total dimension of trivial cohomology: 38.

Length associated polynomial: 38.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,5} + \omega^{4,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,4,6} = -\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$;

$d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7} + \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5} - \omega^{4,6}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,5} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,6} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,4,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,5} + \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 2\omega^{1,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} - \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7} + 2\omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,6,7} + 2\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,6,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,5,7} + 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = -\omega^{1,3,5,7} + \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} =$

$-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7}=2\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+\omega^{2,3,5,7}-\omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7}=\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6}=\omega^{1,3,5,6}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,3,5,7}-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{1,3,6,7}-\omega^{2,3,5,7}+2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6}=\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{2,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_2}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7}\rangle+e^{-3\beta_2}\ \langle\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{1,3}-\omega^{2,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}+\omega^{3,4}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7}-\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}-\omega^{1,4,5}\rangle+e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,7}\rangle+e^{-\beta_1-5\beta_2}\ \langle\omega^{2,3,6}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,6}+\omega^{2,4,5}\rangle+2e^{-2\beta_1-4\beta_2}\ \langle\omega^{1,3,6}-\omega^{2,4,6},\omega^{2,3,5}+\omega^{2,4,6}\rangle+e^{-4\beta_1-3\beta_2}\ \langle 2\omega^{1,3,7}+\omega^{1,5,6}-\omega^{2,4,7}\rangle+e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,6,7}+\omega^{2,5,7}\rangle+e^{-3\beta_1-4\beta_2}\ \langle\omega^{2,3,7}+\omega^{3,4,5}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-6\beta_2}\ \langle(-\omega^{2,3,6,7}+\omega^{3,4,5,6})/2\rangle+e^{-2\beta_1-6\beta_2}\ \langle-\omega^{2,3,4,6}\rangle+e^{-7\beta_1-3\beta_2}\ \langle-\omega^{1,4,5,7}\rangle+e^{-5\beta_1-5\beta_2}\ \langle(-\omega^{1,3,6,7}-\omega^{2,3,5,7})/2\rangle+2e^{-6\beta_1-4\beta_2}\ \langle(-\omega^{1,3,5,7}+\omega^{1,4,6,7}+2\omega^{2,4,5,7})/3,(\omega^{1,3,5,7}+2\omega^{1,4,6,7}+\omega^{2,4,5,7})/3\rangle+e^{-4\beta_1-5\beta_2}\ \langle(\omega^{1,3,5,6}+\omega^{2,3,4,7}-2\omega^{2,4,5,6})/6\rangle+e^{-3\beta_1-5\beta_2}\ \langle(\omega^{1,3,4,6}+\omega^{2,3,4,5})/2\rangle+e^{-5\beta_1-4\beta_2}\ \langle(\omega^{1,2,6,7}+\omega^{1,4,5,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-4\beta_1-7\beta_2}\ \langle-\omega^{2,3,4,5,6}\rangle+e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle+e^{-7\beta_1-6\beta_2}\ \langle(\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7})/2\rangle+e^{-6\beta_1-6\beta_2}\ \langle(\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/2\rangle+e^{-7\beta_1-5\beta_2}\ \langle(\omega^{1,2,5,6,7}-\omega^{1,3,4,5,7})/2\rangle+e^{-5\beta_1-6\beta_2}\ \langle(-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-7\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-8\beta_1-7\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-8\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-8\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+2Y+4Y^2+5Y^3+4Y^4+2Y^5+Y^6)(1+Y).$

5.4.24 Algebra $\mathcal{G}_{7,2.20}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & \xi_3^2 & 0 & 0\\ \xi_1^5 & 0 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & \xi_3^5 & 4\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & -\xi_1^5+\xi_3^6 & \xi_1^3+\xi_3^5 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_3^5+\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=0, ad\ x_5=-(\delta_1^6+\delta_3^7), ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{3\beta_1}\ \langle x_5\rangle+e^{4\beta_1}\ \langle x_6\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{5\beta_1}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{-\beta_1+\beta_2}\ \langle \delta_1^2,\delta_3^4\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+3e^{\beta_1}\ \langle\delta_2^4,\delta_1^3,\delta_3^5\rangle+2e^0\ \langle t_2,t_1\rangle+e^{-2\beta_1+\beta_2}\ \langle\delta_3^2\rangle+2e^{3\beta_1}\ \langle\delta_1^6,\delta_3^7\rangle+e^{4\beta_1}\ \langle\delta_1^7\rangle+e^{5\beta_1-\beta_2}\ \langle\delta_2^7\rangle+e^{4\beta_1-\beta_2}\ \langle\delta_2^6\rangle+2e^{2\beta_1}\ \langle\delta_1^5,\delta_3^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_2^4,\delta_3^2]=\delta_3^4$; $[\delta_2^4,\delta_2^6]=-\delta_2^7$; $[\delta_3^4,\delta_2^6]=-\delta_3^7$; $[\delta_3^4,\delta_1^3]=\delta_1^4$; $[\delta_3^2,\delta_2^7]=-\delta_3^7$; $[\delta_3^2,\delta_2^6]=-\delta_3^6$; $[\delta_3^2,\delta_1^5]=\delta_1^4$; $[\delta_3^2,\delta_1^3]=\delta_1^2$; $[\delta_3^2,\delta_3^5]=\delta_3^4$; $[\delta_1^6,\delta_1^3]=-\delta_1^7$; $[\delta_1^6,\delta_3^5]=-\delta_1^7$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^3]=-\delta_2^7$; $[\delta_2^6,\delta_3^5]=-\delta_2^7$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_1^5,\delta_3^5]=-(\delta_3^7+\delta_1^6)$; $[\delta_3^6,\delta_1^3]=-\delta_3^7+\delta_1^6$; $[\delta_1^3,\delta_3^5]=-\delta_1^5$.

ch($H^1_{\mathcal{G}}$)= $2e^{-\beta_1+\beta_2}\ \langle\delta_1^2,\delta_3^4\rangle+2e^0\ \langle t_2,t_1\rangle+e^{-2\beta_1+\beta_2}\ \langle\delta_3^2\rangle+e^{5\beta_1-\beta_2}\ \langle\delta_2^7\rangle+e^{3\beta_1}\ \langle\delta_1^6\rangle+e^{4\beta_1-\beta_2}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_3^6\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_3^2,\delta_2^6]=-\delta_3^6$; $[\delta_3^2,\delta_1^3]=\delta_1^2$; $[\delta_3^2,\delta_2^4]=-\delta_3^4$; $[\delta_2^6,\delta_1^3]=-\delta_2^7$; $[\delta_2^6,\delta_2^4]=\delta_2^7$; $[\delta_3^4,\delta_2^6]=\delta_1^6$; $[\delta_3^2,\delta_2^7]=\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $6e^{-2\beta_1}\ \langle x_2\omega^{1,4},-x_2\omega^{2,3}+x_4\omega^{3,4},-x_1\omega^{1,3}+x_2\omega^{2,3}+2x_5\omega^{1,6}+x_6\omega^{3,6},x_2\omega^{2,3}+x_4\omega^{2,5},x_1\omega^{1,3}-x_2\omega^{2,3}-x_5\omega^{1,6}+x_6\omega^{1,7},x_3\omega^{1,5}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2},x_4\omega^{2,3},x_3\omega^{1,3}+x_6\omega^{3,5}\rangle+2e^{-5\beta_1+\beta_2}\ \langle x_2\omega^{1,6},x_2\omega^{3,5}+x_4\omega^{3,6}\rangle+e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,5}\rangle+2e^{-6\beta_1+\beta_2}\ \langle x_2(\omega^{1,7}+\omega^{3,6})+x_4\omega^{3,7},x_2(\omega^{1,7}+\omega^{3,6})+x_4\omega^{5,6}\rangle+3e^{-3\beta_1}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})+2x_3(\omega^{1,6}-\omega^{3,5})+x_4\omega^{4,5},x_5(\omega^{1,7}+$

$\omega^{3,6}) + x_6\omega^{3,7}, x_1\omega^{1,5} + x_3(\omega^{1,6} - \omega^{3,5}) + x_4\omega^{2,6}\rangle + 2e^{2\beta_1-\beta_2}\ \langle -x_5\omega^{1,2} + x_6\omega^{2,3}, x_6\omega^{1,4}\rangle + 2e^{\beta_1-\beta_2}\ \langle x_5(\omega^{1,4} - \omega^{2,3}) + x_6\omega^{3,4}, x_3\omega^{1,2} + x_5\omega^{2,3} + x_6\omega^{2,5}\rangle + e^{3\beta_1-\beta_2}\ \langle x_6\omega^{1,2}\rangle + e^{4\beta_1-2\beta_2}\ \langle x_7\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle -1/5x_1\omega^{1,2} + 2/5x_3\omega^{2,3} + 3/5x_5\omega^{2,5} + 4/5x_6\omega^{2,6} + x_7\omega^{2,7}, x_4\omega^{2,4}, -3/5x_1\omega^{1,2} + x_3(\omega^{1,4} + 1/5\omega^{2,3}) - 1/5x_5(\omega^{2,5} + 5\omega^{3,4}) - 3/5x_6(\omega^{2,6} - 5/3\omega^{4,5})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{2\beta_1-2\beta_2} + e^{\beta_1-2\beta_2} + e^{-\beta_1-2\beta_2} + 6e^{-2\beta_1-\beta_2} + 6e^{-3\beta_1-\beta_2} + 3e^{-4\beta_1-\beta_2} + 4e^{-5\beta_1} + 6e^{-6\beta_1} + 5e^{-7\beta_1} + 2e^{-8\beta_1+\beta_2} + e^{-8\beta_1} + 2e^{-9\beta_1+\beta_2} + e^{-10\beta_1+\beta_2} + e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-2\beta_1-2\beta_2} + 3e^{-3\beta_1-2\beta_2} + 2e^{-4\beta_1-2\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + 5e^{-6\beta_1-\beta_2} + 7e^{-7\beta_1-\beta_2} + 4e^{-8\beta_1-\beta_2} + 2e^{-8\beta_1} + e^{-9\beta_1-\beta_2} + 5e^{-9\beta_1} + 4e^{-10\beta_1} + 2e^{-11\beta_1} + e^{-12\beta_1+\beta_2} + 2e^{-13\beta_1+\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-6\beta_1-2\beta_2} + 2e^{-7\beta_1-2\beta_2} + 3e^{-8\beta_1-2\beta_2} + 3e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + 4e^{-10\beta_1-\beta_2} + 3e^{-11\beta_1-\beta_2} + 2e^{-12\beta_1-\beta_2} + e^{-13\beta_1-\beta_2} + 3e^{-13\beta_1} + 3e^{-14\beta_1} + e^{-15\beta_1+\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-9\beta_1-2\beta_2} + e^{-10\beta_1-2\beta_2} + e^{-11\beta_1-2\beta_2} + e^{-12\beta_1-2\beta_2} + e^{-13\beta_1-\beta_2} + e^{-13\beta_1-2\beta_2} + 3e^{-14\beta_1-\beta_2} + e^{-14\beta_1-2\beta_2} + e^{-15\beta_1-\beta_2} + e^{-15\beta_1} + e^{-16\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-14\beta_1-2\beta_2} + e^{-15\beta_1-2\beta_2} + e^{-16\beta_1-\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 59, 128, 158, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 26, 40, 41, 28, 13, 3)$

S$=2(8\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 20, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 10, 10, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6} - \omega^{3,5}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$;

$d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-2\beta_1-\beta_2} \langle\omega^{1,4}, \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-5\beta_1} \langle\omega^{1,6}\rangle + e^{-6\beta_1} \langle\omega^{1,7} + \omega^{3,6}\rangle + e^{-7\beta_1} \langle\omega^{3,7} - \omega^{5,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-8\beta_1} \langle\omega^{1,3,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,2,6}\rangle + 2e^{-7\beta_1-\beta_2} \langle\omega^{2,3,7} - \omega^{2,5,6}, \omega^{1,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + 2e^{-6\beta_1-\beta_2} \langle\omega^{1,2,7}+\omega^{3,4,5}, \omega^{2,3,6}+\omega^{3,4,5}\rangle + e^{-9\beta_1} \langle\omega^{1,5,7}\rangle + e^{-10\beta_1} \langle\omega^{1,6,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-9\beta_1-\beta_2} \langle\omega^{1,3,4,7}, \omega^{1,2,5,7}\rangle + 2e^{-10\beta_1-\beta_2} \langle\omega^{1,2,6,7}-\omega^{3,4,5,6}, \omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{-13\beta_1} \langle\omega^{1,5,6,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,2,4,7} - \omega^{2,3,4,6}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{2,3,4,7} + \omega^{2,4,5,6}\rangle + e^{-11\beta_1-\beta_2} \langle\omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{3,4,5,7}\rangle + e^{-14\beta_1} \langle\omega^{3,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-9\beta_1-2\beta_2} \langle\omega^{1,2,3,4,7}\rangle + e^{-13\beta_1-\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-15\beta_1} \langle\omega^{1,3,5,6,7}\rangle + 2e^{-14\beta_1-\beta_2} \langle\omega^{1,4,5,6,7}, \omega^{2,3,5,6,7}\rangle + e^{-10\beta_1-2\beta_2} \langle\omega^{1,2,4,5,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-14\beta_1-2\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-16\beta_1-\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-15\beta_1-2\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-16\beta_1-2\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 20, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 10, 10, 7, 3, 1)$

Total dimension of trivial cohomology: 42.

Length associated polynomial: 42.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$;

$d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{2,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} - \omega^{2,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} - \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} =$

$\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7} - \omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} - \omega^{2,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 2e^{-2\beta_1-\beta_2} \langle\omega^{1,4}, \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-5\beta_1} \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-6\beta_1} \langle\omega^{1,7} + \omega^{3,6}\rangle + e^{-7\beta_1} \langle\omega^{3,7} - \omega^{5,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-8\beta_1} \langle\omega^{1,3,7} - \omega^{1,5,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,3,4}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,5}\rangle + 2e^{-7\beta_1-\beta_2} \langle\omega^{2,3,7} - \omega^{2,5,6}, \omega^{1,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + 2e^{-6\beta_1-\beta_2} \langle\omega^{1,2,7} - 2\omega^{2,3,6} - \omega^{3,4,5}, \omega^{1,4,6} - \omega^{2,3,6} - \omega^{3,4,5}\rangle + e^{-9\beta_1} \langle\omega^{1,5,7} + \omega^{3,5,6}\rangle + e^{-10\beta_1} \langle\omega^{1,6,7} - \omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-14\beta_1} \langle-\omega^{3,5,6,7}\rangle + e^{-8\beta_1-2\beta_2} \langle(-\omega^{2,3,4,7} - \omega^{2,4,5,6})/2\rangle + e^{-13\beta_1} \langle-\omega^{1,5,6,7}\rangle + e^{-11\beta_1-\beta_2} \langle(\omega^{1,4,6,7} - \omega^{2,3,6,7} - \omega^{3,4,5,7})/3\rangle + 2e^{-9\beta_1-\beta_2} \langle(-\omega^{1,2,5,7} + 2\omega^{1,3,4,7} + 3\omega^{1,4,5,6} + \omega^{2,3,5,6})/5, (-2\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6})/5\rangle + 2e^{-10\beta_1-\beta_2} \langle(\omega^{1,4,5,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6})/3, (-\omega^{1,2,6,7} - 2\omega^{2,3,5,7} - \omega^{3,4,5,6})/3\rangle + e^{-7\beta_1-2\beta_2} \langle(\omega^{1,2,4,7} - \omega^{2,3,4,6})/2\rangle + e^{-6\beta_1-2\beta_2} \langle(\omega^{1,2,4,6} + \omega^{2,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = 2e^{-14\beta_1-\beta_2} \langle\omega^{2,3,5,6,7}, \omega^{1,4,5,6,7}\rangle + e^{-15\beta_1} \langle-\omega^{1,3,5,6,7}\rangle + e^{-13\beta_1-\beta_2} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-11\beta_1-2\beta_2} \langle(\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-10\beta_1-2\beta_2} \langle(\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-9\beta_1-2\beta_2} \langle(-\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-15\beta_1-2\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-16\beta_1-\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1-2\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-16\beta_1-2\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.4.25 Algebra $\mathcal{G}_{7,2.21}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 4\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^3+\xi_2^6 & \xi_1^2+\xi_3^6 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4+\delta_3^7, ad\ x_3=-(\delta_1^5+\delta_2^7), ad\ x_4=-\delta_1^6, ad\ x_5=0, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{3\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{4\beta_1}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{4\beta_1}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{\beta_2}\ \langle\delta_1^4,\delta_3^7\rangle+2e^{3\beta_1}\ \langle\delta_1^5,\delta_2^7\rangle+e^{4\beta_1-\beta_2}\ \langle\delta_2^5\rangle+2e^{\beta_1}\ \langle\delta_3^5,\delta_2^4\rangle+e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^6\rangle+2e^{-\beta_1+\beta_2}\ \langle\delta_1^2,\delta_3^6\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^3]=-\delta_1^5$; $[\delta_1^4,\delta_1^3]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^5,\delta_1^2]=-\delta_1^7$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_2^5,\delta_1^2]=-\delta_2^7+\delta_1^5$; $[\delta_2^5,\delta_3^6]=-\delta_2^7$; $[\delta_3^5,\delta_2^3]=\delta_2^5$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_3^5,\delta_1^2]=-\delta_3^7$; $[\delta_3^5,\delta_3^6]=-\delta_3^7$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_2^4]=\delta_2^5$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_1^3,\delta_3^6]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_2^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_3^6,\delta_2^4]=-\delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{4\beta_1-\beta_2}\ \langle\delta_2^5\rangle+e^{3\beta_1-\beta_2}\ \langle\delta_2^3\rangle+e^{3\beta_1}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1}\ \langle\delta_1^3,\delta_2^6\rangle+2e^{-\beta_1+\beta_2}\ \langle\delta_1^2,\delta_3^6\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^2]=2\delta_1^5$; $[\delta_2^3,\delta_1^4]=\delta_1^5$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_2^4]=\delta_2^5$; $[\delta_2^5,\delta_3^6]=\delta_1^5$; $[\delta_1^3,\delta_2^4]=\delta_1^5$; $[\delta_3^6,\delta_2^4]=-\delta_1^4$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$.

$\mathrm{ch}(H^2_{\mathcal{G}})=4e^{-3\beta_1}\ \langle x_1\omega^{1,3}+x_2(\omega^{1,6}-2\omega^{2,3})+x_4\omega^{3,4},-x_1\omega^{1,3}-2x_2(\omega^{1,6}-1/2\omega^{2,3})+x_6\omega^{3,6},-x_2\omega^{2,3}+x_4\omega^{1,7},x_5\omega^{3,5}\rangle+e^{3\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle+5e^{-\beta_2}\ \langle x_3\omega^{2,3}+x_5\omega^{2,5},-x_1\omega^{1,2}+2x_3(\omega^{1,6}-1/2\omega^{2,3})+x_6\omega^{2,6},-x_3\omega^{2,3}+x_5\omega^{1,7},x_1\omega^{1,2}-x_3(\omega^{1,6}-\omega^{2,3})+x_4\omega^{2,4},-x_3\omega^{2,3}+x_5\omega^{3,4}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}+x_5\omega^{2,6}\rangle+$

$e^{\beta_1-2\beta_2}\ \langle -x_5(\omega^{2,7}-\omega^{4,6})\rangle + 3e^{-2\beta_1}\ \langle x_3\omega^{1,5}+x_4\omega^{2,3}+x_6\omega^{2,5}, x_4\omega^{1,6}, -x_2\omega^{1,4} - x_3\omega^{1,5}+x_7\omega^{4,5}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,4}+x_3(\omega^{1,7}-\omega^{3,4})+x_4\omega^{2,6}-x_5\omega^{3,6}+x_6\omega^{2,7}, -x_1\omega^{1,4}+x_2\omega^{2,4}-x_3(\omega^{1,7}+3\omega^{2,5}+2\omega^{3,4})-6x_5(\omega^{3,6}+1/2\omega^{4,5})+x_7\omega^{4,7}\rangle + e^{-5\beta_1+\beta_2}\ \langle -x_2\omega^{1,5}+x_6\omega^{3,5}\rangle + e^{-4\beta_1}\ \langle -x_2(\omega^{1,7}-\omega^{3,4})+x_4\omega^{3,6}+x_6\omega^{3,7}\rangle + 4e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_6\omega^{2,3}, -x_2\omega^{1,2}+x_3\omega^{1,3}, x_5\omega^{1,5}, x_4\omega^{1,4}\rangle + e^{2\beta_1-\beta_2}\ \langle x_3\omega^{1,2}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + 2e^{-4\beta_1+\beta_2}\ \langle x_2\omega^{1,3}, x_4\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-3\beta_2}+4e^{-\beta_1-2\beta_2}+e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+4e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2}+8e^{-4\beta_1-\beta_2}+2e^{-5\beta_1-\beta_2}+4e^{-5\beta_1}+e^{-6\beta_1-\beta_2}+2e^{-6\beta_1}+4e^{-7\beta_1}+e^{-8\beta_1+\beta_2}+e^{-8\beta_1}+e^{-2\beta_2}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-2\beta_1-4\beta_2}+4e^{-3\beta_1-3\beta_2}+4e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-3\beta_2}+4e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+6e^{-6\beta_1-2\beta_2}+5e^{-7\beta_1-\beta_2}+3e^{-7\beta_1-2\beta_2}+5e^{-8\beta_1-\beta_2}+e^{-8\beta_1}+2e^{-9\beta_1-\beta_2}+e^{-9\beta_1}+e^{-10\beta_1-\beta_2}+e^{-10\beta_1}+e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-6\beta_1-4\beta_2}+5e^{-7\beta_1-3\beta_2}+3e^{-8\beta_1-2\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}+2e^{-9\beta_1-2\beta_2}+2e^{-9\beta_1-3\beta_2}+5e^{-10\beta_1-2\beta_2}+2e^{-11\beta_1-\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-12\beta_1-\beta_2}+e^{-13\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-4\beta_2}+e^{-9\beta_1-4\beta_2}+2e^{-10\beta_1-3\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-2\beta_2}+2e^{-11\beta_1-3\beta_2}+e^{-12\beta_1-4\beta_2}+e^{-13\beta_1-2\beta_2}+e^{-13\beta_1-3\beta_2}+e^{-14\beta_1-2\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-11\beta_1-4\beta_2}+e^{-13\beta_1-4\beta_2}+e^{-14\beta_1-3\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,16,60,129,158,115,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,11,27,42,42,28,13,3)$

S=$2(7\beta_1+2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,20,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,10,10,7,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,3}$; $d\omega^{1,7}=\omega^{1,2,3}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}+\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{1,4,5}$; $d\omega^{5,7}=-\omega^{1,3,7}-\omega^{1,5,6}+\omega^{2,3,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{2,3,6}$; $d\omega^{1,4,7}=-\omega^{1,2,3,4}$; $d\omega^{1,5,7}=-\omega^{1,2,3,5}$; $d\omega^{1,6,7}=-\omega^{1,2,3,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}+\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,4,5,6}-$

$\omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} - \omega^{2,3,5,6}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-3\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-5\beta_1} \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + 2e^{-4\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-7\beta_1} \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}, \omega^{1,4,6}\rangle + e^{-8\beta_1} \langle\omega^{1,3,5}\rangle + 2e^{-7\beta_1-\beta_2} \langle\omega^{1,3,7}, \omega^{1,5,6}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-8\beta_1-\beta_2} \langle\omega^{1,5,7} + \omega^{3,4,5}\rangle + 2e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7} + \omega^{3,4,6}, \omega^{2,3,7} + \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-8\beta_1-2\beta_2} \langle\omega^{1,2,5,7}, \omega^{1,4,5,6}\rangle + e^{-11\beta_1-\beta_2} \langle\omega^{1,3,5,7}\rangle + 2e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7} - \omega^{2,4,5,6}, \omega^{2,3,4,7} + \omega^{2,4,5,6}\rangle + 2e^{-10\beta_1-2\beta_2} \langle\omega^{1,5,6,7} - \omega^{3,4,5,6}, \omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-9\beta_1-2\beta_2} \langle\omega^{1,3,6,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + 2e^{-10\beta_1-3\beta_2} \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{1,2,3,5,7}\rangle + e^{-13\beta_1-2\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-11\beta_1-3\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-9\beta_1-4\beta_2} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-4\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-14\beta_1-3\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-13\beta_1-4\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-14\beta_1-4\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 20, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 10, 10, 7, 3, 1)$

Total dimension of trivial cohomology: 42.

Length associated polynomial: 42.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,7} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$;

$d^{\star}\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^{\star}\omega^{1,3,7} = -\omega^{5,7}$; $d^{\star}\omega^{1,4,5} = \omega^{5,6}$; $d^{\star}\omega^{1,4,6} = -\omega^{4,7}$; $d^{\star}\omega^{1,4,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,5,6} = -\omega^{5,7}$; $d^{\star}\omega^{2,3,4} = \omega^{4,7}$; $d^{\star}\omega^{2,3,5} = \omega^{5,7}$; $d^{\star}\omega^{2,3,6} = \omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{1,4,7} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,7} + \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{1,6,7} - \omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^{\star}\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^{\star}\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^{\star}\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = -\omega^{4,5,7}$; $d^{\star}\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,4,5,7} + \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,6} + \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,7} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,7} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,3,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,3,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,7} - \omega^{1,4,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,6,7} + \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,7} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{1,6,7} + \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,6,7} - \omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 4\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 4\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{2,3,4,6}$;

$\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,4,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,6,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,3,6,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,5,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,4,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,5,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-5\beta_1}\ \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + 2e^{-4\beta_1-\beta_2}\ \langle\omega^{1,7} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-7\beta_1}\ \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-8\beta_1}\ \langle\omega^{1,3,5}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} + \omega^{2,3,4}, \omega^{1,4,6} + \omega^{2,3,4}\rangle + 2e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,7} + \omega^{2,3,5}, \omega^{1,5,6} + \omega^{2,3,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,4,7} + \omega^{2,3,6} + 2\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-8\beta_1-\beta_2}\ \langle\omega^{1,5,7} + \omega^{3,4,5}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,6,7} + \omega^{3,4,6}, \omega^{2,3,7} + \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2}\ \langle-\omega^{2,4,6,7}\rangle + 2e^{-10\beta_1-2\beta_2}\ \langle(-\omega^{1,5,6,7} + \omega^{2,3,5,7} + 2\omega^{3,4,5,6})/3, (-\omega^{1,5,6,7} - 2\omega^{2,3,5,7} - \omega^{3,4,5,6})/3\rangle + 2e^{-7\beta_1-3\beta_2}\ \langle(\omega^{1,4,6,7} - \omega^{2,3,4,7} - 2\omega^{2,4,5,6})/3, (\omega^{1,4,6,7} + 2\omega^{2,3,4,7} + \omega^{2,4,5,6})/3\rangle + e^{-9\beta_1-2\beta_2}\ \langle(-2\omega^{1,3,6,7} - \omega^{1,4,5,7} + \omega^{2,3,5,6})/6\rangle + e^{-11\beta_1-\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,2,6,7} - \omega^{2,3,4,6})/2\rangle + 2e^{-8\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,7} - \omega^{1,4,5,6} + 2\omega^{2,3,4,5})/3, (-\omega^{1,2,5,7} + 2\omega^{1,4,5,6} - \omega^{2,3,4,5})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_1-4\beta_2}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-11\beta_1-3\beta_2}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-13\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle + 2e^{-10\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - 2\omega^{2,3,4,5,6})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,6})/3\rangle + e^{-7\beta_1-4\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle +$

$e^{-11\beta_1-2\beta_2}$ $\langle(\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})=$ $e^{-13\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ $+$ $e^{-14\beta_1-3\beta_2}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ $+$ $e^{-11\beta_1-4\beta_2}$ $\langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})=$ $e^{-14\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1+Y+3Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

5.4.26 Algebra $\mathcal{G}_{7,2.22}$

Adjoint cohomology

$[x_1, x_3] = x_4$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_3^4 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_1^2 & 2\xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & 0 & \xi_3^6 & \xi_3^4 & 0 & 4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4 + \xi_3^6 & 0 & \xi_3^4 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_3^4, ad\ x_2 = \delta_3^5, ad\ x_3 = -(\delta_2^5 + \delta_1^4), ad\ x_4 = -(\delta_1^6 + \delta_3^7), ad\ x_5 = 0, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T{=}\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G}){=}\ e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + e^{3\beta_1}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{4\beta_1}\ \langle x_6\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}){=}\ e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}){=}\ e^{-2\beta_1+\beta_2}\ \langle \delta_3^2\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5\rangle + 3e^{2\beta_1}\ \langle \delta_2^5, \delta_1^4, \delta_3^6\rangle + e^{\beta_2}\ \langle \delta_3^5\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^{0}\ \langle t_2, t_1\rangle + 2e^{3\beta_1}\ \langle \delta_1^6, \delta_3^7\rangle + e^{4\beta_1}\ \langle \delta_1^7\rangle + e^{5\beta_1-\beta_2}\ \langle \delta_2^7\rangle + e^{\beta_1}\ \langle \delta_3^4\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^2, \delta_2^5] = -\delta_3^5$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^4] = \delta_1^5$; $[\delta_1^2, \delta_3^4] = \delta_3^5$; $[\delta_1^6, \delta_3^4] = -\delta_1^7$; $[\delta_1^4, \delta_3^6] = -\delta_1^7$; $[\delta_1^4, \delta_3^4] = -(\delta_3^7 + \delta_1^6)$.

$\mathrm{ch}(H^1_{\mathcal{G}}){=}\ e^{-2\beta_1+\beta_2}\ \langle \delta_3^2\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^{0}\ \langle t_2, t_1\rangle + e^{5\beta_1-\beta_2}\ \langle \delta_2^7\rangle + e^{3\beta_1}\ \langle \delta_1^6\rangle + 2e^{2\beta_1}\ \langle \delta_1^4, \delta_3^6\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^2, \delta_2^7] = \delta_1^6$; $[\delta_1^2, \delta_1^4] = \delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}}){=}\ e^{-4\beta_1}\ \langle x_2\omega^{3,5}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_5\omega^{1,5}, x_2\omega^{1,2} - x_3\omega^{1,3} + x_4\omega^{1,4}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_6\omega^{3,4}\rangle + e^{-4\beta_1+\beta_2}\ \langle -x_2\omega^{1,4} + x_5\omega^{1,7}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle + 4e^{-2\beta_1}\ \langle x_2\omega^{2,3}, -x_1\omega^{1,3} + 2x_4\omega^{1,6} + x_6\omega^{3,6}, x_1\omega^{1,3} - x_4\omega^{1,6} + x_6\omega^{1,7}, x_1\omega^{1,3} - x_4(\omega^{1,6} - \omega^{3,4})\rangle + 2e^{-5\beta_1+\beta_2}\ \langle x_2\omega^{3,4}, x_2\omega^{1,6} + x_5\omega^{4,6}\rangle + 2e^{-3\beta_1}\ \langle x_1\omega^{1,4} + x_2(\omega^{1,5} + \omega^{2,4}) + x_3(\omega^{1,6} - \omega^{3,4}) - x_5(\omega^{2,7} - \omega^{4,5}), x_4(\omega^{1,7} + \omega^{3,6}) + x_6\omega^{3,7}\rangle + 3e^{-\beta_2}\ \langle 1/5x_1\omega^{1,2} - 2/5x_3\omega^{2,3} + x_4(\omega^{1,5} + 2/5\omega^{2,4}) + 1/5x_6(\omega^{2,6} + 5\omega^{3,5}), -1/5x_1\omega^{1,2} + 2/5x_3\omega^{2,3} + 3/5x_4\omega^{2,4} + 4/5x_6\omega^{2,6} + x_7\omega^{2,7}, x_5\omega^{2,5}\rangle + e^{2\beta_1-\beta_2}\ \langle -x_4\omega^{1,2} + x_6\omega^{2,3}\rangle + e^{3\beta_1-2\beta_2}\ \langle x_7\omega^{2,5}\rangle + 2e^{\beta_1-\beta_2}\ \langle x_3\omega^{1,2} + x_4\omega^{2,3} + x_6\omega^{2,4}, -x_4\omega^{2,3} + x_6\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2}+e^{-\beta_1-\beta_2}+e^{-\beta_1-2\beta_2}+2e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+4e^{-3\beta_1-\beta_2}+5e^{-4\beta_1-\beta_2}+e^{-5\beta_1-\beta_2}+4e^{-5\beta_1}+3e^{-6\beta_1}+5e^{-7\beta_1}+2e^{-8\beta_1+\beta_2}+e^{-8\beta_1}+e^{-9\beta_1+\beta_2}+e^{-10\beta_1+\beta_2}+e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-3\beta_1-2\beta_2}+3e^{-4\beta_1-2\beta_2}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2}+2e^{-6\beta_1-\beta_2}+e^{-6\beta_1-2\beta_2}+6e^{-7\beta_1-\beta_2}+3e^{-8\beta_1-\beta_2}+2e^{-8\beta_1}+2e^{-9\beta_1-\beta_2}+2e^{-9\beta_1}+e^{-10\beta_1-\beta_2}+4e^{-10\beta_1}+e^{-11\beta_1+\beta_2}+e^{-11\beta_1}+e^{-12\beta_1+\beta_2}+e^{-12\beta_1}+e^{-13\beta_1}+e^{-14\beta_1+\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-7\beta_1-2\beta_2}+2e^{-8\beta_1-2\beta_2}+3e^{-9\beta_1-2\beta_2}+3e^{-10\beta_1-\beta_2}+2e^{-10\beta_1-2\beta_2}+3e^{-11\beta_1-\beta_2}+3e^{-12\beta_1-\beta_2}+e^{-13\beta_1-\beta_2}+2e^{-13\beta_1}+3e^{-14\beta_1}+e^{-15\beta_1+\beta_2}+e^{-15\beta_1-\beta_2}+e^{-16\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-10\beta_1-2\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-12\beta_1-2\beta_2}+e^{-13\beta_1-\beta_2}+e^{-13\beta_1-2\beta_2}+e^{-14\beta_1-\beta_2}+e^{-14\beta_1-2\beta_2}+e^{-15\beta_1-\beta_2}+e^{-15\beta_1-2\beta_2}+e^{-15\beta_1}+2e^{-16\beta_1-\beta_2}+e^{-17\beta_1}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-15\beta_1-2\beta_2}+e^{-16\beta_1-2\beta_2}+e^{-17\beta_1-\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,14,56,125,156,115,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,9,21,34,36,26,13,3)$

S=$17\beta_1+2\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,19,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,9,9,7,3,1)$

$d\omega^4=-\omega^{1,3}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,3,4}$; $d\omega^{2,4}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,3,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,3,6}$; $d\omega^{4,7}=-\omega^{1,3,7}-\omega^{1,4,6}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,7}=-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,3,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,3,6,7}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,5,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,6,7}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,7}=-\omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_2}\ \langle\omega^{1,2}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}+\omega^{2,4}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-5\beta_1}\ \langle\omega^{1,6}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{3,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,7}+\omega^{3,6}\rangle + e^{-7\beta_1}\ \langle\omega^{3,7}-\omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-8\beta_1}\ \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,2,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,2,7}-\omega^{2,3,6}\rangle + 2e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,6}+\omega^{2,4,6},\omega^{2,3,7}-\omega^{2,4,6}\rangle + e^{-9\beta_1}\ \langle\omega^{1,4,7}\rangle + e^{-10\beta_1}\ \langle\omega^{1,6,7}-\omega^{3,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= 2e^{-10\beta_1-\beta_2}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7},\omega^{1,3,5,7}\rangle + e^{-13\beta_1}\ \langle\omega^{1,4,6,7}\rangle + e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,5,6}\rangle + e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,2,5,7}-\omega^{2,3,5,6}\rangle + e^{-9\beta_1-2\beta_2}\ \langle\omega^{2,3,5,7}-\omega^{2,4,5,6}\rangle + e^{-11\beta_1-\beta_2}\ \langle\omega^{1,4,5,7}-\omega^{2,3,6,7}\rangle + e^{-12\beta_1-\beta_2}\ \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}+\omega^{3,4,5,7}\rangle + e^{-14\beta_1}\ \langle\omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-10\beta_1-2\beta_2}\ \langle\omega^{1,2,3,5,7}\rangle + e^{-13\beta_1-\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-11\beta_1-2\beta_2}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-12\beta_1-2\beta_2}\ \langle\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-14\beta_1-\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-16\beta_1-\beta_2}\ \langle\omega^{3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-15\beta_1-2\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-17\beta_1-\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-16\beta_1-2\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-17\beta_1-2\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+2Y+4Y^2+2Y^3+Y^4)(1+Y^2)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,19,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,9,9,7,3,1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,3}=-\omega^4$; $d^\star\omega^{1,4}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,3}=-\omega^5$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{1,5}-\omega^{2,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,6}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,5}=-\omega^{4,5}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{1,3,7}=-\omega^{4,7}$; $d^\star\omega^{1,4,5}=\omega^{5,6}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,4,7}=-\omega^{6,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{2,3,6}=-\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{5,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,4,5}$; $d^\star\omega^{1,2,3,6}=\omega^{1,5,6}-\omega^{2,3,7}-\omega^{2,4,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7}=-\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{2,5,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,5,7}=-\omega^{4,5,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7}=$

$\omega^{4,5,7}$; $d^{\star}\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = -\omega^{2,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,4,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} - \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} + \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} + \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} - \omega^{1,3,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} =$

$2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_2}\ \langle\omega^{1,2}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-5\beta_1}\ \langle\omega^{1,6} - \omega^{3,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{3,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,7} + \omega^{3,6}\rangle + e^{-7\beta_1}\ \langle\omega^{3,7} - \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-8\beta_1}\ \langle\omega^{1,3,7} - \omega^{1,4,6}\rangle + e^{-5\beta_1-\beta_2}\ \langle\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{2,3,5}\rangle + e^{-6\beta_1-\beta_2}\ \langle 2\omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,3,6}\rangle + 2e^{-7\beta_1-\beta_2}\ \langle\omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} - \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-9\beta_1}\ \langle\omega^{1,4,7} + \omega^{3,4,6}\rangle + e^{-10\beta_1}\ \langle\omega^{1,6,7} - \omega^{3,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-14\beta_1}\ \langle\omega^{3,4,6,7}\rangle + e^{-9\beta_1-2\beta_2}\ \langle(\omega^{2,3,5,7} - \omega^{2,4,5,6})/2\rangle + e^{-12\beta_1-\beta_2}\ \langle(-\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7})/3\rangle + e^{-13\beta_1}\ \langle\omega^{1,4,6,7}\rangle + e^{-11\beta_1-\beta_2}\ \langle(\omega^{1,4,5,7} - \omega^{2,3,6,7} + 2\omega^{3,4,5,6})/6\rangle + 2e^{-10\beta_1-\beta_2}\ \langle(\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,3,4,7})/3, (\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle + e^{-8\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,7} + \omega^{2,3,5,6})/2\rangle + e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,6} + \omega^{2,3,4,5})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-16\beta_1-\beta_2}\ \langle\omega^{3,4,5,6,7}\rangle + e^{-14\beta_1-\beta_2}\ \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-15\beta_1}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-12\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-13\beta_1-\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle + e^{-11\beta_1-2\beta_2}\ \langle(\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-10\beta_1-2\beta_2}\ \langle(\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-16\beta_1-2\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-17\beta_1-\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-15\beta_1-2\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-17\beta_1-2\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.4.27 Algebra $\mathcal{G}_{7,2.23}$

Adjoint cohomology

$[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 0 & 0 & 2\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_2^3 & 2\xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_1^5 & \xi_1^4 & \xi_4^6 & 0 & \xi_1^1 + 2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_3^5 & \xi_4^6 & 2(\xi_1^1 + \xi_2^2) \end{pmatrix}$$

$ad\ x_1 = \delta_4^6, ad\ x_2 = \delta_3^5, ad\ x_3 = -\delta_2^5 + \delta_4^7, ad\ x_4 = -(\delta_1^6 + \delta_3^7), ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{2\beta_1}\ \langle x_3 \rangle + e^{2\beta_2}\ \langle x_4 \rangle + e^{2\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+2\beta_2}\ \langle x_6 \rangle + e^{2\beta_1+2\beta_2}\ \langle x_7 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+2\beta_2}\ \langle x_7 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{2\beta_1}\ \langle \delta_2^5, \delta_4^7 \rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^3 \rangle + 2e^{2\beta_2}\ \langle \delta_1^6, \delta_3^7 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5 \rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_1^4 \rangle + e^{\beta_1+2\beta_2}\ \langle \delta_1^7 \rangle + e^{2\beta_1+\beta_2}\ \langle \delta_2^7 \rangle + e^{\beta_2}\ \langle \delta_3^5 \rangle + e^{\beta_1}\ \langle \delta_4^6 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_3^5] = -\delta_2^7$; $[\delta_2^3, \delta_1^4] = -\delta_1^5$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^5] = \delta_4^7 - \delta_2^5$; $[\delta_1^6, \delta_4^6] = -\delta_1^7$; $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_1^5, \delta_4^6] = -\delta_2^7$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^4, \delta_4^6] = -(\delta_3^7 + \delta_1^6)$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_1-\beta_2}\ \langle \delta_2^3 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5 \rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_1^4 \rangle + e^{2\beta_2}\ \langle \delta_1^6 \rangle + e^{2\beta_1}\ \langle \delta_2^5 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_1^4] = -\delta_1^5$.

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_2}\ \langle x_1\omega^{4,6} \rangle + e^{-4\beta_1}\ \langle x_2\omega^{3,5} \rangle + e^{2\beta_1-3\beta_2}\ \langle x_3\omega^{2,4} \rangle + e^{-2\beta_1-\beta_2}\ \langle -x_2(\omega^{2,5} - \omega^{3,4}) + x_3\omega^{3,5} \rangle + e^{-3\beta_1+2\beta_2}\ \langle x_4\omega^{1,3} \rangle + e^{-\beta_1-2\beta_2}\ \langle -x_1(\omega^{1,6} - \omega^{3,4}) + x_4\omega^{4,6} \rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2} + 2x_3\omega^{1,3} - 2x_4\omega^{1,4} + x_5\omega^{1,5}, x_2\omega^{1,2} - x_3\omega^{1,3} + x_4\omega^{1,4} + x_6\omega^{2,5}, x_4\omega^{1,4} + x_6\omega^{3,4} \rangle + 3e^{-2\beta_2}\ \langle -x_2\omega^{2,4} - 2x_3\omega^{2,5} + x_5\omega^{4,5}, -x_2\omega^{2,4} - x_3(\omega^{2,5} - \omega^{3,4}), -x_1\omega^{1,4} - x_2\omega^{2,4} - x_3(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{2,7} \rangle + e^{\beta_1-3\beta_2}\ \langle x_1\omega^{2,4} + x_5\omega^{4,6} \rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} - 2x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} + x_3\omega^{2,3} - x_4\omega^{2,4} + x_5\omega^{1,6}, x_3\omega^{2,3} + x_5\omega^{3,4} \rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3} + x_6\omega^{3,5} \rangle + 3e^{-2\beta_1}\ \langle -x_1\omega^{1,3} + 2x_4\omega^{1,6} + x_6\omega^{3,6}, x_1\omega^{1,3} - x_4(\omega^{1,6} - \omega^{3,4}), x_1\omega^{1,3} + x_2\omega^{2,3} - x_4(\omega^{1,6} + \omega^{2,5}) + x_6\omega^{1,7} \rangle + e^{-\beta_1+\beta_2}\ \langle x_4\omega^{1,2} + x_5\omega^{1,3} \rangle + e^{\beta_1-\beta_2}\ \langle -x_3\omega^{1,2} + x_5\omega^{1,4} \rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-5\beta_2}+e^{-\beta_1-\beta_2}+e^{-\beta_1-2\beta_2}+2e^{-\beta_1-3\beta_2}+3e^{-\beta_1-4\beta_2}+e^{-2\beta_1-\beta_2}+4e^{-2\beta_1-2\beta_2}+2e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-4\beta_2}+2e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2}+2e^{-3\beta_1}+3e^{-4\beta_1-\beta_2}+e^{-4\beta_1-2\beta_2}+e^{-4\beta_1}+e^{-5\beta_1+\beta_2}+e^{-5\beta_1}+2e^{-3\beta_2}+e^{-4\beta_2}+e^{-5\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-\beta_1-5\beta_2}+e^{-\beta_1-6\beta_2}+e^{-\beta_1-7\beta_2}+2e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-5\beta_2}+e^{-2\beta_1-7\beta_2}+2e^{-3\beta_1-3\beta_2}+3e^{-3\beta_1-4\beta_2}+2e^{-3\beta_1-5\beta_2}+e^{-3\beta_1-6\beta_2}+2e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-7\beta_2}+2e^{-3\beta_1-7\beta_2}+e^{-3\beta_1-8\beta_2}+2e^{-4\beta_1-5\beta_2}+3e^{-4\beta_1-6\beta_2}+e^{-4\beta_1-8\beta_2}+2e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+2e^{-5\beta_1-6\beta_2}+e^{-5\beta_1-7\beta_2}+3e^{-6\beta_1-4\beta_2}+2e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-7\beta_2}+e^{-7\beta_1-2\beta_2}+2e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-7\beta_1-6\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-4\beta_1-7\beta_2}+e^{-4\beta_1-8\beta_2}+2e^{-5\beta_1-8\beta_2}+3e^{-6\beta_1-6\beta_2}+e^{-6\beta_1-7\beta_2}+e^{-6\beta_1-8\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-6\beta_2}+2e^{-7\beta_1-7\beta_2}+e^{-8\beta_1-4\beta_2}+2e^{-8\beta_1-5\beta_2}+e^{-8\beta_1-6\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-6\beta_1-8\beta_2}+e^{-7\beta_1-8\beta_2}+e^{-8\beta_1-6\beta_2}+e^{-8\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,13,58,123,156,118,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\leq j\leq 7$: $(1,7,22,34,34,29,17,4)$

S=$8(\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,4,10,18,24,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\leq j\leq 7$: $(1,4,7,7,7,7,4,1)$

$d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,7}=-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=\omega^{2,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=$

$-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-2\beta_2} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-3\beta_1} \langle\omega^{1,3}\rangle + 2e^{-2\beta_1-2\beta_2} \langle\omega^{1,6},\omega^{2,5}\rangle + e^{-3\beta_2} \langle\omega^{2,4}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{3,5}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,2,7} - \omega^{2,3,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{1,4,6}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,5,6} + \omega^{2,3,7}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,4,7} - \omega^{2,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,5,7} - \omega^{2,3,5,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,2,6,7} + \omega^{2,3,4,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,6,7}-\omega^{2,3,5,7}\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{1,4,6,7}-\omega^{2,4,5,7}\rangle + e^{-3\beta_1-7\beta_2} \langle\omega^{2,4,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,4,5,7} - \omega^{2,3,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,3,5,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{1,2,4,6,7}\rangle + 2e^{-6\beta_1-6\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}, \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-8\beta_2} \langle\omega^{2,4,5,6,7}\rangle + e^{-7\beta_1-7\beta_2} \langle\omega^{3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-6\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-8\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 3Y + 4Y^2 + 3Y^3 + 4Y^4 + 3Y^5 + Y^6)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 10, 18, 24, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 7, 7, 7, 7, 4, 1)$

Total dimension of trivial cohomology: 38.

Length associated polynomial: 38.

Riemann Hyp.: yes

$d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} + \omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}-\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7}$;

$d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$;

$\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,4,6,7}$;

$\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-2\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-3\beta_2} \langle\omega^{2,4}\rangle + 2e^{-2\beta_1-2\beta_2} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} - \omega^{3,4}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{3,5}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-5\beta_1-\beta_2} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,6} - \omega^{2,3,5}\rangle + e^{-3\beta_1-3\beta_2} \langle 2\omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,3,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{1,4,6} - \omega^{2,4,5}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{3,4,5}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-3\beta_1-7\beta_2} \langle -\omega^{2,4,6,7}\rangle + e^{-4\beta_1-6\beta_2} \langle(-\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-5\beta_1-5\beta_2} \langle(\omega^{1,4,5,7} - \omega^{2,3,6,7} + 2\omega^{3,4,5,6})/6\rangle + e^{-6\beta_1-4\beta_2} \langle(\omega^{1,3,6,7} - \omega^{2,3,5,7})/2\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-5\beta_2} \langle(2\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle + e^{-5\beta_1-4\beta_2} \langle(-2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{2,3,5,6})/6\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-7\beta_1-7\beta_2} \langle\omega^{3,4,5,6,7}\rangle + e^{-5\beta_1-8\beta_2} \langle -\omega^{2,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle -\omega^{1,3,5,6,7}\rangle + 2e^{-6\beta_1-6\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (-\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7})/3\rangle + e^{-4\beta_1-7\beta_2} \langle -\omega^{1,2,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle -\omega^{1,2,3,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-7\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-8\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-6\beta_2} \langle -\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-8\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + 4Y^2 + 3Y^3 + 4Y^4 + 3Y^5 + Y^6)(1 + Y)$.

5.4.28 Algebra $\mathcal{G}_{7,2.24}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_5]=-x_7$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^3 & 2\xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_1^4+\xi_3^5 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6-\xi_2^5 & -\xi_2^4 & 2\xi_1^3+\xi_2^4 & 3\xi_1^1+\xi_2^2\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-(\delta_1^6+\delta_3^7), ad\ x_5=\delta_1^7, ad\ x_6=-\delta_1^7, ad\ x_7=0$.

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+2e^{2\beta_1+\beta_2}\ \langle x_5,x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $3e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_1^6,\delta_3^7\rangle+2e^{\beta_2}\ \langle\delta_1^4,\delta_3^5\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+2e^{2\beta_1}\ \langle\delta_2^5,\delta_2^6\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5,\delta_2^4]=\delta_1^7$; $[\delta_1^6,\delta_1^3]=-2\delta_1^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=\delta_1^7$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_1^3]=-2\delta_3^7+\delta_1^6+\delta_1^5$; $[\delta_1^4,\delta_2^4]=-(\delta_3^7+\delta_1^6)$; $[\delta_3^5,\delta_1^3]=-2\delta_3^7+\delta_1^6+\delta_1^5$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_2^6,\delta_1^3]=-2\delta_2^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^5$.

ch($H^1_{\mathcal{G}}$)= $2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_1^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{\beta_1}\ \langle\delta_1^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^3]=3\delta_1^6+\delta_1^5$; $[\delta_2^6,\delta_1^3]=-2\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}\rangle+5e^{-2\beta_1}\ \langle -x_2\omega^{1,4}+x_5\omega^{1,7},-x_2\omega^{2,3}+x_4\omega^{1,5}+x_6\omega^{3,5},-x_1\omega^{1,3}+2x_2\omega^{2,3}-x_4(\omega^{1,5}-2\omega^{1,6})+x_6\omega^{3,6},x_1\omega^{1,3}-3x_2\omega^{2,3}+x_4(\omega^{1,5}-\omega^{1,6}+\omega^{3,4}),x_1\omega^{1,3}-2x_2\omega^{2,3}+x_4(\omega^{1,5}-\omega^{1,6})+x_6\omega^{1,7}\rangle+e^{-\beta_1-\beta_2}\ \langle 2x_1\omega^{1,4}-2x_2\omega^{2,4}-x_3(\omega^{1,5}-\omega^{1,6}-\omega^{3,4})+x_5(\omega^{4,5}-\omega^{4,6})\rangle+3e^{-\beta_2}\ \langle -x_1\omega^{1,2}-x_3(\omega^{1,4}-\omega^{2,3})-x_4\omega^{2,4}+x_5\omega^{2,6}+x_6\omega^{2,5},-x_3(\omega^{1,4}+\omega^{2,3})+x_4\omega^{2,4}+x_5\omega^{2,6}+x_6\omega^{2,6},-x_1\omega^{1,2}+2x_3\omega^{2,3}-x_4\omega^{2,4}+x_7\omega^{2,7}\rangle+e^{-3\beta_1}\ \langle x_4(\omega^{1,7}-$

$\omega^{3,5}+\omega^{3,6})+x_6\omega^{3,7}\rangle+2e^{\beta_1-\beta_2}\ \langle x_3\omega^{1,2}+x_6\omega^{2,4},x_7\omega^{2,5}\rangle+4e^{-\beta_1}\ \langle x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4},x_2\omega^{1,2}+x_5\omega^{1,5},x_5\omega^{1,6},-x_4\omega^{2,3}+x_6\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$3e^{-\beta_1-2\beta_2}+2e^{-\beta_1-3\beta_2}+e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+4e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+7e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+2e^{-5\beta_1-\beta_2}+2e^{-5\beta_1}+e^{-6\beta_1}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$4e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-5\beta_1-\beta_2}+5e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+3e^{-6\beta_1-\beta_2}+4e^{-6\beta_1-2\beta_2}+2e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+2e^{-8\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-5\beta_1-3\beta_2}+3e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}+4e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-5\beta_2}+4e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+3e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1-3\beta_2}+2e^{-9\beta_1-2\beta_2}+e^{-10\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$2e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-8\beta_1-4\beta_2}+2e^{-8\beta_1-5\beta_2}+e^{-9\beta_1-3\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-3\beta_2}+e^{-10\beta_1-4\beta_2}+e^{-11\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-5\beta_2}+e^{-11\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,13,54,123,155,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,8,18,30,33,24,12,3)$

S=$11\beta_1+5\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,19,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,8,8,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=\omega^{1,5}-\omega^{1,6}-\omega^{3,4}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=\omega^{1,2,5}-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=\omega^{1,3,5}-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}+\omega^{1,4,5}-\omega^{1,4,6}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=\omega^{1,2,3,5}-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=\omega^{1,2,4,5}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}+\omega^{1,3,4,5}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=\omega^{1,2,3,4,5}-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$;

$d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{3,4}\rangle + 2e^{-4\beta_1-\beta_2} \langle\omega^{3,5}, \omega^{1,7} + \omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,3,5}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{2,3,5}, \omega^{1,2,7} - \omega^{2,3,6}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}, \omega^{2,4,6}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-3\beta_2} \langle\omega^{1,2,4,7}\rangle + 2e^{-8\beta_1-2\beta_2} \langle\omega^{1,3,5,7}, \omega^{1,3,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,5,7} - \omega^{2,3,5,6}\rangle + 2e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}, \omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,5,7} - \omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}, \omega^{1,2,4,5,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-9\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)^2(1+Y^2)(1+Y)$. res nclhar

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,5} = \omega^7$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,5} = \omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7} - \omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} + \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,4,7} + \omega^{2,5,6}$;

$d^{\star}\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^{\star}\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{2,6,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{1,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7}$; $d^{\star}\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{3,5,7} - \omega^{3,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{4,5,7} - \omega^{4,6,7}$; $d^{\star}\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = -\omega^{2,5,7}$; $d^{\star}\omega^{2,3,4,6} = -\omega^{2,6,7} + \omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^{\star}\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} - \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,4,5,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7} - \omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = 2\omega^{1,5} - \omega^{1,6}$; $\Delta\omega^{1,6} = -\omega^{1,5} + \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 3\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7} + \omega^{5,6}$; $\Delta\omega^{5,6} = \omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7} + \omega^{6,7}$; $\Delta\omega^{6,7} = \omega^{5,7} + 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,2,6}$; $\Delta\omega^{1,2,6} = -\omega^{1,2,5} + 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{1,3,6}$; $\Delta\omega^{1,3,6} = -\omega^{1,3,5} + \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = 3\omega^{1,4,5} - \omega^{1,4,6}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} - \omega^{1,4,5} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{1,5,6} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,4,7} + 3\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = \omega^{1,2,7} + 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 3\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = \omega^{2,4,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,5,7} + 3\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,7} - \omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 3\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,5,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} + \omega^{3,6,7}$; $\Delta\omega^{3,6,7} = \omega^{3,5,7} + 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7} + \omega^{4,6,7}$; $\Delta\omega^{4,6,7} = \omega^{4,5,7} + 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5} - \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,6} = -\omega^{1,2,3,5} + 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,4,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,7} + 3\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 3\omega^{1,3,4,5} - \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,3,4,5} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,7} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 3\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{2,3,4,5}$;

$\Delta\omega^{2,3,4,6} = \omega^{1,2,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 3\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,5,7} + \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} + \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,5,7} + 3\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,5,7}+\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,4,5,7}-\omega^{2,3,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,5,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5} - \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,6} = -\omega^{1,2,3,4,5} + 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,7}+3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 3\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,5,7} + 2\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5} + 2\omega^{1,6} - \omega^{3,4}\rangle + 2e^{-4\beta_1-\beta_2} \langle\omega^{3,5}, \omega^{1,7} + \omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,3,5} + \omega^{1,3,6}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{2,3,5}, 2\omega^{1,2,7}-\omega^{1,4,5}-3\omega^{1,4,6}-\omega^{2,3,6}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,5}, \omega^{2,4,6}\rangle + e^{-5\beta_1-2\beta_2} \langle 5\omega^{1,4,7} - 2\omega^{1,5,6} + \omega^{2,3,7} - \omega^{3,4,5} + 3\omega^{3,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{1,6,7} - \omega^{3,4,7} + 2\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-6\beta_1-4\beta_2} \langle(\omega^{2,4,5,7} - \omega^{2,4,6,7})/2\rangle + e^{-5\beta_1-4\beta_2} \langle-\omega^{2,4,5,6}\rangle + 2e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}, (\omega^{1,4,5,7} + 3\omega^{2,3,5,7} - \omega^{2,3,6,7} + 2\omega^{3,4,5,6})/15\rangle + 2e^{-8\beta_1-2\beta_2} \langle-\omega^{1,3,6,7}, \omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2} \langle(-3\omega^{1,2,5,7} - \omega^{1,2,6,7} + \omega^{1,4,5,6} - 2\omega^{2,3,4,7} + 5\omega^{2,3,5,6})/40\rangle + e^{-5\beta_1-3\beta_2} \langle(2\omega^{1,2,4,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5} - \omega^{2,3,4,6})/7\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-8\beta_1-5\beta_2} \langle-\omega^{2,4,5,6,7}\rangle + e^{-10\beta_1-3\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6,7} + 2\omega^{2,3,4,5,7} - \omega^{2,3,4,6,7})/6\rangle + 2e^{-7\beta_1-4\beta_2} \langle-\omega^{1,2,4,6,7}, (\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-11\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-11\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y).$

5.4.29 Algebra $\mathcal{G}_{7,2.25}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_6$; $[x_3, x_4] = -x_7$; $[x_3, x_5] = -x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ \xi_1^4 & -2\xi_1^3 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & -(\xi_1^4+\xi_1^5) & \xi_1^3 & -\xi_1^3+\xi_2^5 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^6 & -2\xi_1^3+\xi_2^5 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5, ad\ x_3 = -(\delta_2^6 + \delta_4^7), ad\ x_4 = \delta_3^7, ad\ x_5 = -\delta_1^6 + \delta_3^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + 2e^{\beta_1+\beta_2}\ \langle x_4, x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{-\beta_1+\beta_2}\ \langle \delta_3^4, \delta_1^2\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_1^6, \delta_3^7\rangle + 2e^{\beta_2}\ \langle \delta_1^4, \delta_1^5\rangle + e^{2\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + 2e^{2\beta_1}\ \langle \delta_4^7, \delta_2^6\rangle + 2e^{\beta_1}\ \langle \delta_1^3, \delta_2^5\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = \delta_3^7 - \delta_1^6$; $[\delta_1^2, \delta_1^3] = -\delta_1^5 + 2\delta_1^4$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^6, \delta_1^3] = 2\delta_1^7$; $[\delta_1^6, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^4, \delta_1^3] = -2(\delta_3^7 + \delta_1^6)$; $[\delta_1^4, \delta_2^5] = \delta_3^7$; $[\delta_1^5, \delta_2^6] = -\delta_1^7$; $[\delta_1^5, \delta_1^3] = -2\delta_3^7$; $[\delta_1^5, \delta_2^5] = \delta_3^7 - \delta_1^6$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_4^7, \delta_1^3] = -2\delta_2^7$; $[\delta_2^6, \delta_1^3] = 2\delta_2^7$; $[\delta_1^3, \delta_2^5] = -(\delta_4^7 + \delta_2^6)$.

ch($H^1_{\mathcal{G}}$)= $2e^{-\beta_1+\beta_2}\ \langle \delta_3^4, \delta_1^2\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_1^2, \delta_1^3] = 2\delta_1^4$; $[\delta_2^6, \delta_1^3] = 2\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle + e^{\beta_1-2\beta_2}\ \langle x_3(\omega^{2,4}+\omega^{2,5})\rangle + e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}\rangle + 4e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3\omega^{1,3} + x_4(\omega^{1,4} + \omega^{1,5}), x_2\omega^{1,2} + x_5\omega^{2,3}, x_2\omega^{1,2} - x_3\omega^{1,3} + x_4\omega^{1,5} + x_5\omega^{1,4}, x_4\omega^{2,3}\rangle + 4e^{-\beta_2}\ \langle -x_3\omega^{2,3} + x_4(\omega^{2,4} + \omega^{2,5}), -x_1\omega^{1,2} + x_3\omega^{2,3} + x_4\omega^{2,5} - x_5\omega^{2,5} + x_7\omega^{2,7}, x_3(\omega^{1,4} + \omega^{1,5} - \omega^{2,3}) + x_5(\omega^{2,4} + \omega^{2,5}), x_3(\omega^{1,4} + 2\omega^{1,5}) - 2x_4\omega^{2,5} + 2x_5\omega^{2,5} + x_6\omega^{4,5}\rangle + e^{-3\beta_1}\ \langle -x_2(\omega^{1,6} - \omega^{3,5}) + x_5\omega^{3,6}\rangle + 3e^{-\beta_1-\beta_2}\ \langle x_1(\omega^{1,4} + \omega^{1,5} - \omega^{2,3}) - 2x_2(\omega^{2,4} + \omega^{2,5}) - x_3(\omega^{1,6} - 2\omega^{3,4} - \omega^{3,5}) -$

$x_5(\omega^{2,6}-\omega^{4,5}), -x_1(\omega^{1,4}+2\omega^{1,5}-2\omega^{2,3})+x_2(\omega^{2,4}+4\omega^{2,5})+2x_3(\omega^{1,6}-\omega^{3,4}-\omega^{3,5})+2x_5\omega^{2,6}+x_6\omega^{4,6}, x_1(\omega^{1,5}-\omega^{2,3})-2x_2\omega^{2,5}-x_3(\omega^{1,6}-\omega^{3,5})-x_5\omega^{2,6}+x_7\omega^{4,7}\rangle+5e^{-2\beta_1}\ \langle -x_2\omega^{1,4}+x_5\omega^{3,4}, -x_2(\omega^{1,5}+\omega^{2,3})+x_5\omega^{3,5}, x_1\omega^{1,3}+x_2(\omega^{1,5}-2\omega^{2,3})+x_5\omega^{1,6}, x_4\omega^{3,4}, -x_1\omega^{1,3}-x_2(\omega^{1,5}-\omega^{2,3})-x_4(\omega^{1,6}-\omega^{3,5})\rangle+e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=4e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+4e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+5e^{-3\beta_1-\beta_2}+2e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+5e^{-4\beta_1-\beta_2}+e^{-4\beta_1}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1}+e^{-6\beta_1}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-4\beta_2}+e^{-3\beta_1-2\beta_2}+4e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+4e^{-4\beta_1-2\beta_2}+3e^{-4\beta_1-3\beta_2}+6e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+3e^{-6\beta_1-\beta_2}+3e^{-6\beta_1-2\beta_2}+3e^{-7\beta_1-\beta_2}+e^{-7\beta_1-2\beta_2}+e^{-8\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_1-4\beta_2}+2e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-5\beta_1-5\beta_2}+2e^{-6\beta_1-3\beta_2}+3e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-2\beta_2}+5e^{-7\beta_1-3\beta_2}+2e^{-7\beta_1-4\beta_2}+4e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1-3\beta_2}+2e^{-9\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-6\beta_1-5\beta_2}+2e^{-7\beta_1-4\beta_2}+2e^{-7\beta_1-5\beta_2}+e^{-8\beta_1-3\beta_2}+4e^{-8\beta_1-4\beta_2}+e^{-8\beta_1-5\beta_2}+3e^{-9\beta_1-3\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-10\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-8\beta_1-5\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,14,56,123,155,117,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,8,21,32,33,27,16,4)$

S=$5(2\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,9,19,23,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,6,7,7,6,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,5}-\omega^{2,3}$; $d\omega^7=-\omega^{1,6}+\omega^{3,4}+\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=-\omega^{1,3,4}-\omega^{1,3,5}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,6}-\omega^{2,3,4}-\omega^{2,3,5}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=-\omega^{1,4,5}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,4,6}+\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,2,7}-\omega^{1,5,6}-\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,3,7}-\omega^{3,4,6}-\omega^{3,5,6}$; $d\omega^{1,2,7}=\omega^{1,2,3,4}+\omega^{1,2,3,5}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=-\omega^{1,3,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}+\omega^{1,3,4,6}+\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}+\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}+\omega^{2,3,4,6}+\omega^{2,3,5,6}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,4,6}=-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,7}=\omega^{1,2,3,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=\omega^{1,2,4,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,4,5,7}+\omega^{2,3,4,7}-\omega^{3,4,5,6}$; $d\omega^{5,6,7}=$

$-\omega^{1,2,6,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-\beta_1-\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + 2e^{-2\beta_1-\beta_2} \langle\omega^{1,4}, \omega^{1,5}\rangle + 2e^{-\beta_1-2\beta_2} \langle\omega^{2,4}, \omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} + \omega^{2,3,6} + \omega^{3,4,5}\rangle + 2e^{-5\beta_1-2\beta_2} \langle\omega^{1,4,7} - \omega^{3,4,6}, \omega^{1,5,7} + \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-7\beta_1-2\beta_2} \langle\omega^{1,3,4,7}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,3,6,7}\rangle + 2e^{-5\beta_1-3\beta_2} \langle\omega^{1,2,4,7} + \omega^{2,3,4,6}, \omega^{1,2,5,7} - \omega^{2,3,4,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7} + 2\omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}\rangle + 2e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,6,7}, \omega^{1,3,5,6,7}\rangle + 2e^{-8\beta_1-4\beta_2} \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}, \omega^{2,3,5,6,7}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-9\beta_1-4\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 2Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 19, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 7, 7, 6, 4, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{3,5} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = -\omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{1,7} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} =$

$-\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{2,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{4,7} - \omega^{5,7}$; $d^\star\omega^{3,4,6} = -\omega^{6,7}$; $d^\star\omega^{3,5,6} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,2,7} - \omega^{1,5,6} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,4,7} + \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,4,7} + \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,6,7} + \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,4,7} - \omega^{1,2,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,6,7} - \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,6,7} - \omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 3\omega^{2,7} - \omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{3,4} + \omega^{3,5}$; $\Delta\omega^{3,5} = \omega^{3,4} + 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{1,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,7} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 3\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4} + \omega^{1,3,5}$; $\Delta\omega^{1,3,5} = \omega^{1,3,4} + 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,2,7} + 2\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,7} + \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,7} + 2\omega^{1,5,7} + \omega^{2,3,7} + \omega^{3,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} - \omega^{1,4,5} + 2\omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{2,3,4} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,7} + \omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + 2\omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} - \omega^{2,5,7}$; $\Delta\omega^{2,5,7} = -\omega^{2,4,7} + 2\omega^{2,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + 3\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,7} + \omega^{2,3,7} + 2\omega^{3,4,6} + \omega^{3,5,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,5,7} + \omega^{3,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7} + 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} + \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,2,5,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6}$;

$\Delta\omega^{1,2,6,7} = 3\omega^{1,2,6,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,3,7} + 2\omega^{1,3,4,6} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,4,6} + 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,5,7} + 2\omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} - \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} - \omega^{1,5,6,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,4,6,7} + 2\omega^{1,5,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,7} - \omega^{1,4,5,6} + 2\omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,6,7} - \omega^{1,4,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,5,7} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = 2\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7} - \omega^{2,5,6,7}$; $\Delta\omega^{2,5,6,7} = -\omega^{2,4,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,5,7} + \omega^{2,3,5,7} + 3\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7} - \omega^{1,2,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,5,7} + 3\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{-\beta_1-\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + 2e^{-2\beta_1-\beta_2} \langle\omega^{1,4}, \omega^{1,5} - \omega^{2,3}\rangle + 2e^{-\beta_1-2\beta_2} \langle\omega^{2,4}, \omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} + 2\omega^{3,4} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-6\beta_1-\beta_2} \langle\omega^{1,3,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6} - 2\omega^{1,4,5} - 2\omega^{2,3,4} + \omega^{2,3,5}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-2\beta_2} \langle 2\omega^{1,2,7} + \omega^{1,4,6} - 3\omega^{1,5,6} + 5\omega^{2,3,6} + \omega^{3,4,5}\rangle + 2e^{-5\beta_1-2\beta_2} \langle 2\omega^{1,4,7} + \omega^{2,3,7} - 2\omega^{3,4,6} + \omega^{3,5,6}, \omega^{1,5,7} - \omega^{2,3,7} + \omega^{3,4,6} - \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-4\beta_2} \langle -\omega^{2,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle(\omega^{1,4,6,7} + 2\omega^{1,5,6,7} - 2\omega^{2,3,6,7} - \omega^{3,4,5,7})/10\rangle + e^{-8\beta_1-2\beta_2} \langle -\omega^{1,3,6,7}\rangle + e^{-7\beta_1-2\beta_2} \langle -\omega^{1,3,4,7}\rangle + e^{-6\beta_1-3\beta_2} \langle(\omega^{1,2,6,7} - 5\omega^{1,4,5,7} - 3\omega^{2,3,4,7} - \omega^{2,3,5,7} + 2\omega^{3,4,5,6})/40\rangle + 2e^{-5\beta_1-3\beta_2} \langle(\omega^{1,2,5,7} - \omega^{2,3,4,6} + 2\omega^{2,3,5,6})/6, (-3\omega^{1,2,4,7} - \omega^{1,2,5,7} - 3\omega^{1,4,5,6} - 5\omega^{2,3,4,6} + 4\omega^{2,3,5,6})/12\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_1-5\beta_2} \langle -\omega^{2,4,5,6,7}\rangle + 2e^{-8\beta_1-4\beta_2} \langle\omega^{2,3,5,6,7}, (-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + 2e^{-9\beta_1-3\beta_2} \langle -\omega^{1,3,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle(\omega^{1,2,4,6,7} + 2\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-9\beta_1-4\beta_2} \langle -\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 2Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

5.4.30 Algebra $\mathcal{G}_{7,2.26}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$; $[x_3,x_5]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & \xi_1^2/2 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & \xi_1^2/2 & 0 & 0\\ \xi_1^5 & 0 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & \xi_2^4-\xi_1^5 & 3\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 & -\xi_1^4 & \xi_1^2 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-\delta_3^7, ad\ x_5=-(\delta_1^6+\delta_3^6+\delta_2^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

ch($\mathcal{G}$)= $2e^{\beta_3}\ \langle x_1,x_3\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_2+\beta_3}\ \langle x_4\rangle+e^{2\beta_3}\ \langle x_5\rangle+e^{3\beta_3}\ \langle x_6\rangle+e^{\beta_2+2\beta_3}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1}\ \langle x_6\rangle+e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_2}\ \langle\delta_3^4,\delta_1^4\rangle+3e^{2\beta_3}\ \langle\delta_1^6,\delta_3^6,\delta_2^7\rangle+e^{-\beta_2+3\beta_3}\ \langle\delta_2^6\rangle+2e^{\beta_3}\ \langle\delta_2^4,\delta_1^5\rangle+2e^{\beta_2+\beta_3}\ \langle\delta_1^7,\delta_3^7\rangle+e^{\beta_2-\beta_3}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4,\delta_1^5]=\delta_3^7$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+1/2\delta_3^6+\delta_1^6$; $[\delta_3^6,\delta_1^2]=-\delta_3^7$; $[\delta_2^4,\delta_1^5]=\delta_2^7+\delta_3^6+\delta_1^6$; $[\delta_2^4,\delta_1^4]=\delta_3^7$; $[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_2^7,\delta_1^2]=1/2\delta_3^7+\delta_1^7$; $[\delta_1^5,\delta_1^2]=-1/2\delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_2}\ \langle\delta_3^4\rangle+e^{-\beta_2+3\beta_3}\ \langle\delta_2^6\rangle+e^{\beta_2+\beta_3}\ \langle\delta_1^7\rangle+2e^{2\beta_3}\ \langle\delta_1^6,\delta_3^6\rangle+e^{\beta_2-\beta_3}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^6,\delta_1^2]=3/2\delta_3^6+2\delta_1^6$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$.

ch($H^2_{\mathcal{G}}$)= $2e^{\beta_2-3\beta_3}\ \langle x_2\omega^{3,5}+x_4\omega^{3,6},x_2\omega^{1,5}\rangle+3e^{-2\beta_3}\ \langle -x_1\omega^{3,5}+x_4\omega^{3,7},2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})-2x_3\omega^{3,5}+x_4\omega^{4,5},x_2\omega^{1,4}+x_3\omega^{1,5}\rangle+e^{-\beta_2-\beta_3}\ \langle -2x_1\omega^{1,4}+2x_2\omega^{2,4}+3x_3(\omega^{1,4}+1/3\omega^{2,5}+1/3\omega^{3,4})+x_5\omega^{4,5}\rangle+e^{-2\beta_2+2\beta_3}\ \langle x_6\omega^{2,4}\rangle+4e^{-\beta_2}\ \langle -x_1(\omega^{1,2}-2\omega^{2,3})-x_3\omega^{2,3}+x_4\omega^{2,4}+x_6\omega^{2,6},x_1(\omega^{1,2}-3\omega^{2,3})+2x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5},x_1(\omega^{1,2}-2\omega^{2,3})+2x_3\omega^{2,3}-3x_4\omega^{2,4}-x_5\omega^{3,4}+x_6\omega^{1,7},-x_1\omega^{2,3}+x_6\omega^{3,7}\rangle+2e^{\beta_2-2\beta_3}\ \langle x_2\omega^{1,3}+x_7\omega^{1,6},x_4\omega^{3,5}\rangle+5e^{-\beta_3}\ \langle -x_1\omega^{1,3}+x_2(\omega^{1,2}+1/2\omega^{2,3})+x_3\omega^{1,3}-1/2x_4\omega^{2,5}+x_7\omega^{1,7},x_1\omega^{1,3}-1/2x_2\omega^{2,3}+1/2x_4\omega^{2,5}+x_6\omega^{1,6},-x_1\omega^{1,3}+1/2x_2\omega^{2,3}-1/2x_4\omega^{2,5}+x_6\omega^{3,6},-1/2x_2\omega^{2,3}+1/2x_4(\omega^{2,5}+$

$2\omega^{3,4}), x_1\omega^{1,3} - 1/2x_2\omega^{2,3} - x_3\omega^{1,3} + x_4(\omega^{1,4} + 1/2\omega^{2,5})\rangle + e^{-\beta_2+\beta_3}\ \langle x_5(\omega^{1,2} - \omega^{2,3})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_2-4\beta_3} + e^{\beta_2-5\beta_3} + e^{\beta_2-6\beta_3} + e^{-\beta_2-\beta_3} + 5e^{-\beta_2-2\beta_3} + 5e^{-\beta_2-3\beta_3} + e^{-\beta_2-4\beta_3} + 5e^{-2\beta_2-\beta_3} + e^{-2\beta_2-2\beta_3} + e^{-3\beta_2} + 3e^{-3\beta_3} + 4e^{-4\beta_3} + 3e^{-5\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{\beta_2-7\beta_3} + 2e^{-\beta_2-4\beta_3} + 7e^{-\beta_2-5\beta_3} + 3e^{-\beta_2-6\beta_3} + e^{-2\beta_2-2\beta_3} + 3e^{-2\beta_2-3\beta_3} + 6e^{-2\beta_2-4\beta_3} + 2e^{-2\beta_2-5\beta_3} + e^{-3\beta_2-\beta_3} + 3e^{-3\beta_2-3\beta_3} + 5e^{-6\beta_3} + 2e^{-7\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-\beta_2-6\beta_3} + 2e^{-\beta_2-7\beta_3} + 3e^{-\beta_2-8\beta_3} + e^{-\beta_2-9\beta_3} + 2e^{-2\beta_2-5\beta_3} + 6e^{-2\beta_2-6\beta_3} + 2e^{-2\beta_2-7\beta_3} + e^{-2\beta_2-8\beta_3} + 2e^{-3\beta_2-4\beta_3} + 3e^{-3\beta_2-5\beta_3} + e^{-3\beta_2-6\beta_3} + e^{-7\beta_3} + e^{-9\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-\beta_2-9\beta_3} + e^{-\beta_2-10\beta_3} + e^{-2\beta_2-7\beta_3} + e^{-2\beta_2-8\beta_3} + 3e^{-2\beta_2-9\beta_3} + e^{-3\beta_2-6\beta_3} + e^{-3\beta_2-7\beta_3} + 3e^{-3\beta_2-8\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-2\beta_2-10\beta_3} + 2e^{-3\beta_2-9\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 13, 55, 124, 157, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 8, 19, 32, 36, 26, 12, 3)$

S=$10\beta_1 + 3\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{3,5}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,3,5}$; $d\omega^{1,7} = \omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}+\omega^{2,3,5}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5} - \omega^{3,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6} - \omega^{3,5,7}$; $d\omega^{1,2,6} = -\omega^{1,2,3,5}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = \omega^{1,3,4,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{1,3,5,7}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7}-\omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,4,5,6} - \omega^{3,4,5,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6} + \omega^{1,3,4,5,7}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,5,7}$;

$d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1} \langle\omega^1, \omega^3\rangle + e^{-\beta_2} \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-3\beta_1} \langle\omega^{1,5}\rangle + e^{-4\beta_1} \langle\omega^{1,6} + \omega^{3,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7} + 2\omega^{2,6} + 2\omega^{3,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + 2e^{-6\beta_1} \langle\omega^{1,5,6}, \omega^{3,5,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,7}\rangle + 2e^{-4\beta_1-\beta_2} \langle\omega^{1,2,6} + \omega^{3,4,5}, \omega^{2,3,6} + \omega^{3,4,5}\rangle + 2e^{-5\beta_1-\beta_2} \langle\omega^{1,4,6} + \omega^{2,5,6} + \omega^{3,4,6}, \omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle(2\omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{2,5,7} + \omega^{3,4,7})/2\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-5\beta_1-2\beta_2} \langle\omega^{1,2,4,6} - \omega^{2,3,4,6}, \omega^{1,2,5,7} + \omega^{2,3,4,6}\rangle + 2e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,7}, \omega^{2,3,4,7}\rangle + e^{-7\beta_1} \langle\omega^{1,3,5,6}\rangle + e^{-7\beta_1-\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-6\beta_1-\beta_2} \langle\omega^{1,2,5,6}\rangle + 2e^{-6\beta_1-2\beta_2} \langle\omega^{2,3,6,7} + \omega^{3,4,5,7}, \omega^{1,2,6,7} - \omega^{2,4,5,6} + \omega^{3,4,5,7}\rangle + e^{-8\beta_1-\beta_2} \langle\omega^{3,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-9\beta_1-\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-2\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,4,5,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,2,4,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = 2e^{-9\beta_1-3\beta_2} \langle\omega^{1,2,4,5,6,7}, \omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-2\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-3\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,6} - \omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = \omega^{2,6} - \omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,6} + \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{3,5,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,6} + \omega^{1,3,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{2,4,6}$;

$d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,7} = \omega^{1,6,7} - \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,6} - \omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{2,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{3,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,6} + \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,2,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{1,4,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,2,4,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{3,6}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + 2\omega^{2,6} - \omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,5} + \omega^{3,5}$; $\Delta\omega^{3,6} = -\omega^{1,6} + \omega^{3,6}$; $\Delta\omega^{3,7} = -\omega^{2,6} + \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 3\omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,6} + 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5} - \omega^{2,3,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{1,3,7} + \omega^{1,4,5} + \omega^{2,3,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = -\omega^{1,2,6} + 2\omega^{1,3,7} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} + \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{1,5,7} - \omega^{3,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + 2\omega^{1,5,7} - \omega^{2,5,6} + \omega^{3,5,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} - \omega^{3,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,2,6} - \omega^{1,3,7} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + 2\omega^{2,4,6} - \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + 2\omega^{2,5,6} - \omega^{3,5,7}$; $\Delta\omega^{2,5,7} = -\omega^{2,4,6} + 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,5} - \omega^{2,3,6} + 3\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,4,6} + 3\omega^{3,4,6} - \omega^{3,5,7}$; $\Delta\omega^{3,4,7} = -\omega^{2,4,6} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,7} = \omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,4,6} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = -\omega^{1,6,7} + 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 4\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 4\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{1,3,5,7}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,4,6} + 3\omega^{1,2,5,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,2,6,7} = 3\omega^{1,2,6,7} + \omega^{1,4,5,7} + \omega^{2,3,6,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6} - \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{2,4,5,6} + \omega^{3,4,5,7}$; $\Delta\omega^{1,4,6,7} =$

$2\omega^{1,4,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,5} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,4,6} - \omega^{1,3,4,7} + 2\omega^{2,3,4,6} - \omega^{2,3,5,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,2,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + 2\omega^{2,4,5,6} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7} - \omega^{2,4,5,6} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{1,4,6,7} + 3\omega^{3,4,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = -\omega^{1,2,3,4,6} + 3\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6} + 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = 2e^{-\beta_1}\ \langle\omega^1, \omega^3\rangle + e^{-\beta_2}\ \langle\omega^2\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-\beta_2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1}\ \langle\omega^{1,5} - \omega^{3,5}\rangle + e^{-4\beta_1}\ \langle\omega^{1,6} + \omega^{3,6}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,7} + 2\omega^{2,6} + 2\omega^{3,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle + 2e^{-6\beta_1}\ \langle\omega^{1,5,6}, \omega^{3,5,6}\rangle + 2e^{-4\beta_1-\beta_2}\ \langle\omega^{1,2,6} - \omega^{1,4,5} - \omega^{2,3,6}, \omega^{1,3,7} - \omega^{1,4,5} + 2\omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,7}\rangle + e^{-4\beta_1-2\beta_2}\ \langle 2\omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{2,5,7} + \omega^{3,4,7}\rangle + 2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,4,6} + \omega^{2,5,6} + \omega^{3,4,6} + 2\omega^{3,5,7}, \omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,4,6} - 3\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-8\beta_1-\beta_2}\ \langle -\omega^{3,5,6,7}\rangle + 2e^{-4\beta_1-3\beta_2}\ \langle\omega^{2,3,4,7}, \omega^{1,2,4,7}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle(\omega^{1,2,6,7} + 5\omega^{1,4,5,7} - 8\omega^{2,3,6,7} - \omega^{2,4,5,6} - 7\omega^{3,4,5,7})/20, (3\omega^{1,2,6,7} - 5\omega^{1,4,5,7} - 4\omega^{2,3,6,7} - 3\omega^{2,4,5,6} - \omega^{3,4,5,7})/20\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,4,5,6}\rangle + e^{-7\beta_1}\ \langle -\omega^{1,3,5,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{1,3,5,7} + 2\omega^{2,3,5,6})/10\rangle + 2e^{-5\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,7} - \omega^{1,3,4,7} - 2\omega^{2,3,4,6} - 3\omega^{2,3,5,7})/5, (5\omega^{1,2,4,6} - \omega^{1,2,5,7} - \omega^{1,3,4,7} - 7\omega^{2,3,4,6} - 8\omega^{2,3,5,7})/20\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-8\beta_1-2\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-9\beta_1-\beta_2}\ \langle -\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,5,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,2,3,6,7} - 2\omega^{1,2,4,5,6} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/10\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = 2e^{-9\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6,7}, \omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-2\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

5.4.31 Algebra $\mathcal{G}_{7,2.27}$

Adjoint cohomology

$[x_1,x_2]=x_5$; $[x_1,x_3]=x_7$; $[x_1,x_5]=x_6$; $[x_2,x_4]=x_7$; $[x_2,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 2\xi_2^2 & -\xi_1^2 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^5 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^3-\xi_1^4-\xi_1^5 & \xi_1^2 & \xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^5+\delta_3^7, ad\ x_2=\delta_4^7-\delta_1^5, ad\ x_3=-\delta_1^7, ad\ x_4=-\delta_2^7, ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_2}\ \langle x_3\rangle+2e^{\beta_1+\beta_2}\ \langle x_4,x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+2\beta_2}\ \langle\delta_1^3\rangle+4e^{\beta_1}\ \langle\delta_2^4,\delta_4^6,\delta_2^5,\delta_3^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^6,\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1-\beta_2}\ \langle\delta_3^6\rangle+e^{2\beta_2}\ \langle\delta_1^7\rangle+4e^{\beta_2}\ \langle\delta_4^7,\delta_2^3,\delta_1^4,\delta_1^5\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3,\delta_3^6]=-\delta_1^6$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_2^4,\delta_4^6]=-\delta_2^6$; $[\delta_2^4,\delta_4^7]=-\delta_2^7$; $[\delta_2^4,\delta_1^2]=\delta_1^4+\delta_2^3$; $[\delta_1^6,\delta_1^2]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=-\delta_2^7+\delta_1^6$; $[\delta_3^6,\delta_2^3]=\delta_2^6$; $[\delta_3^6,\delta_1^2]=-(\delta_3^7+\delta_4^6)$; $[\delta_4^6,\delta_1^4]=\delta_1^6$; $[\delta_4^6,\delta_1^2]=-\delta_4^7$; $[\delta_2^5,\delta_2^3]=-\delta_2^7$; $[\delta_2^5,\delta_1^4]=\delta_2^7$; $[\delta_2^5,\delta_1^5]=\delta_2^7+\delta_1^6$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_2^7,\delta_1^2]=\delta_1^7$; $[\delta_3^7,\delta_2^3]=\delta_2^7$; $[\delta_3^7,\delta_1^2]=-\delta_4^7$; $[\delta_4^7,\delta_1^4]=\delta_1^7$; $[\delta_2^3,\delta_1^5]=\delta_1^7$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_1^4,\delta_1^5]=-\delta_1^7$; $[\delta_1^4,\delta_1^2]=\delta_1^3$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+2\beta_2}\ \langle\delta_1^3\rangle+3e^{\beta_1}\ \langle\delta_2^4,\delta_4^6,\delta_2^5\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{2\beta_1-\beta_2}\ \langle\delta_3^6\rangle+3e^{\beta_2}\ \langle\delta_2^3,\delta_1^4,\delta_1^5\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^4,\delta_4^6]=-\delta_2^6$; $[\delta_2^4,\delta_1^2]=\delta_1^4+\delta_2^3$; $[\delta_3^6,\delta_2^3]=\delta_2^6$; $[\delta_3^6,\delta_1^2]=\delta_2^5-\delta_4^6$; $[\delta_2^5,\delta_1^2]=\delta_1^5$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_4^6,\delta_1^2]=-\delta_1^5$; $[\delta_1^4,\delta_1^2]=\delta_1^3$.

ch($H^2_{\mathcal{G}}$)= $4e^{-2\beta_1}\ \langle x_2(\omega^{1,4}+\omega^{1,5}),x_2\omega^{1,5}+x_3(\omega^{1,7}+\omega^{2,6}-\omega^{4,5})+x_4\omega^{1,6},x_2\omega^{1,5}+x_3(\omega^{1,7}+\omega^{2,6}-\omega^{4,5})+x_5\omega^{1,6},x_2\omega^{1,5}+x_7\omega^{4,6}\rangle+2e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,4},x_3\omega^{1,5}\rangle+e^{-3\beta_1+\beta_2}\ \langle x_3\omega^{1,6}\rangle+5e^{-\beta_1}\ \langle -x_2\omega^{1,2}-x_3\omega^{2,5}+x_4\omega^{1,4},x_3\omega^{1,3},x_2\omega^{1,2}+x_3(\omega^{2,4}+\omega^{2,5}),x_3\omega^{2,5}+x_4\omega^{1,5},-x_2\omega^{1,2}-x_3\omega^{2,5}+x_5\omega^{1,4}\rangle+3e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,3},x_5\omega^{2,3}+x_6\omega^{2,7},x_6\omega^{3,4}\rangle+5e^{-2\beta_2}\ \langle -x_1\omega^{2,5}+x_4(\omega^{2,7}+$

$\omega^{3,5}), x_1(\omega^{1,3}-3\omega^{2,5})-2x_2\omega^{2,3}-2x_4\omega^{3,4}+x_5\omega^{3,5}, -x_1(\omega^{1,3}-\omega^{2,5})+x_2\omega^{2,3}+x_4\omega^{3,4}+x_6\omega^{3,6}, -x_1\omega^{2,4}+2x_4\omega^{3,4}-x_5\omega^{3,4}+x_6\omega^{4,7}, x_1(\omega^{1,3}+\omega^{2,4})-x_4\omega^{3,4}+x_5\omega^{2,7}+x_6\omega^{5,7}\rangle+6e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,5}-x_2\omega^{2,5}-x_3(\omega^{2,7}+\omega^{3,5})-x_4(\omega^{1,7}+\omega^{2,6}-\omega^{4,5}), x_1(\omega^{1,4}+3\omega^{1,5})+3x_2(\omega^{1,3}-2/3\omega^{2,4}-\omega^{2,5})+2x_3\omega^{3,4}+x_5\omega^{4,5}, -x_1(\omega^{1,4}+\omega^{1,5})-x_2(\omega^{1,3}-\omega^{2,4}-\omega^{2,5})+x_6\omega^{4,6}, x_1\omega^{1,5}+x_2\omega^{1,3}+x_3\omega^{3,4}+x_5\omega^{2,6}+x_6\omega^{5,6}, x_2(\omega^{1,3}-\omega^{2,4}-\omega^{2,5})+x_3(\omega^{2,7}+\omega^{3,4}+\omega^{3,5})+x_5\omega^{1,7}, -x_1\omega^{1,5}-x_2(\omega^{1,3}-\omega^{2,5})-2x_3\omega^{3,4}+x_7\omega^{4,7}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_6\omega^{2,3}\rangle+6e^{-\beta_2}\ \langle -x_1\omega^{1,2}-x_3\omega^{2,3}+x_6\omega^{2,6}, x_3\omega^{2,3}+x_4\omega^{1,3}, x_4\omega^{2,4}, x_4\omega^{2,5}, x_3\omega^{2,3}+x_5\omega^{1,3}, x_1\omega^{1,2}+x_3\omega^{2,3}+x_5\omega^{2,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-4\beta_2}+6e^{-\beta_1-2\beta_2}+9e^{-\beta_1-3\beta_2}+4e^{-2\beta_1-\beta_2}+10e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+7e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+e^{-4\beta_1-\beta_2}+2e^{-4\beta_1}+4e^{-3\beta_2}+3e^{-4\beta_2}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-4\beta_2}+4e^{-\beta_1-5\beta_2}+e^{-\beta_1-6\beta_2}+e^{-2\beta_1-3\beta_2}+9e^{-2\beta_1-4\beta_2}+3e^{-2\beta_1-5\beta_2}+11e^{-3\beta_1-3\beta_2}+5e^{-3\beta_1-4\beta_2}+6e^{-4\beta_1-2\beta_2}+5e^{-4\beta_1-3\beta_2}+e^{-5\beta_1-\beta_2}+2e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_2}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-2\beta_1-6\beta_2}+e^{-2\beta_1-7\beta_2}+5e^{-3\beta_1-5\beta_2}+3e^{-3\beta_1-6\beta_2}+e^{-3\beta_1-7\beta_2}+6e^{-4\beta_1-4\beta_2}+6e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2}+3e^{-5\beta_1-3\beta_2}+5e^{-5\beta_1-4\beta_2}+2e^{-5\beta_1-5\beta_2}+e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-4\beta_2}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-7\beta_2}+2e^{-4\beta_1-6\beta_2}+2e^{-4\beta_1-7\beta_2}+e^{-4\beta_1-8\beta_2}+3e^{-5\beta_1-5\beta_2}+4e^{-5\beta_1-6\beta_2}+e^{-5\beta_1-7\beta_2}+e^{-6\beta_1-4\beta_2}+2e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-6\beta_2}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-5\beta_1-7\beta_2}+e^{-5\beta_1-8\beta_2}+e^{-6\beta_1-6\beta_2}+e^{-6\beta_1-7\beta_2}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,17,65,132,164,119,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,12,33,50,51,38,18,4)$

S=$2(3\beta_1+4\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,21,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,8,11,11,8,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,5}$; $d\omega^7=-\omega^{1,3}-\omega^{2,4}-\omega^{2,5}$; $d\omega^{1,7}=\omega^{1,2,4}+\omega^{1,2,5}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,3}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}$; $d\omega^{3,7}=-\omega^{2,3,4}-\omega^{2,3,5}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=-\omega^{1,4,5}$; $d\omega^{4,7}=\omega^{1,3,4}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,2,6}$; $d\omega^{5,7}=-\omega^{1,2,7}+\omega^{1,3,5}+\omega^{2,4,5}$; $d\omega^{6,7}=\omega^{1,3,6}-\omega^{1,5,7}+\omega^{2,4,6}+\omega^{2,5,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}+\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,4,6}-\omega^{1,2,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,4,7}=\omega^{1,2,3,4}$; $d\omega^{2,5,7}=\omega^{1,2,3,5}$; $d\omega^{2,6,7}=\omega^{1,2,3,6}-\omega^{1,2,5,7}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,4,6}=-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,7}=$

$\omega^{1,2,3,7}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}+\omega^{2,3,4,6}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}$; $d\omega^{4,5,7}=\omega^{1,2,4,7}-\omega^{1,3,4,5}$; $d\omega^{4,6,7}=-\omega^{1,3,4,6}-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,2,6,7}-\omega^{1,3,5,6}-\omega^{2,4,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7}=-\omega^{1,2,3,4,6}-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7}=-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=\omega^{1,2,3,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_2}\ \langle\omega^3\rangle+e^{-\beta_1-\beta_2}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= 2e^{-\beta_1-2\beta_2}\ \langle\omega^{1,3},\omega^{2,4}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle+e^{-3\beta_2}\ \langle\omega^{2,3}\rangle+2e^{-\beta_1-3\beta_2}\ \langle\omega^{3,4},\omega^{2,7}+\omega^{3,5}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,7}+\omega^{2,6}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 3e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6},\omega^{1,4,7}+\omega^{2,4,6},\omega^{1,5,7}-\omega^{2,4,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6},\omega^{1,5,6}\rangle+e^{-\beta_1-4\beta_2}\ \langle\omega^{2,3,4}\rangle+e^{-\beta_1-5\beta_2}\ \langle\omega^{2,3,7}\rangle+e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7}\rangle+3e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}+\omega^{2,5,7},\omega^{1,3,7}-\omega^{2,5,7}+\omega^{3,4,5},\omega^{2,4,7}+\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= 3e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,2,6,7},\omega^{1,3,4,6},\omega^{1,3,5,6}\rangle+3e^{-3\beta_1-5\beta_2}\ \langle\omega^{1,3,4,7}+\omega^{2,4,5,7},\omega^{1,3,5,7}-\omega^{2,4,5,7},\omega^{2,3,4,6}-\omega^{2,4,5,7}\rangle+e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle+e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,4,6,7}+\omega^{1,5,6,7}\rangle+2e^{-2\beta_1-6\beta_2}\ \langle\omega^{2,3,4,7},\omega^{2,3,5,7}\rangle+e^{-4\beta_1-5\beta_2}\ \langle\omega^{1,3,6,7}-\omega^{2,5,6,7}-\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7},\omega^{1,2,4,6,7}\rangle+2e^{-5\beta_1-6\beta_2}\ \langle\omega^{1,3,4,6,7}+\omega^{2,4,5,6,7},\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}\rangle+e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle+e^{-4\beta_1-6\beta_2}\ \langle\omega^{1,2,3,6,7}\rangle+e^{-3\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle+e^{-4\beta_1-7\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-7\beta_2}\ \langle\omega^{1,2,3,5,6,7}\rangle+e^{-6\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle+e^{-6\beta_1-7\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle+e^{-5\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,21,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,8,11,11,8,4,1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^7$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,7} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{4,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6} + \omega^{5,7}$; $d^\star\omega^{1,3,6} = \omega^{6,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} + \omega^{2,4,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{2,3,6} + \omega^{2,5,7}$; $d^\star\omega^{1,2,3,6} = \omega^{2,6,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6} - \omega^{4,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7} + \omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7} - \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7} - \omega^{1,3,5,7} - \omega^{2,3,4,6} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} + \omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{3,4,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4} + \omega^{2,5}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4} + \omega^{2,5}$; $\Delta\omega^{2,5} = \omega^{1,3} + \omega^{2,4} + \omega^{2,5}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} - \omega^{3,5}$; $\Delta\omega^{3,5} = -\omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{5,7}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{3,6} - \omega^{4,7} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} + \omega^{1,2,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,4} + 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} - \omega^{1,3,5} - \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4} - \omega^{2,4,5}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,7} + 2\omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,5,7} + \omega^{2,4,6} + \omega^{2,5,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{2,3,6} + \omega^{2,4,7} + \omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = -\omega^{1,3,6} - \omega^{1,4,7} + 2\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,5} = \omega^{2,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6} - \omega^{2,5,7}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} - \omega^{1,3,4} + \omega^{1,3,5} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} - \omega^{1,4,7} + 2\omega^{2,4,6} + \omega^{2,5,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} + \omega^{2,4,7} - \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{1,3,6} - \omega^{1,5,7} + \omega^{2,4,6} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,3,7} - \omega^{2,3,6} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,7} - \omega^{2,4,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6} + \omega^{4,5,7}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7} - \omega^{3,5,7}$; $\Delta\omega^{3,5,6} = \omega^{2,6,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{3,4,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{3,4,6} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7} - \omega^{5,6,7}$; $\Delta\omega^{5,6,7} = -\omega^{4,6,7} + 3\omega^{5,6,7}$;

$\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4} + \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,4} + 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,5,7}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,4,6} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = -\omega^{1,2,3,6} + \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} - \omega^{1,3,5,7} - \omega^{2,3,4,6} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,6,7} + \omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,3,4,7} + 2\omega^{1,3,5,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{2,4,6,7} + \omega^{2,5,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,3,4,6} + \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{1,5,6,7}$; $\Delta\omega^{1,5,6,7} = -\omega^{1,4,6,7} + \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + 2\omega^{2,3,4,6} + \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} - \omega^{1,3,4,6} + \omega^{1,3,5,6} - \omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,3,4,7} + \omega^{1,3,5,7} + \omega^{2,3,4,6} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} + 2\omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = \omega^{1,3,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,6,7} + \omega^{2,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7} - \omega^{3,5,6,7}$; $\Delta\omega^{3,5,6,7} = -\omega^{3,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6} + \omega^{1,2,3,5,6} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,4,6} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,3,4,6} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $\Delta\omega^{1,3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_2} \langle\omega^3\rangle + e^{-\beta_1-\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-3\beta_2} \langle\omega^{2,3}\rangle + 2e^{-\beta_1-2\beta_2} \langle\omega^{1,3} - \omega^{2,5}, \omega^{2,4} - \omega^{2,5}\rangle + 2e^{-\beta_1-3\beta_2} \langle\omega^{3,4}, \omega^{2,7} + \omega^{3,5}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,7} + \omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}, \omega^{1,5,6}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{2,3,4} - \omega^{2,3,5}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,7} + \omega^{1,3,4} + \omega^{2,4,5}\rangle + 3e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,6} - \omega^{2,5,6}, \omega^{1,4,7} + \omega^{2,4,6} - \omega^{2,5,6}, \omega^{1,5,7} - \omega^{2,4,6} + 2\omega^{2,5,6}\rangle + 3e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6} + \omega^{2,5,7}, \omega^{1,3,7} - \omega^{2,5,7} + \omega^{3,4,5}, \omega^{2,4,7} + \omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = 2e^{-2\beta_1-6\beta_2} \langle-\omega^{2,3,5,7}, \omega^{2,3,4,7}\rangle + e^{-5\beta_1-4\beta_2} \langle(-\omega^{1,4,6,7} - \omega^{1,5,6,7})/2\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-5\beta_2} \langle(-\omega^{1,3,6,7} + \omega^{2,5,6,7} + \omega^{3,4,5,6})/3\rangle + 3e^{-3\beta_1-5\beta_2} \langle(\omega^{1,3,4,7} - \omega^{1,3,5,7} - \omega^{2,3,4,6} + 3\omega^{2,4,5,7})/4, (\omega^{1,3,5,7} - \omega^{2,3,4,6} + 2\omega^{2,3,5,6})/3, (-3\omega^{1,3,4,7} - \omega^{1,3,5,7} - 5\omega^{2,3,4,6} + 4\omega^{2,3,5,6} + 3\omega^{2,4,5,7})/12\rangle + 3e^{-4\beta_1-4\beta_2} \langle(\omega^{1,2,6,7} + 3\omega^{1,3,4,6} + \omega^{1,3,5,6} - 5\omega^{1,4,5,7} - 2\omega^{2,4,5,6})/8, (\omega^{1,2,6,7} - \omega^{1,3,4,6} + \omega^{1,3,5,6} - \omega^{1,4,5,7} - 2\omega^{2,4,5,6})/4, (3\omega^{1,2,6,7} + \omega^{1,3,4,6} - 5\omega^{1,3,5,6} + \omega^{1,4,5,7} + 2\omega^{2,4,5,6})/8\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-4\beta_1-7\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + 2e^{-5\beta_1-6\beta_2}\ \langle(-\omega^{1,3,4,6,7}+\omega^{1,3,5,6,7}-2\omega^{2,4,5,6,7})/3, (-\omega^{1,3,4,6,7}-2\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7})/3\rangle + 2e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,5,6,7}, (-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6})/2\rangle + e^{-4\beta_1-6\beta_2}\ \langle(-\omega^{1,2,3,6,7}-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6})/3\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-5\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-6\beta_1-7\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_2}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-6\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^3.$

5.4.32 Algebra $\mathcal{G}_{7,2.28}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & -\xi_1^2 & -\xi_1^1 + 2\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_1^2 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & -\xi_1^2 & 2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^5 + \xi_2^6 & -\xi_1^4 + \xi_3^6 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5, ad\ x_3 = -\delta_1^6 + \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{-\beta_1+2\beta_2}\ \langle x_3 \rangle + e^{2\beta_1}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{2\beta_2}\ \langle x_6 \rangle + e^{\beta_1+2\beta_2}\ \langle x_7 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+2\beta_2}\ \langle x_7 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{3\beta_1-2\beta_2}\ \langle \delta_3^4 \rangle + 3e^{\beta_1}\ \langle \delta_2^5, \delta_1^4, \delta_3^6 \rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4 \rangle + 2e^{-\beta_1+2\beta_2}\ \langle \delta_1^6, \delta_4^7 \rangle + e^{-2\beta_1+2\beta_2}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{2\beta_2}\ \langle \delta_1^7 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_2^7 \rangle + e^{2\beta_1}\ \langle \delta_3^7 \rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_2^6 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^3] = \delta_3^6 + \delta_1^4$; $[\delta_3^4, \delta_1^2] = -\delta_2^4$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_2^5, \delta_1^2] = \delta_2^6 + \delta_1^5$; $[\delta_2^5, \delta_1^5] = \delta_2^7$; $[\delta_2^5, \delta_2^6] = -\delta_2^7$; $[\delta_2^4, \delta_1^3] = \delta_2^6 + \delta_1^5$; $[\delta_2^4, \delta_1^2] = \delta_3^6 - 2\delta_2^5 + \delta_1^4$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_2^4, \delta_1^5] = \delta_3^7$; $[\delta_2^4, \delta_2^6] = -\delta_3^7$; $[\delta_1^6, \delta_1^4] = \delta_1^7$; $[\delta_1^6, \delta_3^6] = -\delta_1^7$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^4] = -(\delta_4^7 + \delta_1^6)$; $[\delta_1^3, \delta_3^6] = \delta_4^7 - \delta_1^6$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_3^7] = \delta_2^7$; $[\delta_1^2, \delta_1^5] = \delta_4^7 - \delta_1^6$; $[\delta_1^2, \delta_2^6] = -(\delta_4^7 + \delta_1^6)$; $[\delta_1^2, \delta_1^4] = \delta_1^5$; $[\delta_1^2, \delta_3^6] = \delta_2^6$; $[\delta_4^7, \delta_1^4] = \delta_1^7$; $[\delta_1^5, \delta_2^6] = -\delta_1^7$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_2^6, \delta_3^6] = -\delta_2^7$; $[\delta_1^4, \delta_3^6] = -\delta_3^7$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1-2\beta_2}\ \langle \delta_3^4 \rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4 \rangle + e^{-2\beta_1+2\beta_2}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_1^6 \rangle + e^{\beta_2}\ \langle \delta_2^6 \rangle + 2e^{\beta_1}\ \langle \delta_1^4, \delta_2^5 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^3] = \delta_1^4 - \delta_2^5$; $[\delta_3^4, \delta_1^2] = -\delta_2^4$; $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_2^4, \delta_1^2] = -3\delta_2^5 + \delta_1^4$; $[\delta_1^3, \delta_1^4] = -2\delta_1^6$; $[\delta_1^2, \delta_2^6] = -2\delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_2^6$.

ch($H^2_{\mathcal{G}}$)= $4e^{-2\beta_1}\ \langle x_2\omega^{1,5}, -2x_1\omega^{1,4} + x_2\omega^{2,4} - 2x_3(\omega^{1,6} - \omega^{3,4}) + x_5\omega^{4,5}, -x_3(\omega^{1,6} + \omega^{3,4}) + x_6\omega^{4,6}, x_1\omega^{1,4} - x_2\omega^{2,4} + x_3(\omega^{1,6} - \omega^{2,5} - 2\omega^{3,4}) + x_6\omega^{1,7} \rangle + e^{-4\beta_1+2\beta_2}\ \langle x_3\omega^{1,4} \rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5}, x_2\omega^{1,4} - x_3\omega^{2,4} +$

$x_6\omega^{4,5}\rangle + 4e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2} + x_6\omega^{2,5}, x_5\omega^{2,4}, x_3\omega^{1,3} + x_6\omega^{3,4}\rangle + 5e^{-\beta_2}\ \langle -x_2\omega^{1,3} + x_4\omega^{1,5}, x_1\omega^{1,2} + 2x_3\omega^{2,3} - 2x_4\omega^{2,4} + x_6\omega^{2,6}, x_4\omega^{2,4} + x_5\omega^{3,4}, x_5\omega^{2,5}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{2,4} + x_5\omega^{1,6}\rangle + e^{3\beta_1-3\beta_2}\ \langle x_4\omega^{2,3}\rangle + 2e^{2\beta_1-3\beta_2}\ \langle -x_1\omega^{2,3} + x_4(\omega^{2,6} + \omega^{3,5}), x_1\omega^{2,3} + x_5\omega^{3,6}\rangle + e^{3\beta_1-4\beta_2}\ \langle x_4\omega^{3,6}\rangle + 4e^{\beta_1-2\beta_2}\ \langle -2x_1\omega^{1,3} + x_2\omega^{2,3} + 2x_4\omega^{1,6} + x_5\omega^{3,5}, x_1\omega^{1,3} - x_4\omega^{1,6} + x_5\omega^{2,6}, x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} - x_4(\omega^{1,6} - \omega^{2,5})\rangle + e^{-2\beta_2}\ \langle x_1(\omega^{1,6} - \omega^{3,4}) - x_2(\omega^{2,6}+\omega^{3,5}) - x_3\omega^{3,6} - x_5(\omega^{2,7}-\omega^{5,6})\rangle + 2e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,2} + x_6\omega^{2,4}, -x_3\omega^{1,2} + x_5\omega^{1,4}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,5} + \omega^{2,4}) - x_2(\omega^{1,6} - \omega^{3,4}) + x_3(\omega^{2,6} + \omega^{3,5}) + x_5(\omega^{1,7} - 2\omega^{4,6}) + x_6\omega^{2,7}, -2x_1\omega^{1,5} - 3x_2(\omega^{1,6} - 2/3\omega^{2,5} - 1/3\omega^{3,4}) + x_3(\omega^{2,6} + \omega^{3,5}) - x_5\omega^{4,6} + x_6\omega^{5,6}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_4\omega^{1,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{3\beta_1-5\beta_2} + e^{2\beta_1-4\beta_2} + 2e^{2\beta_1-5\beta_2} + 2e^{\beta_1-3\beta_2} + 4e^{\beta_1-4\beta_2} + e^{\beta_1-5\beta_2} + 6e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-3\beta_2} + 6e^{-2\beta_1-\beta_2} + 4e^{-2\beta_1-2\beta_2} + 4e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1} + e^{-4\beta_1+\beta_2} + 2e^{-4\beta_1} + e^{-5\beta_1+\beta_2} + 6e^{-3\beta_2} + e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{2\beta_1-7\beta_2} + 2e^{\beta_1-5\beta_2} + 2e^{\beta_1-6\beta_2} + e^{\beta_1-7\beta_2} + e^{-\beta_1-3\beta_2} + 5e^{-\beta_1-4\beta_2} + 4e^{-\beta_1-5\beta_2} + 5e^{-2\beta_1-3\beta_2} + 4e^{-2\beta_1-4\beta_2} + 2e^{-3\beta_1-2\beta_2} + 6e^{-3\beta_1-3\beta_2} + e^{-4\beta_1-\beta_2} + 3e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + 2e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-6\beta_1-\beta_2} + e^{-4\beta_2} + 5e^{-5\beta_2} + e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{\beta_1-8\beta_2} + e^{-\beta_1-5\beta_2} + 4e^{-\beta_1-6\beta_2} + 2e^{-\beta_1-7\beta_2} + 4e^{-2\beta_1-5\beta_2} + 3e^{-2\beta_1-6\beta_2} + 2e^{-3\beta_1-4\beta_2} + 5e^{-3\beta_1-5\beta_2} + e^{-4\beta_1-3\beta_2} + 4e^{-4\beta_1-4\beta_2} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + 3e^{-7\beta_2} + e^{-8\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=2e^{-\beta_1-8\beta_2} + 3e^{-2\beta_1-7\beta_2} + e^{-2\beta_1-8\beta_2} + 3e^{-3\beta_1-6\beta_2} + e^{-3\beta_1-7\beta_2} + e^{-4\beta_1-5\beta_2} + 2e^{-4\beta_1-6\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-4\beta_2} + e^{-8\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-2\beta_1-8\beta_2} + e^{-3\beta_1-8\beta_2} + e^{-4\beta_1-7\beta_2} + e^{-5\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 16, 63, 131, 163, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 10, 30, 47, 49, 37, 18, 4)$

S=$4(\beta_1 + 2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 20, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 10, 10, 8, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} =$

$\omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{\beta_1-2\beta_2} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1} \langle\omega^{1,4}\rangle + 2e^{-2\beta_1-\beta_2} \langle\omega^{1,5},\omega^{2,4}\rangle + 2e^{-\beta_1-2\beta_2} \langle\omega^{1,6},\omega^{2,5}\rangle + e^{\beta_1-3\beta_2} \langle\omega^{2,3}\rangle + e^{-3\beta_2} \langle\omega^{2,6}+\omega^{3,5}\rangle + e^{\beta_1-4\beta_2} \langle\omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-\beta_2} \langle\omega^{1,4,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{1,2,6}\rangle + e^{-4\beta_2} \langle\omega^{1,3,6}\rangle + e^{\beta_1-5\beta_2} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + 2e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,7}-\omega^{3,4,5},\omega^{2,4,6}+\omega^{3,4,5}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{1,3,7}-\omega^{2,5,6}\rangle + e^{-5\beta_2} \langle\omega^{2,3,7}+\omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,5,7}-\omega^{2,4,7}-\omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,5,7}+\omega^{2,4,5,6}\rangle + 2e^{-2\beta_1-5\beta_2} \langle\omega^{1,2,6,7}+\omega^{3,4,5,6},\omega^{2,3,4,7}-\omega^{3,4,5,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{1,2,3,7}\rangle + e^{-\beta_1-6\beta_2} \langle\omega^{1,3,6,7}-\omega^{2,3,5,7}\rangle + e^{-7\beta_2} \langle\omega^{2,3,6,7}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{1,4,6,7}-\omega^{2,4,5,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,5,6,7}+\omega^{2,4,6,7}+\omega^{3,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,4,5,7}\rangle + 2e^{-3\beta_1-6\beta_2} \langle\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7},\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,2,4,6,7}\rangle + 2e^{-2\beta_1-7\beta_2} \langle\omega^{1,3,5,6,7},\omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-\beta_1-8\beta_2} \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-2\beta_1-8\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-4\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+3Y+5Y^2+5Y^3+5Y^4+3Y^5+Y^6)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,11,20,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,8,10,10,8,4,1)$

Total dimension of trivial cohomology: 46.

Length associated polynomial: 46.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{2,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{3,4,6}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{4,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} - \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,5,7} - \omega^{2,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} =$

$2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,4,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{\beta_1-2\beta_2}\ \langle\omega^3\rangle + e^{-2\beta_1}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-3\beta_1}\ \langle\omega^{1,4}\rangle + 2e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}, \omega^{2,4}\rangle + e^{\beta_1-3\beta_2}\ \langle\omega^{2,3}\rangle + 2e^{-\beta_1-2\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} - \omega^{3,4}\rangle + e^{-3\beta_2}\ \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{\beta_1-4\beta_2}\ \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-4\beta_1-\beta_2}\ \langle\omega^{1,4,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{-4\beta_2}\ \langle\omega^{1,3,6} - \omega^{2,3,5}\rangle + e^{\beta_1-5\beta_2}\ \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,4,6} - \omega^{2,4,5}\rangle + 2e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7} + 2\omega^{2,4,6} + \omega^{3,4,5}, \omega^{1,5,6} + \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-4\beta_2}\ \langle\omega^{1,3,7} - \omega^{2,5,6} + 2\omega^{3,4,6}\rangle + e^{-5\beta_2}\ \langle\omega^{2,3,7} + \omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,5,7} - \omega^{2,4,7} - \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-7\beta_2}\ \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1-5\beta_2}\ \langle(-\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7})/3\rangle + e^{-4\beta_1-4\beta_2}\ \langle(-\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-5\beta_1-3\beta_2}\ \langle-\omega^{1,4,5,7}\rangle + e^{-\beta_1-6\beta_2}\ \langle(\omega^{1,3,6,7} - \omega^{2,3,5,7})/2\rangle + 2e^{-2\beta_1-5\beta_2}\ \langle(\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{3,4,5,6})/3, (\omega^{1,2,6,7} + 2\omega^{2,3,4,7} - \omega^{3,4,5,6})/3\rangle + e^{-3\beta_1-4\beta_2}\ \langle(-2\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,4,5,6})/6\rangle + e^{-4\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,7} + \omega^{1,4,5,6})/2\rangle + e^{-\beta_1-5\beta_2}\ \langle(\omega^{1,2,3,7} + \omega^{1,3,5,6} - \omega^{2,3,4,6})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-\beta_1-8\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle + 2e^{-2\beta_1-7\beta_2}\ \langle-\omega^{2,3,4,6,7}, -\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + 2e^{-3\beta_1-6\beta_2}\ \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (-\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7})/3\rangle + e^{-4\beta_1-5\beta_2}\ \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,7}\rangle$.

$\operatorname{ch}(H^6_{\mathfrak{C}})= e^{-3\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-2\beta_1-8\beta_2} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\operatorname{ch}(H^7_{\mathfrak{C}})= e^{-4\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y+5Y^2+5Y^3+5Y^4+3Y^5+Y^6)(1+Y)$.

5.4.33 Algebra $\mathcal{G}_{7,2.29}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 \\ 0 & -2\xi_1^2 & \xi_3^4 & -\xi_1^1 + 2\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & -\xi_1^2 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^5 & 2\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^5 & \xi_1^2 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5 + \delta_3^6, ad\ x_3 = -\delta_2^6 + \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{2\beta_1}\ \langle x_3\rangle + e^{-\beta_1+2\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-3\beta_1+2\beta_2}\ \langle \delta_3^4\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_1^6, \delta_2^7\rangle + 2e^{2\beta_1}\ \langle \delta_2^6, \delta_4^7\rangle + 2e^{\beta_2}\ \langle \delta_3^6, \delta_1^5\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_4^6\rangle + e^{\beta_1}\ \langle \delta_2^5\rangle + e^{2\beta_2}\ \langle \delta_1^7\rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_3^7\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = -\delta_2^7 + \delta_1^6$; $[\delta_3^6, \delta_1^2] = -\delta_3^7$; $[\delta_4^6, \delta_1^2] = -\delta_4^7 - 2\delta_2^6$; $[\delta_2^5, \delta_1^5] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^2] = -\delta_3^6 + \delta_1^5$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_4^7, \delta_1^2] = -2\delta_2^7$; $[\delta_1^5, \delta_1^2] = \delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-3\beta_1+2\beta_2}\ \langle \delta_3^4\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_4^6\rangle + e^{2\beta_2}\ \langle \delta_1^7\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^6\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{\beta_2}\ \langle \delta_1^5\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^6, \delta_1^2] = -3\delta_2^6$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_3^4, \delta_4^6] = -\delta_1^5$; $[\delta_2^6, \delta_1^2] = 2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{3\beta_1-3\beta_2}\ \langle x_3\omega^{2,4}\rangle + e^{-4\beta_1+2\beta_2}\ \langle x_4\omega^{1,3}\rangle + 4e^{-\beta_2}\ \langle -x_3\omega^{2,3} + x_4\omega^{2,4}, x_1\omega^{1,2} - x_2\omega^{1,4} - x_3(\omega^{1,5} - \omega^{2,3}) + x_5\omega^{3,4}, 2x_1\omega^{1,2} - x_2\omega^{1,4} - x_3\omega^{1,5} + x_6\omega^{1,7}, -x_1\omega^{1,2} + x_6\omega^{2,6}\rangle + 4e^{-2\beta_1}\ \langle -2x_1\omega^{1,3} - 3x_2(\omega^{1,5} - 1/3\omega^{2,3}) + x_5\omega^{3,5}, x_4\omega^{3,4}, x_1\omega^{1,3} + x_2(\omega^{1,5} - \omega^{2,3}) + x_4\omega^{2,5}, x_2\omega^{1,5} + x_6\omega^{3,6}\rangle + 3e^{\beta_1-2\beta_2}\ \langle x_1\omega^{1,4} - 2x_2\omega^{2,4} - 3x_3(\omega^{2,5} - 2/3\omega^{3,4}) + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3(\omega^{2,5} - 2\omega^{3,4}) + x_6\omega^{4,6}, -x_3\omega^{3,4} + x_6\omega^{2,7}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{1,4} + x_5\omega^{2,4}\rangle + e^{2\beta_1-3\beta_2}\ \langle -x_1\omega^{2,4} + x_6\omega^{4,7}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,5}\rangle + e^{-2\beta_1+\beta_2}\ \langle -x_4\omega^{1,2} + x_7\omega^{1,6}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2} - x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_7\omega^{1,7}, x_3\omega^{1,3} - x_4\omega^{1,4} + x_5\omega^{2,3}, x_3\omega^{1,3} -$

$x_4\omega^{1,4} + x_6\omega^{1,6}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_4\omega^{2,3}, -x_2\omega^{1,3} + x_4\omega^{1,5}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,5} - \omega^{2,3}) + x_2\omega^{2,5}, -x_1\omega^{2,3} + x_6\omega^{3,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{2\beta_1-4\beta_2} + e^{2\beta_1-5\beta_2} + 2e^{\beta_1-3\beta_2} + 2e^{\beta_1-4\beta_2} + 4e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-3\beta_2} + 3e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + 2e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 2e^{-2\beta_2} + 3e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{\beta_1-6\beta_2} + 3e^{-\beta_1-4\beta_2} + 2e^{-\beta_1-5\beta_2} + 2e^{-2\beta_1-3\beta_2} + 5e^{-2\beta_1-4\beta_2} + 7e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + 5e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2} + 2e^{-5\beta_1-\beta_2} + 3e^{-5\beta_1-2\beta_2} + 2e^{-6\beta_1-\beta_2} + e^{-7\beta_1} + 3e^{-5\beta_2} + e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = 2e^{-\beta_1-6\beta_2} + e^{-\beta_1-7\beta_2} + 3e^{-2\beta_1-5\beta_2} + 2e^{-2\beta_1-6\beta_2} + e^{-2\beta_1-7\beta_2} + 2e^{-3\beta_1-4\beta_2} + 2e^{-3\beta_1-5\beta_2} + e^{-3\beta_1-6\beta_2} + e^{-4\beta_1-3\beta_2} + 3e^{-4\beta_1-4\beta_2} + 2e^{-4\beta_1-5\beta_2} + 2e^{-5\beta_1-3\beta_2} + 2e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2} + 2e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-2\beta_1-7\beta_2} + e^{-3\beta_1-6\beta_2} + 2e^{-3\beta_1-7\beta_2} + 2e^{-4\beta_1-6\beta_2} + e^{-4\beta_1-7\beta_2} + 2e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2} + 3e^{-6\beta_1-5\beta_2} + 2e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-4\beta_1-7\beta_2} + e^{-5\beta_1-7\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-7\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 59, 126, 159, 117, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 24, 38, 40, 31, 16, 4)$

$S = 6\beta_1 + 7\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 21, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 9, 9, 6, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = -\omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = \omega^{2,3,4,5,6}$;

$d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^3\rangle + e^{\beta_1-2\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1} \langle\omega^{1,3}\rangle + e^{-2\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{\beta_1-3\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-5\beta_1-\beta_2} \langle\omega^{1,3,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,7} + \omega^{2,3,6}, \omega^{1,5,6} - \omega^{2,3,6}\rangle + 2e^{-3\beta_1-3\beta_2} \langle\omega^{2,3,7}, \omega^{1,5,7} - \omega^{3,4,6}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{1,4,7} + \omega^{2,4,6}\rangle + e^{-5\beta_2} \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,4,6} - \omega^{3,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,5,7} - \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle + 2e^{-3\beta_1-4\beta_2} \langle\omega^{1,3,4,7} + \omega^{2,3,4,6}, \omega^{1,4,5,6} + \omega^{2,3,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,3,5,6}\rangle + 2e^{-2\beta_1-5\beta_2} \langle\omega^{1,4,5,7}, \omega^{2,3,4,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-\beta_1-6\beta_2} \langle\omega^{2,4,5,7}\rangle + e^{-4\beta_1-4\beta_2} \langle\omega^{1,2,6,7} + \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-7\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)^2(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 21, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 9, 9, 6, 4, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$;

$d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7}+\omega^{1,3,4,7}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7}+\omega^{2,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7}-\omega^{2,3,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} + \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}+\omega^{2,3,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6}+3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7}-\omega^{1,4,5,6}+2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,4,5,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$;

$\Delta\omega^{2,3,6,7}=\omega^{1,5,6,7}+\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6}=-\omega^{1,4,5,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{2,3,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{1,4,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{A}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle+e^{\beta_1-2\beta_2}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{A}})= e^{-3\beta_1}\ \langle\omega^{1,3}\rangle+e^{-2\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}-\omega^{2,3}\rangle+e^{\beta_1-3\beta_2}\ \langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}-\omega^{3,4}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{A}})= e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,7}+\omega^{2,3,6},\omega^{1,5,6}-\omega^{2,3,6}\rangle+e^{-\beta_1-4\beta_2}\ \langle\omega^{1,4,7}+\omega^{2,4,6}\rangle+e^{-5\beta_2}\ \langle\omega^{2,4,7}\rangle+2e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,3,7}+\omega^{2,5,6},\omega^{1,5,7}+2\omega^{2,5,6}-\omega^{3,4,6}\rangle+e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7}-2\omega^{1,4,6}+\omega^{3,4,5}\rangle+e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,5,7}-\omega^{3,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{A}})= e^{-\beta_1-6\beta_2}\ \langle\omega^{2,4,5,7}\rangle+2e^{-2\beta_1-5\beta_2}\ \langle(-\omega^{1,4,5,7}+\omega^{2,3,4,7}-2\omega^{2,4,5,6})/3,(\omega^{1,4,5,7}+2\omega^{2,3,4,7}-\omega^{2,4,5,6})/3\rangle+e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,3,5,7}+\omega^{2,3,5,6})/2\rangle+e^{-6\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6}\rangle+2e^{-3\beta_1-4\beta_2}\ \langle(-\omega^{1,2,5,7}-\omega^{1,3,4,7}+3\omega^{1,4,5,6}+\omega^{2,3,4,6})/4,(\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,4,5,6}-\omega^{2,3,4,6})/4\rangle+e^{-4\beta_1-4\beta_2}\ \langle(\omega^{1,2,6,7}+2\omega^{2,3,5,7}+\omega^{3,4,5,6})/6\rangle+e^{-4\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6}+\omega^{1,3,4,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{A}})= e^{-3\beta_1-7\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle+e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle+e^{-4\beta_1-6\beta_2}\ \langle(-\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7})/2\rangle+e^{-7\beta_1-4\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle+e^{-5\beta_1-5\beta_2}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2\rangle+e^{-3\beta_1-6\beta_2}\ \langle(\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{A}})= e^{-5\beta_1-7\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle+e^{-6\beta_1-6\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle+e^{-4\beta_1-7\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle+e^{-7\beta_1-5\beta_2}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{A}})= e^{-6\beta_1-7\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+3Y+Y^2)(1+Y^2)^2(1+Y).$

5.4.34 Algebra $\mathcal{G}_{7,2.30}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_3^4 & 5\xi_1^1 - \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_1^4 & -\xi_1^3 & \xi_2^5 & 4\xi_1^1 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^5 + \xi_2^6 & \xi_1^2 + \xi_2^5 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5, ad\ x_3 = \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{5\beta_1-\beta_3}\ \langle x_4\rangle + e^{3\beta_1}\ \langle x_5\rangle + e^{4\beta_1}\ \langle x_6\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-5\beta_1+2\beta_3}\ \langle \delta_4^3\rangle + e^{5\beta_1-2\beta_3}\ \langle \delta_3^4\rangle + 2e^0\ \langle t_3, t_1\rangle + 2e^{3\beta_1}\ \langle \delta_1^6, \delta_2^7\rangle + e^{4\beta_1-\beta_3}\ \langle \delta_1^4\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{4\beta_1}\ \langle \delta_1^7\rangle + e^{5\beta_1-\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_3}\ \langle \delta_4^7\rangle + 2e^{2\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + 2e^{\beta_1}\ \langle \delta_1^2, \delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_4^3, \delta_3^7] = -\delta_4^7$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_1^6, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^4, \delta_1^2] = -\delta_3^7$; $[\delta_1^4, \delta_2^5] = -\delta_3^7$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^2] = \delta_4^7$; $[\delta_1^3, \delta_2^5] = \delta_4^7$; $[\delta_2^7, \delta_1^2] = \delta_1^7$; $[\delta_1^5, \delta_2^6] = -\delta_1^7$; $[\delta_1^5, \delta_2^5] = -(\delta_2^7 + \delta_1^6)$; $[\delta_2^6, \delta_1^2] = -\delta_2^7 + \delta_1^6$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-5\beta_1+2\beta_3}\ \langle \delta_4^3\rangle + e^{5\beta_1-2\beta_3}\ \langle \delta_3^4\rangle + 2e^0\ \langle t_3, t_1\rangle + e^{4\beta_1-\beta_3}\ \langle \delta_1^4\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{3\beta_1}\ \langle \delta_1^6\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_2^6, \delta_1^2] = 2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-6\beta_1+2\beta_3}\ \langle x_3\omega^{1,4}\rangle + e^{-7\beta_1+2\beta_3}\ \langle x_3\omega^{2,4}\rangle + e^{4\beta_1-2\beta_3}\ \langle x_4\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_3\omega^{1,3} + x_6\omega^{3,4}, x_2\omega^{1,2} + x_6\omega^{2,5}\rangle + e^{3\beta_1-2\beta_3}\ \langle x_4\omega^{2,3}\rangle + 4e^{-2\beta_1}\ \langle -x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + 2x_5\omega^{1,6} + x_6\omega^{2,6}, x_1\omega^{1,2} - x_3\omega^{2,3} - x_5(\omega^{1,6} + \omega^{3,4}) + x_6\omega^{1,7}, x_2\omega^{1,5}\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} - x_4(\omega^{1,6} - 3\omega^{2,5}) + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + 2x_4(\omega^{1,6} - 1/2\omega^{2,5}) + x_6\omega^{3,6}, x_4\omega^{3,4}\rangle + 3e^{-5\beta_1+\beta_3}\ \langle x_1\omega^{1,4} - 2x_2\omega^{2,4} + x_3(\omega^{1,6} - 3\omega^{2,5}) + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} -$

$2x_3(\omega^{1,6} - 1/2\omega^{2,5}) + x_6\omega^{4,6}, x_3\omega^{3,4}\rangle + e^{2\beta_1-\beta_3}\ \langle -x_4\omega^{1,2} + x_6\omega^{2,3}\rangle + e^{-3\beta_1+\beta_3}\ \langle x_3\omega^{1,2} + x_6\omega^{2,4}\rangle + e^{\beta_1-\beta_3}\ \langle -x_2\omega^{1,3} + x_4\omega^{1,5}\rangle + e^{-4\beta_1+\beta_3}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle -x_1\omega^{1,5} - x_2(\omega^{1,6} - \omega^{2,5})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1-\beta_3}+e^{-\beta_1-2\beta_3}+3e^{-2\beta_1-\beta_3}+e^{-2\beta_1-2\beta_3}+3e^{-3\beta_1-\beta_3}+2e^{-4\beta_1-\beta_3}+4e^{-5\beta_1}+e^{-6\beta_1+\beta_3}+3e^{-6\beta_1}+3e^{-7\beta_1+\beta_3}+2e^{-7\beta_1}+3e^{-8\beta_1+\beta_3}+e^{-8\beta_1}+2e^{-9\beta_1+\beta_3}+e^{-10\beta_1+2\beta_3}+e^{-11\beta_1+2\beta_3}+e^{-12\beta_1+2\beta_3}+e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-3\beta_1-2\beta_3}+e^{-4\beta_1-2\beta_3}+e^{-5\beta_1-\beta_3}+e^{-5\beta_1-2\beta_3}+3e^{-6\beta_1-\beta_3}+4e^{-7\beta_1-\beta_3}+2e^{-8\beta_1-\beta_3}+4e^{-8\beta_1}+4e^{-9\beta_1}+e^{-10\beta_1+\beta_3}+2e^{-10\beta_1}+3e^{-11\beta_1+\beta_3}+4e^{-12\beta_1+\beta_3}+e^{-13\beta_1+2\beta_3}+2e^{-13\beta_1+\beta_3}+e^{-14\beta_1+2\beta_3}+e^{-15\beta_1+2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-8\beta_1-\beta_3}+e^{-8\beta_1-2\beta_3}+e^{-9\beta_1-\beta_3}+e^{-9\beta_1-2\beta_3}+3e^{-10\beta_1-\beta_3}+e^{-11\beta_1-\beta_3}+e^{-11\beta_1}+e^{-12\beta_1-\beta_3}+2e^{-12\beta_1}+e^{-13\beta_1+\beta_3}+e^{-13\beta_1-\beta_3}+3e^{-13\beta_1}+e^{-14\beta_1+\beta_3}+2e^{-14\beta_1}+3e^{-15\beta_1+\beta_3}+e^{-16\beta_1+\beta_3}+e^{-17\beta_1+\beta_3}+e^{-18\beta_1+2\beta_3}+e^{-18\beta_1+\beta_3}+e^{-19\beta_1+2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-10\beta_1-2\beta_3}+2e^{-13\beta_1-\beta_3}+2e^{-14\beta_1-\beta_3}+3e^{-15\beta_1}+e^{-16\beta_1}+e^{-17\beta_1}+2e^{-18\beta_1+\beta_3}+e^{-18\beta_1}+2e^{-19\beta_1+\beta_3}+e^{-20\beta_1+2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-15\beta_1-\beta_3}+e^{-18\beta_1}+e^{-19\beta_1}+e^{-20\beta_1+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 15, 56, 125, 156, 117, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 9, 22, 34, 36, 28, 16, 4)$

S=$20\beta_1$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 20, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 8, 8, 6, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = -\omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6}$; $d\omega^{5,7} = -\omega^{1,2,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} =$

$-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-5\beta_1+\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-6\beta_1+\beta_3} \langle\omega^{1,4}\rangle + 2e^{-5\beta_1} \langle\omega^{1,6},\omega^{2,5}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{2,3}\rangle + e^{-7\beta_1+\beta_3} \langle\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-5\beta_1-\beta_3} \langle\omega^{1,3,6}\rangle + e^{-6\beta_1-\beta_3} \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-10\beta_1+\beta_3} \langle\omega^{1,4,6}\rangle + e^{-11\beta_1+\beta_3} \langle\omega^{1,4,7}+\omega^{2,4,6}\rangle + e^{-8\beta_1} \langle\omega^{1,2,7}-\omega^{3,4,5}\rangle + e^{-9\beta_1} \langle\omega^{1,5,7}-\omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_3} \langle\omega^{2,3,7}+\omega^{3,5,6}\rangle + e^{-12\beta_1+\beta_3} \langle\omega^{2,4,7}+\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-11\beta_1} \langle\omega^{1,3,4,7}+\omega^{2,3,4,6}\rangle + e^{-8\beta_1-\beta_3} \langle\omega^{1,2,3,7}\rangle + e^{-13\beta_1+\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-12\beta_1} \langle\omega^{2,3,4,7}-\omega^{3,4,5,6}\rangle + e^{-9\beta_1-\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-10\beta_1-\beta_3} \langle\omega^{1,3,6,7}-\omega^{2,3,5,7}\rangle + e^{-14\beta_1+\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-15\beta_1+\beta_3} \langle\omega^{1,4,6,7}-\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-15\beta_1} \langle\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7},\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle + e^{-13\beta_1-\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-18\beta_1+\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-14\beta_1-\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-19\beta_1+\beta_3} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-15\beta_1-\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-20\beta_1+\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-18\beta_1} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-19\beta_1} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-20\beta_1} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y+3Y^2+5Y^3+3Y^4+3Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,9,20,23,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,6,8,8,6,4,1)$

Total dimension of trivial cohomology: 38.

Length associated polynomial: 38.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7}-\omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7}+\omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}-\omega^{2,4,6}$;

$d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 3\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,5,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} - \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,4,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$;

$\Delta\omega^{2,3,5,7}=\omega^{1,3,6,7}+\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6}=-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{2,3,4,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,5,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=\omega^{1,2,3,4,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6}=2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,4,7}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-2\beta_1}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-5\beta_1+\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+e^{-6\beta_1+\beta_3}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-7\beta_1+\beta_3}\ \langle\omega^{2,4}\rangle+2e^{-5\beta_1}\ \langle\omega^{1,6}-\omega^{3,4},\omega^{2,5}-\omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-5\beta_1-\beta_3}\ \langle\omega^{1,3,6}-\omega^{2,3,5}\rangle + e^{-6\beta_1-\beta_3}\ \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-10\beta_1+\beta_3}\ \langle\omega^{1,4,6}-\omega^{2,4,5}\rangle + e^{-11\beta_1+\beta_3}\ \langle\omega^{1,4,7}+\omega^{2,4,6}\rangle + e^{-8\beta_1}\ \langle\omega^{1,2,7}-2\omega^{1,5,6}-\omega^{3,4,5}\rangle + e^{-9\beta_1}\ \langle\omega^{1,5,7}+2\omega^{2,5,6}-\omega^{3,4,6}\rangle + e^{-7\beta_1-\beta_3}\ \langle\omega^{2,3,7}+\omega^{3,5,6}\rangle + e^{-12\beta_1+\beta_3}\ \langle\omega^{2,4,7}+\omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-15\beta_1+\beta_3}\ \langle(-\omega^{1,4,6,7}+\omega^{2,4,5,7})/2\rangle + e^{-14\beta_1+\beta_3}\ \langle(-\omega^{1,4,5,7}-\omega^{2,4,5,6})/2\rangle+e^{-10\beta_1-\beta_3}\ \langle(\omega^{1,3,6,7}-\omega^{2,3,5,7})/2\rangle+e^{-9\beta_1-\beta_3}\ \langle(\omega^{1,3,5,7}+\omega^{2,3,5,6})/2\rangle+e^{-12\beta_1}\ \langle(-\omega^{1,2,6,7}-2\omega^{2,3,4,7}+\omega^{3,4,5,6})/6\rangle + e^{-11\beta_1}\ \langle(\omega^{1,2,5,7}-2\omega^{1,3,4,7}-\omega^{2,3,4,6})/6\rangle+e^{-13\beta_1+\beta_3}\ \langle(\omega^{1,2,4,7}+\omega^{1,4,5,6})/2\rangle+e^{-8\beta_1-\beta_3}\ \langle(-\omega^{1,2,3,7}-\omega^{1,3,5,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-19\beta_1+\beta_3}\ \langle-\omega^{2,4,5,6,7}\rangle + e^{-14\beta_1-\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-18\beta_1+\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-13\beta_1-\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle + 2e^{-15\beta_1}\ \langle(-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+2\omega^{2,3,4,5,7})/3,(-\omega^{1,2,5,6,7}+2\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7})/3\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-19\beta_1}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-18\beta_1}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-20\beta_1+\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-15\beta_1-\beta_3}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-20\beta_1}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+3Y+3Y^2+5Y^3+3Y^4+3Y^5+Y^6)(1+Y).$

5.4.35 Algebra $\mathcal{G}_{7,2.31}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_3] = x_5$; $[x_2, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & 3\xi_1^1 - \xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & 0 & \xi_2^4 & 0 & 2\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 + \xi_2^6 & \xi_3^5 & \xi_2^4 & 3\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4 + \delta_3^5, ad\ x_3 = -\delta_2^5, ad\ x_4 = -\delta_1^6, ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T{=}\langle t_1, t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{3\beta_1-\beta_2}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{3\beta_1}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{3\beta_1-2\beta_2}\ \langle \delta_2^3\rangle + 2e^{\beta_2}\ \langle \delta_1^4, \delta_3^5\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_2^5\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^6\rangle + e^{2\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + e^{2\beta_2}\ \langle \delta_3^7\rangle + 2e^{2\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^5] = -\delta_2^5$; $[\delta_1^4, \delta_1^5] = \delta_1^7$; $[\delta_1^4, \delta_2^6] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_2^5, \delta_3^5] = -\delta_2^7$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $e^{3\beta_1-2\beta_2}\ \langle \delta_2^3\rangle + e^{2\beta_2}\ \langle \delta_3^7\rangle + 2e^{2\beta_1}\ \langle \delta_1^5, \delta_2^6\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{2\beta_1-3\beta_2}\ \langle x_3\omega^{2,4}\rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2} + 2x_3\omega^{1,3} + x_5\omega^{1,5}, x_2\omega^{1,2} - x_3\omega^{1,3} + x_6\omega^{2,5}, x_4\omega^{1,4}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_3\omega^{1,2}\rangle + 2e^{-2\beta_2}\ \langle x_3\omega^{2,5}, x_1\omega^{2,4} - x_3\omega^{1,6} + x_5\omega^{4,6}\rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} + 2x_4\omega^{2,4} + x_6\omega^{2,6}, x_3\omega^{2,3}, -x_1\omega^{1,2} - x_4\omega^{2,4} + x_5\omega^{1,6}\rangle + e^{-4\beta_1+2\beta_2}\ \langle x_2\omega^{1,3} + x_6\omega^{3,5}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_1\omega^{1,3} - 3x_4(\omega^{1,5} - 2/3\omega^{3,4}) + x_6\omega^{3,6}, -x_1\omega^{1,3} - x_2\omega^{2,3} + 3x_4(\omega^{1,5} - 1/3\omega^{3,4}) + x_7\omega^{3,7}\rangle + e^{-\beta_1+\beta_2}\ \langle x_5\omega^{1,3} + x_7\omega^{3,4}\rangle + e^{-3\beta_1+2\beta_2}\ \langle x_4\omega^{1,3}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,4} - 2x_2\omega^{2,4} - 3x_3(\omega^{1,5} - \omega^{3,4}) - 3x_4\omega^{2,6} - 4x_5(\omega^{1,7} + 1/4\omega^{4,5}) - 3x_6\omega^{2,7} + x_7\omega^{4,7}\rangle + e^{-2\beta_1}\ \langle x_4\omega^{1,6}\rangle + e^{-3\beta_1}\ \langle -x_2\omega^{2,5} + x_3\omega^{3,5} + x_4\omega^{1,7}\rangle$.

ch($H^3_{\mathcal{G}}$)=$3e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-3\beta_2} + e^{-2\beta_1-3\beta_2} + 4e^{-3\beta_1-\beta_2} + 4e^{-3\beta_1-2\beta_2} + 2e^{-4\beta_1+\beta_2} + 2e^{-4\beta_1-\beta_2} + 2e^{-4\beta_1} + e^{-5\beta_1+\beta_2} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1} + 3e^{-6\beta_1+\beta_2} + e^{-7\beta_1+2\beta_2} + e^{-7\beta_1+\beta_2} + e^{-4\beta_2}$.

ch($H^4_{\mathcal{G}}$)=$e^{-3\beta_1-3\beta_2} + 2e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2} + 4e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 3e^{-6\beta_1-\beta_2} + 4e^{-6\beta_1-2\beta_2} + e^{-6\beta_1} + e^{-7\beta_1+\beta_2} + e^{-7\beta_1-\beta_2} + 3e^{-7\beta_1} + e^{-8\beta_1+\beta_2} + e^{-8\beta_1-\beta_2} + e^{-8\beta_1} + e^{-9\beta_1+\beta_2} + e^{-9\beta_1} + e^{-10\beta_1+\beta_2}$.

ch($H^5_{\mathcal{G}}$)=$e^{-4\beta_1-5\beta_2} + e^{-6\beta_1-3\beta_2} + 3e^{-6\beta_1-4\beta_2} + e^{-6\beta_1-5\beta_2} + e^{-7\beta_1-\beta_2} + 2e^{-7\beta_1-2\beta_2} + 2e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-2\beta_2} + e^{-8\beta_1-3\beta_2} + e^{-8\beta_1-4\beta_2} + 3e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + e^{-10\beta_1-\beta_2} + 2e^{-10\beta_1} + e^{-11\beta_1} + e^{-12\beta_1}$.

ch($H^6_{\mathcal{G}}$)=$e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-9\beta_1-3\beta_2} + 2e^{-9\beta_1-4\beta_2} + e^{-10\beta_1-\beta_2} + e^{-10\beta_1-2\beta_2} + e^{-10\beta_1-3\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-\beta_2} + e^{-12\beta_1-2\beta_2} + e^{-13\beta_1-\beta_2}$.

ch($H^7_{\mathcal{G}}$)=$e^{-10\beta_1-4\beta_2} + e^{-12\beta_1-3\beta_2} + e^{-13\beta_1-2\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 54, 122, 155, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 18, 29, 32, 24, 12, 3)$

S=$13\beta_1 + 3\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{2,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5}$; $d\omega^{1,5} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{2,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = \omega^{1,4,5} - \omega^{2,3,6}$; $d\omega^{5,7} = -\omega^{1,5,6} - \omega^{2,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{2,5,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = \omega^{1,2,3,6}$; $d\omega^{1,5,7} = \omega^{1,2,3,7}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = \omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathcal{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-3\beta_1+\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1+\beta_2}\ \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle + e^{-4\beta_1}\ \langle\omega^{1,5} - \omega^{3,4}\rangle + e^{-6\beta_1+\beta_2}\ \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}, \omega^{1,4,6}\rangle + e^{-7\beta_1+\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,6,7} - \omega^{2,5,7}\rangle + e^{-7\beta_1}\ \langle\omega^{1,3,7} - \omega^{3,4,5}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,6} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-10\beta_1}\ \langle\omega^{1,3,5,7}\rangle + 2e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,6,7} + \omega^{3,4,5,6}, \omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-7\beta_1-\beta_2}\ \langle\omega^{1,3,4,6}\rangle + e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,2,6,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle\omega^{1,4,6,7} - \omega^{2,4,5,7}\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-10\beta_1-\beta_2}\ \langle\omega^{1,2,3,5,7}\rangle + e^{-12\beta_1-\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-2\beta_2}\ \langle\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-9\beta_1-4\beta_2}\ \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-13\beta_1-2\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-13\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$;

$d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = \omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{1,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,5,7} + 3\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 3\omega^{1,2,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$;

$\Delta\omega^{2,3,4,5,7} = \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-3\beta_1+\beta_2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1+\beta_2}\ \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{2,5}\rangle + e^{-4\beta_1}\ \langle\omega^{1,5} - \omega^{3,4}\rangle + e^{-6\beta_1+\beta_2}\ \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{2,7} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-7\beta_1+\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,3,6} - \omega^{2,3,5}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} - \omega^{2,4,5}, \omega^{1,4,6} - \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,6,7} - \omega^{2,5,7}\rangle + e^{-7\beta_1}\ \langle\omega^{1,3,7} - \omega^{3,4,5}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,6} - \omega^{2,3,7} - 2\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2}\ \langle-\omega^{2,4,6,7}\rangle + e^{-7\beta_1-3\beta_2}\ \langle(-\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + 2e^{-9\beta_1-\beta_2}\ \langle(\omega^{1,3,6,7} + \omega^{2,3,5,7} + 2\omega^{3,4,5,6})/3, (\omega^{1,3,6,7} - 2\omega^{2,3,5,7} - \omega^{3,4,5,6})/3\rangle + e^{-10\beta_1}\ \langle\omega^{1,3,5,7}\rangle + e^{-7\beta_1-\beta_2}\ \langle(-\omega^{1,3,4,6} + \omega^{2,3,4,5})/2\rangle + e^{-6\beta_1-3\beta_2}\ \langle(-\omega^{1,2,6,7} - \omega^{2,4,5,6})/2\rangle + e^{-7\beta_1-2\beta_2}\ \langle(2\omega^{1,2,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_1-4\beta_2}\ \langle-\omega^{2,4,5,6,7}\rangle + e^{-12\beta_1-\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-2\beta_2}\ \langle(-\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-9\beta_1-3\beta_2}\ \langle(-\omega^{1,2,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-7\beta_1-4\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle + e^{-10\beta_1-\beta_2}\ \langle(\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-12\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-13\beta_1-2\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-13\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

5.4.36 Algebra $\mathcal{G}_{7,2.32}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & (2\xi_1^1+\xi_2^2)/2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & 0 & \xi_3^5 & 0 & (4\xi_1^1+\xi_2^2)/2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & -\xi_1^3 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & -\xi_1^5+\xi_3^6 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{(2\beta_1+\beta_2)/2}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{(4\beta_1+\beta_2)/2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{\beta_1}\ \langle\delta_3^5,\delta_2^4\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_2/2}\ \langle\delta_1^3\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{(4\beta_1+\beta_2)/2}\ \langle\delta_3^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+2e^{(2\beta_1+\beta_2)/2}\ \langle\delta_1^5,\delta_3^6\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_3^5,\delta_1^3]=\delta_3^6+\delta_1^5$; $[\delta_3^5,\delta_1^5]=\delta_3^7$; $[\delta_3^5,\delta_3^6]=-\delta_3^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_2^6]=\delta_3^7$; $[\delta_1^3,\delta_1^5]=-\delta_1^6$; $[\delta_1^3,\delta_3^6]=-\delta_1^6$; $[\delta_1^3,\delta_2^4]=\delta_3^6$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_3^6,\delta_2^4]=-\delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_2/2}\ \langle\delta_1^3\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{(2\beta_1+\beta_2)/2}\ \langle\delta_3^6\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_2^4]=\delta_3^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{-3\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,5}\rangle+2e^{-2\beta_1}\ \langle x_3\omega^{1,5},x_2\omega^{1,4}\rangle+e^{-2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{-3\beta_1}\ \langle x_2\omega^{1,6}\rangle+e^{-\beta_1-\beta_2}\ \langle x_3(\omega^{2,5}+\omega^{3,4})+2x_5(\omega^{3,6}+1/2\omega^{4,5})\rangle+e^{\beta_1-1/2\beta_2}\ \langle -x_5\omega^{1,2}+x_6\omega^{2,3}\rangle+e^{-1/2\beta_2}\ \langle x_3\omega^{1,2}+x_4\omega^{2,3}+x_6\omega^{2,5}\rangle+3e^{-\beta_2}\ \langle x_1\omega^{1,2}+1/2x_3\omega^{2,3}+2x_4\omega^{2,4}-1/2x_5\omega^{2,5}+x_6\omega^{2,6},-x_1\omega^{1,2}-x_4\omega^{2,4}+x_5\omega^{2,5}+x_7\omega^{2,7},-1/2x_3\omega^{2,3}+1/2x_5(\omega^{2,5}+2\omega^{3,4})\rangle+3e^{-\beta_1-1/2\beta_2}\ \langle x_1\omega^{1,3}-3x_2\omega^{2,3}+2x_4\omega^{3,4}+x_6\omega^{3,6},x_5\omega^{3,5},-x_1\omega^{1,3}+2x_2\omega^{2,3}-x_4\omega^{3,4}+x_5\omega^{1,6}\rangle+2e^{-2\beta_1-1/2\beta_2}\ \langle -1/3x_1\omega^{1,5}+2/3x_2(\omega^{2,5}+\omega^{3,4})-1/3x_3(\omega^{1,6}+2\omega^{3,5})+x_4(\omega^{3,6}+1/3\omega^{4,5})+x_5\omega^{1,7}+x_6\omega^{3,7},-x_1\omega^{1,5}+x_2(\omega^{2,5}+\omega^{3,4})-2x_3(\omega^{1,6}-$

$1/2\omega^{3,5}) + x_4\omega^{3,6} + x_6\omega^{5,6}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2}, x_3\omega^{1,3} + x_6\omega^{3,5}\rangle + e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 2e^{-2\beta_1-3\beta_2/2} + e^{-2\beta_1-5\beta_2/2} + 4e^{-4\beta_1-3\beta_2/2} + 2e^{-6\beta_1-\beta_2/2} + 2e^{-6\beta_1-3\beta_2/2} + 2e^{-8\beta_1-\beta_2/2} + e^{-8\beta_1-3\beta_2/2} + 2e^{-10\beta_1-\beta_2/2} + e^{-12\beta_1-\beta_2/2} + 2e^{-\beta_1-2\beta_2} + 2e^{-2\beta_1-2\beta_2} + 3e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-5\beta_1-\beta_2} + e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-4\beta_1-5\beta_2/2} + 2e^{-6\beta_1-5\beta_2/2} + e^{-6\beta_1-7\beta_2/2} + 3e^{-8\beta_1-5\beta_2/2} + e^{-8\beta_1-7\beta_2/2} + 2e^{-10\beta_1-3\beta_2/2} + 2e^{-10\beta_1-5\beta_2/2} + 4e^{-12\beta_1-3\beta_2/2} + 2e^{-14\beta_1-3\beta_2/2} + e^{-16\beta_1-3\beta_2/2} + 3e^{-3\beta_1-3\beta_2} + 3e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + 2e^{-5\beta_1-2\beta_2} + 3e^{-6\beta_1-2\beta_2} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-8\beta_1-7\beta_2/2} + 2e^{-10\beta_1-7\beta_2/2} + e^{-12\beta_1-5\beta_2/2} + e^{-12\beta_1-7\beta_2/2} + e^{-12\beta_1-9\beta_2/2} + 2e^{-14\beta_1-5\beta_2/2} + e^{-14\beta_1-7\beta_2/2} + 3e^{-16\beta_1-5\beta_2/2} + e^{-18\beta_1-5\beta_2/2} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2} + 2e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-2\beta_2} + e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-14\beta_1-9\beta_2/2} + e^{-16\beta_1-7\beta_2/2} + e^{-16\beta_1-9\beta_2/2} + 2e^{-18\beta_1-7\beta_2/2} + e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-4\beta_2} + e^{-8\beta_1-5\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + e^{-10\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-18\beta_1-9\beta_2/2} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 14, 55, 122, 156, 113, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 8, 20, 30, 33, 24, 11, 3)$

S=$5(2\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 20, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 8, 8, 5, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6} - \omega^{3,5}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$;

$d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{(-2\beta_1-\beta_2)/2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{(-6\beta_1-\beta_2)/2} \langle\omega^{1,5}\rangle + e^{(-2\beta_1-3\beta_2)/2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{(-4\beta_1-3\beta_2)/2} \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{(-10\beta_1-3\beta_2)/2} \langle\omega^{1,3,7}\rangle + e^{(-12\beta_1-3\beta_2)/2} \langle\omega^{1,5,7}\rangle + e^{(-4\beta_1-5\beta_2)/2} \langle\omega^{2,3,4}\rangle + e^{(-6\beta_1-5\beta_2)/2} \langle\omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}+\omega^{3,4,5}\rangle + e^{(-8\beta_1-5\beta_2)/2} \langle\omega^{2,3,7}+\omega^{3,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-7\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{(-12\beta_1-5\beta_2)/2} \langle\omega^{1,2,5,7}\rangle + e^{(-14\beta_1-5\beta_2)/2} \langle\omega^{1,3,6,7}\rangle + e^{(-16\beta_1-5\beta_2)/2} \langle\omega^{1,5,6,7}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{(-8\beta_1-7\beta_2)/2} \langle\omega^{2,3,4,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{(-10\beta_1-7\beta_2)/2} \langle\omega^{2,3,4,7}+\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{(-16\beta_1-7\beta_2)/2} \langle\omega^{1,2,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{(-18\beta_1-7\beta_2)/2} \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7}\rangle + e^{(-14\beta_1-9\beta_2)/2} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{(-18\beta_1-9\beta_2)/2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+3Y^2+5Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,9,20,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,5,8,8,5,3,1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}-\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$;

$d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7} - \omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}+\omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}-\omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}+\omega^{1,4,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5}+2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}-\omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}+\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$;

$\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{(-2\beta_1-\beta_2)/2}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{(-6\beta_1-\beta_2)/2}\ \langle\omega^{1,5}\rangle + e^{(-2\beta_1-3\beta_2)/2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{(-4\beta_1-3\beta_2)/2}\ \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{(-10\beta_1-3\beta_2)/2}\ \langle\omega^{1,3,7} - \omega^{1,5,6}\rangle + e^{(-12\beta_1-3\beta_2)/2}\ \langle\omega^{1,5,7}\rangle + e^{(-4\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,4}\rangle + e^{(-6\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7} - 2\omega^{1,4,6} + \omega^{3,4,5}\rangle + e^{(-8\beta_1-5\beta_2)/2}\ \langle\omega^{2,3,7} + \omega^{3,4,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,6,7} - \omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{(-10\beta_1-7\beta_2)/2}\ \langle(-\omega^{2,3,4,7} - \omega^{2,4,5,6})/2\rangle + e^{(-8\beta_1-7\beta_2)/2}\ \langle-\omega^{2,3,4,6}\rangle + e^{(-16\beta_1-5\beta_2)/2}\ \langle-\omega^{1,5,6,7}\rangle + e^{(-14\beta_1-5\beta_2)/2}\ \langle(-\omega^{1,3,6,7} - \omega^{1,4,5,7})/2\rangle + e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,2,6,7} + 2\omega^{2,3,5,7} + \omega^{3,4,5,6})/6\rangle + e^{(-12\beta_1-5\beta_2)/2}\ \langle(-\omega^{1,2,5,7} + \omega^{1,4,5,6})/2\rangle + e^{-4\beta_1-3\beta_2}\ \langle(\omega^{1,2,4,6} + \omega^{2,3,4,5})/2\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{(-14\beta_1-9\beta_2)/2}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{(-18\beta_1-7\beta_2)/2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{(-16\beta_1-7\beta_2)/2}\ \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-7\beta_1-4\beta_2}\ \langle(\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-9\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{(-18\beta_1-9\beta_2)/2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 2Y + 3Y^2 + 5Y^3 + 3Y^4 + 2Y^5 + Y^6)(1+Y).$

5.4.37 Algebra $\mathcal{G}_{7,2.33}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & (3\xi_1^1)/2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & (3\xi_1^1+2\xi_2^2)/2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_1^5 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & -\xi_2^5 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_3^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{(3\beta_1)/2}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{(3\beta_1+2\beta_2)/2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_2}\ \langle\delta_1^4,\delta_3^5\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{(\beta_1+2\beta_2)/2}\ \langle\delta_1^5\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{(3\beta_1+2\beta_2)/2}\ \langle\delta_3^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{(3\beta_1)/2}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_3^5,\delta_2^5]=\delta_3^7$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^5]=\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_3^7$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_2}\ \langle\delta_1^4\rangle+e^{(\beta_1+2\beta_2)/2}\ \langle\delta_1^5\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-5/2\beta_1+\beta_2}\ \langle x_2\omega^{1,3}\rangle+e^{-2\beta_2}\ \langle x_3\omega^{2,5}\rangle+3e^{-\beta_1}\ \langle 2x_2\omega^{1,2}-x_3\omega^{1,3}+x_5\omega^{1,5},x_4\omega^{1,4},-x_2\omega^{1,2}+x_3\omega^{1,3}+x_6\omega^{3,5}\rangle+e^{-3\beta_1}\ \langle -x_2\omega^{3,5}+x_4\omega^{1,7}\rangle+e^{-3/2\beta_1-\beta_2}\ \langle -x_1(\omega^{1,5}-\omega^{3,4})+2x_2\omega^{2,5}-x_3(\omega^{1,6}+2\omega^{3,5})+x_4\omega^{4,5}+x_5\omega^{4,6}\rangle+e^{-2\beta_1}\ \langle x_4\omega^{1,6}\rangle+e^{1/2\beta_1-\beta_2}\ \langle x_3\omega^{1,2}+x_6\omega^{2,5}\rangle+2e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_4\omega^{2,4}+x_6\omega^{2,6},-x_1\omega^{1,2}+x_3\omega^{2,3}-x_4\omega^{2,4}+x_7\omega^{2,7}\rangle+3e^{-3/2\beta_1}\ \langle x_1\omega^{1,3}-3x_2\omega^{2,3}+2x_4\omega^{3,4}+x_6\omega^{3,6},-x_2\omega^{2,3}+x_4\omega^{1,5},-x_1\omega^{1,3}+2x_2\omega^{2,3}-x_4\omega^{3,4}+x_5\omega^{1,6}\rangle+e^{-5/2\beta_1}\ \langle -x_2(\omega^{1,5}-\omega^{3,4})+x_4\omega^{3,6}+x_5\omega^{1,7}+x_6\omega^{3,7}\rangle+e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle$.

ch($H^3_{\mathcal{G}}$)=$3e^{-3\beta_1-4\beta_2/2}+e^{-3\beta_1-6\beta_2/2}+3e^{-5\beta_1-2\beta_2/2}+e^{-5\beta_1-4\beta_2/2}+e^{-5\beta_1/2}+e^{-7\beta_1-2\beta_2/2}+e^{-7\beta_1/2}+2e^{-9\beta_1-2\beta_2/2}+e^{-9\beta_1/2}+e^{-11\beta_1/2}+2e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+e^{-2\beta_1-3\beta_2}+3e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2}+2e^{-4\beta_1-\beta_2}+e^{-4\beta_1}$.

ch($H^4_{\mathcal{G}}$)=$e^{-5\beta_1-4\beta_2/2}+2e^{-5\beta_1-6\beta_2/2}+e^{-7\beta_1-6\beta_2/2}+e^{-7\beta_1-8\beta_2/2}+2e^{-9\beta_1-4\beta_2/2}+2e^{-9\beta_1-6\beta_2/2}+2e^{-11\beta_1-2\beta_2/2}+2e^{-11\beta_1-4\beta_2/2}+e^{-13\beta_1-2\beta_2/2}+$

$e^{-13\beta_1-4\beta_2/2}+e^{-15\beta_1-2\beta_2/2}+2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-6\beta_1-\beta_2}+3e^{-6\beta_1-2\beta_2}+e^{-7\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-9\beta_1-8\beta_2/2}+2e^{-11\beta_1-6\beta_2/2}+e^{-11\beta_1-8\beta_2/2}+e^{-13\beta_1-8\beta_2/2}+e^{-13\beta_1-10\beta_2/2}+e^{-15\beta_1-4\beta_2/2}+3e^{-15\beta_1-6\beta_2/2}+e^{-17\beta_1-4\beta_2/2}+e^{-4\beta_1-3\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-5\beta_2}+2e^{-7\beta_1-2\beta_2}+e^{-7\beta_1-3\beta_2}+e^{-8\beta_1-2\beta_2}+e^{-9\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-15\beta_1-8\beta_2/2}+2e^{-15\beta_1-10\beta_2/2}+e^{-17\beta_1-6\beta_2/2}+e^{-17\beta_1-8\beta_2/2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}+e^{-8\beta_1-5\beta_2}+e^{-9\beta_1-3\beta_2}+e^{-9\beta_1-4\beta_2}+e^{-10\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-17\beta_1-10\beta_2/2}+e^{-9\beta_1-5\beta_2}+e^{-10\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,12,53,120,153,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,6,16,26,28,21,11,3)$

S=$5(2\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,7,7,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}-\omega^{3,5}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,3,5}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{3,4,5}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}$; $d\omega^{6,7}=-\omega^{1,4,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}$; $d\omega^{1,6,7}=-\omega^{1,3,5,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}+\omega^{2,3,4,5}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7}=-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7}=\omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,6,7}$; $d\omega^{1,2,4,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{(-3\beta_1)/2}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{(-5\beta_1)/2}\ \langle\omega^{1,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+e^{(-3\beta_1-4\beta_2)/2}\ \langle\omega^{2,5}\rangle+e^{(-5\beta_1-2\beta_2)/2}\ \langle\omega^{1,5}-\omega^{3,4}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{(-11\beta_1-2\beta_2)/2}\ \langle\omega^{1,3,7}\rangle + e^{(-5\beta_1-4\beta_2)/2}\ \langle\omega^{1,2,5}\rangle + e^{(-5\beta_1-6\beta_2)/2}\ \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,2,7}+\omega^{3,4,5}\rangle + e^{(-9\beta_1-4\beta_2)/2}\ \langle\omega^{1,5,6}-\omega^{3,4,6}\rangle + e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,6,7}-\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{(-15\beta_1-4\beta_2)/2}\ \langle\omega^{1,3,6,7}\rangle + e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6}\rangle + e^{(-11\beta_1-6\beta_2)/2}\ \langle\omega^{1,2,5,7}\rangle + e^{-6\beta_1-3\beta_2}\ \langle\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle + e^{(-9\beta_1-8\beta_2)/2}\ \langle\omega^{2,4,5,6}\rangle + e^{(-15\beta_1-6\beta_2)/2}\ \langle\omega^{1,5,6,7}-\omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{(-17\beta_1-6\beta_2)/2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-9\beta_1-3\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7}\rangle + e^{(-15\beta_1-8\beta_2)/2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{(-15\beta_1-10\beta_2)/2}\ \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{(-17\beta_1-10\beta_2)/2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+Y^2)^2(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,9,19,23,17,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,3,5,7,7,5,3,1)$

Total dimension of trivial cohomology: 32.

Length associated polynomial: 32.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5}-\omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6}-\omega^{2,3,7}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}-\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7}-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$;

$d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^{\star}\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{A}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{(-3\beta_1)/2}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{(-5\beta_1)/2} \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{(-3\beta_1-4\beta_2)/2} \langle\omega^{2,5}\rangle + e^{(-5\beta_1-2\beta_2)/2} \langle\omega^{1,5}-\omega^{3,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{(-11\beta_1-2\beta_2)/2} \langle\omega^{1,3,7}\rangle + e^{(-5\beta_1-4\beta_2)/2} \langle\omega^{1,2,5}+\omega^{2,3,4}\rangle + e^{(-5\beta_1-6\beta_2)/2} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}-2\omega^{1,4,6}+\omega^{3,4,5}\rangle + e^{(-9\beta_1-4\beta_2)/2} \langle\omega^{1,5,6}-\omega^{2,3,7}-2\omega^{3,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{(-9\beta_1-8\beta_2)/2} \langle-\omega^{2,4,5,6}\rangle + e^{(-15\beta_1-6\beta_2)/2} \langle(-\omega^{1,5,6,7}+\omega^{3,4,6,7})/2\rangle + e^{(-15\beta_1-4\beta_2)/2} \langle-\omega^{1,3,6,7}\rangle + e^{-7\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-3\beta_2} \langle(\omega^{1,2,6,7}+2\omega^{2,3,5,7}+\omega^{3,4,5,6})/6\rangle + e^{(-11\beta_1-6\beta_2)/2} \langle(2\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7})/6\rangle + e^{-4\beta_1-3\beta_2} \langle(\omega^{1,2,4,6}+\omega^{2,3,4,5})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{(-15\beta_1-10\beta_2)/2} \langle-\omega^{2,4,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{(-17\beta_1-6\beta_2)/2} \langle\omega^{1,3,4,6,7}\rangle + e^{(-15\beta_1-8\beta_2)/2} \langle(-\omega^{1,2,5,6,7}-\omega^{2,3,4,6,7})/2\rangle + e^{-7\beta_1-4\beta_2} \langle(\omega^{1,2,4,6,7}+\omega^{2,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{(-17\beta_1-10\beta_2)/2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y^2)^2(1+Y)^3$.

5.4.38 Algebra $\mathcal{G}_{7,2.34}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_7$; $[x_2,x_4]=x_6$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & (\xi_1^1+\xi_2^2)/2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & 0 & \xi_3^5 & 0 & (3\xi_1^1+\xi_2^2)/2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^4 & -\xi_1^5 & 0 & 2\xi_1^1+\xi_2^2\end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_2^6+\delta_1^7), ad\ x_5=-\delta_3^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{(\beta_1+\beta_2)/2}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{(3\beta_1+\beta_2)/2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=2e^{\beta_1}\ \langle\delta_3^5,\delta_2^4\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_2^6,\delta_1^7\rangle+e^{(\beta_1+3\beta_2)/2}\ \langle\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{2\beta_1}\ \langle\delta_2^7\rangle+e^{(3\beta_1+\beta_2)/2}\ \langle\delta_3^7\rangle+e^{(\beta_1+\beta_2)/2}\ \langle\delta_1^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^5,\delta_1^5]=\delta_3^7$; $[\delta_1^4,\delta_2^4]=-(\delta_1^7+\delta_2^6)$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{(\beta_1+3\beta_2)/2}\ \langle\delta_3^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^7\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})=2e^{-2\beta_1}\ \langle x_3\omega^{1,5},x_2\omega^{1,4}\rangle+3e^{-1/2\beta_1-1/2\beta_2}\ \langle x_1\omega^{1,3}+x_2\omega^{2,3}+3x_3\omega^{1,2}+3x_4(\omega^{2,5}+1/3\omega^{3,4}),-x_2\omega^{2,3}-x_3\omega^{1,2}-x_4\omega^{2,5}+x_6\omega^{3,6},-x_1\omega^{1,3}-x_3\omega^{1,2}-x_4\omega^{2,5}+x_5\omega^{1,4}\rangle+e^{-3/2\beta_1-1/2\beta_2}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})+3x_3(\omega^{1,4}-2/3\omega^{3,5})+x_4\omega^{4,5}\rangle+3e^{-\beta_2}\ \langle -2x_1\omega^{1,2}-x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5},x_6\omega^{2,6},x_1\omega^{1,2}+x_3\omega^{2,3}+2x_4\omega^{2,4}+x_7\omega^{2,7}\rangle+2e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4}+2x_2\omega^{2,4}+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5},x_1\omega^{1,4}-x_2\omega^{2,4}+x_5\omega^{3,7}\rangle+e^{-3/2\beta_1+3/2\beta_2}\ \langle x_6\omega^{1,5}\rangle+2e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_6\omega^{1,6},-x_2\omega^{1,2}-x_3\omega^{1,3}-x_4\omega^{3,5}+x_6\omega^{2,7}\rangle+e^{-3/2\beta_1+1/2\beta_2}\ \langle -x_2\omega^{1,3}+x_6\omega^{3,7}\rangle+e^{\beta_1-2\beta_2}\ \langle x_7\omega^{2,6}\rangle+e^{1/2\beta_1-3/2\beta_2}\ \langle -x_1\omega^{2,3}+x_5\omega^{2,4}\rangle+e^{-2\beta_2}\ \langle x_1\omega^{2,4}+x_5\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1-3\beta_2/2}+e^{-\beta_1-5\beta_2/2}+e^{-\beta_1-7\beta_2/2}+3e^{-3\beta_1-3\beta_2/2}+2e^{-3\beta_1-5\beta_2/2}+3e^{-5\beta_1-\beta_2/2}+2e^{-5\beta_1-3\beta_2/2}+e^{-7\beta_1+\beta_2/2}+e^{-7\beta_1-\beta_2/2}+2e^{-\beta_1-2\beta_2}+2e^{-\beta_1-3\beta_2}+$

$3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+e^{-4\beta_1-\beta_2}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-3\beta_1-7\beta_2/2}+e^{-3\beta_1-9\beta_2/2}+e^{-5\beta_1-5\beta_2/2}+4e^{-5\beta_1-7\beta_2/2}+e^{-5\beta_1-9\beta_2/2}+e^{-7\beta_1-3\beta_2/2}+4e^{-7\beta_1-5\beta_2/2}+e^{-7\beta_1-7\beta_2/2}+3e^{-9\beta_1-3\beta_2/2}+e^{-9\beta_1-5\beta_2/2}+e^{-11\beta_1-3\beta_2/2}+3e^{-2\beta_1-4\beta_2}+4e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-4\beta_1}+e^{-5\beta_1-2\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-5\beta_1-9\beta_2/2}+e^{-7\beta_1-7\beta_2/2}+2e^{-7\beta_1-9\beta_2/2}+e^{-7\beta_1-11\beta_2/2}+e^{-9\beta_1-5\beta_2/2}+2e^{-9\beta_1-7\beta_2/2}+2e^{-9\beta_1-9\beta_2/2}+2e^{-11\beta_1-5\beta_2/2}+e^{-11\beta_1-7\beta_2/2}+e^{-13\beta_1-7\beta_2/2}+e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-3\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-9\beta_1-11\beta_2/2}+e^{-11\beta_1-9\beta_2/2}+e^{-11\beta_1-11\beta_2/2}+2e^{-13\beta_1-9\beta_2/2}+e^{-5\beta_1-5\beta_2}+e^{-5\beta_1-6\beta_2}+e^{-6\beta_1-3\beta_2}+2e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-13\beta_1-11\beta_2/2}+e^{-6\beta_1-6\beta_2}+e^{-7\beta_1-5\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,12,55,121,157,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,7,18,29,33,25,11,3)$

$\mathrm{S}=7\beta_1+6\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{2,4}$; $d\omega^7=-\omega^{1,4}-\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,3,5}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,7}=-\omega^{1,2,4}+\omega^{2,3,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,3,7}+\omega^{1,4,5}$; $d\omega^{6,7}=\omega^{1,4,6}-\omega^{2,4,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,4,7}=\omega^{1,3,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,3,5,6}$; $d\omega^{2,3,7}=-\omega^{1,2,3,4}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=\omega^{2,3,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}+\omega^{1,2,4,5}$; $d\omega^{2,6,7}=\omega^{1,2,4,6}-\omega^{2,3,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}$; $d\omega^{3,5,6}=\omega^{2,3,4,5}$; $d\omega^{3,5,7}=\omega^{1,3,4,5}$; $d\omega^{3,6,7}=\omega^{1,3,4,6}-\omega^{2,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,5,6}+\omega^{2,4,5,7}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,5,6}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,3,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7}=-\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=\omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$;

$d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{(-\beta_1-\beta_2)/2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{(-5\beta_1-\beta_2)/2} \langle\omega^{1,5}\rangle + e^{(-\beta_1-3\beta_2)/2} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{(-3\beta_1-3\beta_2)/2} \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{(-7\beta_1-3\beta_2)/2} \langle\omega^{1,3,7}\rangle + e^{(-9\beta_1-3\beta_2)/2} \langle\omega^{1,5,7}\rangle + e^{(-3\beta_1-7\beta_2)/2} \langle\omega^{2,3,6}\rangle + e^{(-5\beta_1-5\beta_2)/2} \langle\omega^{2,3,7}+\omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{(-5\beta_1-7\beta_2)/2} \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{(-7\beta_1-5\beta_2)/2} \langle\omega^{1,5,6}+\omega^{2,5,7}+\omega^{3,4,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{(-9\beta_1-5\beta_2)/2} \langle\omega^{1,2,5,7}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,7}+\omega^{2,3,5,7}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{(-11\beta_1-5\beta_2)/2} \langle\omega^{1,4,5,7}\rangle + e^{(-5\beta_1-9\beta_2)/2} \langle\omega^{2,3,4,6}\rangle + e^{(-7\beta_1-7\beta_2)/2} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{(-7\beta_1-9\beta_2)/2} \langle\omega^{2,3,6,7}-\omega^{2,4,5,6}\rangle + e^{(-9\beta_1-7\beta_2)/2} \langle\omega^{1,3,6,7}-\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-6\beta_1-3\beta_2} \langle\omega^{1,3,4,5,7}\rangle + e^{(-11\beta_1-9\beta_2)/2} \langle\omega^{1,2,5,6,7}\rangle + e^{(-13\beta_1-9\beta_2)/2} \langle\omega^{1,4,5,6,7}\rangle + e^{(-9\beta_1-11\beta_2)/2} \langle\omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{(-13\beta_1-11\beta_2)/2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-7\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,3,5,9,9,5,3,1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}-\omega^{2,7}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}+\omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{2,6,7}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}$; $d^\star\omega^{1,2,5,6} =$

$-\omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{1,4,7} + \omega^{3,5,7}$; $d^{\star}\omega^{1,3,4,6} = \omega^{3,6,7} + \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{1,6,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{2,4,7} + \omega^{3,5,6}$; $d^{\star}\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^{\star}\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,4,6,7} - \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} - \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,7}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,7} = -\omega^{1,6} + 2\omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,4} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} - \omega^{2,3,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} + \omega^{3,5,7}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} - \omega^{2,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,6} + \omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,7} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} + 2\omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,4,6} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,4,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + 2\omega^{1,3,4,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,4,6} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$;

$\Delta\omega^{3,5,6,7} = \omega^{1,4,6,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,5,6} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{(-\beta_1-\beta_2)/2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{(-5\beta_1-\beta_2)/2} \langle\omega^{1,5}\rangle + e^{(-\beta_1-3\beta_2)/2} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{(-3\beta_1-3\beta_2)/2} \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,4}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{(-7\beta_1-3\beta_2)/2} \langle\omega^{1,3,7}+\omega^{1,4,5}\rangle + e^{(-9\beta_1-3\beta_2)/2} \langle\omega^{1,5,7}\rangle + e^{(-3\beta_1-7\beta_2)/2} \langle\omega^{2,3,6}\rangle + e^{(-5\beta_1-5\beta_2)/2} \langle\omega^{1,3,6}+2\omega^{2,3,7}+\omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{(-5\beta_1-7\beta_2)/2} \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{(-7\beta_1-5\beta_2)/2} \langle\omega^{1,5,6}+\omega^{2,5,7}+\omega^{3,4,7}\rangle + e^{-3\beta_1-3\beta_2} \langle 2\omega^{1,4,6}+\omega^{2,4,7}-\omega^{3,5,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,7}-\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{(-7\beta_1-9\beta_2)/2} \langle(\omega^{2,3,6,7}-\omega^{2,4,5,6})/2\rangle + e^{(-5\beta_1-9\beta_2)/2} \langle-\omega^{2,3,4,6}\rangle + e^{(-11\beta_1-5\beta_2)/2} \langle-\omega^{1,4,5,7}\rangle + e^{(-9\beta_1-7\beta_2)/2} \langle(-\omega^{1,3,6,7}+2\omega^{1,4,5,6}+\omega^{2,4,5,7})/6\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{(-9\beta_1-5\beta_2)/2} \langle(-\omega^{1,2,5,7}-\omega^{1,3,4,7})/2\rangle + e^{(-7\beta_1-7\beta_2)/2} \langle(\omega^{1,2,5,6}+\omega^{1,3,4,6}+\omega^{2,3,4,7})/3\rangle + e^{-4\beta_1-3\beta_2} \langle(-\omega^{1,2,4,7}-\omega^{1,3,5,6}-2\omega^{2,3,5,7})/6\rangle + e^{-3\beta_1-4\beta_2} \langle(\omega^{1,2,4,6}+\omega^{2,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{(-9\beta_1-11\beta_2)/2} \langle-\omega^{2,3,4,6,7}\rangle + e^{(-13\beta_1-9\beta_2)/2} \langle\omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle-\omega^{1,3,4,5,7}\rangle + e^{(-11\beta_1-9\beta_2)/2} \langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2\rangle + e^{-5\beta_1-5\beta_2} \langle(\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-6\beta_1-6\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-7\beta_1-5\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{(-13\beta_1-11\beta_2)/2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-7\beta_1-6\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+3Y^2+6Y^3+3Y^4+2Y^5+Y^6)(1+Y)$.

5.4.39 Algebra $\mathcal{G}_{7,2.35}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -\xi_1^1+2\xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & 0 & 2\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^5-\xi_1^4 & \xi_2^4 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 & -\xi_1^4 & 0 & 3\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_2^6+\delta_3^7), ad\ x_5=-(\delta_1^6+\delta_2^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{-\beta_1+2\beta_2}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{2\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{3\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{2\beta_2}\ \langle\delta_1^6,\delta_2^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_2^6,\delta_3^7\rangle+e^{2\beta_1}\ \langle\delta_3^6\rangle+2e^{\beta_2}\ \langle\delta_2^5,\delta_1^4\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle+e^{-\beta_1+3\beta_2}\ \langle\delta_1^7\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^4]=\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^5]=\delta_2^7+\delta_1^6$; $[\delta_2^4,\delta_1^4]=\delta_3^7+\delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $e^{2\beta_1}\ \langle\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_2^5\rangle+2e^0\ \langle t_1,t_2\rangle+e^{-\beta_1+3\beta_2}\ \langle\delta_1^7\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1+\beta_2}\ \langle x_3\omega^{1,4}\rangle+2e^{2\beta_1-3\beta_2}\ \langle x_1\omega^{2,3}+x_4\omega^{3,5},-x_1\omega^{2,3}+x_6\omega^{3,7}\rangle+2e^{-2\beta_2}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})-2x_3\omega^{3,5}+x_4\omega^{4,5},-x_1\omega^{2,4}+x_5\omega^{2,7}+x_6\omega^{4,7}\rangle+2e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4}-3x_2(\omega^{1,5}-2/3\omega^{2,4})+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5},-x_1\omega^{1,4}+x_2\omega^{2,4}+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{2,6}+x_6\omega^{4,6}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle+2e^{\beta_1-2\beta_2}\ \langle -x_1\omega^{1,3}+x_4\omega^{3,4}+x_6\omega^{3,6},1/3x_2\omega^{2,3}-1/3x_4(\omega^{2,5}+2\omega^{3,4})+x_6\omega^{2,7}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,2}+x_7\omega^{1,6}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}+2x_3\omega^{1,3}-x_4\omega^{1,4}+x_7\omega^{1,7},-x_3\omega^{1,3}+x_4\omega^{1,4}+x_5\omega^{2,4},-x_3\omega^{1,3}+x_4\omega^{1,4}+x_6\omega^{1,6}\rangle+3e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5},2x_1\omega^{1,2}+x_2\omega^{1,3}+2x_3\omega^{2,3}-x_4(\omega^{1,5}+2\omega^{2,4})+x_6\omega^{1,7},-x_1\omega^{1,2}-x_3\omega^{2,3}+x_4\omega^{2,4}+x_6\omega^{2,6}\rangle+e^{-2\beta_1}\ \langle x_2\omega^{1,4}+x_3\omega^{1,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{2\beta_1-5\beta_2}+e^{2\beta_1-6\beta_2}+e^{\beta_1-3\beta_2}+4e^{\beta_1-4\beta_2}+e^{\beta_1-5\beta_2}+3e^{-\beta_1-2\beta_2}+3e^{-\beta_1-3\beta_2}+e^{-\beta_1-4\beta_2}+3e^{-2\beta_1-\beta_2}+2e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1}+e^{-4\beta_1-\beta_2}+2e^{-3\beta_2}+e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{2\beta_1-8\beta_2}+2e^{\beta_1-6\beta_2}+4e^{\beta_1-7\beta_2}+e^{\beta_1-8\beta_2}+4e^{-\beta_1-5\beta_2}+e^{-\beta_1-6\beta_2}+e^{-2\beta_1-3\beta_2}+5e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-5\beta_2}+3e^{-3\beta_1-3\beta_2}+2e^{-3\beta_1-4\beta_2}+e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-5\beta_2}+4e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{\beta_1-8\beta_2}+e^{\beta_1-9\beta_2}+e^{\beta_1-10\beta_2}+e^{-\beta_1-6\beta_2}+2e^{-\beta_1-7\beta_2}+e^{-\beta_1-8\beta_2}+e^{-2\beta_1-5\beta_2}+3e^{-2\beta_1-6\beta_2}+e^{-2\beta_1-7\beta_2}+e^{-3\beta_1-4\beta_2}+2e^{-3\beta_1-5\beta_2}+e^{-3\beta_1-6\beta_2}+e^{-3\beta_1-7\beta_2}+e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2}+2e^{-7\beta_2}+3e^{-8\beta_2}+e^{-9\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-\beta_1-8\beta_2}+e^{-\beta_1-9\beta_2}+e^{-\beta_1-10\beta_2}+2e^{-2\beta_1-9\beta_2}+e^{-3\beta_1-7\beta_2}+2e^{-3\beta_1-8\beta_2}+e^{-4\beta_1-7\beta_2}+e^{-10\beta_2}+e^{-11\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-\beta_1-11\beta_2}+e^{-2\beta_1-10\beta_2}+e^{-3\beta_1-9\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,12,55,121,157,113,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,7,18,29,33,25,11,3)$

S$=2\beta_1+11\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,9,21,23,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,5,9,9,5,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}-\omega^{1,4,5}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,3,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,5,7}-\omega^{2,4,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,6}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,6}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,5,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=\omega^{2,3,4,5}$; $d\omega^{3,6,7}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=$

$\omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{\beta_1-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{\beta_1-3\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,5}\rangle + e^{\beta_1-4\beta_2} \langle\omega^{3,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{1,7} + 2\omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{\beta_1-6\beta_2} \langle\omega^{2,3,7}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{1,4,7} + \omega^{2,4,6}, \omega^{1,5,6} - \omega^{2,4,6}\rangle + e^{-5\beta_2} \langle\omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{1,5,7} - \omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-6\beta_2} \langle\omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}\rangle + e^{\beta_1-7\beta_2} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-\beta_1-6\beta_2} \langle\omega^{1,3,5,6} - \omega^{2,3,4,6}\rangle + 2e^{-7\beta_2} \langle\omega^{1,3,5,7}, \omega^{2,3,4,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{\beta_1-8\beta_2} \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{1,2,6,7} - \omega^{2,4,5,6}\rangle + e^{-8\beta_2} \langle\omega^{2,3,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-3\beta_1-7\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-3\beta_1-8\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-\beta_1-8\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-\beta_1-9\beta_2} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-10\beta_2} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-3\beta_1-9\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-2\beta_1-10\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-\beta_1-11\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-2\beta_1-11\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 3Y^2 + 6Y^3 + 3Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 9, 21, 23, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 5, 9, 9, 5, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} =$

$-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $d^\star\omega^{1,2,4,7}=\omega^{1,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}-\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7}=-\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{2,4,5,7}=\omega^{5,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{1,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{1,5,6,7}-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{1,3,5,6,7}-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^{4}=\omega^{4}$; $\Delta\omega^{5}=\omega^{5}$; $\Delta\omega^{6}=2\omega^{6}$; $\Delta\omega^{7}=2\omega^{7}$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,5}=\omega^{1,5}+\omega^{2,4}$; $\Delta\omega^{1,6}=\omega^{1,6}$; $\Delta\omega^{1,7}=2\omega^{1,7}-\omega^{2,6}$; $\Delta\omega^{2,4}=\omega^{1,5}+\omega^{2,4}$; $\Delta\omega^{2,5}=2\omega^{2,5}$; $\Delta\omega^{2,6}=-\omega^{1,7}+\omega^{2,6}+\omega^{4,5}$; $\Delta\omega^{2,7}=\omega^{2,7}-\omega^{3,6}$; $\Delta\omega^{3,4}=2\omega^{3,4}$; $\Delta\omega^{3,6}=-\omega^{2,7}+2\omega^{3,6}$; $\Delta\omega^{3,7}=\omega^{3,7}$; $\Delta\omega^{4,5}=\omega^{2,6}+2\omega^{4,5}$; $\Delta\omega^{4,6}=2\omega^{4,6}$; $\Delta\omega^{4,7}=2\omega^{4,7}-\omega^{5,6}$; $\Delta\omega^{5,6}=-\omega^{4,7}+2\omega^{5,6}$; $\Delta\omega^{5,7}=2\omega^{5,7}$; $\Delta\omega^{6,7}=4\omega^{6,7}$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=3\omega^{1,2,5}$; $\Delta\omega^{1,2,6}=\omega^{1,2,6}+\omega^{1,4,5}$; $\Delta\omega^{1,2,7}=2\omega^{1,2,7}-\omega^{1,3,6}$; $\Delta\omega^{1,3,4}=2\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=\omega^{1,3,5}+\omega^{2,3,4}$; $\Delta\omega^{1,3,6}=-\omega^{1,2,7}+2\omega^{1,3,6}$; $\Delta\omega^{1,3,7}=2\omega^{1,3,7}-\omega^{2,3,6}$; $\Delta\omega^{1,4,5}=\omega^{1,2,6}+\omega^{1,4,5}$; $\Delta\omega^{1,4,7}=\omega^{1,4,7}-\omega^{1,5,6}-\omega^{2,4,6}$; $\Delta\omega^{1,5,6}=-\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $\Delta\omega^{1,5,7}=2\omega^{1,5,7}+\omega^{2,4,7}-\omega^{2,5,6}$; $\Delta\omega^{1,6,7}=3\omega^{1,6,7}$; $\Delta\omega^{2,3,4}=\omega^{1,3,5}+2\omega^{2,3,4}$; $\Delta\omega^{2,3,5}=\omega^{2,3,5}$; $\Delta\omega^{2,3,6}=-\omega^{1,3,7}+\omega^{2,3,6}-\omega^{3,4,5}$; $\Delta\omega^{2,4,5}=3\omega^{2,4,5}$; $\Delta\omega^{2,4,6}=-\omega^{1,4,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $\Delta\omega^{2,4,7}=\omega^{1,5,7}+\omega^{2,4,7}-\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,5,6}=-\omega^{1,5,7}-\omega^{2,4,7}+2\omega^{2,5,6}$; $\Delta\omega^{2,5,7}=2\omega^{2,5,7}-\omega^{3,5,6}$; $\Delta\omega^{2,6,7}=2\omega^{2,6,7}+\omega^{4,5,7}$; $\Delta\omega^{3,4,5}=-\omega^{2,3,6}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6}=-\omega^{2,4,7}+3\omega^{3,4,6}$; $\Delta\omega^{3,4,7}=2\omega^{3,4,7}-\omega^{3,5,6}$; $\Delta\omega^{3,5,6}=-\omega^{2,5,7}-\omega^{3,4,7}+\omega^{3,5,6}$; $\Delta\omega^{3,6,7}=3\omega^{3,6,7}$; $\Delta\omega^{4,5,6}=2\omega^{4,5,6}$; $\Delta\omega^{4,5,7}=\omega^{2,6,7}+2\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=3\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4}=3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6}=2\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7}=2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5}=3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6}=-\omega^{1,2,4,7}+2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7}=3\omega^{1,2,5,7}-\omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7}=2\omega^{1,2,6,7}+\omega^{1,4,5,7}$; $\Delta\omega^{1,3,4,5}=-\omega^{1,2,3,6}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,4,7}+2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7}=2\omega^{1,3,4,7}-\omega^{1,3,5,6}-\omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6}=-\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{1,3,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7}=\omega^{1,3,5,7}+\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7}=3\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,7}=\omega^{1,2,6,7}+\omega^{1,4,5,7}-\omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}+\omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5}=3\omega^{2,3,4,5}$;

$\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7}+\omega^{1,3,5,6}+2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7}+\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7}+2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{\beta_1-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{\beta_1-3\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,5} - \omega^{2,4}\rangle + e^{\beta_1-4\beta_2} \langle\omega^{3,5}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{1,7} + 2\omega^{2,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})= e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,6} - \omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{\beta_1-6\beta_2} \langle\omega^{2,3,7}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{1,4,7} + \omega^{2,4,6}, \omega^{1,5,6} - \omega^{2,4,6}\rangle + e^{-5\beta_2} \langle\omega^{1,3,7} + 2\omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{1,5,7} - 3\omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-6\beta_2} \langle\omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}\rangle + e^{\beta_1-7\beta_2} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}})= e^{-8\beta_2} \langle(-\omega^{2,3,6,7} - \omega^{3,4,5,7})/2\rangle + e^{\beta_1-8\beta_2} \langle-\omega^{2,3,5,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,4,5,6}\rangle + 2e^{-7\beta_2} \langle(\omega^{1,3,5,7} + \omega^{2,3,4,7} + 2\omega^{2,3,5,6})/3, (-\omega^{1,3,5,7} + 2\omega^{2,3,4,7} + \omega^{2,3,5,6})/3\rangle + e^{-2\beta_1-6\beta_2} \langle(\omega^{1,2,6,7} - 2\omega^{1,4,5,7} - \omega^{2,4,5,6})/6\rangle + e^{-\beta_1-6\beta_2} \langle(\omega^{1,2,5,7} + \omega^{1,3,4,7} + 3\omega^{1,3,5,6} - \omega^{2,3,4,6})/12\rangle + e^{-2\beta_1-5\beta_2} \langle(2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6})/6\rangle + e^{-3\beta_1-4\beta_2} \langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}})= e^{-10\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-8\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-\beta_1-9\beta_2} \langle(\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-3\beta_1-7\beta_2} \langle-\omega^{1,2,4,6,7}\rangle + e^{-\beta_1-8\beta_2} \langle(-\omega^{1,2,3,6,7} - 2\omega^{1,3,4,5,7} - \omega^{2,3,4,5,6})/6\rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}})= e^{-\beta_1-11\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-2\beta_1-10\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-9\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}})= e^{-2\beta_1-11\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 3Y^2 + 6Y^3 + 3Y^4 + 2Y^5 + Y^6)(1 + Y)$.

5.4.40 Algebra $\mathcal{G}_{7,2.36}$

Adjoint cohomology

$[x_1,x_3]=x_5$; $[x_1,x_4]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_4]=-x_5$; $[x_3,x_6]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & \xi_4^1 & 0 & 0 & 0\\ 0 & \xi_2^2 & -\xi_4^1 & 0 & 0 & 0 & 0\\ 0 & 0 & 2(\xi_1^1-\xi_2^2) & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_3^4 & 3(\xi_1^1-\xi_2^2) & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & 3\xi_1^1-2\xi_2^2 & \xi_2^1-\xi_3^4 & 0\\ -(\xi_2^1+\xi_3^4) & \xi_2^6 & \xi_3^6 & \xi_3^1 & 0 & 2\xi_1^1-\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_2^1 & \xi_2^6 & 4\xi_1^1-3\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_3^5+\delta_4^7, ad\ x_2=-\delta_4^5+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=\delta_2^5-\delta_1^7, ad\ x_5=0, ad\ x_6=\delta_3^7, ad\ x_7=0.$

Maximal torus : T=$\langle t_1,t_2\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1-2\beta_2}\ \langle x_3\rangle+e^{3\beta_1-3\beta_2}\ \langle x_4\rangle+e^{3\beta_1-2\beta_2}\ \langle x_5\rangle+e^{2\beta_1-\beta_2}\ \langle x_6\rangle+e^{4\beta_1-3\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1-2\beta_2}\ \langle x_5\rangle+e^{4\beta_1-3\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-2\beta_1+3\beta_2}\ \langle\delta_4^1\rangle+2e^{2\beta_1-2\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{3\beta_1-3\beta_2}\ \langle\delta_2^5,\delta_1^7\rangle+2e^{\beta_1}\ \langle\delta_3^5,\delta_4^7\rangle+2e^{\beta_2}\ \langle\delta_4^5,\delta_3^6\rangle+2e^{\beta_1-\beta_2}\ \langle\delta_3^4,\delta_2^1\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^1\rangle+e^{4\beta_1-4\beta_2}\ \langle\delta_2^7\rangle+e^{2\beta_1-\beta_2}\ \langle\delta_3^7\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_4^1,\delta_1^5]=-\delta_4^5$; $[\delta_4^1,\delta_2^5]=\delta_3^5$; $[\delta_4^1,\delta_3^4]=\delta_3^1$; $[\delta_4^1,\delta_1^7]=-\delta_4^7$; $[\delta_4^1,\delta_2^7]=\delta_3^7$; $[\delta_4^1,\delta_2^1]=\delta_3^1$; $[\delta_4^1,\delta_2^6]=\delta_3^6$; $[\delta_1^5,\delta_3^1]=\delta_3^5$; $[\delta_1^5,\delta_2^1]=\delta_1^7+\delta_2^5$; $[\delta_2^5,\delta_2^1]=\delta_2^7$; $[\delta_3^5,\delta_2^1]=\delta_3^7$; $[\delta_4^5,\delta_3^4]=\delta_3^5$; $[\delta_4^5,\delta_2^1]=\delta_4^7$; $[\delta_3^4,\delta_3^6]=-\delta_3^5$; $[\delta_3^4,\delta_3^1]=-2\delta_3^6-\delta_4^5$; $[\delta_3^4,\delta_4^7]=-\delta_3^7$; $[\delta_3^4,\delta_2^1]=-\delta_2^6+2\delta_1^5$; $[\delta_3^4,\delta_2^6]=\delta_1^7-\delta_2^5$; $[\delta_3^6,\delta_2^1]=-\delta_3^5$; $[\delta_3^6,\delta_2^6]=-\delta_3^7$; $[\delta_3^1,\delta_1^7]=-\delta_3^7$; $[\delta_3^1,\delta_2^1]=\delta_3^6-\delta_4^5$; $[\delta_3^1,\delta_2^6]=-\delta_4^7$; $[\delta_1^7,\delta_2^1]=\delta_2^7$; $[\delta_2^1,\delta_2^6]=\delta_1^7+\delta_2^5$.

ch($H^1_{\mathcal{G}}$)= $e^{-2\beta_1+3\beta_2}\ \langle\delta_4^1\rangle+2e^{\beta_1-\beta_2}\ \langle\delta_3^4,\delta_2^1\rangle+e^{\beta_2}\ \langle\delta_4^5\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^1\rangle+e^{3\beta_1-3\beta_2}\ \langle\delta_2^5\rangle+e^{4\beta_1-4\beta_2}\ \langle\delta_2^7\rangle+e^{\beta_1}\ \langle\delta_3^5\rangle+e^{2\beta_1-2\beta_2}\ \langle\delta_1^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^1,\delta_3^4]=\delta_3^1$; $[\delta_4^1,\delta_2^5]=\delta_3^5$; $[\delta_4^1,\delta_2^1]=\delta_3^1$; $[\delta_4^1,\delta_1^5]=-\delta_4^5$; $[\delta_3^4,\delta_4^5]=-\delta_3^5$; $[\delta_3^4,\delta_3^1]=-3\delta_4^5$; $[\delta_3^4,\delta_2^1]=3\delta_1^5$; $[\delta_4^5,\delta_2^1]=-\delta_3^5$; $[\delta_3^1,\delta_1^5]=-\delta_3^5$; $[\delta_2^5,\delta_2^1]=\delta_2^7$; $[\delta_2^1,\delta_1^5]=-2\delta_2^5$.

ch($H^2_{\mathcal{G}}$)= $e^{-4\beta_1+5\beta_2}\ \langle x_1\omega^{3,4}\rangle+e^{-5\beta_1+6\beta_2}\ \langle x_2\omega^{3,4}\rangle+5e^{-3\beta_1+3\beta_2}\ \langle -x_1(\omega^{1,4}+\omega^{3,6})-x_2\omega^{2,4}+x_3\omega^{3,4}, -x_1\omega^{1,4}+x_2\omega^{1,3}, -x_1\omega^{1,4}+x_5\omega^{4,5}, x_1\omega^{3,6}-x_2\omega^{2,4}+x_6\omega^{4,6}, x_1\omega^{1,4}+x_5\omega^{3,7}\rangle+e^{2\beta_1-4\beta_2}\ \langle x_4\omega^{1,2}\rangle+e^{\beta_1-3\beta_2}\ \langle -x_3\omega^{1,2}+x_4\omega^{2,6}\rangle+5e^{-2\beta_1+2\beta_2}\ \langle x_1(\omega^{1,3}+\omega^{2,4})+x_4\omega^{3,4}, -x_1\omega^{2,4}+x_2\omega^{2,3}, -x_1\omega^{2,4}+x_5\omega^{4,6}, x_1\omega^{1,3}+$

$x_6\omega^{1,4}, -x_1\omega^{2,4} + x_5\omega^{3,5}\rangle + e^{-2\beta_1+\beta_2}\ \langle -2x_2\omega^{2,6} - x_3(\omega^{1,4} - 2\omega^{3,6}) + x_4(\omega^{4,6} - \omega^{3,5}) + 3x_5(\omega^{1,7} - 1/3\omega^{5,6}) + x_6\omega^{1,5}\rangle + 4e^{-\beta_1}\ \langle 2x_2\omega^{1,2} - x_3\omega^{1,3} - 2x_4\omega^{1,4} + x_6\omega^{1,6}, x_1\omega^{2,6} + x_2\omega^{1,2} - x_3(\omega^{1,3} + \omega^{2,4}) + x_4\omega^{3,6}, x_1\omega^{2,6} - x_3\omega^{2,4} + 2x_4\omega^{1,4} + x_5\omega^{2,7}, x_4\omega^{1,4} + x_5\omega^{1,5}\rangle + 4e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_4\omega^{2,4} + x_5\omega^{1,6}, x_1\omega^{1,2} + x_4(\omega^{1,3} + \omega^{2,4}), -x_1\omega^{1,2} - x_4\omega^{2,4} + x_5\omega^{2,5}, x_1\omega^{1,2} + x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + 2e^{-4\beta_1+4\beta_2}\ \langle x_2\omega^{3,6} + x_5\omega^{4,7}, -x_1(\omega^{4,6} - \omega^{3,5}) - x_2(\omega^{1,4} - \omega^{3,6}) + x_6\omega^{4,5}\rangle + e^{-5\beta_1+5\beta_2}\ \langle x_1(\omega^{4,5}+\omega^{3,7}) + x_2(\omega^{4,6}-\omega^{3,5}) + x_6\omega^{4,7}\rangle + 2e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}, x_4\omega^{2,3} + x_7\omega^{2,5}\rangle + e^{-\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=2e^{-\beta_1-\beta_2}+2e^{-\beta_1-2\beta_2}+e^{-2\beta_1-\beta_2}+4e^{-2\beta_1}+2e^{-3\beta_1+2\beta_2}+5e^{-3\beta_1+\beta_2}+5e^{-4\beta_1+3\beta_2}+3e^{-4\beta_1+2\beta_2}+e^{-5\beta_1+5\beta_2}+7e^{-5\beta_1+4\beta_2}+e^{-6\beta_1+6\beta_2}+6e^{-6\beta_1+5\beta_2}+2e^{-7\beta_1+7\beta_2}+2e^{-7\beta_1+6\beta_2}+2e^{-8\beta_1+8\beta_2}+e^{-9\beta_1+9\beta_2}+e^{-3\beta_2}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-3\beta_1-\beta_2}+e^{-3\beta_1}+3e^{-4\beta_1+\beta_2}+2e^{-4\beta_1}+e^{-5\beta_1+3\beta_2}+4e^{-5\beta_1+2\beta_2}+e^{-5\beta_1+\beta_2}+5e^{-6\beta_1+4\beta_2}+4e^{-6\beta_1+3\beta_2}+7e^{-7\beta_1+5\beta_2}+2e^{-7\beta_1+4\beta_2}+2e^{-8\beta_1+7\beta_2}+6e^{-8\beta_1+6\beta_2}+3e^{-9\beta_1+8\beta_2}+3e^{-9\beta_1+7\beta_2}+3e^{-10\beta_1+9\beta_2}+2e^{-11\beta_1+10\beta_2}+e^{-12\beta_1+11\beta_2}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-6\beta_1+2\beta_2}+e^{-7\beta_1+4\beta_2}+3e^{-7\beta_1+3\beta_2}+e^{-7\beta_1+2\beta_2}+3e^{-8\beta_1+5\beta_2}+2e^{-8\beta_1+4\beta_2}+e^{-8\beta_1+3\beta_2}+5e^{-9\beta_1+6\beta_2}+2e^{-9\beta_1+5\beta_2}+e^{-10\beta_1+8\beta_2}+4e^{-10\beta_1+7\beta_2}+e^{-10\beta_1+6\beta_2}+3e^{-11\beta_1+9\beta_2}+2e^{-11\beta_1+8\beta_2}+3e^{-12\beta_1+10\beta_2}+e^{-12\beta_1+9\beta_2}+2e^{-13\beta_1+11\beta_2}+e^{-14\beta_1+12\beta_2}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-9\beta_1+5\beta_2}+e^{-9\beta_1+4\beta_2}+e^{-10\beta_1+6\beta_2}+2e^{-10\beta_1+5\beta_2}+2e^{-11\beta_1+7\beta_2}+e^{-11\beta_1+6\beta_2}+e^{-12\beta_1+9\beta_2}+3e^{-12\beta_1+8\beta_2}+2e^{-13\beta_1+10\beta_2}+e^{-13\beta_1+9\beta_2}+2e^{-14\beta_1+11\beta_2}+e^{-15\beta_1+12\beta_2}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-12\beta_1+7\beta_2}+e^{-13\beta_1+8\beta_2}+e^{-14\beta_1+10\beta_2}+e^{-15\beta_1+11\beta_2}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 62, 132, 164, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 29, 47, 51, 38, 18, 4)$

S=$5(3\beta_1 - 2\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

$d\omega^5 = -\omega^{1,3} + \omega^{2,4}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = -\omega^{1,4} + \omega^{3,6}$; $d\omega^{1,5} = -\omega^{1,2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = -\omega^{1,3,6}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,4} - \omega^{2,3,6}$; $d\omega^{3,5} = \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,4}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{4,7} = \omega^{3,4,6}$; $d\omega^{5,6} = -\omega^{1,3,6}+\omega^{2,3,5}+\omega^{2,4,6}$; $d\omega^{5,7} = -\omega^{1,3,7}+\omega^{1,4,5}+\omega^{2,4,7}+\omega^{3,5,6}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{2,3,7}$; $d\omega^{1,2,7} = \omega^{1,2,3,6}$; $d\omega^{1,3,5} = -\omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,3,4,6}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} - \omega^{1,2,4,6}$; $d\omega^{1,5,7} = -\omega^{1,2,4,7} - \omega^{1,3,5,6}$;

$d\omega^{1,6,7} = \omega^{1,2,3,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{2,3,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} + \omega^{1,2,4,5} - \omega^{2,3,5,6}$; $d\omega^{2,6,7} = \omega^{1,2,4,6}$; $d\omega^{3,5,6} = \omega^{2,3,4,6}$; $d\omega^{3,5,7} = \omega^{1,3,4,5} + \omega^{2,3,4,7}$; $d\omega^{3,6,7} = \omega^{1,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,3,4,7} + \omega^{3,4,5,6}$; $d\omega^{4,6,7} = \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $d\omega^{1,2,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,5,7} = -\omega^{1,3,4,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} - \omega^{1,2,4,6,7}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,5,6} + \omega^{2,3,4,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1+2\beta_2} \langle\omega^3\rangle + e^{-3\beta_1+3\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-3\beta_1+2\beta_2} \langle\omega^{1,3}\rangle + e^{-3\beta_1+\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-2\beta_1} \langle\omega^{2,6}\rangle + e^{-5\beta_1+5\beta_2} \langle\omega^{3,4}\rangle + e^{-4\beta_1+3\beta_2} \langle\omega^{1,4}\rangle + e^{-6\beta_1+5\beta_2} \langle\omega^{3,7} + \omega^{4,5}\rangle + e^{-5\beta_1+4\beta_2} \langle\omega^{3,5} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-4\beta_1+\beta_2} \langle\omega^{1,2,5}\rangle + e^{-3\beta_1} \langle\omega^{1,2,6}\rangle + 2e^{-7\beta_1+5\beta_2} \langle\omega^{1,3,7}, \omega^{1,4,5}\rangle + 2e^{-6\beta_1+4\beta_2} \langle\omega^{1,3,5} + \omega^{2,4,5}, \omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-5\beta_1+3\beta_2} \langle\omega^{2,3,5}\rangle + e^{-5\beta_1+2\beta_2} \langle\omega^{1,2,7} + \omega^{2,5,6}\rangle + e^{-8\beta_1+7\beta_2} \langle\omega^{3,4,5}\rangle + e^{-9\beta_1+8\beta_2} \langle\omega^{3,4,7}\rangle + e^{-8\beta_1+6\beta_2} \langle\omega^{1,4,7} + \omega^{3,6,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-6\beta_1+2\beta_2} \langle\omega^{1,2,5,6}\rangle + e^{-7\beta_1+3\beta_2} \langle\omega^{1,2,6,7}\rangle + 2e^{-8\beta_1+5\beta_2} \langle\omega^{1,2,4,7} - \omega^{2,4,5,6}, \omega^{2,3,6,7} - \omega^{2,4,5,6}\rangle + 2e^{-9\beta_1+6\beta_2} \langle\omega^{1,3,6,7}, \omega^{1,4,5,6} - \omega^{2,3,5,7}\rangle + e^{-10\beta_1+7\beta_2} \langle\omega^{1,3,5,7} - \omega^{1,4,6,7}\rangle + e^{-7\beta_1+4\beta_2} \langle\omega^{1,2,4,5}\rangle + e^{-10\beta_1+8\beta_2} \langle\omega^{1,3,4,7}\rangle + e^{-12\beta_1+10\beta_2} \langle\omega^{3,4,5,7}\rangle + e^{-11\beta_1+9\beta_2} \langle\omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-9\beta_1+5\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-10\beta_1+6\beta_2} \langle\omega^{1,2,3,5,7}\rangle + e^{-10\beta_1+5\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-13\beta_1+10\beta_2} \langle\omega^{1,3,4,5,7}\rangle + e^{-12\beta_1+9\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-11\beta_1+7\beta_2} \langle\omega^{1,2,4,5,7} + \omega^{2,3,5,6,7}\rangle + e^{-12\beta_1+8\beta_2} \langle\omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}\rangle + e^{-14\beta_1+11\beta_2} \langle\omega^{3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-12\beta_1+7\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-13\beta_1+8\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-15\beta_1+11\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1+10\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-15\beta_1+10\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)(1 + Y^2)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = \omega^5$; $d^\star\omega^{3,6} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,4} = -\omega^{1,5} - \omega^{2,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{1,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{3,5} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,6} = -\omega^{2,7}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,6} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{5,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{1,2,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,5,6} + \omega^{2,6,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{1,5,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,4,7} + \omega^{3,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,4,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{3,5,7} + \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,5,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,6,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,5,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,3,5,7} - \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,5,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,5,7} - \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$;

$\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,3} = \omega^{1,3} - \omega^{2,4}$; $\Delta\omega^{1,4} = \omega^{1,4} - \omega^{3,6}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,7}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{5,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{1,5} + 2\omega^{2,7}$; $\Delta\omega^{3,5} = \omega^{3,5} + \omega^{4,6}$; $\Delta\omega^{3,6} = -\omega^{1,4} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{3,5} + \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{1,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 4\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4} + \omega^{2,3,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} - \omega^{2,5,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{1,4,6} + \omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,5} - \omega^{2,4,7} - \omega^{3,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,7} + \omega^{1,4,5} + \omega^{2,4,7} + \omega^{3,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,3,5} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{3,6,7} - \omega^{4,5,6}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,6,7}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,3,6} = \omega^{1,2,4} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{1,3,5} + 2\omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,3,6} + \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,3,7} + \omega^{1,4,5} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,2,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + 3\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = -\omega^{1,5,6} + \omega^{2,6,7}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = -\omega^{1,3,7} + \omega^{1,4,5} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} + \omega^{4,6,7}$; $\Delta\omega^{3,6,7} = -\omega^{1,4,7} + \omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = -\omega^{1,4,7} + \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} =$

$2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{3,5,7} + \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,5} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + \omega^{1,2,4,5} - \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,5} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,6} = 3\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} - \omega^{2,3,6,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,3,5,7} - \omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,6,7} + 2\omega^{1,4,5,6} - \omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,3,5,7} + \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = -\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,4,5} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,3,7} - \omega^{1,2,4,5} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,3,5,7} - \omega^{1,4,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,5,6,7} = \omega^{1,4,5,7} + 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,3,5,7} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,4,5,6} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1+2\beta_2} \langle\omega^3\rangle + e^{-3\beta_1+3\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-3\beta_1+2\beta_2} \langle\omega^{1,3} + \omega^{2,4}\rangle + e^{-3\beta_1+\beta_2} \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-2\beta_1} \langle\omega^{2,6}\rangle + e^{-5\beta_1+5\beta_2} \langle\omega^{3,4}\rangle + e^{-4\beta_1+3\beta_2} \langle\omega^{1,4} + \omega^{3,6}\rangle + e^{-6\beta_1+5\beta_2} \langle\omega^{3,7} + \omega^{4,5}\rangle + e^{-5\beta_1+4\beta_2} \langle\omega^{3,5} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-4\beta_1+\beta_2} \langle\omega^{1,2,5}\rangle + e^{-3\beta_1} \langle\omega^{1,2,6}\rangle + 2e^{-6\beta_1+4\beta_2} \langle\omega^{1,3,5} + \omega^{2,4,5}, \omega^{1,4,6} + \omega^{2,3,7} + 2\omega^{2,4,5}\rangle + e^{-5\beta_1+3\beta_2} \langle\omega^{2,3,5} - \omega^{2,4,6}\rangle + e^{-5\beta_1+2\beta_2} \langle\omega^{1,2,7} + \omega^{2,5,6}\rangle + e^{-8\beta_1+7\beta_2} \langle\omega^{3,4,5}\rangle + e^{-9\beta_1+8\beta_2} \langle\omega^{3,4,7}\rangle + 2e^{-7\beta_1+5\beta_2} \langle 2\omega^{1,3,7} + \omega^{2,4,7} + \omega^{3,5,6}, 2\omega^{1,4,5} - \omega^{2,4,7} - \omega^{3,5,6}\rangle + e^{-8\beta_1+6\beta_2} \langle\omega^{1,4,7} + \omega^{3,6,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-11\beta_1+9\beta_2} \langle\omega^{3,4,6,7}\rangle + e^{-12\beta_1+10\beta_2} \langle -\omega^{3,4,5,7}\rangle + 2e^{-9\beta_1+6\beta_2} \langle(-\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7} - 3\omega^{2,4,6,7})/4, (-\omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,3,5,7} + \omega^{2,4,6,7})/4\rangle + e^{-10\beta_1+7\beta_2} \langle(-\omega^{1,3,5,7} + \omega^{1,4,6,7})/2\rangle + e^{-10\beta_1+8\beta_2} \langle(-\omega^{1,3,4,7} + \omega^{3,4,5,6})/2\rangle + e^{-7\beta_1+3\beta_2} \langle\omega^{1,2,6,7}\rangle + e^{-6\beta_1+2\beta_2} \langle\omega^{1,2,5,6}\rangle + 2e^{-8\beta_1+5\beta_2} \langle(\omega^{1,2,4,7} - \omega^{1,3,5,6} + \omega^{2,3,6,7} - 3\omega^{2,4,5,6})/8, (-\omega^{1,2,4,7} + \omega^{1,3,5,6} +$

$3\omega^{2,3,6,7} - \omega^{2,4,5,6})/8\rangle + e^{-7\beta_1+4\beta_2}\ \langle(\omega^{1,2,4,5} + \omega^{2,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-14\beta_1+11\beta_2}\ \langle\omega^{3,4,5,6,7}\rangle + e^{-12\beta_1+8\beta_2}\ \langle(-\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-12\beta_1+9\beta_2}\ \langle(\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-13\beta_1+10\beta_2}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-10\beta_1+5\beta_2}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-11\beta_1+7\beta_2}\ \langle(\omega^{1,2,4,5,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-9\beta_1+5\beta_2}\ \langle(\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6})/2\rangle + e^{-10\beta_1+6\beta_2}\ \langle(\omega^{1,2,3,5,7} - \omega^{1,2,4,6,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-14\beta_1+10\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-15\beta_1+11\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-13\beta_1+8\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1+7\beta_2}\ \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-15\beta_1+10\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

5.4.41 Algebra $\mathcal{G}_{7,2.37}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_5$; $[x_1, x_6] = x_7$; $[x_2, x_4] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3+\xi_2^4 & 2\xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_1^3 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5+\xi_2^6 & \xi_1^3-\xi_1^4 & 2\xi_2^3+\xi_2^4 & 2(\xi_1^1+\xi_2^2) \end{pmatrix}$$

$ad\ x_1 = \delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -(\delta_1^5 + \delta_2^6 + \delta_3^7), ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0$.

Maximal torus : $T{=}\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G}){=}\ e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + 2e^{\beta_1+\beta_2}\ \langle x_3, x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}){=}\ e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}){=}\ e^{2\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_2}\ \langle\delta_1^6\rangle + e^{\beta_1+2\beta_2}\ \langle\delta_1^7\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_2^7\rangle + 3e^{\beta_1+\beta_2}\ \langle\delta_3^7, \delta_1^5, \delta_2^6\rangle + 2e^{\beta_2}\ \langle\delta_1^3, \delta_1^4\rangle + 2e^{\beta_1}\ \langle\delta_2^3, \delta_2^4\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^3] = -\delta_2^7$; $[\delta_2^5, \delta_1^4] = \delta_2^7$; $[\delta_1^6, \delta_2^3] = -2\delta_1^7$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_1^5, \delta_1^3] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = -\delta_2^7$; $[\delta_2^6, \delta_1^4] = \delta_1^7$; $[\delta_2^6, \delta_2^3] = -2\delta_2^7$; $[\delta_1^3, \delta_2^3] = -2\delta_3^7 + \delta_2^6$; $[\delta_1^3, \delta_2^4] = -\delta_1^5$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^4, \delta_2^4] = -(\delta_3^7 + \delta_2^6 + \delta_1^5)$.

$\mathrm{ch}(H^1_{\mathcal{G}}){=}\ e^{2\beta_1}\ \langle\delta_2^5\rangle + e^{2\beta_2}\ \langle\delta_1^6\rangle + e^{\beta_1+\beta_2}\ \langle\delta_2^6\rangle + e^{\beta_2}\ \langle\delta_1^3\rangle + e^{\beta_1}\ \langle\delta_2^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_2^3] = 3\delta_2^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}){=}\ 2e^{-2\beta_1}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5}, x_4\omega^{1,5} + x_6(\omega^{1,7} + \omega^{3,5})\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle + 2e^{-2\beta_2}\ \langle -x_1(\omega^{2,3} + \omega^{2,4}) - x_3\omega^{2,6} + x_4\omega^{2,6}, x_1(\omega^{2,3} + \omega^{2,4}) + x_3\omega^{2,6} + x_5(\omega^{2,7} + \omega^{3,6})\rangle + e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle + 3e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,3} - 2/3x_2(\omega^{2,3} - \omega^{2,4}) + 2/3x_4(\omega^{2,5} + 1/2\omega^{3,4}) + 2/3x_5\omega^{4,5} + x_6\omega^{3,6}, x_1\omega^{1,3} + x_2\omega^{2,4} - x_4(\omega^{1,6} + \omega^{3,4}) + x_5(\omega^{1,7} + \omega^{4,5}) + x_6\omega^{2,7}, 1/3x_2(\omega^{2,3} + 2\omega^{2,4}) + 2/3x_4(\omega^{2,5} + 1/2\omega^{3,4}) + x_5(\omega^{3,5} + 2/3\omega^{4,5})\rangle + e^{-\beta_1+\beta_2}\ \langle x_6\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle x_3\omega^{1,4}, -x_3\omega^{1,3} + x_4\omega^{1,4} + x_6\omega^{1,6}, x_3\omega^{1,3} + x_4\omega^{1,3} + x_6\omega^{3,4}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + 3e^{-\beta_2}\ \langle x_3(\omega^{2,3} + 2\omega^{2,4}) - x_4\omega^{2,4} + x_5\omega^{2,5}, -x_3(\omega^{2,3} + \omega^{2,4}) + x_4(\omega^{2,3} + \omega^{2,4}), -x_3(\omega^{2,3} + 2\omega^{2,4}) + 2x_4\omega^{2,4} + x_5\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-\beta_1-2\beta_2}+4e^{-\beta_1-3\beta_2}+2e^{-\beta_1-4\beta_2}+e^{-2\beta_1-\beta_2}+2e^{-2\beta_1-2\beta_2}+4e^{-2\beta_1-3\beta_2}+4e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2}+e^{-3\beta_1}+2e^{-4\beta_1-\beta_2}+e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-5\beta_2}+2e^{-3\beta_1-3\beta_2}+4e^{-3\beta_1-4\beta_2}+3e^{-3\beta_1-5\beta_2}+e^{-3\beta_1-6\beta_2}+2e^{-4\beta_1-2\beta_2}+4e^{-4\beta_1-3\beta_2}+2e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2}+2e^{-5\beta_1-2\beta_2}+3e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-4\beta_1-5\beta_2}+4e^{-4\beta_1-6\beta_2}+e^{-4\beta_1-7\beta_2}+2e^{-5\beta_1-4\beta_2}+3e^{-5\beta_1-5\beta_2}+2e^{-5\beta_1-6\beta_2}+e^{-5\beta_1-7\beta_2}+4e^{-6\beta_1-4\beta_2}+2e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-5\beta_1-6\beta_2}+e^{-5\beta_1-7\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-6\beta_1-6\beta_2}+2e^{-6\beta_1-7\beta_2}+e^{-6\beta_1-8\beta_2}+e^{-7\beta_1-5\beta_2}+2e^{-7\beta_1-6\beta_2}+e^{-7\beta_1-7\beta_2}+e^{-8\beta_1-6\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-7\beta_1-7\beta_2}+e^{-7\beta_1-8\beta_2}+e^{-8\beta_1-7\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,13,53,121,154,114,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,7,17,27,30,23,12,3)$

S=$8(\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,18,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,7,7,6,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}-\omega^{1,4}$; $d\omega^6=-\omega^{2,4}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,5}=-\omega^{1,2,3}-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,5}=-\omega^{1,3,4}$; $d\omega^{3,6}=-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}-\omega^{1,4,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,3,7}-\omega^{1,4,7}-\omega^{1,5,6}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{2,4,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,5}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}-\omega^{1,2,4,7}-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,3,6,7}-\omega^{1,4,6,7}+\omega^{2,4,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,3,6,7}-\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7}=$

$\omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_1-\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,7} + 2\omega^{3,5} + \omega^{4,5}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,7} + \omega^{3,6} - \omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,5}, \omega^{1,4,5}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6}, \omega^{2,4,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,2,7} + \omega^{2,4,5}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,3,7} - \omega^{3,4,5}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{2,3,7} + \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,5,7}, \omega^{1,4,5,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,5,7} + \omega^{1,3,5,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{2,3,4,7} + \omega^{2,4,5,6}\rangle + 2e^{-4\beta_1-6\beta_2} \langle\omega^{2,3,6,7}, \omega^{2,4,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,3,6,7} + \omega^{2,3,5,7} + 2\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-6\beta_1-5\beta_2} \langle\omega^{1,2,3,5,7}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{1,3,4,5,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-5\beta_1-7\beta_2} \langle\omega^{2,3,4,6,7}\rangle + e^{-6\beta_1-7\beta_2} \langle\omega^{2,3,5,6,7} + \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-7\beta_1-7\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-8\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-8\beta_1-8\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 3Y^3 + 4Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 18, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 7, 7, 6, 3, 1)$

Total dimension of trivial cohomology: 34.

Length associated polynomial: 34.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^5$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} - \omega^{3,5} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7} - \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} +$

$\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,5,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,6,7} = -\omega^{3,5,6,7} + \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{2,3,5,6,7} + \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{1,4}$; $\Delta\omega^{1,4} = \omega^{1,3} + \omega^{1,4}$; $\Delta\omega^{1,6} = 2\omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{3,5}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = 3\omega^{2,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} - \omega^{3,6} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{3,5} - \omega^{4,5}$; $\Delta\omega^{3,6} = -\omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,5} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = 4\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} + \omega^{1,2,4}$; $\Delta\omega^{1,2,4} = \omega^{1,2,3} + 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,6} + \omega^{1,4,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = 3\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = -\omega^{1,2,7} + 3\omega^{1,3,6} + \omega^{1,4,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,4,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,3,6} + 2\omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,3,7} + 2\omega^{1,4,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7} + \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + 2\omega^{2,3,5} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,5} + 3\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7} - \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 3\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 4\omega^{2,5,7} - \omega^{3,5,6} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} - \omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} - \omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 3\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} - \omega^{2,5,7} + 2\omega^{3,5,6} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7} - \omega^{4,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} - \omega^{3,5,6} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,4,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,3,6} + 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,3,7} + 2\omega^{1,2,4,7} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 3\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{1,4,5,6}$;

$\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,4,7} + 4\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 3\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,5,7}+2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7}+\omega^{1,4,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,3,6,7} + 2\omega^{1,4,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + \omega^{1,2,5,6} + 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} + 2\omega^{2,3,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,7} = 2\omega^{2,3,5,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} - \omega^{2,3,5,6} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,5,7} + 3\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = 3\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7}-\omega^{2,3,5,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7}+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7} - \omega^{4,5,6,7}$; $\Delta\omega^{4,5,6,7} = -\omega^{3,5,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 4\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7}-\omega^{1,2,4,5,7}+2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 3\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7} - \omega^{2,4,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = -\omega^{2,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_1-\beta_2}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3} - \omega^{1,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,7} + 2\omega^{3,5} + \omega^{4,5}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,7} + \omega^{3,6} - \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5}, \omega^{1,4,5}\rangle + 2e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}, \omega^{2,4,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle 5\omega^{1,2,7} + 3\omega^{1,3,6} - 4\omega^{1,4,6} - 3\omega^{2,3,5} - \omega^{2,4,5}\rangle + e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,3,7} - \omega^{1,4,7} - \omega^{1,5,6} - \omega^{3,4,5}\rangle + e^{-3\beta_1-4\beta_2}\ \langle 3\omega^{2,3,7} + \omega^{2,4,7} - \omega^{2,5,6} + 2\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= 2e^{-4\beta_1-6\beta_2}\ \langle-\omega^{2,4,6,7}, \omega^{2,3,6,7}\rangle + 2e^{-6\beta_1-4\beta_2}\ \langle-\omega^{1,4,5,7}, \omega^{1,3,5,7}\rangle + e^{-5\beta_1-5\beta_2}\ \langle(\omega^{1,3,6,7} - 3\omega^{1,4,6,7} + 4\omega^{2,3,5,7} + 3\omega^{2,4,5,7} + 5\omega^{3,4,5,6})/60\rangle + e^{-4\beta_1-5\beta_2}\ \langle(-\omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{2,3,5,6} - \omega^{2,4,5,6})/4\rangle + e^{-5\beta_1-4\beta_2}\ \langle(-2\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6} + 3\omega^{1,4,5,6})/15\rangle.$

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-6\beta_1-7\beta_2}\ \langle(-\omega^{2,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-5\beta_1-7\beta_2}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-7\beta_1-6\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-5\beta_2}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-5\beta_1-6\beta_2}\ \langle(\omega^{1,2,3,6,7} - 2\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/6\rangle + e^{-6\beta_1-5\beta_2}\ \langle(\omega^{1,2,3,5,7} + \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-7\beta_1-8\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-7\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-7\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-8\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+2Y+4Y^2+3Y^3+4Y^4+2Y^5+Y^6)(1+Y).$

5.4.42 Algebra $\mathcal{G}_{7,2.38}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_6$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_7$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & 0 & 2\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^3-\xi_1^4 & \xi_4^6 & \xi_5^6 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^3+\xi_2^5 & \xi_4^7 & \xi_5^7 & 0 & \xi_1^1+2\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^3+\delta_5^7, ad\ x_2=-\delta_1^3+\delta_4^6, ad\ x_3=-(\delta_1^6+\delta_2^7), ad\ x_4=-\delta_2^6, ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{2\beta_1}\ \langle x_4\rangle+e^{2\beta_2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{2\beta_1-\beta_2}\ \langle\delta_2^4,\delta_5^6\rangle+2e^{-\beta_1+2\beta_2}\ \langle\delta_1^5,\delta_4^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^6,\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+3e^{\beta_1}\ \langle\delta_2^3,\delta_1^4,\delta_5^7\rangle+3e^{\beta_2}\ \langle\delta_4^6,\delta_1^3,\delta_2^5\rangle+e^{2\beta_2}\ \langle\delta_1^7\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4,\delta_4^6]=-\delta_2^6$; $[\delta_2^4,\delta_4^7]=-\delta_2^7$; $[\delta_1^5,\delta_5^6]=-\delta_1^6$; $[\delta_1^5,\delta_5^7]=-\delta_1^7$; $[\delta_2^3,\delta_1^4]=\delta_2^6$; $[\delta_2^3,\delta_1^3]=\delta_2^7+\delta_1^6$; $[\delta_2^3,\delta_2^5]=-\delta_2^7$; $[\delta_1^4,\delta_4^6]=-\delta_1^6$; $[\delta_1^4,\delta_1^3]=-\delta_1^6$; $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_5^6,\delta_2^5]=\delta_2^6$; $[\delta_1^3,\delta_2^5]=-\delta_1^7$; $[\delta_2^5,\delta_5^7]=-\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= 2e^{2\beta_1-\beta_2}\ \langle\delta_2^4,\delta_5^6\rangle+2e^{-\beta_1+2\beta_2}\ \langle\delta_1^5,\delta_4^7\rangle+2e^{\beta_1}\ \langle\delta_1^4,\delta_2^3\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+2e^{\beta_2}\ \langle\delta_1^3,\delta_2^5\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^3]=-\delta_1^6$; $[\delta_2^4,\delta_4^7]=\delta_1^6$; $[\delta_1^5,\delta_5^6]=-\delta_1^6$; $[\delta_2^5,\delta_2^3]=-\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=\ 4e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,3}-\omega^{2,4})-x_2(\omega^{1,5}-\omega^{2,3})+x_3\omega^{4,5}, x_2\omega^{2,3}+x_3\omega^{2,6}+x_4(\omega^{1,6}+\omega^{3,4}), x_1\omega^{1,3}-x_3\omega^{1,7}-x_5(\omega^{2,7}-\omega^{3,5}), x_1\omega^{1,3}+x_2\omega^{1,5}+x_3\omega^{2,6}+x_6(\omega^{3,6}+\omega^{4,7})\rangle+5e^{-\beta_1}\ \langle x_3\omega^{2,4}+x_4\omega^{1,4}, x_2\omega^{1,2}+x_3\omega^{2,4}+x_5\omega^{2,3}, x_5\omega^{1,5}, -x_3\omega^{2,4}+x_6\omega^{1,6}, -x_2\omega^{1,2}+x_3(\omega^{1,3}-\omega^{2,4})\rangle+e^{2\beta_1-3\beta_2}\ \langle x_4\omega^{2,5}\rangle+4e^{-2\beta_2}\ \langle 1/2x_1(\omega^{1,5}-\omega^{2,3})+x_3(\omega^{2,7}-1/2\omega^{3,5})+x_4(\omega^{1,7}+\omega^{2,6}), 1/2x_1(\omega^{1,5}-3\omega^{2,3})-x_2\omega^{2,5}-1/2x_3\omega^{3,5}+x_4\omega^{4,5}, -2x_1\omega^{2,3}-x_2\omega^{2,5}-x_3\omega^{3,5}+x_6\omega^{5,6}, x_1\omega^{1,5}+x_3\omega^{2,7}+x_6\omega^{3,7}\rangle+2e^{\beta_1-3\beta_2}\ \langle -x_1\omega^{2,5}-x_4(\omega^{2,7}-\omega^{3,5}), -x_1\omega^{2,5}+x_6\omega^{5,7}\rangle+e^{-3\beta_1+2\beta_2}\ \langle x_5\omega^{1,4}\rangle+5e^{-\beta_2}\ \langle x_3\omega^{1,5}+x_5\omega^{2,5}, -x_1\omega^{1,2}+$

$x_3\omega^{1,5} + x_4\omega^{1,3}, x_4\omega^{2,4}, x_1\omega^{1,2} - x_3\omega^{1,5} + x_6\omega^{1,7}, x_1\omega^{1,2} - x_3(\omega^{1,5} - \omega^{2,3})\rangle + 4e^{-2\beta_1}\ \langle 1/2x_2(\omega^{1,3} - \omega^{2,4}) + x_3(\omega^{1,6} + 1/2\omega^{3,4}) + x_5(\omega^{1,7} + \omega^{2,6}), x_1\omega^{1,4} + 3/2x_2(\omega^{1,3} - 1/3\omega^{2,4}) + 1/2x_3\omega^{3,4} + x_5\omega^{4,5}, x_2\omega^{1,3} + x_6\omega^{4,6}, -x_1\omega^{1,4} - 2x_2\omega^{1,3} - x_3\omega^{3,4} + x_7\omega^{4,7}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle -x_2\omega^{1,4} + x_5(\omega^{1,6} + \omega^{3,4}), -x_2\omega^{1,4} + x_7\omega^{4,6}\rangle + 2e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}, x_3\omega^{2,5} + x_4\omega^{1,5}\rangle + 2e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,3}, x_3\omega^{1,4} + x_5\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-4\beta_2} + e^{\beta_1-5\beta_2} + 4e^{-\beta_1-2\beta_2} + 7e^{-\beta_1-3\beta_2} + 4e^{-2\beta_1-\beta_2} + 9e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 7e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + 3e^{-3\beta_1} + e^{-4\beta_1+\beta_2} + 3e^{-4\beta_1} + e^{-5\beta_1+\beta_2} + 3e^{-3\beta_2} + 3e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=3e^{-\beta_1-5\beta_2} + e^{-\beta_1-6\beta_2} + e^{-2\beta_1-3\beta_2} + 7e^{-2\beta_1-4\beta_2} + 3e^{-2\beta_1-5\beta_2} + e^{-3\beta_1-2\beta_2} + 9e^{-3\beta_1-3\beta_2} + 4e^{-3\beta_1-4\beta_2} + 7e^{-4\beta_1-2\beta_2} + 4e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2} + 3e^{-5\beta_1-\beta_2} + 3e^{-5\beta_1-2\beta_2} + e^{-6\beta_1-\beta_2} + e^{-6\beta_1} + e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-2\beta_1-6\beta_2} + e^{-2\beta_1-7\beta_2} + 4e^{-3\beta_1-5\beta_2} + 2e^{-3\beta_1-6\beta_2} + e^{-3\beta_1-7\beta_2} + 5e^{-4\beta_1-4\beta_2} + 5e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-6\beta_2} + 4e^{-5\beta_1-3\beta_2} + 5e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + 2e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-7\beta_2} + e^{-4\beta_1-6\beta_2} + 2e^{-4\beta_1-7\beta_2} + 3e^{-5\beta_1-5\beta_2} + 2e^{-5\beta_1-6\beta_2} + e^{-5\beta_1-7\beta_2} + e^{-6\beta_1-4\beta_2} + 2e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-7\beta_1-3\beta_2} + 2e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-5\beta_1-7\beta_2} + e^{-6\beta_1-7\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-7\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 65, 131, 164, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 32, 49, 50, 38, 18, 4)$

S=$7(\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,5}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{5,6} = \omega^{1,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = \omega^{2,3,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,3,4,5}$; $d\omega^{4,5,7} = -\omega^{2,3,4,5}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = \omega^{1,3,5,7} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} =$

$\omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_1} \langle\omega^4\rangle + e^{-2\beta_2} \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,3}\rangle + e^{-3\beta_2} \langle\omega^{2,5}\rangle + 2e^{-2\beta_1-2\beta_2} \langle\omega^{1,7} + \omega^{2,6}, \omega^{4,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} + \omega^{3,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,7} - \omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 3e^{-3\beta_1-3\beta_2} \langle\omega^{1,3,7}, \omega^{2,3,6}, \omega^{1,5,6} - \omega^{3,4,5}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,7}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,6} - \omega^{2,4,6}, \omega^{1,4,7} + \omega^{2,4,6}\rangle + 2e^{-2\beta_1-4\beta_2} \langle\omega^{1,5,7} + \omega^{2,5,6}, \omega^{2,3,7} + \omega^{2,5,6}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 3e^{-4\beta_1-4\beta_2} \langle\omega^{1,2,6,7}, \omega^{1,3,5,6} - \omega^{2,4,5,6}, \omega^{1,4,5,7} + \omega^{2,4,5,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,3,4,6}\rangle + 2e^{-5\beta_1-3\beta_2} \langle\omega^{1,3,4,7}, \omega^{1,4,5,6}\rangle + 2e^{-3\beta_1-5\beta_2} \langle\omega^{1,3,5,7}, \omega^{2,3,5,6}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{2,3,5,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{1,3,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,6}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,5,6,7} - \omega^{2,3,6,7} - \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,3,6,7}, \omega^{1,3,4,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-6\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,3,4,6,7}\rangle + e^{-5\beta_1-7\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-7\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,7} - \omega^{2,6}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = \omega^{4,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,7}$;

$d^{\star}\omega^{2,3,5} = \omega^{5,7}$; $d^{\star}\omega^{2,3,6} = \omega^{6,7}$; $d^{\star}\omega^{2,4,5} = \omega^{5,6}$; $d^{\star}\omega^{2,4,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,7} + \omega^{2,4,6}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{1,6,7}$; $d^{\star}\omega^{1,2,3,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,7} - \omega^{3,4,5}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{3,4,6}$; $d^{\star}\omega^{1,2,4,7} = \omega^{1,6,7} - \omega^{3,4,7}$; $d^{\star}\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{3,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{3,4,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^{\star}\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^{\star}\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,7}$; $d^{\star}\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^{\star}\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{1,4,6,7}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{1,5,6,7} + \omega^{2,3,6,7}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = \omega^{2,4,6,7} - \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,4} = -\omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{2,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{5,6}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = -\omega^{3,6} + 2\omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,2,7} + 2\omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,5,6} - \omega^{2,3,6} + \omega^{2,4,7}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6} + \omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7} + 2\omega^{1,5,6} + \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,6} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,3,6} - \omega^{2,4,7}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} - \omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} - \omega^{2,3,6} + 2\omega^{2,4,7} + \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,5,6}$; $\Delta\omega^{3,4,5} = \omega^{1,5,6} + \omega^{2,4,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,6,7} + 2\omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{2,6,7} + 2\omega^{3,5,6} - \omega^{4,5,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,5,6} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} - \omega^{1,2,4,7}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6}$; $\Delta\omega^{1,2,4,5} = 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,4,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,6,7} + 2\omega^{1,3,5,6} - \omega^{1,4,5,7} + \omega^{2,4,5,6}$;

$\Delta\omega^{1,3,5,7}=\omega^{1,3,5,7}-\omega^{2,3,5,6}+\omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}+\omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,3,4,7}+\omega^{1,4,5,6}-\omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}+\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,2,5,6}+2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=-\omega^{1,3,4,7}-\omega^{1,4,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,2,6,7}-\omega^{1,4,5,7}+2\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6}=-\omega^{1,3,5,7}+\omega^{2,3,5,6}-\omega^{2,4,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,5,6,7}+\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6}=\omega^{1,3,5,6}-\omega^{1,4,5,7}+\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,3,5,7}-\omega^{2,3,5,6}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{1,3,6,7}+2\omega^{2,4,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{2,4,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6}=2\omega^{1,2,3,5,6}-\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7}=\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,4,7}+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,5,6}+2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}-\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=-\omega^{1,3,4,5,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=-\omega^{1,4,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^4\rangle+e^{-2\beta_2}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-3\beta_1}\ \langle\omega^{1,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{1,5}-\omega^{2,3}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3}-\omega^{2,4}\rangle+e^{-3\beta_2}\ \langle\omega^{2,5}\rangle+2e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,7}+\omega^{2,6},\omega^{4,5}\rangle+e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}+\omega^{3,4}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,7}-\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-5\beta_1-\beta_2}\ \langle\omega^{1,4,6}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,6}-\omega^{1,4,5}-\omega^{2,3,4}\rangle+e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7}-\omega^{1,3,5}+\omega^{2,4,5}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,6}-\omega^{2,4,6},\omega^{1,4,7}+\omega^{2,4,6}\rangle+2e^{-2\beta_1-4\beta_2}\ \langle\omega^{1,5,7}+\omega^{2,5,6},\omega^{2,3,7}+\omega^{2,5,6}\rangle+e^{-\beta_1-5\beta_2}\ \langle\omega^{2,5,7}\rangle+3e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,7}-\omega^{2,4,7}+\omega^{3,4,5},\omega^{1,5,6}+\omega^{2,4,7}-2\omega^{3,4,5},\omega^{2,3,6}+\omega^{2,4,7}-\omega^{3,4,5}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=e^{-2\beta_1-6\beta_2}\ \langle-\omega^{2,3,5,7}\rangle+e^{-4\beta_1-5\beta_2}\ \langle(\omega^{1,5,6,7}-\omega^{2,3,6,7}-\omega^{3,4,5,7})/3\rangle+e^{-5\beta_1-4\beta_2}\ \langle(-\omega^{1,3,6,7}+\omega^{2,4,6,7}+\omega^{3,4,5,6})/3\rangle+2e^{-3\beta_1-5\beta_2}\ \langle(-\omega^{1,3,5,7}+\omega^{2,3,5,6}+2\omega^{2,4,5,7})/3,(\omega^{1,3,5,7}+2\omega^{2,3,5,6}+\omega^{2,4,5,7})/3\rangle+2e^{-5\beta_1-3\beta_2}\ \langle(-\omega^{1,3,4,7}-\omega^{1,4,5,6}-2\omega^{2,3,4,6})/3,(-\omega^{1,3,4,7}+2\omega^{1,4,5,6}+\omega^{2,3,4,6})/3\rangle+e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,3,4,6}\rangle+3e^{-4\beta_1-4\beta_2}\ \langle(\omega^{1,3,5,6}-\omega^{1,4,5,7}+\omega^{2,3,4,7}-3\omega^{2,4,5,6})/4,(-4\omega^{1,2,6,7}+\omega^{1,3,5,6}+3\omega^{1,4,5,7}+5\omega^{2,3,4,7}-3\omega^{2,4,5,6})/12,(-\omega^{1,3,5,6}-3\omega^{1,4,5,7}-\omega^{2,3,4,7}-\omega^{2,4,5,6})/4\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=e^{-4\beta_1-7\beta_2}\ \langle\omega^{2,3,5,6,7}\rangle+e^{-6\beta_1-5\beta_2}\ \langle(-\omega^{1,4,5,6,7}-\omega^{2,3,4,6,7})/2\rangle+e^{-5\beta_1-6\beta_2}\ \langle(\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7})/2\rangle+e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle+2e^{-5\beta_1-5\beta_2}\ \langle(-\omega^{1,3,4,5,7}-\omega^{2,3,4,5,6})/2,\omega^{1,2,3,6,7}\rangle+e^{-4\beta_1-6\beta_2}\ \langle(\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7})/2\rangle+e^{-6\beta_1-4\beta_2}\ \langle(\omega^{1,2,4,6,7}+\omega^{1,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-6\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_2} \langle-\omega^{1,2,3,5,6,7}\rangle + e^{-7\beta_1-5\beta_2} \langle\omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-7\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

5.4.43 Algebra $\mathcal{G}_{7,2.39}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_5]=x_7$; $[x_2,x_3]=x_6$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & 2\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & \xi_3^2 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & 3\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^4 & \xi_1^2+\xi_3^4 & 2\xi_1^1+\xi_2^2 & \xi_3^2\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_3^5 & 0 & 4\xi_1^1\end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_1^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{2\beta_1}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{3\beta_1}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{4\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=\ 2e^{\beta_2}\ \langle\delta_1^4,\delta_3^6\rangle+3e^{2\beta_1}\ \langle\delta_1^5,\delta_2^6,\delta_3^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+3e^{\beta_1}\ \langle\delta_1^3,\delta_2^4,\delta_3^5\rangle+2e^{-\beta_1+\beta_2}\ \langle\delta_1^2,\delta_3^4\rangle+2e^0\ \langle t_2,t_1\rangle+e^{-2\beta_1+\beta_2}\ \langle\delta_3^2\rangle+e^{3\beta_1}\ \langle\delta_1^7\rangle+e^{4\beta_1-\beta_2}\ \langle\delta_2^7\rangle+e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^3]=\delta_1^6$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_1^4,\delta_2^5]=-\delta_1^7$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_1^5,\delta_3^4]=-\delta_1^6$; $[\delta_1^5,\delta_3^2]=-\delta_1^4$; $[\delta_1^5,\delta_3^5]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_2^6,\delta_3^2]=\delta_3^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_2^6$; $[\delta_1^3,\delta_3^4]=-(\delta_3^6+\delta_1^4)$; $[\delta_1^3,\delta_3^2]=-\delta_1^2$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_3^5]=-\delta_1^5$; $[\delta_2^4,\delta_1^2]=\delta_1^4$; $[\delta_2^4,\delta_3^4]=\delta_3^6$; $[\delta_2^4,\delta_3^2]=\delta_3^4$; $[\delta_2^4,\delta_2^5]=-\delta_2^7$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^5]=\delta_2^6-\delta_1^5$; $[\delta_1^2,\delta_3^5]=\delta_3^6$; $[\delta_3^4,\delta_2^5]=-\delta_3^7+\delta_2^6$; $[\delta_3^4,\delta_3^5]=\delta_3^6$; $[\delta_3^2,\delta_1^7]=\delta_1^6$; $[\delta_3^2,\delta_2^7]=-\delta_3^7+\delta_2^6$; $[\delta_3^2,\delta_3^7]=\delta_3^6$; $[\delta_3^2,\delta_2^5]=-\delta_3^5+\delta_2^4$; $[\delta_3^2,\delta_3^5]=\delta_3^4$; $[\delta_2^5,\delta_3^5]=-\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=\ 2e^{2\beta_1}\ \langle\delta_1^5,\delta_3^7\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+2e^{\beta_1}\ \langle\delta_1^3,\delta_2^4\rangle+2e^{-\beta_1+\beta_2}\ \langle\delta_1^2,\delta_3^4\rangle+2e^0\ \langle t_2,t_1\rangle+e^{-2\beta_1+\beta_2}\ \langle\delta_3^2\rangle+e^{4\beta_1-\beta_2}\ \langle\delta_2^7\rangle+e^{3\beta_1-\beta_2}\ \langle\delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^5]=-2\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^3,\delta_3^4]=-2\delta_1^4$; $[\delta_1^3,\delta_3^2]=-\delta_1^2$; $[\delta_1^3,\delta_2^4]=\delta_1^5$; $[\delta_3^4,\delta_2^5]=-\delta_3^7-\delta_1^5$; $[\delta_3^4,\delta_2^4]=-\delta_1^4$; $[\delta_1^5,\delta_3^2]=-\delta_1^4$; $[\delta_3^2,\delta_2^7]=-\delta_3^7-\delta_1^5$; $[\delta_3^2,\delta_3^7]=\delta_1^4$; $[\delta_3^2,\delta_2^5]=2\delta_2^4$; $[\delta_3^2,\delta_2^4]=-\delta_3^4$; $[\delta_2^5,\delta_2^4]=\delta_2^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})=6e^{-2\beta_1}\ \langle-x_2(\omega^{1,4}+\omega^{2,3})+x_4\omega^{3,4},x_5\omega^{1,7},-x_2\omega^{2,3}+x_4\omega^{1,6},x_2\omega^{2,3}+x_3\omega^{1,5}+x_4\omega^{2,5},x_1\omega^{1,3}+x_2(\omega^{1,4}-\omega^{2,3})+x_5\omega^{3,5},-x_1\omega^{1,3}-x_2(\omega^{1,4}-$

$\omega^{2,3}) + x_7\omega^{3,7}\rangle + 2e^{-4\beta_1+\beta_2}\ \langle -x_2\omega^{1,5} + x_4\omega^{3,5}, x_4\omega^{1,7}\rangle + 3e^{-3\beta_1}\ \langle -x_2(\omega^{1,6} - \omega^{3,4}) + x_4\omega^{3,6}, -x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) - 3x_3(\omega^{1,7} - 1/3\omega^{3,5}) - 3x_4(\omega^{2,7} - 1/3\omega^{4,5}), 2x_1\omega^{1,5} + x_2(\omega^{1,6} + \omega^{2,5}) - 2x_3\omega^{3,5} + 3x_4\omega^{2,7} + 3x_6(\omega^{4,7} + 1/3\omega^{5,6})\rangle + 2e^{-5\beta_1+\beta_2}\ \langle -x_2(\omega^{1,7} - \omega^{3,5}) + x_4\omega^{3,7}, x_2\omega^{1,7} + x_6\omega^{5,7}\rangle + 5e^{-\beta_2}\ \langle x_1\omega^{1,2} - x_3(\omega^{1,4} - 2\omega^{2,3}) + x_5\omega^{2,5}, -x_3\omega^{2,3} + x_5\omega^{1,6}, -x_1\omega^{1,2} + x_3(\omega^{1,4} - \omega^{2,3}) + x_7\omega^{2,7}, -x_3\omega^{2,3} + x_5\omega^{3,4} + x_7\omega^{3,6}, x_3\omega^{1,4} + x_4\omega^{2,4}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4} + 2x_2\omega^{2,4} + 3x_3(\omega^{1,6} + 1/3\omega^{2,5} - 2/3\omega^{3,4}) + 3x_4\omega^{2,6} + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_3(\omega^{2,5} + \omega^{3,4}) - x_4\omega^{2,6} + x_5\omega^{2,7} - x_6\omega^{4,6} + x_7\omega^{4,7}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3\omega^{1,3}, x_5\omega^{1,5}, x_2\omega^{1,2} + x_4\omega^{2,3}, x_4\omega^{1,4}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,3}, x_4\omega^{1,5}\rangle + e^{3\beta_1-2\beta_2}\ \langle x_7\omega^{2,4}\rangle + 3e^{\beta_1-\beta_2}\ \langle x_3\omega^{1,2}, x_5\omega^{1,4}, -x_5\omega^{2,3} + x_7\omega^{3,4}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,4} + x_7\omega^{2,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{\beta_1-3\beta_2} + 5e^{-\beta_1-2\beta_2} + 4e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2} + 9e^{-3\beta_1-\beta_2} + 6e^{-4\beta_1-\beta_2} + 2e^{-4\beta_1} + 8e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 4e^{-6\beta_1} + 2e^{-7\beta_1+\beta_2} + e^{-8\beta_1+\beta_2} + 2e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-\beta_1-3\beta_2} + e^{-2\beta_1-3\beta_2} + 2e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-3\beta_2} + 7e^{-4\beta_1-2\beta_2} + 2e^{-5\beta_1-\beta_2} + 6e^{-5\beta_1-2\beta_2} + 8e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2} + 7e^{-7\beta_1-\beta_2} + 2e^{-8\beta_1-\beta_2} + 4e^{-8\beta_1} + 4e^{-9\beta_1} + e^{-10\beta_1+\beta_2} + e^{-10\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-4\beta_1-3\beta_2} + 2e^{-5\beta_1-3\beta_2} + 2e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1-3\beta_2} + 4e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-\beta_2} + 6e^{-8\beta_1-2\beta_2} + 4e^{-9\beta_1-\beta_2} + 2e^{-9\beta_1-2\beta_2} + 4e^{-10\beta_1-\beta_2} + e^{-10\beta_1} + e^{-11\beta_1-\beta_2} + e^{-11\beta_1} + e^{-12\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = 2e^{-8\beta_1-3\beta_2} + e^{-9\beta_1-2\beta_2} + e^{-9\beta_1-3\beta_2} + 2e^{-10\beta_1-2\beta_2} + e^{-10\beta_1-3\beta_2} + 2e^{-11\beta_1-2\beta_2} + e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-\beta_2} + e^{-12\beta_1-2\beta_2} + e^{-13\beta_1-\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-11\beta_1-3\beta_2} + e^{-12\beta_1-3\beta_2} + e^{-13\beta_1-2\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 17, 63, 131, 163, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 31, 47, 49, 33, 13, 3)$

$\mathrm{S} = 13\beta_1 + 3\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 23, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 13, 13, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{1,5,6} - \omega^{2,3,7}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} =$

$-\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{1,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = 2e^{-5\beta_1}\ \langle\omega^{1,7}, \omega^{3,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + 2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = 2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,6}, \omega^{1,2,7}\rangle + 2e^{-6\beta_1-\beta_2}\ \langle\omega^{1,4,7}, \omega^{1,5,6} + \omega^{3,4,5}\rangle + e^{-8\beta_1}\ \langle\omega^{1,5,7}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6} + \omega^{2,4,5}, \omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + 2e^{-7\beta_1-\beta_2}\ \langle\omega^{1,6,7} - \omega^{3,4,7} - \omega^{3,5,6}, \omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}\rangle + e^{-9\beta_1}\ \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6}\rangle + 2e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,2,4,7}, \omega^{1,3,4,6}\rangle + 2e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,2,6,7}, \omega^{1,4,5,6} - \omega^{2,3,5,6}\rangle + e^{-8\beta_1-\beta_2}\ \langle\omega^{1,2,5,7}\rangle + e^{-10\beta_1}\ \langle\omega^{1,3,5,7}\rangle + 2e^{-9\beta_1-\beta_2}\ \langle\omega^{1,3,6,7}, \omega^{1,4,5,7}\rangle + 2e^{-8\beta_1-2\beta_2}\ \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}, \omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{2,3,4,6}\rangle + e^{-10\beta_1-\beta_2}\ \langle\omega^{1,5,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = 2e^{-8\beta_1-3\beta_2}\ \langle\omega^{1,2,4,6,7}, \omega^{2,3,4,5,6}\rangle + 2e^{-10\beta_1-2\beta_2}\ \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-12\beta_1-\beta_2}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-11\beta_1-2\beta_2}\ \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-11\beta_1-3\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-13\beta_1-2\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-13\beta_1-3\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 4Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 23, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 13, 13, 7, 3, 1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{4}$; $d^{\star}\omega^{1,3} = -\omega^{5}$; $d^{\star}\omega^{1,4} = -\omega^{6}$; $d^{\star}\omega^{1,5} = -\omega^{7}$; $d^{\star}\omega^{2,3} = -\omega^{6}$; $d^{\star}\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^{\star}\omega^{1,2,4} = -\omega^{2,6}$; $d^{\star}\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^{\star}\omega^{1,2,6} = -\omega^{4,6}$; $d^{\star}\omega^{1,2,7} = -\omega^{4,7}$; $d^{\star}\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^{\star}\omega^{1,3,5} = -\omega^{3,7}$; $d^{\star}\omega^{1,3,6} = -\omega^{5,6}$; $d^{\star}\omega^{1,3,7} = -\omega^{5,7}$; $d^{\star}\omega^{1,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^{\star}\omega^{1,4,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,5,6} = \omega^{6,7}$; $d^{\star}\omega^{2,3,4} = \omega^{4,6}$; $d^{\star}\omega^{2,3,5} = \omega^{5,6}$; $d^{\star}\omega^{2,3,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^{\star}\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{2,4,7} + \omega^{2,5,6}$; $d^{\star}\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{3,4,7} + \omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^{\star}\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,4,5,6} - \omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} - \omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 3\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} - \omega^{1,3,6} + 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6} + \omega^{2,3,7}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} - \omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{2,4,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} =$

$3\omega^{1,2,3,7}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5}=-\omega^{1,2,3,6}+2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7}=\omega^{1,2,4,7}-\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,6}=-\omega^{1,2,4,7}+2\omega^{1,2,5,6}-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7}=\omega^{1,2,5,7}-\omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5}=-\omega^{1,2,3,7}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,5,6}+\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7}=-\omega^{1,2,5,7}+2\omega^{1,3,4,7}-\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6}=-\omega^{1,3,4,7}+\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}-\omega^{1,4,5,7}-\omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6}=-\omega^{1,2,6,7}+2\omega^{1,4,5,6}+\omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=\omega^{1,5,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,5,6}-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,2,6,7}+2\omega^{2,3,4,7}-\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6}=\omega^{1,4,5,6}-\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+\omega^{1,4,5,7}+\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,4,6,7}+2\omega^{2,3,6,7}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6}=\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=-\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}-\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6}=-\omega^{1,2,3,4,7}+2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}-\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}})= 2e^{-5\beta_1}\ \langle\omega^{1,7},\omega^{3,5}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}-\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{3,4},\omega^{2,5}+\omega^{3,4}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{2,7}-\omega^{3,6}-\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-8\beta_1}\ \langle\omega^{1,5,7}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,6}+\omega^{2,3,4}\rangle+2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,2,7}-\omega^{1,4,5}+\omega^{2,3,5},\omega^{1,3,6}+\omega^{2,3,5}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6}+\omega^{2,4,5},\omega^{2,3,6}+\omega^{2,4,5}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,4,6}\rangle+2e^{-6\beta_1-\beta_2}\ \langle\omega^{1,4,7}-\omega^{2,3,7}-\omega^{3,4,5},\omega^{1,5,6}+\omega^{2,3,7}+2\omega^{3,4,5}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{2,4,7}+\omega^{2,5,6}+\omega^{3,4,6}\rangle+2e^{-7\beta_1-\beta_2}\ \langle\omega^{1,6,7}-\omega^{3,4,7}-\omega^{3,5,6},\omega^{2,5,7}+\omega^{3,4,7}+\omega^{3,5,6}\rangle+e^{-9\beta_1}\ \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-5\beta_1-3\beta_2}\ \langle-\omega^{2,3,4,6}\rangle+e^{-10\beta_1-\beta_2}\ \langle(-\omega^{1,5,6,7}-\omega^{3,4,5,7})/2\rangle+2e^{-8\beta_1-2\beta_2}\ \langle(\omega^{1,4,6,7}-2\omega^{2,3,6,7}-\omega^{2,4,5,7}+2\omega^{3,4,5,6})/5,(2\omega^{1,4,6,7}+\omega^{2,3,6,7}+3\omega^{2,4,5,7}-\omega^{3,4,5,6})/5\rangle+2e^{-9\beta_1-\beta_2}\ \langle(-\omega^{1,3,6,7}+\omega^{1,4,5,7}-2\omega^{2,3,5,7})/3,(-\omega^{1,3,6,7}-2\omega^{1,4,5,7}+\omega^{2,3,5,7})/3\rangle+e^{-10\beta_1}\ \langle\omega^{1,3,5,7}\rangle+2e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,4,5,6}+\omega^{2,3,4,7}+2\omega^{2,3,5,6})/3,(\omega^{1,2,6,7}+\omega^{2,3,4,7}+\omega^{2,3,5,6})/3\rangle+e^{-8\beta_1-\beta_2}\ \langle(-\omega^{1,2,5,7}-\omega^{1,3,4,7}-\omega^{1,3,5,6})/3\rangle+2e^{-6\beta_1-2\beta_2}\ \langle(-\omega^{1,2,4,7}-\omega^{1,2,5,6}+2\omega^{1,3,4,6}+3\omega^{2,3,4,5})/5,(\omega^{1,2,4,7}+\omega^{1,2,5,6}+3\omega^{1,3,4,6}+2\omega^{2,3,4,5})/5\rangle+e^{-4\beta_1-3\beta_2}\ \langle-\omega^{1,2,4,6}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= 2e^{-8\beta_1-3\beta_2} \left\langle -\omega^{2,3,4,5,6}, -\omega^{1,2,4,6,7}\right\rangle + e^{-11\beta_1-2\beta_2} \left\langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\right\rangle + e^{-12\beta_1-\beta_2} \left\langle -\omega^{1,3,5,6,7}\right\rangle + 2e^{-10\beta_1-2\beta_2} \left\langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3\right\rangle + e^{-9\beta_1-2\beta_2} \left\langle(-\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\right\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-12\beta_1-3\beta_2} \left\langle\omega^{2,3,4,5,6,7}\right\rangle + e^{-13\beta_1-2\beta_2} \left\langle -\omega^{1,3,4,5,6,7}\right\rangle + e^{-11\beta_1-3\beta_2} \left\langle\omega^{1,2,4,5,6,7}\right\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-13\beta_1-3\beta_2} \left\langle\omega^{1,2,3,4,5,6,7}\right\rangle.$

$P(Y) = (1 + 4Y^2 + Y^4)(1 + Y)^3.$

5.4.44 Algebra $\mathcal{G}_{7,2.40}$

Adjoint cohomology

$[x_1, x_2] = x_3$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_1^1 + \xi_2^2 & \xi_1^2 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^3 - \xi_1^4 & \xi_4^5 & 2\xi_1^1 + \xi_2^2 & 0 & 2\xi_1^2 \\ \xi_1^6 & \xi_2^6 & -\xi_1^3 & \xi_4^6 & \xi_1^2 & \xi_1^1 + 2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_2^4 & \xi_4^7 & 0 & 0 & 3\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_2^3 + \delta_4^7, ad\ x_2 = -\delta_1^3 + \delta_4^5, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = -(\delta_2^5 + \delta_1^7), ad\ x_5 = 0, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_1+\beta_2}\ \langle x_3\rangle + e^{2\beta_1}\ \langle x_4\rangle + e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{3\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_1+\beta_2}\ \langle \delta_1^5, \delta_2^6\rangle + 2e^{2\beta_1}\ \langle \delta_2^5, \delta_1^7\rangle + 3e^{\beta_1}\ \langle \delta_2^3, \delta_1^4, \delta_4^7\rangle + 2e^{\beta_2}\ \langle \delta_4^5, \delta_1^3\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_4^6\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_2, t_1\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_2^7\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_2^5, \delta_1^2] = -\delta_2^6 + \delta_1^5$; $[\delta_2^3, \delta_1^4] = \delta_2^5$; $[\delta_2^3, \delta_1^3] = \delta_2^6 + \delta_1^5$; $[\delta_2^3, \delta_1^2] = \delta_4^5 + \delta_1^3$; $[\delta_2^3, \delta_2^4] = -\delta_2^7$; $[\delta_1^4, \delta_4^5] = -\delta_1^5$; $[\delta_1^4, \delta_1^3] = -\delta_1^5$; $[\delta_1^4, \delta_4^6] = -\delta_1^6$; $[\delta_1^4, \delta_1^2] = -(\delta_4^5 + \delta_1^3)$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_4^5, \delta_1^2] = -\delta_4^6$; $[\delta_4^5, \delta_2^4] = \delta_2^5$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_1^3, \delta_1^2] = -\delta_4^6$; $[\delta_1^3, \delta_2^4] = -\delta_1^7$; $[\delta_4^6, \delta_2^4] = \delta_2^6$; $[\delta_1^2, \delta_1^7] = 2\delta_1^5$; $[\delta_1^2, \delta_2^7] = -\delta_1^7 + 2\delta_2^5$; $[\delta_1^2, \delta_2^4] = -\delta_4^7 - \delta_1^4 + \delta_2^3$; $[\delta_1^2, \delta_4^7] = 2\delta_4^5$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= 2e^{\beta_1}\ \langle \delta_1^4, \delta_2^3\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^5\rangle + e^{\beta_2}\ \langle \delta_1^3\rangle + e^{-\beta_1+2\beta_2}\ \langle \delta_4^6\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + 2e^0\ \langle t_2, t_1\rangle + e^{2\beta_1}\ \langle \delta_2^5\rangle + e^{3\beta_1-\beta_2}\ \langle \delta_2^7\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^3] = -\delta_1^5$; $[\delta_1^4, \delta_4^6] = -\delta_1^6$; $[\delta_1^4, \delta_1^2] = -2\delta_1^3$; $[\delta_1^4, \delta_2^3] = -\delta_2^5$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^3, \delta_1^2] = -\delta_4^6$; $[\delta_1^3, \delta_2^4] = \delta_2^5$; $[\delta_4^6, \delta_2^4] = -\delta_1^5$; $[\delta_1^2, \delta_2^5] = -2\delta_1^5$; $[\delta_1^2, \delta_2^7] = 3\delta_2^5$; $[\delta_1^2, \delta_2^4] = 2\delta_2^3 - \delta_1^4$; $[\delta_1^2, \delta_2^3] = -2\delta_1^3$; $[\delta_2^4, \delta_2^3] = \delta_2^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 5e^{-2\beta_1}\ \langle 2x_1\omega^{1,4} + 3x_2(\omega^{1,3} - 1/3\omega^{2,4}) + x_3\omega^{3,4}, x_1\omega^{1,4} + x_2\omega^{1,3} + x_3\omega^{2,7} + x_4\omega^{1,7}, x_2\omega^{1,3} + x_5\omega^{4,5}, x_1\omega^{1,4} + x_2(\omega^{1,3} - \omega^{2,4}) + x_6\omega^{4,6}, x_1\omega^{1,4} + x_3\omega^{2,7} + x_5\omega^{3,7}\rangle + 4e^{-\beta_1-\beta_2}\ \langle x_1(\omega^{1,3} - \omega^{2,4}) + x_3\omega^{2,5} + x_4(\omega^{1,5} +$

$\omega^{3,4}), -x_1\omega^{2,4} + x_5\omega^{4,6}, -x_1(\omega^{1,3} - 2\omega^{2,4}) + 2x_2\omega^{2,3} + x_3\omega^{2,5} + x_4(\omega^{1,5} + \omega^{2,7}) + x_7\omega^{3,7}, 2x_1\omega^{2,4} + x_2\omega^{2,3} + x_3\omega^{2,5} + x_5\omega^{3,5}\rangle + 2e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,4} + x_6\omega^{1,5}, x_5\omega^{1,7}\rangle + 4e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_3\omega^{2,4} + x_6\omega^{1,6}, -x_3\omega^{2,4} + x_5\omega^{1,5}, x_3\omega^{2,4} + x_4\omega^{1,4}, x_3\omega^{2,4} + x_5\omega^{2,7}\rangle + e^{-3\beta_1+2\beta_2}\ \langle x_6\omega^{1,7}\rangle + 5e^{-\beta_2}\ \langle -x_3\omega^{2,3} + x_5\omega^{1,6}, x_3\omega^{2,3} + x_4\omega^{1,3}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_4\omega^{2,4}, x_3\omega^{2,3} + x_5\omega^{2,5}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_7\omega^{2,7}\rangle + 2e^{-3\beta_1+\beta_2}\ \langle x_5\omega^{4,7}, -x_2\omega^{1,4} + x_6\omega^{4,5}\rangle + e^{-4\beta_1+2\beta_2}\ \langle x_6\omega^{4,7}\rangle + 2e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}, x_4\omega^{2,3} + x_7\omega^{2,5}\rangle + e^{2\beta_1-3\beta_2}\ \langle x_7\omega^{2,6}\rangle + 2e^{-2\beta_2}\ \langle -x_1\omega^{2,3} + x_3\omega^{2,6} + x_4(\omega^{1,6} + \omega^{2,5}) + x_7\omega^{3,5}, x_3\omega^{2,6} + x_5\omega^{3,6}\rangle + e^{\beta_1-3\beta_2}\ \langle x_4\omega^{2,6} + x_7\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-4\beta_2} + 3e^{-\beta_1-2\beta_2} + 4e^{-\beta_1-3\beta_2} + 3e^{-2\beta_1-\beta_2} + 6e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2} + 7e^{-3\beta_1-\beta_2} + 3e^{-3\beta_1-2\beta_2} + e^{-3\beta_1} + e^{-4\beta_1+\beta_2} + 4e^{-4\beta_1-\beta_2} + 4e^{-4\beta_1} + e^{-5\beta_1+2\beta_2} + e^{-5\beta_1+\beta_2} + 3e^{-5\beta_1} + e^{-6\beta_1+\beta_2} + 2e^{-3\beta_2} + e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-\beta_1-5\beta_2} + 4e^{-2\beta_1-4\beta_2} + 6e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-4\beta_2} + 6e^{-4\beta_1-2\beta_2} + 5e^{-4\beta_1-3\beta_2} + 4e^{-5\beta_1-\beta_2} + 6e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + 4e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2} + 2e^{-6\beta_1} + e^{-7\beta_1+\beta_2} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-3\beta_1-5\beta_2} + e^{-4\beta_1-4\beta_2} + 2e^{-4\beta_1-5\beta_2} + e^{-5\beta_1-3\beta_2} + 5e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-2\beta_2} + 4e^{-6\beta_1-3\beta_2} + 3e^{-6\beta_1-4\beta_2} + 3e^{-7\beta_1-2\beta_2} + 3e^{-7\beta_1-3\beta_2} + e^{-8\beta_1-\beta_2} + 2e^{-8\beta_1-2\beta_2} + e^{-9\beta_1-\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-5\beta_1-5\beta_2} + 2e^{-6\beta_1-5\beta_2} + 2e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2} + e^{-8\beta_1-3\beta_2} + 2e^{-8\beta_1-4\beta_2} + e^{-8\beta_1-5\beta_2} + e^{-9\beta_1-2\beta_2} + e^{-9\beta_1-4\beta_2} + e^{-10\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-8\beta_1-5\beta_2} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 16, 63, 132, 159, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 12, 30, 48, 46, 29, 13, 3)$

S=$5(2\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = -\omega^{1,4}$; $d\omega^{1,5} = \omega^{1,2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5} - \omega^{2,3,4}$; $d\omega^{3,6} = -\omega^{1,2,6}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,4}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,4,6}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{1,4,5} - \omega^{2,4,7}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{2,3,7}$; $d\omega^{1,3,5} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5} + \omega^{1,2,4,6}$; $d\omega^{1,5,7} = \omega^{1,2,4,7}$; $d\omega^{1,6,7} = \omega^{1,2,3,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} + \omega^{1,2,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} - \omega^{2,3,4,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,5} - \omega^{2,3,4,7}$;

$d\omega^{3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,3,4,7}$; $d\omega^{4,6,7} = \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7} - \omega^{2,4,6,7}$; $d\omega^{1,3,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,6,7}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6} - \omega^{2,3,4,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_1}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1}\ \langle\omega^{1,7}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + 2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5} + \omega^{3,4}, \omega^{2,7} - \omega^{3,4}\rangle + e^{-5\beta_1}\ \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}, \omega^{2,3,5}\rangle + 2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,7}, \omega^{1,4,5}\rangle + e^{-6\beta_1}\ \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}\rangle + 2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5} - \omega^{2,4,5}, \omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,5}\rangle + e^{-4\beta_1-3\beta_2}\ \langle\omega^{2,6,7} + \omega^{3,4,6}\rangle + e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,7} + \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,2,5,7}\rangle + 2e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6} - \omega^{2,4,5,6}, \omega^{2,3,6,7} - \omega^{2,4,5,6}\rangle + 2e^{-7\beta_1-2\beta_2}\ \langle\omega^{1,3,5,7} - \omega^{2,4,5,7}, \omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle + 2e^{-6\beta_1-3\beta_2}\ \langle\omega^{1,3,6,7}, \omega^{1,4,5,6} - \omega^{2,3,5,7}\rangle + e^{-8\beta_1-\beta_2}\ \langle\omega^{1,4,5,7}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,2,6,7}\rangle + e^{-4\beta_1-4\beta_2}\ \langle\omega^{1,2,5,6}\rangle + e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle + 2e^{-7\beta_1-4\beta_2}\ \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,5,6}\rangle + e^{-9\beta_1-2\beta_2}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-6\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-8\beta_1-3\beta_2}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-5\beta_2}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 44.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^5$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,4} = \omega^{1,5} - \omega^{2,7} - \omega^{3,4}$;

$d^\star\omega^{1,2,5} = -\omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,5} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,6} = -\omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{5,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,5}-\omega^{1,4,6}-\omega^{2,3,7}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = \omega^{1,5,6} + \omega^{2,6,7} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,5,7} - \omega^{3,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = \omega^{3,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,5,7}+\omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,3,5,6}+\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,5,7}-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,5,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{1,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^5 = 2\omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,7} - \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = -\omega^{1,5} + \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,4} = -\omega^{1,5} + \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = 2\omega^{3,5} - \omega^{4,6}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,5} + \omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}+\omega^{2,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,7} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,7}+\omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}-\omega^{2,3,5}+\omega^{2,4,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}-\omega^{1,4,5}+\omega^{2,4,7}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,7} + \omega^{1,4,5} - \omega^{2,4,7}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,5} + 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{2,6,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{3,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $\Delta\omega^{2,3,4} = \omega^{1,2,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5} - \omega^{2,4,6}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,5} + 2\omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = \omega^{1,3,5} - \omega^{1,4,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} - \omega^{2,3,5} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} - \omega^{1,4,5} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + 2\omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = \omega^{1,5,6} + \omega^{2,6,7} - \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,5,6} - \omega^{2,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{1,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 3\omega^{3,5,7} - \omega^{4,6,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = -\omega^{3,5,7} + \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}-\omega^{1,2,4,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,5} + 3\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} =$

$2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7}=\omega^{1,2,5,7}-\omega^{1,3,4,5}+\omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}-\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5}=-\omega^{1,2,5,7}+\omega^{1,3,4,5}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,6,7}+2\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7}=\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6}=\omega^{1,3,5,6}+\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7}=\omega^{1,3,5,7}-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,3,6,7}+2\omega^{1,4,5,6}+\omega^{2,4,6,7}$; $\Delta\omega^{1,4,6,7}=-\omega^{1,3,5,7}+\omega^{1,4,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}-\omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7}=\omega^{1,2,5,7}-\omega^{1,3,4,5}+2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+2\omega^{2,3,5,7}-\omega^{2,4,6,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,3,5,6}+\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6}=\omega^{1,3,5,6}+\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{1,3,5,7}-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,3,5,7}+\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,6}=\omega^{2,5,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=-\omega^{1,5,6,7}+\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7}=\omega^{1,2,3,5,7}-\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,6,7}+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=-\omega^{1,2,3,5,7}+2\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}+\omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}+\omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=2\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-2\beta_1}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1}\ \langle\omega^{1,7}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,3}-\omega^{2,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle+2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}+\omega^{3,4},\omega^{2,7}-\omega^{3,4}\rangle+e^{-5\beta_1}\ \langle\omega^{4,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-6\beta_1}\ \langle\omega^{1,4,7}\rangle+e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5}-\omega^{2,4,5},\omega^{1,4,6}+\omega^{2,3,7}+2\omega^{2,4,5}\rangle+2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}-\omega^{2,4,6},\omega^{2,3,5}+\omega^{2,4,6}\rangle+2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,3,7}-\omega^{2,4,7},\omega^{1,4,5}+\omega^{2,4,7}\rangle+e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,6,7}+\omega^{2,5,7}+\omega^{3,4,5}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{2,6,7}+\omega^{3,4,6}\rangle+e^{-6\beta_1-\beta_2}\ \langle\omega^{1,5,7}+\omega^{3,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,5,6}\rangle+e^{-8\beta_1-\beta_2}\ \langle-\omega^{1,4,5,7}\rangle+2e^{-6\beta_1-3\beta_2}\ \langle(\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,3,5,7}-3\omega^{2,4,6,7})/4,(-\omega^{1,3,6,7}+\omega^{1,4,5,6}-\omega^{2,3,5,7}-\omega^{2,4,6,7})/4\rangle+2e^{-7\beta_1-2\beta_2}\ \langle(-\omega^{1,3,5,7}+\omega^{1,4,6,7}+2\omega^{2,4,5,7})/3,(\omega^{1,3,5,7}+2\omega^{1,4,6,7}+\omega^{2,4,5,7})/3\rangle+2e^{-5\beta_1-4\beta_2}\ \langle(\omega^{1,3,5,6}+\omega^{2,3,6,7}-2\omega^{2,4,5,6})/3,(-\omega^{1,3,5,6}+2\omega^{2,3,6,7}-\omega^{2,4,5,6})/3\rangle+e^{-5\beta_1-3\beta_2}\ \langle(\omega^{1,2,6,7}+\omega^{1,3,4,6}+\omega^{2,3,4,5})/3\rangle+e^{-6\beta_1-2\beta_2}\ \langle(-\omega^{1,2,5,7}-\omega^{1,3,4,5})/2\rangle+e^{-4\beta_1-4\beta_2}\ \langle(\omega^{1,2,5,6}-\omega^{2,3,4,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-6\beta_1-5\beta_2}\ \langle-\omega^{2,3,4,5,6}\rangle+e^{-8\beta_1-4\beta_2}\ \langle(\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7})/2\rangle+e^{-8\beta_1-3\beta_2}\ \langle(\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/2\rangle+e^{-9\beta_1-2\beta_2}\ \langle-\omega^{1,3,4,5,7}\rangle+2e^{-7\beta_1-4\beta_2}\ \langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}-2\omega^{2,3,4,6,7})/3,(-\omega^{1,2,5,6,7}+2\omega^{1,3,4,5,6}-$

$\omega^{2,3,4,6,7})/3\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-5\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2}\ \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-5\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+5Y^2+6Y^3+5Y^4+2Y^5+Y^6)(1+Y)$.

5.4.45 Algebra $\mathcal{G}_{7,2.41}$

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_2^2+\xi_1^1 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^3 & 0 & \xi_2^2+2\xi_1^1 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 0 & 2\xi_2^2+\xi_1^1 & 0\\ \xi_1^7 & \xi_2^7 & -\xi_1^5+\xi_2^6 & \xi_4^7 & -\xi_1^3+\xi_2^4 & \xi_2^3 & 2(\xi_2^2+\xi_1^1) \end{pmatrix}$$

$ad\ x_1=\delta_2^3, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=-\delta_1^6, ad\ x_5=-\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{2\beta_2}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+2\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+2\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1}\ \langle\delta_2^5,\delta_4^7\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+2\beta_2}\ \langle\delta_1^7\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_2^7\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+2e^{\beta_2}\ \langle\delta_1^3,\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_1^4,\delta_2^3]=-\delta_1^6$; $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_2^5,\delta_2^4]=-\delta_2^7$; $[\delta_1^6,\delta_2^3]=-\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_1^5,\delta_2^3]=-\delta_2^7$; $[\delta_2^6,\delta_1^3]=\delta_1^7$; $[\delta_4^7,\delta_2^4]=\delta_2^7$; $[\delta_1^3,\delta_2^4]=-\delta_1^6$; $[\delta_1^3,\delta_2^3]=-(\delta_2^6+\delta_1^5)$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+2\beta_2}\ \langle\delta_1^4\rangle+2e^{2\beta_1}\ \langle\delta_2^5,\delta_4^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_2^4\rangle+2e^0\ \langle t_1,t_2\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-3\beta_1+\beta_2}\ \langle x_4\omega^{1,5}\rangle+3e^{-\beta_2}\ \langle x_3\omega^{1,4}+x_4\omega^{2,4}, x_3\omega^{2,3}+x_5\omega^{2,5}, -x_1\omega^{1,2}+x_3(\omega^{1,4}-2\omega^{2,3})+x_5\omega^{1,6}\rangle+e^{-3\beta_1}\ \langle -x_2\omega^{1,5}+x_4(\omega^{1,7}+\omega^{3,5})\rangle+e^{2\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle+e^{\beta_1-2\beta_2}\ \langle x_3\omega^{2,4}+x_5\omega^{2,6}\rangle+3e^{-2\beta_2}\ \langle x_2\omega^{2,4}+2x_3(\omega^{2,6}-1/2\omega^{3,4})+2x_5(\omega^{2,7}+1/2\omega^{4,5}), x_1\omega^{2,3}-x_3(\omega^{2,6}-\omega^{3,4})-2x_5(\omega^{2,7}-1/2\omega^{3,6}), -x_1\omega^{1,4}-x_2\omega^{2,4}-3x_3(\omega^{2,6}-1/3\omega^{3,4})-3x_5\omega^{2,7}+x_7\omega^{4,7}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,3}\rangle+e^{-2\beta_1}\ \langle x_3\omega^{1,5}+x_4(\omega^{1,6}+\omega^{2,5})\rangle+2e^{-\beta_1-\beta_2}\ \langle x_2\omega^{2,3}+x_3\omega^{2,5}+x_5\omega^{3,5}, -2x_1\omega^{1,3}-x_2\omega^{2,3}+4x_3(\omega^{1,6}+1/4\omega^{2,5})+3x_4(\omega^{2,6}-\omega^{3,4})-x_5\omega^{1,7}-x_6\omega^{3,6}+x_7\omega^{3,7}\rangle+3e^{-\beta_1}\ \langle x_3\omega^{1,3}+x_4\omega^{2,3}, x_4\omega^{1,4}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_5\omega^{1,5}\rangle+e^{\beta_1-\beta_2}\ \langle -x_5\omega^{1,4}+x_7\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-4\beta_2} + 2e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-4\beta_2} + 2e^{-2\beta_1-2\beta_2} + 4e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2} + 3e^{-3\beta_1-\beta_2} + 4e^{-3\beta_1-2\beta_2} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + e^{-5\beta_1-\beta_2} + 2e^{-3\beta_2} + e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-5\beta_2} + 2e^{-\beta_1-6\beta_2} + 3e^{-2\beta_1-4\beta_2} + 3e^{-2\beta_1-5\beta_2} + 2e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-4\beta_2} + 2e^{-3\beta_1-5\beta_2} + e^{-3\beta_1-6\beta_2} + e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1-3\beta_2} + 3e^{-4\beta_1-4\beta_2} + e^{-4\beta_1-5\beta_2} + e^{-5\beta_1-2\beta_2} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-6\beta_2} + 2e^{-3\beta_1-7\beta_2} + e^{-3\beta_1-8\beta_2} + 2e^{-4\beta_1-5\beta_2} + 4e^{-4\beta_1-6\beta_2} + e^{-4\beta_1-7\beta_2} + 2e^{-5\beta_1-4\beta_2} + 3e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2} + 2e^{-6\beta_1-4\beta_2} + 2e^{-6\beta_1-5\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-7\beta_1-4\beta_2} + e^{-7\beta_1-5\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-4\beta_1-7\beta_2} + e^{-4\beta_1-8\beta_2} + e^{-5\beta_1-6\beta_2} + e^{-5\beta_1-9\beta_2} + e^{-6\beta_1-6\beta_2} + 2e^{-6\beta_1-7\beta_2} + e^{-6\beta_1-8\beta_2} + e^{-7\beta_1-5\beta_2} + 2e^{-7\beta_1-6\beta_2} + e^{-7\beta_1-7\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-6\beta_1-9\beta_2} + e^{-7\beta_1-7\beta_2} + e^{-7\beta_1-8\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 13, 54, 122, 155, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 7, 18, 29, 32, 24, 12, 3)$

S=$7\beta_1 + 9\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4} - \omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6} - \omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6}$; $d\omega^{6,7} = -\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} - \omega^{1,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$;

$d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-2\beta_2} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,4}\rangle + e^{-3\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,7} + \omega^{3,5}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= 2e^{-3\beta_1-3\beta_2} \langle\omega^{1,2,7}, \omega^{1,3,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{1,5,7}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-\beta_1-5\beta_2} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,4,7} + \omega^{3,4,5}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{2,4,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-4\beta_2} \langle\omega^{1,2,5,7}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,2,6,7}\rangle + 2e^{-4\beta_1-6\beta_2} \langle\omega^{1,4,6,7} + \omega^{2,4,5,7} - 2\omega^{3,4,5,6}, \omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{2,3,4,6}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,3,6,7} - \omega^{2,3,5,7}\rangle + e^{-3\beta_1-7\beta_2} \langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,3,6,7}\rangle + e^{-6\beta_1-6\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-7\beta_1-6\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-7\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-8\beta_2} \langle\omega^{2,3,4,6,7}\rangle + e^{-6\beta_1-7\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_1-7\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-7\beta_1-8\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-9\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-7\beta_1-9\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 19, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 8, 8, 6, 3, 1)$

Total dimension of trivial cohomology: 36.

Length associated polynomial: 36.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^3$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,6} - \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{3,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{1,2,7} = -\omega^{3,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7} + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{1,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} - \omega^{3,4,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7} - \omega^{3,4,7}$;

$d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{3,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^3 = \omega^3$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,6} = 2\omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7} - \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{2,6} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = 3\omega^{3,7}$; $\Delta\omega^{4,5} = -\omega^{3,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} + \omega^{1,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,2,6} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6} + \omega^{2,3,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{3,5,6}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,2,7} + \omega^{1,4,5} + 2\omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,4,7} + 2\omega^{2,3,7}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = 3\omega^{2,5,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,4,7} - \omega^{2,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,6,7} + 3\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 3\omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,6} = 3\omega^{1,2,5,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,6,7} + 3\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,6} = 2\omega^{1,4,5,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,6,7} = 2\omega^{1,4,6,7} + \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,4,7} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7}$;

$\Delta\omega^{2,3,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,4,5,7} + 2\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,4,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = 3\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + 2\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,4,5,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,4,5,6,7} = \omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-2\beta_2}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{1,4} - \omega^{2,3}\rangle + e^{-3\beta_2}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6} - \omega^{3,4}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,7} + \omega^{3,5}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-3\beta_2}\ \langle\omega^{1,5,7}\rangle + 2e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,2,7} + \omega^{1,4,5} - \omega^{2,3,5}, \omega^{1,3,6} + 2\omega^{1,4,5} - \omega^{2,3,5}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{1,4,6} - 2\omega^{2,3,6} - \omega^{2,4,5}\rangle + e^{-\beta_1-5\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-4\beta_2}\ \langle 2\omega^{1,4,7} - \omega^{2,3,7} + \omega^{2,5,6} + 3\omega^{3,4,5}\rangle + e^{-2\beta_1-5\beta_2}\ \langle\omega^{2,4,7} - \omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-3\beta_1-7\beta_2}\ \langle -\omega^{2,4,6,7}\rangle + e^{-2\beta_1-6\beta_2}\ \langle -\omega^{2,3,4,6}\rangle + 2e^{-4\beta_1-6\beta_2}\ \langle(-\omega^{1,4,6,7} - \omega^{2,4,5,7} + 2\omega^{3,4,5,6})/3, (\omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6})/3\rangle + e^{-5\beta_1-5\beta_2}\ \langle(\omega^{1,3,6,7} + 2\omega^{1,4,5,7} - \omega^{2,3,5,7})/6\rangle + e^{-6\beta_1-4\beta_2}\ \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-5\beta_2}\ \langle(3\omega^{1,2,6,7} - \omega^{1,3,4,7} - \omega^{1,4,5,6} + 2\omega^{2,3,5,6})/15\rangle + e^{-5\beta_1-4\beta_2}\ \langle(\omega^{1,2,5,7} - \omega^{1,3,5,6})/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-4\beta_1-8\beta_2}\ \langle -\omega^{2,3,4,6,7}\rangle + e^{-6\beta_1-7\beta_2}\ \langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle + e^{-7\beta_1-6\beta_2}\ \langle -\omega^{1,3,5,6,7}\rangle + e^{-6\beta_1-6\beta_2}\ \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-4\beta_1-7\beta_2}\ \langle(-\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-5\beta_1-6\beta_2}\ \langle(-\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-6\beta_1-9\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-7\beta_1-8\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-7\beta_2}\ \langle -\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-7\beta_1-9\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + Y^2)^2(1 + Y^2)(1 + Y)$.

5.4.46 Algebra $\mathcal{G}_{7,2.42}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\xi_1^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 3\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3+\xi_2^5 & \xi_3^5 & 2\xi_1^1+\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4 & 0 & 0 & 5\xi_1^1 \end{pmatrix}$$

$ad\ x_1 = \delta_3^5 + \delta_2^4, ad\ x_2 = \delta_3^6 - \delta_1^4, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = -\delta_2^7, ad\ x_5 = -\delta_1^6, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{2\beta_1}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{3\beta_1}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{2\beta_1+\beta_3}\ \langle x_6\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_3}\ \langle x_6\rangle + e^{5\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{\beta_1}\ \langle \delta_2^4, \delta_3^5\rangle + 2e^{\beta_3}\ \langle \delta_1^5, \delta_2^6\rangle + e^{\beta_1+\beta_3}\ \langle \delta_1^6\rangle + 2e^{2\beta_1}\ \langle \delta_3^6, \delta_1^4\rangle + 2e^{-\beta_1+\beta_3}\ \langle \delta_1^3, \delta_2^5\rangle + 2e^{0}\ \langle t_3, t_1\rangle + e^{4\beta_1}\ \langle \delta_1^7\rangle + e^{3\beta_1}\ \langle \delta_2^7\rangle + e^{5\beta_1-\beta_3}\ \langle \delta_3^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_2^4, \delta_2^5] = -\delta_2^6$; $[\delta_2^4, \delta_1^4] = \delta_2^7$; $[\delta_1^5, \delta_3^5] = -\delta_1^6$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$; $[\delta_2^5, \delta_3^5] = -\delta_2^6$; $[\delta_2^5, \delta_1^4] = \delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_3}\ \langle \delta_1^5\rangle + 2e^{-\beta_1+\beta_3}\ \langle \delta_1^3, \delta_2^5\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + 2e^{0}\ \langle t_3, t_1\rangle + e^{4\beta_1}\ \langle \delta_1^7\rangle + e^{5\beta_1-\beta_3}\ \langle \delta_3^7\rangle + e^{2\beta_1}\ \langle \delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_2^4] = \delta_1^5$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_2^5, \delta_2^4] = -\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 3e^{-\beta_3}\ \langle x_2\omega^{2,3} + 2x_4(\omega^{2,5} + 1/2\omega^{3,4}), -x_2\omega^{2,3} - x_4\omega^{2,5} + x_7\omega^{3,7}, -x_1\omega^{1,3} - x_4\omega^{2,5} + x_6\omega^{3,6}\rangle + 2e^{-\beta_1-\beta_3}\ \langle 2x_1(\omega^{1,5} - 3/2\omega^{2,3}) - x_2(\omega^{2,5} + \omega^{3,4}) - 2x_3\omega^{3,5} + x_4\omega^{4,5}, x_1(\omega^{1,5} - \omega^{2,3}) - x_3\omega^{3,5} + x_4\omega^{2,6}\rangle + e^{-3\beta_1+\beta_3}\ \langle x_3\omega^{1,2}\rangle + 5e^{-2\beta_1}\ \langle -x_1\omega^{1,2} + 2x_3\omega^{1,5} + x_5\omega^{2,5}, x_1\omega^{1,2} - x_3(\omega^{1,5} - \omega^{2,3}), x_1\omega^{1,2} - x_3\omega^{1,5} + x_5\omega^{1,6}, x_1\omega^{1,2} - x_2\omega^{1,4} - x_3\omega^{1,5} + x_5\omega^{3,4}, x_4\omega^{2,4}\rangle + 2e^{-3\beta_1}\ \langle x_3(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{2,6}, -x_1\omega^{1,4} + x_2\omega^{2,4} - x_3(\omega^{1,6} - \omega^{3,4}) + 3x_6(\omega^{3,7} + 1/3\omega^{4,6})\rangle + e^{-4\beta_1+\beta_3}\ \langle x_3\omega^{1,4} + x_5\omega^{2,4}\rangle + 3e^{-\beta_1}\ \langle x_3\omega^{1,3} + x_5\omega^{2,3}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2} - x_3\omega^{1,3} + x_7\omega^{1,7}\rangle + e^{-5\beta_1+\beta_3}\ \langle -x_3\omega^{2,4} + x_5\omega^{1,7}\rangle + e^{-6\beta_1+\beta_3}\ \langle x_5\omega^{2,7} +$

$x_6\omega^{4,7}\rangle + e^{-4\beta_1} \ \langle x_4\omega^{2,7}\rangle + e^{2\beta_1-\beta_3} \ \langle x_4\omega^{1,3} + x_7\omega^{1,6}\rangle + e^{4\beta_1-2\beta_3} \ \langle x_7\omega^{3,5}\rangle + e^{\beta_1-\beta_3} \ \langle x_4\omega^{2,3}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{2\beta_1-3\beta_3}+e^{\beta_1-2\beta_3}+3e^{-\beta_1-2\beta_3}+3e^{-2\beta_1-\beta_3}+2e^{-2\beta_1-2\beta_3}+6e^{-3\beta_1-\beta_3}+4e^{-4\beta_1-\beta_3}+3e^{-4\beta_1}+e^{-5\beta_1-\beta_3}+3e^{-5\beta_1}+e^{-6\beta_1-\beta_3}+2e^{-6\beta_1}+4e^{-7\beta_1}+2e^{-8\beta_1+\beta_3}+e^{-9\beta_1+\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{\beta_1-3\beta_3} + e^{-3\beta_1-2\beta_3} + 2e^{-3\beta_1-3\beta_3} + 5e^{-4\beta_1-2\beta_3} + e^{-4\beta_1-3\beta_3} + 2e^{-5\beta_1-\beta_3} + 3e^{-5\beta_1-2\beta_3} + 3e^{-6\beta_1-\beta_3} + e^{-6\beta_1-2\beta_3} + 4e^{-7\beta_1-\beta_3} + 2e^{-7\beta_1-2\beta_3} + 4e^{-8\beta_1-\beta_3} + 2e^{-9\beta_1-\beta_3} + 3e^{-9\beta_1} + 4e^{-10\beta_1} + e^{-11\beta_1+\beta_3} + e^{-11\beta_1}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-4\beta_1-3\beta_3} + 2e^{-5\beta_1-3\beta_3} + e^{-6\beta_1-2\beta_3} + e^{-6\beta_1-3\beta_3} + 2e^{-7\beta_1-2\beta_3} + e^{-7\beta_1-3\beta_3} + 2e^{-8\beta_1-2\beta_3} + 3e^{-9\beta_1-2\beta_3} + e^{-9\beta_1-3\beta_3} + 3e^{-10\beta_1-\beta_3} + 3e^{-10\beta_1-2\beta_3} + 4e^{-11\beta_1-\beta_3} + e^{-11\beta_1} + e^{-12\beta_1-\beta_3} + e^{-12\beta_1} + e^{-13\beta_1}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-7\beta_1-3\beta_3} + 2e^{-10\beta_1-3\beta_3} + 2e^{-11\beta_1-2\beta_3} + e^{-11\beta_1-3\beta_3} + 2e^{-12\beta_1-2\beta_3} + e^{-12\beta_1-3\beta_3} + e^{-13\beta_1-\beta_3} + e^{-13\beta_1-2\beta_3} + e^{-14\beta_1-\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-12\beta_1-3\beta_3} + e^{-13\beta_1-3\beta_3} + e^{-14\beta_1-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 58, 126, 159, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 23, 37, 40, 28, 12, 3)$

S=$14\beta_1 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,3}$; $d\omega^7 = -\omega^{2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,3,7} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7}$; $d\omega^{3,5,7} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$;

$d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch$(H^1_{\mathfrak{C}})$= $e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

ch$(H^2_{\mathfrak{C}})$= $e^{-4\beta_1} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-7\beta_1} \langle\omega^{2,7}\rangle + 2e^{-3\beta_1-\beta_3} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle$.

ch$(H^3_{\mathfrak{C}})$= $e^{-5\beta_1-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-9\beta_1} \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-6\beta_1-\beta_3} \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-7\beta_1-\beta_3} \langle\omega^{1,5,7} - \omega^{2,4,6}\rangle + e^{-10\beta_1} \langle\omega^{2,4,7}\rangle + 2e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,6} + \omega^{3,4,5}, \omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-8\beta_1-\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,6}\rangle$.

ch$(H^4_{\mathfrak{C}})$= $e^{-11\beta_1} \langle\omega^{1,2,4,7}\rangle + 2e^{-10\beta_1-\beta_3} \langle\omega^{1,2,6,7}, \omega^{1,4,5,7}\rangle + e^{-6\beta_1-2\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-7\beta_1-2\beta_3} \langle\omega^{1,4,5,6} + \omega^{2,3,4,6}\rangle + e^{-5\beta_1-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-8\beta_1-2\beta_3} \langle\omega^{1,3,6,7} + \omega^{2,4,5,6}\rangle + e^{-11\beta_1-\beta_3} \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle + e^{-9\beta_1-2\beta_3} \langle\omega^{2,3,6,7} + \omega^{3,4,5,7}\rangle$.

ch$(H^5_{\mathfrak{C}})$= $e^{-13\beta_1-\beta_3} \langle\omega^{1,2,4,6,7}\rangle + 2e^{-11\beta_1-2\beta_3} \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-3\beta_3} \langle\omega^{1,3,4,5,6}\rangle + e^{-12\beta_1-2\beta_3} \langle\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-10\beta_1-3\beta_3} \langle\omega^{2,3,5,6,7}\rangle$.

ch$(H^6_{\mathfrak{C}})$= $e^{-14\beta_1-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-13\beta_1-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

ch$(H^7_{\mathfrak{C}})$= $e^{-14\beta_1-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,7}$; $d^\star\omega^{1,2,5} = -\omega^{2,6} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} =$

$\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{1,5} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 3\omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,6} + \omega^{1,4,5} - \omega^{2,3,4}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,3,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} - \omega^{1,4,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,5,6} + \omega^{2,3,6} - \omega^{3,4,5}$; $\Delta\omega^{2,3,7} = \omega^{1,5,7} + \omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{2,3,7} + 2\omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} + \omega^{4,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{3,5,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{2,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,2,6,7} + \omega^{1,4,5,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} =$

$\omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,4,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{1,5,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 3\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,4,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-2\beta_1} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-4\beta_1} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5} - \omega^{2,3}\rangle + e^{-7\beta_1} \langle\omega^{2,7}\rangle + 2e^{-3\beta_1-\beta_3} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-5\beta_1-\beta_3} \langle\omega^{1,2,6} - \omega^{1,4,5}\rangle + e^{-9\beta_1} \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-2\beta_3} \langle\omega^{1,3,6} + \omega^{2,3,5}\rangle + e^{-6\beta_1-\beta_3} \langle\omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-7\beta_1-\beta_3} \langle\omega^{1,5,7} - 2\omega^{2,3,7} - \omega^{2,4,6}\rangle + e^{-10\beta_1} \langle\omega^{2,4,7}\rangle + 2e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,6} + \omega^{3,4,5}, \omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-8\beta_1-\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-9\beta_1-2\beta_3} \langle(-\omega^{2,3,6,7} - \omega^{3,4,5,7})/2\rangle + e^{-5\beta_1-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-11\beta_1-\beta_3} \langle(\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-8\beta_1-2\beta_3} \langle(-\omega^{1,3,6,7} - 2\omega^{2,3,5,7} - \omega^{2,4,5,6})/6\rangle + e^{-7\beta_1-2\beta_3} \langle(-\omega^{1,3,5,7} - 2\omega^{1,4,5,6} - \omega^{2,3,4,6})/6\rangle + e^{-4\beta_1-3\beta_3} \langle-\omega^{1,3,5,6}\rangle + 2e^{-10\beta_1-\beta_3} \langle(\omega^{1,2,6,7} + \omega^{1,4,5,7} + 2\omega^{2,3,4,7})/3, (\omega^{1,2,6,7} - 2\omega^{1,4,5,7} - \omega^{2,3,4,7})/3\rangle + e^{-6\beta_1-2\beta_3} \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6})/2\rangle + e^{-11\beta_1} \langle\omega^{1,2,4,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-10\beta_1-3\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-12\beta_1-2\beta_3} \langle(-\omega^{1,4,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-7\beta_1-3\beta_3} \langle\omega^{1,3,4,5,6}\rangle + 2e^{-11\beta_1-2\beta_3} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3\rangle + e^{-13\beta_1-\beta_3} \langle-\omega^{1,2,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-13\beta_1-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-12\beta_1-3\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-14\beta_1-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-14\beta_1-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

5.4.47 Algebra $\mathcal{G}_{7,2.43}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_7$; $[x_1, x_5] = x_6$; $[x_3, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & (\xi_1^1+\xi_2^2)/2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & \xi_3^2 & 0 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & (3\xi_1^1+\xi_2^2)/2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & \xi_3^5 & (5\xi_1^1+\xi_2^2)/2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^4 & \xi_3^4-\xi_1^5 & \xi_3^2+\xi_1^3 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_1^7, ad\ x_5 = -(\delta_1^6 + \delta_3^7), ad\ x_6 = 0, ad\ x_7 = 0$.

Maximal torus : $T=\langle t_1, t_2\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{(\beta_1+\beta_2)/2}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{(3\beta_1+\beta_2)/2}\ \langle x_5\rangle + e^{(5\beta_1+\beta_2)/2}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_3}\ \langle x_6\rangle + e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + 2e^{(3\beta_1+\beta_2)/2}\ \langle\delta_1^6, \delta_3^7\rangle + e^{(5\beta_1-\beta_2)/2}\ \langle\delta_2^6\rangle + 2e^{2\beta_1}\ \langle\delta_3^6, \delta_2^7\rangle + 2e^{\beta_1}\ \langle\delta_3^5, \delta_2^4\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^7\rangle + 2e^{(\beta_1+\beta_2)/2}\ \langle\delta_3^4, \delta_1^5\rangle + 2e^{(-\beta_1+\beta_2)/2}\ \langle\delta_3^2, \delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^4, \delta_2^4] = -\delta_1^7$; $[\delta_1^6, \delta_3^2] = -\delta_1^7$; $[\delta_1^6, \delta_1^3] = -\delta_1^7$; $[\delta_2^6, \delta_3^2] = -\delta_2^7 + \delta_3^6$; $[\delta_2^6, \delta_1^3] = -\delta_2^7$; $[\delta_3^6, \delta_3^2] = -\delta_3^7$; $[\delta_3^6, \delta_1^3] = -\delta_3^7 + \delta_1^6$; $[\delta_3^5, \delta_3^4] = -\delta_3^7$; $[\delta_3^5, \delta_1^5] = \delta_3^7 + \delta_1^6$; $[\delta_3^5, \delta_3^2] = -\delta_3^4$; $[\delta_3^5, \delta_1^3] = \delta_1^5$; $[\delta_2^7, \delta_3^2] = \delta_3^7$; $[\delta_3^7, \delta_1^3] = \delta_1^7$; $[\delta_2^4, \delta_3^4] = \delta_3^7$; $[\delta_2^4, \delta_3^2] = \delta_3^4$; $[\delta_3^4, \delta_1^5] = \delta_1^7$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_1^5, \delta_3^2] = -\delta_1^4$; $[\delta_3^2, \delta_1^3] = \delta_1^2$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{(5\beta_1-\beta_2)/2}\ \langle\delta_2^6\rangle + 2e^{2\beta_1}\ \langle\delta_3^6, \delta_2^7\rangle + e^{(3\beta_1+\beta_2)/2}\ \langle\delta_1^6\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle + e^{(\beta_1+\beta_2)/2}\ \langle\delta_3^4\rangle + 2e^{(-\beta_1+\beta_2)/2}\ \langle\delta_3^2, \delta_1^3\rangle + 2e^0\ \langle t_1, t_2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_1^3, \delta_2^6] = \delta_2^7$; $[\delta_1^3, \delta_3^6] = -2\delta_1^6$; $[\delta_3^4, \delta_2^4] = \delta_1^6$; $[\delta_3^2, \delta_2^6] = -\delta_3^6 + \delta_2^7$; $[\delta_3^2, \delta_3^6] = -\delta_1^6$; $[\delta_3^2, \delta_2^7] = \delta_1^6$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-7/2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,6}\rangle + e^{-5/2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,5}\rangle + 3e^{-2\beta_1}\ \langle x_2\omega^{1,4}, x_3\omega^{1,5}, x_2\omega^{3,5} + x_4\omega^{3,6} + x_5\omega^{1,6}\rangle + 3e^{-3/2\beta_1-1/2\beta_2}\ \langle 2x_1\omega^{1,5} - x_2(\omega^{2,5}+\omega^{3,4}) + 3x_3(\omega^{1,4} - 2/3\omega^{3,5}) + x_4\omega^{4,5}, x_3\omega^{3,5} + x_5\omega^{3,6} + x_6\omega^{5,6}, -x_1\omega^{1,5} + x_2(\omega^{2,5}+\omega^{3,4}) - x_3\omega^{1,4} + x_4\omega^{3,7} + x_5\omega^{1,7}\rangle + e^{-3\beta_1}\ \langle x_2(\omega^{1,7}+\omega^{3,6}) - x_3\omega^{1,6} +$

$x_4\omega^{5,6}\rangle + e^{2\beta_1-\beta_2}\ \langle x_6\omega^{2,3}\rangle + e^{3/2\beta_1-3/2\beta_2}\ \langle x_6\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle -2/3x_1\omega^{1,2} - 1/3x_3\omega^{2,3} - 2/3x_4\omega^{2,4} + 1/3x_5\omega^{2,5} + x_6\omega^{2,6}, 1/3x_1\omega^{1,2} - 1/3x_3\omega^{2,3} + 1/3x_4\omega^{2,4} + 1/3x_5(\omega^{2,5} + 3\omega^{3,4}) + x_6\omega^{3,7}, 1/3x_1\omega^{1,2} + 2/3x_3\omega^{2,3} + 4/3x_4\omega^{2,4} + 1/3x_5\omega^{2,5} + x_7\omega^{2,7}\rangle + 2e^{\beta_1-\beta_2}\ \langle -x_5\omega^{2,3} + x_6\omega^{3,4}, x_5\omega^{2,3} + x_6\omega^{2,5}\rangle + 5e^{-1/2\beta_1-1/2\beta_2}\ \langle -x_1\omega^{1,3} + 2x_2\omega^{2,3} - x_4\omega^{3,4} + x_6\omega^{3,6}, -x_3\omega^{1,2} + x_7\omega^{2,6}, x_2\omega^{2,3} + x_3\omega^{1,2} + x_4\omega^{2,5}, x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_4\omega^{3,4} + x_6\omega^{1,7}, -x_2\omega^{2,3} + x_4\omega^{3,4} + x_5\omega^{1,4}\rangle + e^{-3/2\beta_1+1/2\beta_2}\ \langle x_2\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2}, x_3\omega^{1,3} + x_4\omega^{3,5}, x_6\omega^{1,6}\rangle + e^{1/2\beta_1-1/2\beta_2}\ \langle -x_4\omega^{2,3} + x_5\omega^{1,2}\rangle + e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4} - x_2\omega^{2,4} + x_5\omega^{3,7} + x_6\omega^{5,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = 2e^{-\beta_1-3\beta_2/2} + e^{-\beta_1-5\beta_2/2} + 6e^{-3\beta_1-3\beta_2/2} + e^{-5\beta_1-\beta_2/2} + 3e^{-5\beta_1-3\beta_2/2} + 4e^{-7\beta_1-\beta_2/2} + e^{-7\beta_1-3\beta_2/2} + 3e^{-9\beta_1-\beta_2/2} + e^{\beta_1-2\beta_2} + 4e^{-\beta_1-2\beta_2} + 5e^{-2\beta_1-\beta_2} + 5e^{-3\beta_1-\beta_2} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1} + e^{-5\beta_1} + e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = 2e^{-3\beta_1-5\beta_2/2} + 3e^{-5\beta_1-5\beta_2/2} + e^{-7\beta_1-3\beta_2/2} + 4e^{-7\beta_1-5\beta_2/2} + 7e^{-9\beta_1-3\beta_2/2} + 3e^{-11\beta_1-3\beta_2/2} + e^{-13\beta_1-3\beta_2/2} + e^{-\beta_1-3\beta_2} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 3e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2} + 7e^{-4\beta_1-2\beta_2} + 3e^{-5\beta_1-\beta_2} + 2e^{-5\beta_1-2\beta_2} + 2e^{-6\beta_1-\beta_2} + e^{-7\beta_1-\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-7\beta_1-7\beta_2/2} + 2e^{-9\beta_1-5\beta_2/2} + 2e^{-9\beta_1-7\beta_2/2} + 4e^{-11\beta_1-5\beta_2/2} + 3e^{-13\beta_1-5\beta_2/2} + e^{-15\beta_1-3\beta_2/2} + e^{-15\beta_1-5\beta_2/2} + 2e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2} + e^{-5\beta_1-2\beta_2} + 4e^{-5\beta_1-3\beta_2} + 2e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2} + 3e^{-7\beta_1-2\beta_2} + e^{-8\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-11\beta_1-7\beta_2/2} + e^{-13\beta_1-7\beta_2/2} + e^{-13\beta_1-9\beta_2/2} + e^{-15\beta_1-5\beta_2/2} + 2e^{-15\beta_1-7\beta_2/2} + e^{-17\beta_1-5\beta_2/2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2} + 2e^{-8\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-15\beta_1-9\beta_2/2} + e^{-17\beta_1-7\beta_2/2} + e^{-8\beta_1-4\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 60, 129, 159, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 27, 42, 43, 29, 13, 3)$

S=$4\beta_1 + 9\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,4} - \omega^{3,5}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,4} + \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{1,4,5}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{1,5,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} +$

$\omega^{1,2,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,4,6} - \omega^{1,2,5,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{1,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7}$; $d\omega^{3,5,7} = \omega^{1,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,4,6} - \omega^{1,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,6}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{\beta_1-2\beta_3} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1-\beta_3} \langle\omega^{1,6}\rangle + e^{\beta_1-3\beta_3} \langle\omega^{2,3}\rangle + e^{\beta_1-4\beta_3} \langle\omega^{2,4}\rangle + e^{-3\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{1,4}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,7} + \omega^{3,6}\rangle + e^{-\beta_1-3\beta_3} \langle\omega^{2,6} - \omega^{3,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-3\beta_3} \langle\omega^{1,2,6}\rangle + 2e^{-3\beta_1-3\beta_3} \langle\omega^{1,4,6}, \omega^{1,5,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,6}\rangle + e^{\beta_1-5\beta_3} \langle\omega^{2,3,4}\rangle + e^{-4\beta_3} \langle\omega^{1,2,4}\rangle + e^{-5\beta_3} \langle\omega^{2,3,7} + \omega^{2,4,5}\rangle + e^{-\beta_1-4\beta_3} \langle\omega^{2,3,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-5\beta_3} \langle\omega^{2,4,6} + \omega^{2,5,7} + \omega^{3,4,7}\rangle + 2e^{-2\beta_1-4\beta_3} \langle\omega^{1,4,7} - \omega^{3,5,7}, \omega^{2,5,6} + \omega^{3,4,6} + \omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = 2e^{-2\beta_1-5\beta_3} \langle\omega^{1,2,4,6}, \omega^{1,2,5,7}\rangle + 2e^{-\beta_1-6\beta_3} \langle\omega^{1,2,4,7} + \omega^{2,3,5,7}, \omega^{2,3,4,6} + \omega^{2,3,5,7}\rangle + e^{-3\beta_1-4\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-4\beta_1-4\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-5\beta_1-4\beta_3} \langle\omega^{1,5,6,7}\rangle + e^{-7\beta_3} \langle\omega^{2,3,4,7}\rangle + e^{-3\beta_1-5\beta_3} \langle\omega^{1,2,6,7} - \omega^{3,4,5,6}\rangle + e^{-2\beta_1-6\beta_3} \langle\omega^{2,3,6,7} - \omega^{2,4,5,6} + \omega^{3,4,5,7}\rangle + e^{-4\beta_1-5\beta_3} \langle\omega^{1,4,6,7} - \omega^{3,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-3\beta_1-6\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-4\beta_1-6\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-5\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-6\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-2\beta_1-7\beta_3} \langle\omega^{1,2,4,5,7}\rangle + e^{-\beta_1-8\beta_3} \langle\omega^{2,3,4,5,7}\rangle + e^{-3\beta_1-7\beta_3} \langle\omega^{1,2,4,6,7} + \omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-4\beta_1-8\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-9\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-4\beta_1-9\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 44.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^4$; $d^\star\omega^{1,3}=-\omega^5$; $d^\star\omega^{1,4}=-\omega^7$; $d^\star\omega^{1,5}=-\omega^6$; $d^\star\omega^{3,5}=-\omega^7$; $d^\star\omega^{1,2,3}=-\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4}=-\omega^{2,7}$; $d^\star\omega^{1,2,5}=-\omega^{2,6}-\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{4,6}$; $d^\star\omega^{1,2,7}=-\omega^{4,7}$; $d^\star\omega^{1,3,4}=-\omega^{3,7}+\omega^{4,5}$; $d^\star\omega^{1,3,5}=\omega^{1,7}-\omega^{3,6}$; $d^\star\omega^{1,3,6}=-\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{5,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,6}+\omega^{5,7}$; $d^\star\omega^{1,4,6}=\omega^{6,7}$; $d^\star\omega^{1,5,7}=-\omega^{6,7}$; $d^\star\omega^{2,3,5}=\omega^{2,7}$; $d^\star\omega^{3,4,5}=-\omega^{4,7}$; $d^\star\omega^{3,5,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{2,3,7}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,2,7}-\omega^{2,3,6}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7}=-\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{2,4,6}+\omega^{2,5,7}$; $d^\star\omega^{1,2,4,6}=\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7}=-\omega^{2,6,7}-\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5}=\omega^{1,4,7}-\omega^{3,4,6}+\omega^{3,5,7}$; $d^\star\omega^{1,3,4,6}=\omega^{3,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{1,6,7}$; $d^\star\omega^{1,3,5,7}=-\omega^{3,6,7}$; $d^\star\omega^{1,3,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7}=-\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{2,4,7}$; $d^\star\omega^{2,3,5,6}=-\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,2,4,7}-\omega^{2,3,4,6}+\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6}=\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{1,2,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=-\omega^{1,4,6,7}-\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=-\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,2,4,6,7}-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^4=\omega^4$; $\Delta\omega^5=\omega^5$; $\Delta\omega^6=\omega^6$; $\Delta\omega^7=2\omega^7$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,4}=\omega^{1,4}+\omega^{3,5}$; $\Delta\omega^{1,5}=\omega^{1,5}$; $\Delta\omega^{1,7}=\omega^{1,7}-\omega^{3,6}$; $\Delta\omega^{2,5}=\omega^{2,5}-\omega^{3,4}$; $\Delta\omega^{2,6}=\omega^{2,6}+\omega^{4,5}$; $\Delta\omega^{2,7}=2\omega^{2,7}$; $\Delta\omega^{3,4}=-\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,5}=\omega^{1,4}+\omega^{3,5}$; $\Delta\omega^{3,6}=-\omega^{1,7}+\omega^{3,6}$; $\Delta\omega^{3,7}=\omega^{3,7}-\omega^{4,5}$; $\Delta\omega^{4,5}=\omega^{2,6}-\omega^{3,7}+2\omega^{4,5}$; $\Delta\omega^{4,6}=2\omega^{4,6}-\omega^{5,7}$; $\Delta\omega^{4,7}=2\omega^{4,7}$; $\Delta\omega^{5,6}=\omega^{5,6}$; $\Delta\omega^{5,7}=-\omega^{4,6}+2\omega^{5,7}$; $\Delta\omega^{6,7}=3\omega^{6,7}$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}-\omega^{2,3,5}$; $\Delta\omega^{1,2,5}=2\omega^{1,2,5}-\omega^{1,3,4}$; $\Delta\omega^{1,2,6}=\omega^{1,2,6}+\omega^{1,4,5}$; $\Delta\omega^{1,2,7}=2\omega^{1,2,7}+\omega^{2,3,6}$; $\Delta\omega^{1,3,4}=-\omega^{1,2,5}+2\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=2\omega^{1,3,5}$; $\Delta\omega^{1,3,6}=\omega^{1,3,6}$; $\Delta\omega^{1,3,7}=\omega^{1,3,7}-\omega^{1,4,5}$; $\Delta\omega^{1,4,5}=\omega^{1,2,6}-\omega^{1,3,7}+2\omega^{1,4,5}$; $\Delta\omega^{1,4,6}=\omega^{1,4,6}-\omega^{1,5,7}+\omega^{3,5,6}$; $\Delta\omega^{1,4,7}=\omega^{1,4,7}-\omega^{3,4,6}+\omega^{3,5,7}$; $\Delta\omega^{1,5,7}=-\omega^{1,4,6}+\omega^{1,5,7}-\omega^{3,5,6}$; $\Delta\omega^{1,6,7}=\omega^{1,6,7}$; $\Delta\omega^{2,3,5}=-\omega^{1,2,4}+\omega^{2,3,5}$; $\Delta\omega^{2,3,6}=\omega^{1,2,7}+\omega^{2,3,6}-\omega^{3,4,5}$; $\Delta\omega^{2,3,7}=\omega^{2,3,7}-\omega^{2,4,5}$; $\Delta\omega^{2,4,5}=-\omega^{2,3,7}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6}=\omega^{2,4,6}-\omega^{2,5,7}$; $\Delta\omega^{2,4,7}=\omega^{2,4,7}$; $\Delta\omega^{2,5,6}=\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,5,7}=-\omega^{2,4,6}+2\omega^{2,5,7}-\omega^{3,4,7}$; $\Delta\omega^{2,6,7}=3\omega^{2,6,7}+\omega^{4,5,7}$; $\Delta\omega^{3,4,5}=-\omega^{2,3,6}+2\omega^{3,4,5}$; $\Delta\omega^{3,4,6}=-\omega^{1,4,7}-\omega^{2,5,6}+2\omega^{3,4,6}-\omega^{3,5,7}$; $\Delta\omega^{3,4,7}=-\omega^{2,5,7}+\omega^{3,4,7}$; $\Delta\omega^{3,5,6}=\omega^{1,4,6}-\omega^{1,5,7}+\omega^{3,5,6}$; $\Delta\omega^{3,5,7}=\omega^{1,4,7}-\omega^{3,4,6}+\omega^{3,5,7}$; $\Delta\omega^{3,6,7}=2\omega^{3,6,7}+\omega^{4,5,6}$; $\Delta\omega^{4,5,6}=\omega^{3,6,7}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7}=\omega^{2,6,7}+2\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=3\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4}=2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6}=$

$2\omega^{1,2,3,6}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,3,7}=2\omega^{1,2,3,7}-\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5}=-\omega^{1,2,3,7}+2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=\omega^{1,2,4,6}-\omega^{1,2,5,7}-\omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,7}=\omega^{1,2,4,7}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7}=-\omega^{1,2,4,6}+2\omega^{1,2,5,7}-\omega^{1,3,4,7}+\omega^{2,3,5,6}$; $\Delta\omega^{1,2,6,7}=2\omega^{1,2,6,7}+\omega^{1,4,5,7}$; $\Delta\omega^{1,3,4,5}=-\omega^{1,2,3,6}+3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=-\omega^{1,2,5,6}+2\omega^{1,3,4,6}-\omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,7}=-\omega^{1,2,5,7}+\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6}=\omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7}=-\omega^{1,3,4,6}+\omega^{1,3,5,7}$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}+\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6}=\omega^{1,3,6,7}+\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7}=\omega^{1,2,6,7}+\omega^{1,4,5,7}-\omega^{3,4,5,6}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}+\omega^{3,5,6,7}$; $\Delta\omega^{2,3,4,5}=\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,2,4,7}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{2,3,5,6}=-\omega^{1,2,4,6}+\omega^{1,2,5,7}+\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,2,4,7}-\omega^{2,3,4,6}+\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=2\omega^{2,3,6,7}+\omega^{2,4,5,6}-\omega^{3,4,5,7}$; $\Delta\omega^{2,4,5,6}=\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}-\omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,4,5,7}+2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{2,3,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=-\omega^{2,5,6,7}+2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=\omega^{1,4,6,7}+\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=2\omega^{1,2,3,4,6}-\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7}=\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6}=2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7}=-\omega^{1,2,3,4,6}+2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=2\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}-\omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}-\omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6}=2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7}=-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7}=-\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,4,5,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7}=\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=-\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7}=\omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{\beta_1-2\beta_3}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-3\beta_1-\beta_3}\ \langle\omega^{1,6}\rangle+e^{\beta_1-3\beta_3}\ \langle\omega^{2,3}\rangle+e^{\beta_1-4\beta_3}\ \langle\omega^{2,4}\rangle+e^{-3\beta_3}\ \langle\omega^{2,5}+\omega^{3,4}\rangle+e^{-\beta_1-2\beta_3}\ \langle\omega^{1,4}-\omega^{3,5}\rangle+e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,7}+\omega^{3,6}\rangle+e^{-\beta_1-3\beta_3}\ \langle\omega^{2,6}-\omega^{3,7}-\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-3\beta_3}\ \langle\omega^{1,2,6}-\omega^{1,3,7}-\omega^{1,4,5}\rangle+e^{-4\beta_1-2\beta_3}\ \langle\omega^{1,5,6}\rangle+e^{\beta_1-5\beta_3}\ \langle\omega^{2,3,4}\rangle+e^{-4\beta_3}\ \langle\omega^{1,2,4}+\omega^{2,3,5}\rangle+e^{-5\beta_3}\ \langle\omega^{2,3,7}+\omega^{2,4,5}\rangle+e^{-\beta_1-4\beta_3}\ \langle\omega^{1,2,7}-2\omega^{2,3,6}-\omega^{3,4,5}\rangle+e^{-\beta_1-5\beta_3}\ \langle\omega^{2,4,6}+\omega^{2,5,7}+\omega^{3,4,7}\rangle+2e^{-3\beta_1-3\beta_3}\ \langle\omega^{1,4,6}-\omega^{3,5,6},\omega^{1,5,7}+\omega^{3,5,6}\rangle+2e^{-2\beta_1-4\beta_3}\ \langle\omega^{1,4,7}-\omega^{3,5,7},\omega^{2,5,6}+\omega^{3,4,6}+\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-2\beta_1-6\beta_3}\ \langle(-\omega^{2,3,6,7}+\omega^{2,4,5,6}-\omega^{3,4,5,7})/3\rangle+e^{-7\beta_3}\ \langle\omega^{2,3,4,7}\rangle+e^{-5\beta_1-4\beta_3}\ \langle-\omega^{1,5,6,7}\rangle+e^{-4\beta_1-5\beta_3}\ \langle(\omega^{1,4,6,7}-\omega^{3,5,6,7})/2\rangle+e^{-4\beta_1-4\beta_3}\ \langle(-\omega^{1,3,6,7}+\omega^{1,4,5,6})/2\rangle+e^{-3\beta_1-5\beta_3}\ \langle(-\omega^{1,2,6,7}+2\omega^{1,4,5,7}+\omega^{3,4,5,6})/6\rangle+e^{-3\beta_1-4\beta_3}\ \langle(\omega^{1,2,5,6}+\omega^{1,3,4,6}+\omega^{1,3,5,7})/3\rangle+2e^{-\beta_1-6\beta_3}\ \langle(-\omega^{1,2,4,7}-\omega^{2,3,4,6}-2\omega^{2,3,5,7})/3,(\omega^{1,2,4,7}-2\omega^{2,3,4,6}-\omega^{2,3,5,7})/3\rangle+2e^{-2\beta_1-5\beta_3}\ \langle(2\omega^{1,2,4,6}-\omega^{1,2,5,7}-\omega^{1,3,4,7}+3\omega^{2,3,5,6})/5,(-\omega^{1,2,4,6}-2\omega^{1,2,5,7}-2\omega^{1,3,4,7}+\omega^{2,3,5,6})/5\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-\beta_1-8\beta_3}\ \langle\omega^{2,3,4,5,7}\rangle+e^{-5\beta_1-6\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle+e^{-5\beta_1-5\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle+$

$e^{-4\beta_1-6\beta_3}$ $\langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2\rangle$ + $e^{-3\beta_1-7\beta_3}$ $\langle(\omega^{1,2,4,6,7}+\omega^{2,3,5,6,7})/2\rangle$ + $e^{-2\beta_1-7\beta_3}$ $\langle(\omega^{1,2,4,5,7}-\omega^{2,3,4,5,6})/2\rangle$ + $e^{-3\beta_1-6\beta_3}$ $\langle(-\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}-\omega^{1,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})=$ $e^{-3\beta_1-9\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-5\beta_1-7\beta_3}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-4\beta_1-8\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})=$ $e^{-4\beta_1-9\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+5Y^2+6Y^3+5Y^4+2Y^5+Y^6)(1+Y)$.

5.4.48 Algebra $\mathcal{G}_{7,2.44}$

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_5] = x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & -\xi_1^2 & -\xi_1^1 + 2\xi_2^2 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^2 & 2\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & \xi_1^2 & -\xi_1^1 + 3\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^4 + \xi_2^5 & \xi_3^5 & \xi_3^4 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4 + \delta_3^6, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = -\delta_2^7, ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{-\beta_1 + 2\beta_2}\ \langle x_3 \rangle + e^{\beta_1 + \beta_2}\ \langle x_4 \rangle + e^{2\beta_2}\ \langle x_5 \rangle + e^{-\beta_1 + 3\beta_2}\ \langle x_6 \rangle + e^{\beta_1 + 2\beta_2}\ \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{-\beta_1 + 3\beta_2}\ \langle x_6 \rangle + e^{\beta_1 + 2\beta_2}\ \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = 2e^{\beta_1}\ \langle \delta_2^4, \delta_3^5 \rangle + 2e^{-\beta_1 + 2\beta_2}\ \langle \delta_1^5, \delta_2^6 \rangle + e^{-2\beta_1 + 3\beta_2}\ \langle \delta_1^6 \rangle + 3e^{\beta_2}\ \langle \delta_3^6, \delta_1^4, \delta_2^5 \rangle + e^{-2\beta_1 + 2\beta_2}\ \langle \delta_1^3 \rangle + e^{-\beta_1 + \beta_2}\ \langle \delta_1^2 \rangle + e^{2\beta_2}\ \langle \delta_1^7 \rangle + e^{\beta_1 + \beta_2}\ \langle \delta_2^7 \rangle + e^{2\beta_1}\ \langle \delta_3^7 \rangle + e^{2\beta_1 - \beta_2}\ \langle \delta_3^4 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_2^4, \delta_1^2] = \delta_2^5 + \delta_1^4$; $[\delta_2^4, \delta_1^4] = \delta_2^7$; $[\delta_2^4, \delta_2^5] = -\delta_2^7$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_1^6, \delta_3^4] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_2^6, \delta_3^4] = -\delta_2^7$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_3^6, \delta_1^2] = -\delta_2^6$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^4] = -\delta_1^6$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^4] = -(\delta_3^6 + \delta_1^4)$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_3^7] = \delta_2^7$; $[\delta_1^2, \delta_1^4] = -\delta_1^5$; $[\delta_1^2, \delta_2^5] = \delta_2^6 - \delta_1^5$; $[\delta_1^2, \delta_3^5] = \delta_3^6 + \delta_2^5$; $[\delta_1^2, \delta_3^4] = -\delta_3^5 + \delta_2^4$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_3^4] = -\delta_3^7$; $[\delta_2^5, \delta_3^5] = -\delta_2^7$; $[\delta_2^5, \delta_3^4] = \delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-2\beta_1 + 3\beta_2}\ \langle \delta_1^6 \rangle + e^{-\beta_1 + 2\beta_2}\ \langle \delta_1^5 \rangle + e^{-2\beta_1 + 2\beta_2}\ \langle \delta_1^3 \rangle + e^{-\beta_1 + \beta_2}\ \langle \delta_1^2 \rangle + e^{2\beta_1}\ \langle \delta_3^7 \rangle + 2e^{\beta_2}\ \langle \delta_1^4, \delta_2^5 \rangle + e^{\beta_1}\ \langle \delta_2^4 \rangle + e^{2\beta_1 - \beta_2}\ \langle \delta_3^4 \rangle + 2e^0\ \langle t_1, t_2 \rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^2] = 2\delta_1^5$; $[\delta_2^5, \delta_3^4] = \delta_3^7$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^4, \delta_1^3] = \delta_1^6$; $[\delta_1^4, \delta_1^2] = \delta_1^5$; $[\delta_1^4, \delta_3^4] = -\delta_3^7$; $[\delta_1^3, \delta_2^4] = \delta_1^5$; $[\delta_1^3, \delta_3^4] = -2\delta_1^4$; $[\delta_1^2, \delta_2^4] = -\delta_2^5 - \delta_1^4$; $[\delta_1^2, \delta_3^4] = 2\delta_2^4$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 4e^{\beta_1 - 2\beta_2}\ \langle x_2\omega^{2,3} + 2x_4(\omega^{2,5} + 1/2\omega^{3,4}), x_4(\omega^{1,6} + \omega^{2,5}) + x_5\omega^{2,6}, -x_1\omega^{1,3} - x_2\omega^{2,3} - 2x_4\omega^{2,5} + x_7\omega^{3,7}, x_6\omega^{3,6} \rangle + 2e^{2\beta_1 - 3\beta_2}\ \langle x_1\omega^{2,3} + x_4\omega^{3,5}, x_4\omega^{2,6} \rangle + 2e^{-2\beta_2}\ \langle -x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) + x_3\omega^{3,5} - 3x_4(\omega^{2,7} -$

$1/3\omega^{4,5}), 2x_1(\omega^{1,5}+1/2\omega^{2,4})+x_2(\omega^{1,6}+\omega^{2,5})-x_3(\omega^{2,6}+2\omega^{3,5})+3x_4\omega^{2,7}-x_5\omega^{4,6}+x_6\omega^{5,6}\rangle+5e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_3\omega^{2,3}+x_5\omega^{2,5}, -x_3\omega^{2,3}+x_5\omega^{1,6}, -x_1\omega^{1,2}-x_3\omega^{2,3}+x_4\omega^{1,5}, x_4\omega^{2,4}, x_2\omega^{1,3}-x_3\omega^{2,3}+x_5\omega^{3,4}\rangle+2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,4}+2x_2\omega^{2,4}-2x_3(\omega^{2,5}+\omega^{3,4})-3x_4\omega^{1,7}-3x_5(\omega^{2,7}-1/3\omega^{4,5}), x_1\omega^{1,4}+x_2(\omega^{1,5}+\omega^{2,4})-x_3(\omega^{1,6}+2\omega^{2,5}+\omega^{3,4})-2x_4\omega^{1,7}-2x_5\omega^{2,7}+x_6(\omega^{3,7}+\omega^{4,6})\rangle+e^{-3\beta_1+2\beta_2}\ \langle x_6\omega^{1,4}\rangle+4e^{-\beta_1}\ \langle x_3\omega^{1,3}+x_6\omega^{1,6}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_5\omega^{2,4}, x_5\omega^{1,5}\rangle+e^{-3\beta_1+\beta_2}\ \langle x_3\omega^{1,4}+x_6\omega^{1,7}\rangle+2e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,2}, x_5\omega^{1,4}\rangle+2e^{-2\beta_1}\ \langle -x_3\omega^{2,4}+x_5\omega^{1,7}, -x_2\omega^{1,4}-x_3\omega^{1,5}+x_6\omega^{4,5}\rangle+e^{2\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle+e^{3\beta_1-3\beta_2}\ \langle x_7\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{3\beta_1-5\beta_2}+e^{3\beta_1-6\beta_2}+3e^{2\beta_1-5\beta_2}+7e^{\beta_1-4\beta_2}+2e^{\beta_1-5\beta_2}+6e^{-\beta_1-2\beta_2}+4e^{-\beta_1-3\beta_2}+3e^{-2\beta_1-\beta_2}+3e^{-2\beta_1-2\beta_2}+3e^{-3\beta_1-\beta_2}+e^{-3\beta_1}+e^{-4\beta_1}+6e^{-3\beta_2}+2e^{-4\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{3\beta_1-8\beta_2}+e^{2\beta_1-6\beta_2}+3e^{2\beta_1-7\beta_2}+e^{2\beta_1-8\beta_2}+e^{\beta_1-5\beta_2}+4e^{\beta_1-6\beta_2}+2e^{\beta_1-7\beta_2}+4e^{-\beta_1-4\beta_2}+4e^{-\beta_1-5\beta_2}+3e^{-2\beta_1-3\beta_2}+6e^{-2\beta_1-4\beta_2}+4e^{-3\beta_1-3\beta_2}+2e^{-4\beta_1-2\beta_2}+e^{-4\beta_2}+6e^{-5\beta_2}+3e^{-6\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{2\beta_1-8\beta_2}+e^{2\beta_1-9\beta_2}+e^{2\beta_1-10\beta_2}+e^{\beta_1-7\beta_2}+2e^{\beta_1-8\beta_2}+e^{\beta_1-9\beta_2}+3e^{-\beta_1-6\beta_2}+4e^{-\beta_1-7\beta_2}+2e^{-2\beta_1-5\beta_2}+5e^{-2\beta_1-6\beta_2}+e^{-3\beta_1-4\beta_2}+3e^{-3\beta_1-5\beta_2}+e^{-4\beta_1-4\beta_2}+3e^{-7\beta_2}+3e^{-8\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{\beta_1-10\beta_2}+e^{\beta_1-11\beta_2}+3e^{-\beta_1-8\beta_2}+2e^{-\beta_1-9\beta_2}+2e^{-2\beta_1-7\beta_2}+e^{-2\beta_1-8\beta_2}+e^{-3\beta_1-6\beta_2}+e^{-3\beta_1-7\beta_2}+e^{-9\beta_2}+e^{-10\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-\beta_1-10\beta_2}+e^{-2\beta_1-9\beta_2}+e^{-11\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,16,60,130,161,116,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,11,27,43,46,32,14,3)$

S=$\beta_1+11\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,21,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,11,11,7,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{1,3}$; $d\omega^6=-\omega^{2,3}$; $d\omega^7=-\omega^{1,5}-\omega^{2,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}$; $d\omega^{2,5}=-\omega^{1,2,3}$; $d\omega^{2,7}=-\omega^{1,2,5}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,7}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,5}$; $d\omega^{5,6}=-\omega^{1,3,6}+\omega^{2,3,5}$; $d\omega^{5,7}=-\omega^{1,3,7}+\omega^{2,4,5}$; $d\omega^{6,7}=\omega^{1,5,6}-\omega^{2,3,7}+\omega^{2,4,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,3,7}=-\omega^{1,2,3,5}$; $d\omega^{2,4,5}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=-\omega^{1,2,4,5}$; $d\omega^{2,5,6}=-\omega^{1,2,3,6}$; $d\omega^{2,5,7}=-\omega^{1,2,3,7}$; $d\omega^{2,6,7}=\omega^{1,2,5,6}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$;

$d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,7} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{\beta_1-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,5}\rangle + 2e^{\beta_1-4\beta_2} \langle\omega^{2,6}, \omega^{3,5}\rangle + 2e^{-3\beta_2} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{2\beta_1-5\beta_2} \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,7}\rangle + e^{-4\beta_2} \langle\omega^{1,2,6}\rangle + e^{\beta_1-5\beta_2} \langle\omega^{1,3,6}\rangle + e^{2\beta_1-6\beta_2} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{1,5,7} - \omega^{2,4,7}\rangle + 2e^{-5\beta_2} \langle\omega^{1,5,6} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{\beta_1-6\beta_2} \langle\omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{2\beta_1-7\beta_2} \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,4,7}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{1,2,5,7}\rangle + 2e^{-\beta_1-6\beta_2} \langle\omega^{1,3,5,7} - \omega^{2,3,4,7}, \omega^{1,2,6,7}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{\beta_1-7\beta_2} \langle\omega^{1,3,5,6}\rangle + e^{2\beta_1-8\beta_2} \langle\omega^{2,3,5,6}\rangle + e^{-7\beta_2} \langle\omega^{1,3,6,7} + \omega^{2,4,5,6}\rangle + e^{-2\beta_1-6\beta_2} \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle + e^{\beta_1-8\beta_2} \langle\omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle + e^{-\beta_1-7\beta_2} \langle\omega^{1,5,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-3\beta_1-6\beta_2} \langle\omega^{1,2,4,5,7}\rangle + 2e^{-2\beta_1-7\beta_2} \langle\omega^{1,2,4,6,7}, \omega^{1,3,4,5,7}\rangle + 2e^{-\beta_1-8\beta_2} \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-9\beta_2} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{\beta_1-10\beta_2} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-2\beta_1-9\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-\beta_1-10\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-\beta_1-11\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 44.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{4}$; $d^{\star}\omega^{1,3} = -\omega^{5}$; $d^{\star}\omega^{1,5} = -\omega^{7}$; $d^{\star}\omega^{2,3} = -\omega^{6}$; $d^{\star}\omega^{2,4} = -\omega^{7}$; $d^{\star}\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^{\star}\omega^{1,2,4} = \omega^{1,7}$; $d^{\star}\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^{\star}\omega^{1,2,6} = -\omega^{4,6}$; $d^{\star}\omega^{1,2,7} = -\omega^{4,7}$; $d^{\star}\omega^{1,3,4} = \omega^{4,5}$; $d^{\star}\omega^{1,3,5} = -\omega^{3,7}$; $d^{\star}\omega^{1,3,6} = -\omega^{5,6}$; $d^{\star}\omega^{1,3,7} = -\omega^{5,7}$; $d^{\star}\omega^{1,4,5} = -\omega^{4,7}$; $d^{\star}\omega^{1,5,6} = \omega^{6,7}$; $d^{\star}\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,6}$; $d^{\star}\omega^{2,3,5} = \omega^{5,6}$; $d^{\star}\omega^{2,3,7} = -\omega^{6,7}$; $d^{\star}\omega^{2,4,5} = \omega^{5,7}$; $d^{\star}\omega^{2,4,6} = \omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,6} + \omega^{2,4,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^{\star}\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{1,5,7} - \omega^{2,4,7}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{1,6,7}$; $d^{\star}\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = -\omega^{3,4,7}$; $d^{\star}\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^{\star}\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{3,5,7} - \omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^{\star}\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} + \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} - \omega^{3,5,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} =$

$2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 3\omega^{1,2,3,7} - \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{\beta_1-2\beta_2} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{1,5} - \omega^{2,4}\rangle + 2e^{\beta_1-4\beta_2} \langle\omega^{2,6}, \omega^{3,5}\rangle + 2e^{-3\beta_2} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{2\beta_1-5\beta_2} \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-2\beta_1-3\beta_2} \langle\omega^{1,2,7} - \omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,7}\rangle + e^{-4\beta_2} \langle\omega^{1,2,6} - \omega^{1,3,5} + \omega^{2,3,4}\rangle + e^{\beta_1-5\beta_2} \langle\omega^{1,3,6} + \omega^{2,3,5}\rangle + e^{2\beta_1-6\beta_2} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-4\beta_2} \langle\omega^{1,3,7} + 2\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{1,5,7} - \omega^{2,4,7}\rangle + 2e^{-5\beta_2} \langle\omega^{1,5,6} - \omega^{2,4,6} + \omega^{3,4,5}, \omega^{2,3,7} + \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{\beta_1-6\beta_2} \langle\omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{2\beta_1-7\beta_2} \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{\beta_1-8\beta_2} \langle(-\omega^{2,3,6,7} + \omega^{3,4,5,6})/2\rangle + e^{2\beta_1-8\beta_2} \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-7\beta_2} \langle(-\omega^{1,5,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7})/3\rangle + e^{-2\beta_1-6\beta_2} \langle(\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-3\beta_1-5\beta_2} \langle-\omega^{1,4,5,7}\rangle + e^{-7\beta_2} \langle(-\omega^{1,3,6,7} - 2\omega^{2,3,5,7} - \omega^{2,4,5,6})/6\rangle + e^{\beta_1-7\beta_2} \langle(\omega^{1,3,5,6} - \omega^{2,3,4,6})/2\rangle + 2e^{-\beta_1-6\beta_2} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{2,3,4,7})/3, (\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle + e^{-2\beta_1-5\beta_2} \langle(-\omega^{1,2,5,7} - \omega^{1,3,4,7})/2\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,4,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{\beta_1-10\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-9\beta_2} \langle(\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + 2e^{-2\beta_1-7\beta_2} \langle-\omega^{1,3,4,5,7}, -\omega^{1,2,4,6,7}\rangle + 2e^{-\beta_1-8\beta_2} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7})/3\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{1,2,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{L}})= e^{-11\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-\beta_1-10\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-2\beta_1-9\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{L}})= e^{-\beta_1-11\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y).$$

5.4.49 Algebra $\mathcal{G}_{7,2.45}$

Adjoint cohomology

$[x_1,x_2]=x_5$; $[x_1,x_4]=x_7$; $[x_1,x_5]=x_6$; $[x_2,x_3]=x_7$; $[x_2,x_4]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & 3\xi_1^1-\xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & -\xi_1^2 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & -2\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & -\xi_1^4+\xi_2^5 & 2\xi_1^1+\xi_2^2 & -\xi_1^2\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^3+\xi_2^4 & 0 & 3\xi_1^1 \end{pmatrix}$$

$ad\ x_1=\delta_2^5+\delta_4^7, ad\ x_2=-\delta_1^5+\delta_4^6+\delta_3^7, ad\ x_3=-\delta_2^7, ad\ x_4=-(\delta_2^6+\delta_1^7), ad\ x_5=-\delta_1^6, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2\rangle$.

$\mathrm{ch}(\mathcal{G})=\ e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{3\beta_1-\beta_2}\ \langle x_3\rangle+e^{2\beta_1}\ \langle x_4\rangle+e^{\beta_1+\beta_2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=\ e^{3\beta_1-2\beta_2}\ \langle\delta_2^3\rangle+3e^{\beta_2}\ \langle\delta_1^5,\delta_4^6,\delta_3^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_2^6,\delta_1^7\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+3e^{\beta_1}\ \langle\delta_1^4,\delta_2^5,\delta_4^7\rangle+2e^{0}\ \langle t_2,t_1\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{3\beta_1-\beta_2}\ \langle\delta_2^7\rangle+2e^{2\beta_1-\beta_2}\ \langle\delta_1^3,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_1^2]=\delta_2^4+\delta_1^3$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^5,\delta_1^4]=\delta_1^6$; $[\delta_1^5,\delta_2^5]=-\delta_1^6$; $[\delta_1^5,\delta_1^3]=\delta_1^7$; $[\delta_1^5,\delta_2^4]=-\delta_1^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_3^6,\delta_1^3]=\delta_1^6$; $[\delta_4^6,\delta_1^4]=\delta_1^6$; $[\delta_4^6,\delta_1^2]=-\delta_3^6$; $[\delta_4^6,\delta_2^4]=\delta_2^6$; $[\delta_1^4,\delta_2^5]=-\delta_2^6$; $[\delta_1^4,\delta_1^2]=2(\delta_4^6+\delta_1^5)$; $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_2^5,\delta_1^2]=-2\delta_4^6+\delta_1^5$; $[\delta_2^5,\delta_1^3]=\delta_2^7$; $[\delta_2^5,\delta_2^4]=-\delta_2^7$; $[\delta_1^2,\delta_1^7]=-\delta_1^6$; $[\delta_1^2,\delta_2^7]=-(\delta_1^7+\delta_2^6)$; $[\delta_1^2,\delta_3^7]=-\delta_3^6$; $[\delta_1^2,\delta_4^7]=\delta_3^7-\delta_4^6$; $[\delta_1^2,\delta_1^3]=-2\delta_4^7-\delta_1^4$; $[\delta_1^2,\delta_2^4]=2\delta_4^7-2\delta_2^5-\delta_1^4$; $[\delta_3^7,\delta_1^3]=\delta_1^7$; $[\delta_4^7,\delta_2^4]=\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=\ e^{3\beta_1-2\beta_2}\ \langle\delta_2^3\rangle+e^{-\beta_1+2\beta_2}\ \langle\delta_3^6\rangle+2e^{\beta_1}\ \langle\delta_1^4,\delta_2^5\rangle+2e^{0}\ \langle t_2,t_1\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+2e^{\beta_2}\ \langle\delta_1^5,\delta_4^6\rangle+2e^{2\beta_1-\beta_2}\ \langle\delta_1^3,\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3,\delta_1^5]=\delta_2^6$; $[\delta_1^3,\delta_1^2]=-2\delta_2^5+\delta_1^4$; $[\delta_2^3,\delta_3^6]=-\delta_2^6$; $[\delta_2^3,\delta_1^2]=\delta_2^4+\delta_1^3$; $[\delta_1^4,\delta_1^2]=2\delta_4^6+2\delta_1^5$; $[\delta_1^4,\delta_2^5]=-\delta_2^6$; $[\delta_2^4,\delta_1^5]=-\delta_2^6$; $[\delta_2^4,\delta_4^6]=-\delta_2^6$; $[\delta_2^4,\delta_1^2]=4\delta_2^5+\delta_1^4$; $[\delta_4^6,\delta_1^2]=-\delta_3^6$; $[\delta_1^2,\delta_2^5]=2\delta_4^6-\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})=\ e^{2\beta_1-3\beta_2}\ \langle x_3\omega^{2,5}\rangle+2e^{-2\beta_2}\ \langle -x_1\omega^{2,5}+x_3(\omega^{2,7}+\omega^{4,5}),-x_1\omega^{2,5}-x_3(\omega^{1,6}+\omega^{2,7})+x_7\omega^{5,6}\rangle+2e^{\beta_1-2\beta_2}\ \langle x_3\omega^{1,5}+x_4\omega^{2,5},x_3\omega^{2,4}\rangle+$

$3e^{-\beta_1-\beta_2}$ $\langle -x_1(\omega^{1,5}-2\omega^{2,4})+x_2\omega^{2,5}+x_3(\omega^{1,7}-\omega^{3,5})+x_4(\omega^{2,7}+\omega^{4,5}), x_1\omega^{2,4}+x_2\omega^{2,5}+x_3(\omega^{1,7}-\omega^{3,5})+x_4(\omega^{1,6}+\omega^{2,7})+x_5\omega^{2,6}, -x_1(\omega^{1,5}+2\omega^{2,4})+x_2\omega^{2,5}+2x_3(\omega^{1,7}-\omega^{3,5})+3x_4(\omega^{1,6}+\omega^{2,7})-3x_6\omega^{5,6}+x_7\omega^{5,7}\rangle + 2e^{-4\beta_1+2\beta_2}$ $\langle x_2\omega^{1,3}+x_5\omega^{3,4}, -x_2\omega^{1,3}+x_6\omega^{3,7}\rangle + 4e^{-3\beta_1+\beta_2}$ $\langle x_1\omega^{1,3}+3x_2(\omega^{1,4}-2/3\omega^{2,3})-2x_4\omega^{3,4}+x_5\omega^{3,5}, -x_1\omega^{1,3}-3x_2(\omega^{1,4}-1/3\omega^{2,3})+2x_4\omega^{3,4}+x_6\omega^{3,6}, x_2(\omega^{1,4}-\omega^{2,3})-x_4\omega^{3,4}+x_5\omega^{1,7}, x_2\omega^{1,4}-x_4\omega^{3,4}+x_6\omega^{4,7}\rangle + 5e^{-2\beta_1}$ $\langle x_1\omega^{1,4}-2x_2\omega^{2,4}+2x_3\omega^{3,4}+x_5\omega^{4,5}, -x_2\omega^{2,4}+x_3\omega^{3,4}+x_5\omega^{1,6}, -x_1\omega^{1,4}+x_2\omega^{2,4}-x_3\omega^{3,4}+x_6\omega^{4,6}, x_1(\omega^{1,4}-\omega^{2,3})+x_2\omega^{1,5}-x_3\omega^{3,4}+x_4(\omega^{1,7}-\omega^{3,5})+x_5\omega^{2,7}, 2x_1\omega^{2,3}-x_2\omega^{1,5}+2x_3\omega^{3,4}-x_4(\omega^{1,7}-\omega^{3,5})+x_6\omega^{5,7}\rangle + e^{-3\beta_1+2\beta_2}$ $\langle x_5\omega^{1,3}\rangle + 2e^{-2\beta_1+\beta_2}$ $\langle x_4\omega^{1,3}+x_5\omega^{2,3}, x_5\omega^{1,4}\rangle + 5e^{-\beta_2}$ $\langle x_3\omega^{1,4}+x_4\omega^{2,4}, -x_1\omega^{1,2}+x_3\omega^{1,4}+x_4\omega^{1,5}, -x_1\omega^{1,2}+x_3\omega^{1,4}+x_6\omega^{2,6}, x_3\omega^{2,3}, x_1\omega^{1,2}-x_3\omega^{1,4}+x_5\omega^{2,5}\rangle + 5e^{-\beta_1}$ $\langle x_3\omega^{1,3}+x_4\omega^{2,3}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_4\omega^{1,4}, -x_2\omega^{1,2}+x_3\omega^{1,3}+x_6\omega^{2,7}, x_2\omega^{1,2}-x_3\omega^{1,3}+x_5\omega^{2,4}, x_5\omega^{1,5}\rangle + e^{2\beta_1-2\beta_2}$ $\langle x_3\omega^{1,2}\rangle$.

ch$(H^3_{\mathcal{G}})$=$5e^{-\beta_1-2\beta_2}+2e^{-\beta_1-3\beta_2}+4e^{-2\beta_1-\beta_2}+5e^{-2\beta_1-2\beta_2}+9e^{-3\beta_1-\beta_2}+3e^{-3\beta_1}+2e^{-4\beta_1+\beta_2}+7e^{-4\beta_1}+5e^{-5\beta_1+\beta_2}+2e^{-5\beta_1}+2e^{-6\beta_1+2\beta_2}+2e^{-6\beta_1+\beta_2}+e^{-7\beta_1+2\beta_2}+2e^{-3\beta_2}+e^{-4\beta_2}$.

ch$(H^4_{\mathcal{G}})$=$e^{-\beta_1-4\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-4\beta_2}+2e^{-3\beta_1-2\beta_2}+5e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-\beta_2}+7e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+8e^{-5\beta_1-\beta_2}+3e^{-5\beta_1-2\beta_2}+4e^{-6\beta_1-\beta_2}+7e^{-6\beta_1}+3e^{-7\beta_1+\beta_2}+4e^{-7\beta_1}+4e^{-8\beta_1+\beta_2}+e^{-9\beta_1+2\beta_2}+e^{-9\beta_1+\beta_2}$.

ch$(H^5_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+3e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2}+6e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2}+4e^{-7\beta_1-\beta_2}+3e^{-7\beta_1-2\beta_2}+e^{-7\beta_1-3\beta_2}+5e^{-8\beta_1-\beta_2}+e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1}+e^{-9\beta_1+\beta_2}+e^{-9\beta_1-\beta_2}+4e^{-9\beta_1}+e^{-10\beta_1+\beta_2}+e^{-10\beta_1}+e^{-11\beta_1+\beta_2}$.

ch$(H^6_{\mathcal{G}})$=$e^{-6\beta_1-4\beta_2}+2e^{-7\beta_1-3\beta_2}+2e^{-8\beta_1-2\beta_2}+2e^{-8\beta_1-3\beta_2}+2e^{-9\beta_1-\beta_2}+3e^{-9\beta_1-2\beta_2}+2e^{-10\beta_1-\beta_2}+e^{-10\beta_1-2\beta_2}+e^{-11\beta_1-\beta_2}+e^{-11\beta_1}+e^{-12\beta_1}$.

ch$(H^7_{\mathcal{G}})$=$e^{-9\beta_1-3\beta_2}+e^{-10\beta_1-2\beta_2}+e^{-11\beta_1-2\beta_2}+e^{-12\beta_1-\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,17,65,134,165,119,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,12,33,52,54,39,18,4)$

S=$2(6\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,22,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,8,12,12,8,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,5}-\omega^{2,4}$; $d\omega^7=-\omega^{1,4}-\omega^{2,3}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,5}$; $d\omega^{2,7}=-\omega^{1,2,4}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,5}-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{1,3,4}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=-\omega^{1,4,5}$; $d\omega^{4,7}=\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,2,7}+\omega^{1,4,5}+\omega^{2,3,5}$;

$d\omega^{6,7} = \omega^{1,4,6} - \omega^{1,5,7} + \omega^{2,3,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = -\omega^{1,2,3,6} + \omega^{1,2,4,7}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,5,7} = \omega^{1,2,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,4,6} - \omega^{1,2,5,7}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}+\omega^{2,3,4,5}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}+\omega^{1,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,4,6}-\omega^{1,3,5,7}-\omega^{2,3,4,7}$; $d\omega^{4,5,6} = \omega^{1,2,4,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{2,3,4,5}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7} - \omega^{2,3,4,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,5,6} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,3,5,6} - \omega^{1,2,4,5,7}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6} - \omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,6} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-3\beta_1+\beta_2} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1+\beta_2} \langle\omega^{1,3}\rangle + e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-5\beta_1+\beta_2} \langle\omega^{3,4}\rangle + e^{-4\beta_1} \langle\omega^{1,7} - \omega^{3,5}\rangle + 2e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{4,5}, \omega^{2,7} + \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1+\beta_2} \langle\omega^{1,3,7}\rangle + 2e^{-5\beta_1-\beta_2} \langle\omega^{1,4,6}, \omega^{1,5,7} - \omega^{2,3,6}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,2,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + 2e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,6} + \omega^{2,5,7}, \omega^{2,4,6} + \omega^{2,5,7}\rangle + 3e^{-6\beta_1} \langle\omega^{1,3,6} + \omega^{3,4,5}, \omega^{1,4,7} - \omega^{3,4,5}, \omega^{2,3,7} - \omega^{3,4,5}\rangle + e^{-8\beta_1+\beta_2} \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle + 3e^{-6\beta_1-2\beta_2} \langle\omega^{1,2,6,7}, \omega^{1,4,5,6}, \omega^{2,3,5,6}\rangle + 2e^{-8\beta_1} \langle\omega^{1,3,4,6}, \omega^{1,3,5,7}\rangle + e^{-9\beta_1+\beta_2} \langle\omega^{1,3,4,7}\rangle + 2e^{-7\beta_1-\beta_2} \langle\omega^{1,3,5,6} + \omega^{2,3,5,7}, \omega^{1,4,5,7} - \omega^{2,3,5,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-8\beta_1-\beta_2} \langle\omega^{1,4,6,7} + \omega^{3,4,5,6}\rangle + e^{-9\beta_1} \langle\omega^{1,3,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= 2e^{-9\beta_1-\beta_2} \langle\omega^{1,2,3,6,7}, \omega^{1,3,4,5,6}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,2,5,6,7}\rangle + e^{-11\beta_1} \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-2\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-10\beta_1-\beta_2} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-9\beta_1-2\beta_2} \langle\omega^{1,4,5,6,7} - \omega^{2,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-10\beta_1-2\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-12\beta_1-\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-11\beta_1-2\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-12\beta_1-2\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 2Y^2 + Y^3)(1 + 2Y + Y^2 + Y^3)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 22, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 12, 12, 8, 4, 1)$

Total dimension of trivial cohomology: 50.
Length associated polynomial: 50.
Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,3} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,7} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,6} - \omega^{2,7} + \omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6} + \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6} + \omega^{4,7}$; $d^\star\omega^{2,3,5} = \omega^{5,7}$; $d^\star\omega^{2,3,6} = \omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,7} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,6,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,6} + \omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{2,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,6}+\omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6} - \omega^{4,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6} + \omega^{1,4,5,7} - \omega^{2,3,4,6} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,4,6,7} + \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,4,5,6,7}-\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{2,3} = \omega^{1,4} + \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = -\omega^{1,6} + \omega^{2,7} - \omega^{4,5}$; $\Delta\omega^{3,5} = \omega^{1,7} + \omega^{3,5}$; $\Delta\omega^{3,6} = 2\omega^{3,6} - \omega^{4,7}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{1,6} - \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6} - \omega^{5,7}$; $\Delta\omega^{4,7} = -\omega^{3,6} + \omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = -\omega^{4,6} + 3\omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 3\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,4,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} - \omega^{1,4,5} - \omega^{2,3,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,7} - \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7} + 2\omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}-\omega^{1,5,7}+\omega^{2,3,6}-\omega^{2,4,7}$; $\Delta\omega^{1,4,7} = -\omega^{1,3,6}+\omega^{1,4,7}+\omega^{2,3,7}+\omega^{3,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,4,6} - \omega^{2,5,7}$; $\Delta\omega^{1,5,7} = -\omega^{1,4,6} + 2\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} - \omega^{3,5,6} + \omega^{4,5,7}$; $\Delta\omega^{2,3,4} = \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,7} + \omega^{1,4,5} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{1,4,6} + 2\omega^{2,3,6} - \omega^{2,4,7}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,6} + \omega^{1,4,7}+\omega^{2,3,7}+\omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6}+\omega^{2,4,6}-\omega^{2,5,7}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{1,5,7} - \omega^{2,3,6} + \omega^{2,4,7}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} - \omega^{2,4,6} + \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} + \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,6} + \omega^{1,4,7} + \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} =$

$\omega^{3,4,6}-\omega^{3,5,7}$; $\Delta\omega^{3,5,6}=-\omega^{1,6,7}+2\omega^{3,5,6}-\omega^{4,5,7}$; $\Delta\omega^{3,5,7}=-\omega^{3,4,6}+2\omega^{3,5,7}$; $\Delta\omega^{3,6,7}=3\omega^{3,6,7}$; $\Delta\omega^{4,5,6}=\omega^{2,6,7}+\omega^{4,5,6}$; $\Delta\omega^{4,5,7}=\omega^{1,6,7}-\omega^{3,5,6}+2\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=2\omega^{4,6,7}$; $\Delta\omega^{5,6,7}=4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4}=4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5}=2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6}=2\omega^{1,2,3,6}-\omega^{1,2,4,7}+\omega^{2,3,4,5}$; $\Delta\omega^{1,2,3,7}=\omega^{1,2,3,7}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5}=3\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6}=2\omega^{1,2,4,6}-\omega^{1,2,5,7}$; $\Delta\omega^{1,2,4,7}=-\omega^{1,2,3,6}+2\omega^{1,2,4,7}-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7}=-\omega^{1,2,4,6}+\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7}=\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}-\omega^{2,4,5,7}$; $\Delta\omega^{1,3,4,5}=\omega^{1,2,3,7}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6}=\omega^{1,3,4,6}-\omega^{1,3,5,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,5,6}=\omega^{1,3,5,6}-\omega^{1,4,5,7}+\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,7}=-\omega^{1,3,4,6}+\omega^{1,3,5,7}+\omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7}=\omega^{1,3,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{1,4,5,6}=\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}-\omega^{2,4,5,7}$; $\Delta\omega^{1,4,5,7}=-\omega^{1,3,5,6}+2\omega^{1,4,5,7}+\omega^{2,3,5,7}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}+\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7}=2\omega^{1,5,6,7}+\omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5}=\omega^{1,2,3,6}-\omega^{1,2,4,7}+2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6}=\omega^{1,3,5,6}+2\omega^{2,3,4,6}-\omega^{2,3,5,7}$; $\Delta\omega^{2,3,4,7}=-\omega^{1,3,4,6}+\omega^{1,3,5,7}+\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6}=\omega^{1,2,6,7}+\omega^{1,4,5,6}+\omega^{2,3,5,6}-\omega^{2,4,5,7}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,6}+\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{1,4,6,7}+2\omega^{2,3,6,7}-\omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,7}=-\omega^{1,2,6,7}-\omega^{1,4,5,6}-\omega^{2,3,5,6}+\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7}=\omega^{1,5,6,7}+\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7}=\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6}=-\omega^{1,4,6,7}-\omega^{2,3,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=-\omega^{1,3,6,7}+\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=3\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=3\omega^{1,2,3,4,6}-\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6}=\omega^{1,2,3,5,6}-\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,3,5,7}=-\omega^{1,2,3,4,6}+\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=-\omega^{1,2,3,5,6}+2\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=-\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7}=\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7}=\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6}=\omega^{1,2,4,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7}=\omega^{1,2,3,6,7}-\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7}=\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{1,4,5,6,7}+\omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-3\beta_1+\beta_2}\ \langle\omega^3\rangle+e^{-2\beta_1}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-4\beta_1+\beta_2}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1}\ \langle\omega^{1,4}-\omega^{2,3}\rangle+e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}-\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle+e^{-5\beta_1+\beta_2}\ \langle\omega^{3,4}\rangle+e^{-4\beta_1}\ \langle\omega^{1,7}-\omega^{3,5}\rangle+2e^{-3\beta_1-\beta_2}\ \langle\omega^{1,6}-\omega^{4,5},\omega^{2,7}+\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-7\beta_1+\beta_2}\ \langle\omega^{1,3,7}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,2,7}+\omega^{2,3,5}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,6}+\omega^{2,4,5}\rangle+2e^{-5\beta_1-\beta_2}\ \langle\omega^{1,4,6}+\omega^{2,4,7},\omega^{1,5,7}-\omega^{2,3,6}-2\omega^{2,4,7}\rangle+e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,5,6}\rangle+2e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,5,6}+\omega^{2,5,7},\omega^{2,4,6}+\omega^{2,5,7}\rangle+3e^{-6\beta_1}\ \langle\omega^{1,3,6}+\omega^{3,4,5},\omega^{1,4,7}-\omega^{3,4,5},\omega^{2,3,7}-\omega^{3,4,5}\rangle+e^{-8\beta_1+\beta_2}\ \langle\omega^{3,4,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-3\beta_2}\ \langle-\omega^{2,4,5,6}\rangle+e^{-8\beta_1-\beta_2}\ \langle(\omega^{1,4,6,7}+\omega^{3,4,5,6})/2\rangle+e^{-9\beta_1}\ \langle(-\omega^{1,3,6,7}-\omega^{3,4,5,7})/2\rangle+2e^{-7\beta_1-\beta_2}\ \langle(-\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,6}-3\omega^{2,3,5,7})/4,(\omega^{1,3,5,6}+\omega^{1,4,5,7}-\omega^{2,3,4,6}-\omega^{2,3,5,7})/4\rangle+e^{-9\beta_1+\beta_2}\ \langle-\omega^{1,3,4,7}\rangle+$

$2e^{-8\beta_1}$ $\langle(\omega^{1,3,4,6} - \omega^{1,3,5,7} + 2\omega^{2,3,4,7})/3, (\omega^{1,3,4,6} + 2\omega^{1,3,5,7} - \omega^{2,3,4,7})/3\rangle$ + $3e^{-6\beta_1-2\beta_2}$ $\langle(\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,5,6} + 3\omega^{2,4,5,7})/4, (-\omega^{1,2,6,7} - \omega^{1,4,5,6} + 3\omega^{2,3,5,6} + \omega^{2,4,5,7})/4, (-\omega^{1,2,6,7} + 3\omega^{1,4,5,6} - \omega^{2,3,5,6} + \omega^{2,4,5,7})/4\rangle$ + $e^{-4\beta_1-3\beta_2}$ $\langle\omega^{1,2,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{T}})=$ $e^{-8\beta_1-3\beta_2}$ $\langle-\omega^{2,4,5,6,7}\rangle$ + $e^{-9\beta_1-2\beta_2}$ $\langle(-\omega^{1,4,5,6,7} + \omega^{2,3,5,6,7})/2\rangle$ + $e^{-10\beta_1-\beta_2}$ $\langle(\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle$ + $e^{-11\beta_1}$ $\langle\omega^{1,3,4,6,7}\rangle$ + $e^{-7\beta_1-3\beta_2}$ $\langle\omega^{1,2,5,6,7}\rangle$ + $e^{-8\beta_1-2\beta_2}$ $\langle(\omega^{1,2,4,6,7} - \omega^{2,3,4,5,6})/2\rangle$ + $2e^{-9\beta_1-\beta_2}$ $\langle(-\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,3,6,7} + 2\omega^{1,3,4,5,6} + \omega^{2,3,4,5,7})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{T}})=$ $e^{-11\beta_1-2\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-12\beta_1-\beta_2}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-9\beta_1-3\beta_2}$ $\langle\omega^{1,2,4,5,6,7}\rangle$ + $e^{-10\beta_1-2\beta_2}$ $\langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{T}})=$ $e^{-12\beta_1-2\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + Y + 2Y^2 + Y^3)(1 + 2Y + Y^2 + Y^3)(1 + Y)$.

5.5 rank 3 Lie Algebras

5.5.1 Algebra $\mathcal{G}_{7,3.1(i_\lambda)}$ †

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_6$; $[x_2,x_5]=\lambda x_7$; $[x_3,x_4]=x_7(\lambda-1)$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1+\xi_3^3 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & \xi_2^2+\xi_3^3 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6-\xi_1^5\lambda & \xi_1^4(1-\lambda)+\xi_3^6 & \xi_2^4(1-\lambda)+\xi_3^5\lambda & \xi_1^1+\xi_2^2+\xi_3^3 \end{pmatrix}$$

$ad\ x_1=\delta_2^4+\delta_3^5, ad\ x_2=-\delta_1^4+\delta_3^6, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=\delta_3^7(-\lambda+1), ad\ x_5=-\lambda\delta_2^7, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\langle x_1\rangle+e^{\beta_2}\langle x_2\rangle+e^{\beta_3}\langle x_3\rangle+e^{\beta_1+\beta_2}\langle x_4\rangle+e^{\beta_1+\beta_3}\langle x_5\rangle+e^{\beta_2+\beta_3}\langle x_6\rangle+e^{\beta_1+\beta_2+\beta_3}\langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{\beta_1+\beta_2+\beta_3}\langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=3e^0\langle t_1,t_2,t_3\rangle+2e^{\beta_2}\langle\delta_1^4,\delta_3^6\rangle+2e^{\beta_1}\langle\delta_2^4,\delta_3^5\rangle+e^{\beta_1+\beta_2-\beta_3}\langle\delta_3^4\rangle+2e^{\beta_3}\langle\delta_1^5,\delta_2^6\rangle+e^{\beta_1-\beta_2+\beta_3}\langle\delta_2^5\rangle+e^{-\beta_1+\beta_2+\beta_3}\langle\delta_1^6\rangle+e^{\beta_2+\beta_3}\langle\delta_1^7\rangle+e^{\beta_1+\beta_3}\langle\delta_2^7\rangle+e^{\beta_1+\beta_2}\langle\delta_3^7\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_1^5]=\delta_1^7$; $[\delta_1^4,\delta_2^5]=\delta_2^7(1-\lambda)$; $[\delta_1^4,\delta_3^5]=\delta_3^7(1-\lambda)$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_2^4,\delta_1^5]=\lambda\delta_2^7$; $[\delta_2^4,\delta_1^6]=\delta_1^7(1-\lambda)$; $[\delta_2^4,\delta_2^6]=-\lambda\delta_2^7$; $[\delta_2^4,\delta_3^6]=\delta_3^7(1-\lambda)$; $[\delta_3^4,\delta_1^5]=\lambda\delta_3^7$; $[\delta_3^4,\delta_2^6]=-\delta_3^7$; $[\delta_1^5,\delta_3^6]=-\delta_1^7$; $[\delta_2^5,\delta_3^6]=-\delta_2^7$; $[\delta_3^5,\delta_1^6]=\lambda\delta_1^7$; $[\delta_3^5,\delta_2^6]=\lambda\delta_2^7$; $[\delta_3^5,\delta_3^6]=\delta_3^7(-1+\lambda)$.

$\mathrm{ch}(H^1_{\mathcal{G}})=3e^0\langle t_1,t_2,t_3\rangle+e^{\beta_1+\beta_2-\beta_3}\langle\delta_3^4\rangle+e^{\beta_1-\beta_2+\beta_3}\langle\delta_2^5\rangle+e^{\beta_1}\langle\delta_2^4\rangle+e^{-\beta_1+\beta_2+\beta_3}\langle\delta_1^6\rangle+e^{\beta_3}\langle\delta_1^5\rangle+e^{\beta_2}\langle\delta_1^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{-\beta_1+\beta_2-\beta_3}\langle x_2\omega^{1,3}+x_6\lambda\omega^{3,5}\rangle+3e^{-\beta_3}\langle -x_1\lambda\omega^{1,3}+x_2\omega^{2,3}+x_4\omega^{3,4}(\lambda-1)+x_7\lambda\omega^{5,6}, 2x_1\omega^{1,3}(\lambda-1/2)-x_2\lambda\omega^{2,3}+x_4\omega^{2,5}(\lambda-1)-2x_7\omega^{5,6}(\lambda-1/2), x_1\omega^{1,3}(\lambda+1)-x_2\omega^{2,3}(\lambda+1)+x_7(\lambda\omega^{3,7}-2\lambda\omega^{5,6}-\omega^{3,7})\rangle+e^{\beta_2-2\beta_3}\langle x_4\omega^{3,5}\rangle+e^{\beta_1-2\beta_3}\langle x_4\omega^{3,6}\rangle+e^{-2\beta_3}\langle -x_1\lambda^2\omega^{3,5}-x_2\omega^{3,6}+x_4(\lambda\omega^{3,7}-\lambda\omega^{5,6}-\omega^{3,7})\rangle+e^{-\beta_1-\beta_2+\beta_3}\langle x_3\omega^{1,2}+x_6\omega^{2,4}(\lambda-1)\rangle+e^{-2\beta_2+\beta_3}\langle x_5\omega^{2,4}\rangle+3e^{-\beta_2}\langle -x_1\omega^{1,2}(\lambda-$

$1)+x_3\omega^{2,3}+x_5\lambda\omega^{2,5}+x_7\omega^{4,6}(\lambda-1), -x_1\omega^{1,2}(\lambda-1)-x_3\omega^{2,3}(\lambda-1)+x_4\lambda\omega^{2,4}+x_7\omega^{4,6}(\lambda-1), x_1\omega^{1,2}(\lambda-2)+x_3\omega^{2,3}(\lambda-2)+x_7(\lambda\omega^{2,7}-2\lambda\omega^{4,6}+2\omega^{4,6})\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,4}(\lambda^2-2\lambda+1)+x_3\omega^{2,6}+x_5(\lambda\omega^{2,7}-\lambda\omega^{4,6}+\omega^{4,6})\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3}(\lambda-1)+x_4\omega^{2,6}\rangle + e^{-\beta_2-\beta_3}\ \langle -4/3x_1(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}+1/4\lambda\omega^{1,6}-5/4\lambda\omega^{2,5}-3/2\lambda\omega^{3,4}-1/2\omega^{1,6}+1/2\omega^{3,4})-4/3x_2\omega^{2,6}(\lambda^2-\lambda-1/2)+4/3x_3\omega^{3,6}(\lambda^2-\lambda-1/2)-4/3x_4(\lambda^2\omega^{4,6}-3/4\lambda\omega^{4,6}-3/4\omega^{2,7}-1/4\omega^{4,6})+4/3x_5\lambda\omega^{5,6}(\lambda-5/4)-x_7\omega^{6,7}(\lambda-1)\rangle + e^{-2\beta_1+\beta_3}\ \langle x_6\omega^{1,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle + 3e^{-\beta_1}\ \langle -x_2\omega^{1,2}(\lambda-1)+x_3\lambda\omega^{1,3}+x_6\omega^{1,6}+x_7\lambda\omega^{4,5}(\lambda-1), -x_2\lambda\omega^{1,2}+x_3\lambda\omega^{1,3}+x_5\omega^{1,5}+x_7\lambda\omega^{4,5}(\lambda-1), -x_2\omega^{1,2}(\lambda-1)+x_3\omega^{1,3}(\lambda-1)+x_4\omega^{1,4}+x_7\lambda\omega^{4,5}(\lambda-1)\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,4}(\lambda^2-2\lambda+1)+x_3\lambda^2\omega^{1,5}-x_6(\lambda^2\omega^{4,5}-\lambda\omega^{4,5}-\omega^{1,7})\rangle + e^0\ \langle x_6\omega^{2,3}+x_7\omega^{2,5}\rangle + e^{-\beta_1-\beta_2}\ \langle 2/3x_1\omega^{1,4}(\lambda^2-4\lambda+1)-2/3x_2\omega^{2,4}(\lambda^2-4\lambda+1)-1/3x_3(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}-2\lambda\omega^{1,6}-5\lambda\omega^{2,5}-3\lambda\omega^{3,4}-2\omega^{1,6}+2\omega^{3,4})-1/3x_5\lambda\omega^{4,5}(\lambda-5)+x_6(\lambda\omega^{2,7}-5/3\lambda\omega^{4,6}-\omega^{2,7}+1/3\omega^{4,6})+x_7\omega^{4,7}(\lambda-1)\rangle + e^{-\beta_1-\beta_3}\ \langle -2/3x_1\omega^{1,5}(\lambda^2+2\lambda-2)+1/3x_2(\lambda^2\omega^{2,5}+\lambda^2\omega^{3,4}-2\lambda\omega^{1,6}+\lambda\omega^{2,5}+3\lambda\omega^{3,4}+4\omega^{1,6}-4\omega^{3,4})+2/3x_3\omega^{3,5}(\lambda^2+2\lambda-2)-1/3x_4\omega^{4,5}(\lambda^2+3\lambda-4)+x_6(\lambda\omega^{3,7}-5/3\lambda\omega^{5,6}+4/3\omega^{5,6})+x_7\lambda\omega^{5,7}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2-2\beta_3}+3e^{-\beta_1-\beta_2-\beta_3}+2e^{-\beta_1-\beta_2-2\beta_3}+2e^{-\beta_1-2\beta_2-\beta_3}+3e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+3e^{-\beta_1-2\beta_3}+e^{-\beta_1-3\beta_3}+e^{-2\beta_1+\beta_2-2\beta_3}+2e^{-2\beta_1-\beta_2-\beta_3}+3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+\beta_3}+e^{-2\beta_1-2\beta_2}+3e^{-2\beta_1-\beta_3}+e^{-2\beta_1-2\beta_3}+e^{-3\beta_1-\beta_2}+e^{-3\beta_1-\beta_3}+e^{-3\beta_1}+3e^{-\beta_2-2\beta_3}+e^{-\beta_2-3\beta_3}+3e^{-2\beta_2-\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-3\beta_2-\beta_3}+e^{-3\beta_2}+e^{-3\beta_3}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-\beta_1-\beta_2-3\beta_3}+e^{-\beta_1-\beta_2-4\beta_3}+3e^{-\beta_1-2\beta_2-2\beta_3}+2e^{-\beta_1-2\beta_2-3\beta_3}+2e^{-\beta_1-3\beta_2-\beta_3}+2e^{-\beta_1-3\beta_2-2\beta_3}+e^{-\beta_1-4\beta_2-\beta_3}+3e^{-2\beta_1-\beta_2-2\beta_3}+2e^{-2\beta_1-\beta_2-3\beta_3}+3e^{-2\beta_1-2\beta_2-\beta_3}+2e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-2\beta_2}+2e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-2\beta_3}+e^{-2\beta_1-3\beta_3}+2e^{-3\beta_1-\beta_2-\beta_3}+2e^{-3\beta_1-\beta_2-2\beta_3}+2e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-2\beta_3}+e^{-3\beta_1-3\beta_3}+e^{-4\beta_1-\beta_2-\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-2\beta_2-3\beta_3}+e^{-3\beta_2-2\beta_3}+e^{-3\beta_2-3\beta_3}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-\beta_1-3\beta_2-3\beta_3}+e^{-\beta_1-3\beta_2-4\beta_3}+e^{-\beta_1-4\beta_2-3\beta_3}+2e^{-2\beta_1-2\beta_2-3\beta_3}+2e^{-2\beta_1-2\beta_2-4\beta_3}+2e^{-2\beta_1-3\beta_2-2\beta_3}+2e^{-2\beta_1-3\beta_2-3\beta_3}+2e^{-2\beta_1-4\beta_2-2\beta_3}+2e^{-3\beta_1-\beta_2-3\beta_3}+e^{-3\beta_1-\beta_2-4\beta_3}+2e^{-3\beta_1-2\beta_2-2\beta_3}+2e^{-3\beta_1-2\beta_2-3\beta_3}+2e^{-3\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-\beta_3}+e^{-4\beta_1-\beta_2-3\beta_3}+2e^{-4\beta_1-2\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-\beta_3}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-2\beta_1-3\beta_2-4\beta_3}+e^{-2\beta_1-4\beta_2-3\beta_3}+e^{-2\beta_1-4\beta_2-4\beta_3}+e^{-3\beta_1-2\beta_2-4\beta_3}+2e^{-3\beta_1-3\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_2-4\beta_3}+e^{-3\beta_1-4\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-3\beta_3}+e^{-4\beta_1-2\beta_2-3\beta_3}+e^{-4\beta_1-2\beta_2-4\beta_3}+e^{-4\beta_1-3\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-3\beta_3}+e^{-4\beta_1-4\beta_2-2\beta_3}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-3\beta_1-4\beta_2-4\beta_3}+e^{-4\beta_1-3\beta_2-4\beta_3}+e^{-4\beta_1-4\beta_2-3\beta_3}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,15,59,130,159,116,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 9, 25, 42, 44, 30, 14, 3)$

The preceding results are valid for:

$$-(\lambda^2 - \lambda + 1)(\lambda - 1/2)(\lambda + 1)^2(\lambda - 1)^4(\lambda - 2)^2\lambda \neq 0.$$

S=$4(\beta_1 + \beta_2 + \beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 19, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 9, 9, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5}\lambda + \omega^{3,4}(-\lambda + 1)$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,5}\lambda + \omega^{1,3,4}(\lambda - 1)$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}(\lambda - 1)$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}\lambda$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5}\lambda$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}(\lambda - 1)$; $d\omega^{6,7} = -\omega^{2,3,7} + \omega^{2,5,6}\lambda + \omega^{3,4,6}(\lambda - 1)$; $d\omega^{1,2,7} = \omega^{1,2,3,4}(-\lambda + 1)$; $d\omega^{1,3,7} = \omega^{1,2,3,5}\lambda$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}\lambda$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = \omega^{1,3,4,5}(-\lambda + 1)$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,5,6}\lambda + \omega^{1,3,4,6}(-\lambda + 1)$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}(-\lambda + 1)$; $d\omega^{2,6,7} = \omega^{2,3,4,6}(-\lambda + 1)$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}\lambda$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = \omega^{2,3,5,6}\lambda$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}\lambda$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7} + \omega^{3,4,5,6}(-\lambda + 1)$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}(\lambda - 1)$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}(\lambda - 1)$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}\lambda$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}\lambda$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}\lambda$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}(\lambda - 1)$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}(\lambda - 1)$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}\lambda$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}(-\lambda + 1)$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}\lambda$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,6}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,7} + \omega^{2,4,5}(\lambda - 1)\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,7} - \omega^{3,4,5}\lambda\rangle +$

$e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{G}})= e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,4,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,5,7}+\omega^{1,4,5,6}(-\lambda+1)\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,6,7}+\omega^{2,4,5,6}(-\lambda+1)\rangle + e^{-2\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,3,6,7}+\omega^{3,4,5,6}\lambda\rangle + e^{-\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{2,4,6,7}\rangle + e^{-2\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{3,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{G}})= e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,2,5,6,7}+\omega^{2,3,4,5,7}(-\lambda+1)\rangle + e^{-3\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-2\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-2\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{G}})= e^{-4\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{G}})= e^{-4\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+4Y^2+2Y^3+Y^4)(1+Y^2)(1+Y)$.

The preceding results are valid for:

$$(\lambda^2-\lambda+1)(2\lambda-1)(\lambda+1)(\lambda-1)(\lambda-2)\lambda \neq 0.$$

Comments: singular values. The transposition $\tau_1=(1,2)$ (resp:$\tau_2=(2,3)$) on the indices of the generators generates an isomorphism $\tilde\tau_1(resp:\tilde\tau_2):\mathcal{G}_{7,3.1(i_\lambda)}\to\mathcal{G}_{7,3.1(i_{\lambda'})}$ if and only if $\lambda\lambda'=1$ (†) (resp: $\lambda'=1-\lambda$ (‡)). For $\lambda\neq\lambda'$ there exists an isomorphism $f:\mathcal{G}_{7,3.1(i_\lambda)}\to\mathcal{G}_{7,3.1(i_{\lambda'})}$ if and only if λ' is deduced from λ by suitable composition of (†) and (‡), f beeing then the corresponding composition of τ_1 and τ_2 . In particular, the cases $\lambda=0$ and $\lambda=1$ are isomorphic , the cases $\lambda=-1,\lambda=2,\lambda=1/2$ are isomorphic , and the two cases $\lambda^2-\lambda+1=0$ are isomorphic.

changes in adjoint cohomology:

case $\lambda=0$: The only changes are $\dim(Z^0_{\mathcal{G}})$=2 and $\dim(Z^4_{\mathcal{G}})$=161 which imply $\dim(H^0_{\mathcal{G}})$=2, $\dim(H^1_{\mathcal{G}})$=10, $\dim(H^4_{\mathcal{G}})$=46, $\dim(H^5_{\mathcal{G}})$=32. Add in $\mathrm{ch}(H^0_{\mathcal{G}})$ $e^{\beta_1+\beta_3}\langle x_5\rangle$ and in $\mathrm{ch}(H^1_{\mathcal{G}})$ $e^{\beta_1+\beta_3}\langle\delta^7_2\rangle$; $e^{-\beta_1-2\beta_2-3\beta_3}$ and $e^{-3\beta_1-2\beta_2-\beta_3}$ have multiplicity 3 in $\mathrm{ch}(H^4_{\mathcal{G}})$ and appear as new terms in $\mathrm{ch}(H^5_{\mathcal{G}})$.

Comm. relations among the δ^j_i's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta^5_2,\delta^4_1]=-\delta^7_2$.

case $\lambda = -1$:

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1+\xi_3^3 & \xi_2^1 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & \xi_1^2 & \xi_2^2+\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_1^5+\xi_2^6 & 2\xi_1^4+\xi_3^6 & 2\xi_2^4-\xi_3^5 & \xi_1^1+\xi_2^2+\xi_3^3 \end{pmatrix}$$

Add in $\mathrm{ch}(Z^1_{\mathcal{G}})$ and in $\mathrm{ch}(H^1_{\mathcal{G}})$ $e^{\beta_1-\beta_2}\langle\delta_2^1\rangle + e^{\beta_2-\beta_1}\langle\delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^5] = -\delta_3^7$; $[\delta_3^4, \delta_2^6] = -\delta_3^7$; $[\delta_2^5, \delta_1^2] = -\delta_2^6 + \delta_1^5$; $[\delta_2^5, \delta_1^4] = -2\delta_2^7$; $[\delta_2^5, \delta_3^6] = -\delta_2^7$; $[\delta_2^1, \delta_1^6] = -\delta_2^6 + \delta_1^5$; $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_2^1, \delta_1^7] = -\delta_2^7$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_2^6] = \delta_2^5$; $[\delta_2^1, \delta_1^4] = -\delta_2^4$; $[\delta_2^1, \delta_3^6] = \delta_3^5$; $[\delta_1^6, \delta_2^4] = -2\delta_1^7$; $[\delta_1^6, \delta_3^5] = \delta_1^7$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^5] = \delta_1^6$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_3^5] = \delta_3^6$; $[\delta_1^5, \delta_1^4] = -\delta_1^7$; $[\delta_1^5, \delta_3^6] = -\delta_1^7$; $[\delta_1^5, \delta_2^4] = \delta_2^7$; $[\delta_2^6, \delta_1^4] = \delta_1^7$; $[\delta_2^6, \delta_2^4] = -\delta_2^7$; $[\delta_2^6, \delta_3^5] = \delta_2^7$; $[\delta_1^4, \delta_3^5] = 2\delta_3^7$; $[\delta_3^6, \delta_2^4] = -2\delta_3^7$; $[\delta_3^6, \delta_3^5] = 2\delta_3^7$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^5, \delta_1^2] = 2\delta_1^5$; $[\delta_2^1, \delta_1^6] = 2\delta_1^5$; $[\delta_2^1, \delta_1^2] = -t_2 + t_1$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^4] = -\delta_2^4$; $[\delta_1^2, \delta_1^5] = \delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$.

Add in $\mathrm{ch}(H^2_{\mathcal{G}})$ $e^{\beta_1-\beta_2}\langle x_5\omega^{2,3}\rangle + e^{\beta_2-\beta_1}\langle x_6\omega^{1,3}\rangle$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, \underline{17}, 59, \underline{131}, 159, \underline{117}, 45, 7)$ [6]

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, \underline{11}, \underline{27}, \underline{43}, \underline{45}, \underline{31}, \underline{15}, 3)$

$e^{-\beta_1-\beta_2-2\beta_3}$ (resp: $e^{-3\beta_1-3\beta_2-2\beta_3}$) has multiplicity 3 in $\mathrm{ch}(H^3_{\mathcal{G}})$ (resp:$\mathrm{ch}(H^5_{\mathcal{G}})$) and appears as a new term in $\mathrm{ch}(H^4_{\mathcal{G}})$ (resp: $\mathrm{ch}(H^6_{\mathcal{G}})$).

case $\lambda^2 - \lambda + 1 = 0$:

The only change is $\dim(Z^4_{\mathcal{G}})$=160 which implies $\dim(H^4_{\mathcal{G}})$=45 $\dim(H^5_{\mathcal{G}})$=31. $e^{-2\beta_1-2\beta_2-2\beta_3}$ has multiplicity 3 in $\mathrm{ch}(H^4_{\mathcal{G}})$ and appears as a new term in $\mathrm{ch}(H^5_{\mathcal{G}})$.

changes in trivial cohomology:

case $\lambda = 0$:

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{3,6,7}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,2,4,5}\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, \underline{21}, 25, 17, 7, 1)$

[6] underlined dimensions are those differing from the generic ones

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 7, \underline{11}, \underline{11}, 7, 3, 1)$

$P(Y) \ = \ (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$

case $\lambda = -1$:

Add in $\mathrm{ch}(H^2_{\mathfrak{C}})$ $e^{-\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{3,7} - \omega^{5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^5_{\mathfrak{C}})$ $e^{-3\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{1,2,4,5,6}\rangle$; in $\mathrm{ch}(H^3_{\mathfrak{C}})$ $e^{-\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,3,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathfrak{C}})$ $e^{-3\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle$.

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{12}, 19, \underline{26}, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{8}, \underline{10}, \underline{10}, \underline{8}, 3, 1)$

$P(Y) \ = \ (1 + 2Y + 5Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

case $\lambda^2 - \lambda + 1 = 0$

Add in $\mathrm{ch}(H^3_{\mathfrak{C}})$ $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,6,7} + (1-\lambda)\omega^{2,5,7} - \lambda\omega^{3,4,7} - \omega^{4,5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathfrak{C}})$ $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{2,3,4,5}\rangle$

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 11, \underline{20}, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 7, \underline{10}, \underline{10}, 7, 3, 1)$

$P(Y) \ = \ (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 19, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 9, 9, 7, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7\bar\lambda$; $d^\star\omega^{3,4} = \omega^7(-\bar\lambda + 1)$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7}\bar\lambda - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}(\bar\lambda - 1) + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}(\bar\lambda - 1) + \omega^{4,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}\bar\lambda + \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}\bar\lambda$; $d^\star\omega^{2,5,6} = \omega^{6,7}\bar\lambda$; $d^\star\omega^{3,4,5} = \omega^{5,7}(\bar\lambda - 1)$; $d^\star\omega^{3,4,6} = \omega^{6,7}(\bar\lambda - 1)$; $d^\star\omega^{1,2,3,4} = \omega^{1,2,7}(-\bar\lambda + 1) - \omega^{1,4,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}\bar\lambda - \omega^{1,5,6} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}\bar\lambda$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}\bar\lambda - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,5,7}(-\bar\lambda + 1)$; $d^\star\omega^{1,3,4,6} = \omega^{1,6,7}(-\bar\lambda + 1) - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,5} =$

$\omega^{2,5,7}(-\bar\lambda+1)-\omega^{3,4,7}\bar\lambda-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,6,7}(-\bar\lambda+1)$; $d^\star\omega^{2,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}\bar\lambda$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}\bar\lambda$; $d^\star\omega^{3,4,5,6}=\omega^{5,6,7}(-\bar\lambda+1)$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}(\bar\lambda-1)+\omega^{1,3,4,7}\bar\lambda+\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}(\bar\lambda-1)-\omega^{2,3,4,7}+\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}\bar\lambda-\omega^{2,3,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{1,5,6,7}+\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7}=-\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}\bar\lambda-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,5,6,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}(\bar\lambda-1)-\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}(\bar\lambda-1)+\omega^{3,4,6,7}\bar\lambda$; $d^\star\omega^{2,3,4,5,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,2,5,6,7}(-\bar\lambda+1)-\omega^{1,3,4,6,7}\bar\lambda-\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7}=\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^4=\omega^4$; $\Delta\omega^5=\omega^5$; $\Delta\omega^6=\omega^6$; $\Delta\omega^7=\omega^7(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,6}=2\omega^{1,6}+\omega^{2,5}(\lambda-1)+\omega^{3,4}\lambda$; $\Delta\omega^{1,7}=\omega^{1,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+1)-\omega^{4,5}$; $\Delta\omega^{2,3}=\omega^{2,3}$; $\Delta\omega^{2,5}=\omega^{1,6}(\bar\lambda-1)+\omega^{2,5}(\lambda\bar\lambda+1)+\omega^{3,4}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{2,7}=\omega^{2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{4,6}\lambda$; $\Delta\omega^{3,4}=\omega^{1,6}\bar\lambda+\omega^{2,5}(\lambda\bar\lambda-\lambda-1)+\omega^{3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,7}=\omega^{3,7}(\lambda\bar\lambda+1)+\omega^{5,6}(-\lambda+1)$; $\Delta\omega^{4,5}=-\omega^{1,7}+2\omega^{4,5}$; $\Delta\omega^{4,6}=\omega^{2,7}\bar\lambda+2\omega^{4,6}$; $\Delta\omega^{4,7}=\omega^{4,7}(\lambda\bar\lambda+2)$; $\Delta\omega^{5,6}=\omega^{3,7}(-\bar\lambda+1)+2\omega^{5,6}$; $\Delta\omega^{5,7}=\omega^{5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{6,7}=\omega^{6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2,3}=3\omega^{1,2,3}$; $\Delta\omega^{1,2,5}=\omega^{1,2,5}(\lambda\bar\lambda+1)+\omega^{1,3,4}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{1,2,6}=2\omega^{1,2,6}-\omega^{2,3,4}\lambda$; $\Delta\omega^{1,2,7}=\omega^{1,2,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,4,6}\lambda+\omega^{2,4,5}$; $\Delta\omega^{1,3,4}=\omega^{1,2,5}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,3,6}=2\omega^{1,3,6}+\omega^{2,3,5}(\lambda-1)$; $\Delta\omega^{1,3,7}=\omega^{1,3,7}(\lambda\bar\lambda+1)+\omega^{1,5,6}(-\lambda+1)+\omega^{3,4,5}$; $\Delta\omega^{1,4,6}=\omega^{1,2,7}\bar\lambda+2\omega^{1,4,6}+\omega^{2,4,5}(\lambda-1)$; $\Delta\omega^{1,4,7}=\omega^{1,4,7}\lambda\bar\lambda$; $\Delta\omega^{1,5,6}=\omega^{1,3,7}(-\bar\lambda+1)+2\omega^{1,5,6}-\omega^{3,4,5}\lambda$; $\Delta\omega^{1,5,7}=\omega^{1,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{1,6,7}=\omega^{1,6,7}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{2,5,7}(\lambda-1)+\omega^{3,4,7}\lambda+\omega^{4,5,6}$; $\Delta\omega^{2,3,4}=-\omega^{1,2,6}\bar\lambda+\omega^{2,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,5}=\omega^{1,3,6}(\bar\lambda-1)+\omega^{2,3,5}(\lambda\bar\lambda+1)$; $\Delta\omega^{2,3,7}=2\omega^{2,3,7}+\omega^{2,5,6}(-\lambda+1)-\omega^{3,4,6}\lambda$; $\Delta\omega^{2,4,5}=\omega^{1,2,7}+\omega^{1,4,6}(\bar\lambda-1)+\omega^{2,4,5}(\lambda\bar\lambda+1)$; $\Delta\omega^{2,4,7}=\omega^{2,4,7}$; $\Delta\omega^{2,5,6}=\omega^{2,3,7}(-\bar\lambda+1)+\omega^{2,5,6}(\lambda\bar\lambda+1)+\omega^{3,4,6}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{2,5,7}=\omega^{1,6,7}(\bar\lambda-1)+\omega^{2,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)+\omega^{3,4,7}(\lambda\bar\lambda-\bar\lambda-1)+\omega^{4,5,6}\lambda$; $\Delta\omega^{2,6,7}=\omega^{2,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{3,4,5}=\omega^{1,3,7}-\omega^{1,5,6}\bar\lambda+\omega^{3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,6}=-\omega^{2,3,7}\bar\lambda+\omega^{2,5,6}(\lambda\bar\lambda-\lambda-1)+\omega^{3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,7}=\omega^{1,6,7}\bar\lambda+\omega^{2,5,7}(\lambda\bar\lambda-\lambda-1)+\omega^{3,4,7}(\lambda\bar\lambda+2)+\omega^{4,5,6}(\lambda-1)$; $\Delta\omega^{3,5,7}=\omega^{3,5,7}$; $\Delta\omega^{3,6,7}=\omega^{3,6,7}\lambda\bar\lambda$; $\Delta\omega^{4,5,6}=\omega^{1,6,7}+\omega^{2,5,7}\bar\lambda+\omega^{3,4,7}(\bar\lambda-1)+3\omega^{4,5,6}$; $\Delta\omega^{4,5,7}=3\omega^{4,5,7}$; $\Delta\omega^{4,6,7}=\omega^{4,6,7}(\lambda\bar\lambda+2)$; $\Delta\omega^{5,6,7}=\omega^{5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3,4}=\omega^{1,2,3,4}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3,5}=\omega^{1,2,3,5}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,3,6}=3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7}=3\omega^{1,2,3,7}+\omega^{1,2,5,6}(-\lambda+1)-\omega^{1,3,4,6}\lambda-\omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5}=\omega^{1,2,4,5}\lambda\bar\lambda$; $\Delta\omega^{1,2,4,6}=\omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6}=\omega^{1,2,3,7}(-\bar\lambda+1)+\omega^{1,2,5,6}(\lambda\bar\lambda+2)+\omega^{1,3,4,6}(\lambda\bar\lambda-\bar\lambda-1)+\omega^{2,3,4,5}\lambda$; $\Delta\omega^{1,2,5,7}=\omega^{1,2,5,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,4,7}(\lambda\bar\lambda-\bar\lambda-1)+$

$\omega^{1,4,5,6}\lambda$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2) - \omega^{2,3,4,7}\lambda - \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}(\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7}\bar\lambda+\omega^{1,2,5,6}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+3)+\omega^{2,3,4,5}(\lambda-1)$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4,7}(\lambda\bar\lambda+1)+\omega^{1,4,5,6}(\lambda-1)$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}(\lambda\bar\lambda+1)+\omega^{2,3,5,7}(\lambda-1)-\omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7}\bar\lambda+\omega^{1,3,4,7}(\bar\lambda-1)+2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}(\lambda\bar\lambda+1)+\omega^{2,4,5,7}(\lambda-1)$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)-\omega^{3,4,5,7}\lambda$; $\Delta\omega^{2,3,4,5} = -\omega^{1,2,3,7}+\omega^{1,2,5,6}\bar\lambda+\omega^{1,3,4,6}(\bar\lambda-1)+\omega^{2,3,4,5}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7}\bar\lambda+2\omega^{2,3,4,7}+\omega^{2,4,5,6}(\lambda-1)$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}\lambda\bar\lambda$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7}(\bar\lambda-1)+2\omega^{2,3,5,7}-\omega^{3,4,5,6}\lambda$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,6,7}+\omega^{2,3,4,7}(\bar\lambda-1)+\omega^{2,4,5,6}(\lambda\bar\lambda+1)$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7}(\bar\lambda-1)+2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{3,4,6,7}(\lambda\bar\lambda-\bar\lambda-1)$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,6,7}-\omega^{2,3,5,7}\bar\lambda+\omega^{3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7}\bar\lambda+2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7}(\lambda\bar\lambda-\lambda-1)+\omega^{3,4,6,7}(\lambda\bar\lambda+1)$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}(\lambda\bar\lambda-\lambda-\bar\lambda+3)$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}+\omega^{1,2,4,5,6}(\lambda-1)$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}(\lambda\bar\lambda+2)$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}-\omega^{1,3,4,5,6}\lambda$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}+\omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7}(\bar\lambda-1)+\omega^{1,2,4,5,6}(\lambda\bar\lambda+1)$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}(\lambda\bar\lambda-\lambda-\bar\lambda+2)+\omega^{1,3,4,6,7}(\lambda\bar\lambda-\bar\lambda-1)+\omega^{2,3,4,5,7}\lambda$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7}\bar\lambda+\omega^{1,3,4,5,6}(\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7}(\lambda\bar\lambda-\lambda-1)+\omega^{1,3,4,6,7}(\lambda\bar\lambda+1)+\omega^{2,3,4,5,7}(\lambda-1)$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,6,7}+\omega^{2,3,4,5,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+1)$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7}\bar\lambda+\omega^{1,3,4,6,7}(\bar\lambda-1)+2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}(2\lambda\bar\lambda-\lambda-\bar\lambda+2)$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6}(2\bar\lambda-1)+\omega^{2,5}(\bar\lambda-2)-\omega^{3,4}(\bar\lambda+1)\rangle + e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,7}(\bar\lambda+1)+\omega^{1,4,6}(-\lambda\bar\lambda+\bar\lambda-1)+\omega^{2,4,5}(\lambda-2)\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,7}(\bar\lambda-2)+\omega^{1,5,6}(\lambda\bar\lambda-\lambda+1)+\omega^{3,4,5}(\lambda+1)\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,7}(-2\bar\lambda^2-\bar\lambda+1)+\omega^{2,5,6}(\bar\lambda^2-\bar\lambda-2)+\omega^{3,4,6}(-\bar\lambda^2-2\bar\lambda-1)\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-2\beta_2-4\beta_3}\ \langle-\omega^{3,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3}\ \langle(\omega^{1,3,6,7}(-\bar\lambda+2)+\omega^{2,3,5,7}(\lambda\bar\lambda-\lambda+1)+\omega^{3,4,5,6}(\lambda+1))/((\lambda\bar\lambda-\lambda-\bar\lambda+3)(\lambda\bar\lambda+2))\rangle + e^{-2\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{2,4,6,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle(\omega^{1,2,6,7}(\bar\lambda+1)+\omega^{2,3,4,7}(\lambda\bar\lambda-\bar\lambda+1)+\omega^{2,4,5,6}(-\lambda+2))/((\lambda\bar\lambda-\lambda-\bar\lambda+3)(\lambda\bar\lambda+2))\rangle + e^{-\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}\ \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle(\omega^{1,2,5,7}(\lambda+

$1) + \omega^{1,3,4,7}(-\lambda + 2) + \omega^{1,4,5,6}(-2\lambda + 1))/(3(2\lambda\bar{\lambda} - \lambda - \bar{\lambda} + 2)(\bar{\lambda} + 1))\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,4,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-2\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-2\beta_1-4\beta_2-3\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-2\beta_2-4\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle(-\omega^{1,2,5,6,7}(\lambda + 1) + \omega^{1,3,4,6,7}(\lambda - 2) + \omega^{2,3,4,5,7}(2\lambda - 1))/(3(2\lambda\bar{\lambda} - \lambda - \bar{\lambda} + 2))\rangle + e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-3\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-4\beta_3}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-4\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

The preceding results are valid for: $(\bar{\lambda}^2-\bar{\lambda}+1)(\lambda-3\bar{\lambda}-1)(\lambda^2-\lambda+1)(\lambda\bar{\lambda}+\lambda-\bar{\lambda}^2-2\bar{\lambda}+1)(\bar{\lambda}-1)(\lambda-1)(\bar{\lambda})(\lambda)(-(\bar{\lambda}-2))(\bar{\lambda}+1)(\lambda-3\bar{\lambda}-1)(\lambda\bar{\lambda}-\lambda-1)(-(2\bar{\lambda}-1))(2\lambda-1)(-(\bar{\lambda}+1))(\lambda+1)(-(\bar{\lambda}-2))(\lambda-2)(-(\bar{\lambda}-2))(\lambda+1) \neq 0$.

Comments: singular values.

Among the values excluded by the computer, only $\lambda = 0,\ 1$, $\lambda = -1,\ 2,\ 1/2$ and $\lambda^2 - \lambda + 1 = 0$ are singular values. See previous comments for isomorphic cases.

case $\lambda = 0$**:**

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{3,6,7}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,4,5}\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, \underline{21}, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, \underline{11}, \underline{11}, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 40.

Riemann Hyp.: no

$P(Y) = (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y)$.

case $\lambda = -1$**:**

Add in $\mathrm{ch}(H^2_{\mathbb{C}})$ $e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{3,7} - \omega^{5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^5_{\mathbb{C}})$ $e^{-3\beta_1-3\beta_2-2\beta_3}\ \langle(-\omega^{1,2,3,4,7}-\omega^{1,2,4,5,6})/2\rangle$, in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6}+\omega^{2,3,5}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-3\beta_1-3\beta_2-2\beta_3}\ \langle(\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{12}, 19, \underline{26}, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, \underline{8}, \underline{10}, \underline{10}, \underline{8}, 3, 1)$

Total dimension of trivial cohomology: 44.
Length associated polynomial: 40.
Riemann Hyp.: no

$P(Y) \;=\; (1+2Y+5Y^2+2Y^3+Y^4)(1+Y^2)(1+Y).$

case $\lambda^2-\lambda+1=0$:

Add in $\mathrm{ch}(H^3_{\mathbb{C}})$ $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,6,7}+\omega^{2,5,7}(-\lambda+1)-\omega^{3,4,7}\lambda-\omega^{4,5,6}\rangle$ and correspondingly in $\mathrm{ch}(H^4_{\mathbb{C}})$ $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle(\lambda\omega^{1,2,5,6}+\lambda\omega^{1,3,4,6}+\omega^{1,2,3,7}-\omega^{1,2,5,6}+\omega^{2,3,4,5})/4\rangle$.

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,\underline{20},25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,\underline{10},\underline{10},7,3,1)$

Total dimension of trivial cohomology: 42.
Length associated polynomial: 40.
Riemann Hyp.: no

$P(Y) \;=\; (1+Y+3Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y).$

5.5.2 Algebra $\mathcal{G}_{7,3.1(iii)}$

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_6]=x_7$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix}
\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0\\
\xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\
\xi_1^5 & \xi_2^5 & \xi_2^4 & 0 & \xi_1^1+\xi_3^3 & 0 & 0\\
\xi_1^6 & \xi_2^6 & \xi_3^6 & 0 & 0 & \xi_2^2+\xi_3^3 & 0\\
\xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5+\xi_2^6 & -\xi_1^4+\xi_3^6 & \xi_6^7 & \xi_1^1+\xi_2^2+\xi_3^3
\end{pmatrix}$$

$ad\ x_1 = \delta_6^7+\delta_2^4, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_3^7, ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+\beta_3}\ \langle x_5\rangle+e^{\beta_2+\beta_3}\ \langle x_6\rangle+e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{\beta_1+\beta_2-\beta_3}\ \langle\delta^4_3\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta^5_2\rangle + 2e^{\beta_1}\ \langle\delta^4_2,\delta^7_6\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle\delta^6_1\rangle + e^{\beta_2+\beta_3}\ \langle\delta^7_1\rangle + e^{\beta_1+\beta_3}\ \langle\delta^7_2\rangle + e^{\beta_1+\beta_2}\ \langle\delta^7_3\rangle + 2e^{\beta_3}\ \langle\delta^5_1,\delta^6_2\rangle + 2e^{\beta_2}\ \langle\delta^4_1,\delta^6_3\rangle + 3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ^j_i's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta^4_3,\delta^5_1]=\delta^7_3$; $[\delta^4_3,\delta^6_2]=-\delta^7_3$; $[\delta^5_2,\delta^4_1]=\delta^7_2$; $[\delta^5_2,\delta^6_3]=-\delta^7_2$; $[\delta^4_2,\delta^5_1]=\delta^7_2$; $[\delta^4_2,\delta^6_2]=-\delta^7_2$; $[\delta^4_2,\delta^4_1]=\delta^7_3$; $[\delta^4_2,\delta^6_3]=-\delta^7_3$; $[\delta^6_1,\delta^7_6]=-\delta^7_1$; $[\delta^5_1,\delta^6_3]=-\delta^7_1$; $[\delta^6_2,\delta^4_1]=\delta^7_1$; $[\delta^6_2,\delta^7_6]=-\delta^7_2$; $[\delta^6_3,\delta^7_6]=-\delta^7_3$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1+\beta_2-\beta_3}\ \langle\delta^4_3\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta^5_2\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle\delta^6_1\rangle + e^{\beta_3}\ \langle\delta^6_2\rangle + e^{\beta_2}\ \langle\delta^6_3\rangle + e^{\beta_1}\ \langle\delta^4_2\rangle + 3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ^j_i's $i\neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5}\rangle + e^{\beta_2-2\beta_3}\ \langle x_4\omega^{3,5}\rangle + e^{\beta_1-2\beta_3}\ \langle x_4\omega^{3,6}\rangle + e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,5} + 2x_2(\omega^{1,6} - 1/2\omega^{2,5} - 1/2\omega^{3,4}) - 2x_3\omega^{3,5} + x_4\omega^{4,5}\rangle + 3e^{-\beta_2-\beta_3}\ \langle x_1\omega^{1,6} - x_3\omega^{3,6} + x_4\omega^{4,6}, x_1\omega^{1,6} - x_2\omega^{2,6} + x_5\omega^{5,6}, x_1(\omega^{2,5}+\omega^{3,4}) + x_2\omega^{2,6} + x_3\omega^{3,6} + x_4\omega^{2,7} + x_5\omega^{3,7}\rangle + e^{-2\beta_3}\ \langle x_1\omega^{3,5} - x_4(\omega^{3,7}-\omega^{5,6})\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3}+x_4\omega^{2,6}\rangle + e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4}+2x_2\omega^{2,4}-2x_3(\omega^{1,6}-1/2\omega^{2,5}-1/2\omega^{3,4}) + x_5\omega^{4,5}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,4} - x_5(\omega^{2,7}-\omega^{4,6})\rangle + e^{-2\beta_1+\beta_3}\ \langle x_6\omega^{1,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle + e^0\ \langle -x_4\omega^{1,2} + x_6\omega^{2,3}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2} + x_6\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_3\omega^{2,3} - x_4\omega^{2,4} + x_6\omega^{2,6}, x_3\omega^{2,3} - x_4\omega^{2,4} + x_5\omega^{2,5}, -x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_5\omega^{3,4}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3} + x_6\omega^{3,5}\rangle + 3e^{-\beta_3}\ \langle x_4\omega^{1,6} + x_6\omega^{3,6}, -x_1\omega^{1,3} + x_4(\omega^{1,6}+\omega^{3,4}), 2x_1\omega^{1,3} - x_2\omega^{2,3} - 2x_4(\omega^{1,6} - 1/2\omega^{2,5})\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_3\omega^{1,3} + x_6(\omega^{2,5}+\omega^{3,4}), x_6\omega^{1,6}, -x_3\omega^{1,3} + x_4\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-\beta_3} + 2e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 3e^{-\beta_1-2\beta_3} + e^{-\beta_1-3\beta_3} + e^{-2\beta_1+\beta_2-\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2+\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + 3e^{-2\beta_1-\beta_3} + e^{-2\beta_1-2\beta_3} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-\beta_3} + e^{-3\beta_1} + 2e^{-\beta_2-\beta_3} + 3e^{-\beta_2-2\beta_3} + e^{-\beta_2-3\beta_3} + 3e^{-2\beta_2-\beta_3} + e^{-2\beta_2-2\beta_3} + e^{-3\beta_2-\beta_3} + e^{-3\beta_2} + e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-\beta_1-\beta_2-3\beta_3} + e^{-\beta_1-\beta_2-4\beta_3} + 3e^{-\beta_1-2\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-3\beta_3} + 2e^{-\beta_1-3\beta_2-\beta_3} + 2e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-\beta_2-3\beta_3} + 3e^{-2\beta_1-2\beta_2-\beta_3} + 2e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-2\beta_3} + e^{-2\beta_1-3\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-\beta_2-2\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-2\beta_3} + e^{-3\beta_1-3\beta_3} + e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-2\beta_2-2\beta_3} + e^{-2\beta_2-3\beta_3} + e^{-3\beta_2-2\beta_3} + e^{-3\beta_2-3\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-\beta_1-3\beta_2-3\beta_3} + e^{-\beta_1-3\beta_2-4\beta_3} + e^{-\beta_1-4\beta_2-3\beta_3} + 2e^{-2\beta_1-2\beta_2-3\beta_3} + 2e^{-2\beta_1-2\beta_2-4\beta_3} + 2e^{-2\beta_1-3\beta_2-2\beta_3} + 2e^{-2\beta_1-3\beta_2-3\beta_3} + 2e^{-2\beta_1-4\beta_2-2\beta_3} + 2e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-\beta_2-4\beta_3} + 2e^{-3\beta_1-2\beta_2-2\beta_3} + 2e^{-3\beta_1-2\beta_2-3\beta_3} + 2e^{-3\beta_1-3\beta_2-\beta_3} +$

$2e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3}$.

ch$(H^6_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2-4\beta_3} + e^{-2\beta_1-4\beta_2-3\beta_3} + e^{-2\beta_1-4\beta_2-4\beta_3} + e^{-3\beta_1-2\beta_2-4\beta_3} + 2e^{-3\beta_1-3\beta_2-3\beta_3} + e^{-3\beta_1-3\beta_2-4\beta_3} + e^{-3\beta_1-4\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-4\beta_3} + 2e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-3\beta_3} + e^{-4\beta_1-4\beta_2-2\beta_3}$.

ch$(H^7_{\mathcal{G}})$=$e^{-3\beta_1-4\beta_2-4\beta_3} + e^{-4\beta_1-3\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-4\beta_3} + e^{-4\beta_1-4\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 15, 63, 130, 161, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 9, 29, 46, 46, 35, 18, 4)$

S=$4(\mu_1 + \mu_2 + \mu_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 19, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 9, 9, 8, 4, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

ch$(H^1_{\mathbb{C}})$= $e^{-\mu_1} \langle\omega^1\rangle + e^{-\mu_2} \langle\omega^2\rangle + e^{-\mu_3} \langle\omega^3\rangle + e^{-\mu_2-\mu_3} \langle\omega^6\rangle$.

ch$(H^2_{\mathbb{C}})$= $e^{-2\mu_1-\mu_2} \langle\omega^{1,4}\rangle + e^{-2\mu_1-\mu_3} \langle\omega^{1,5}\rangle + e^{-\mu_2-\mu_3} \langle\omega^{2,3}\rangle + e^{-\mu_1-2\mu_2} \langle\omega^{2,4}\rangle + e^{-2\mu_2-\mu_3} \langle\omega^{2,6}\rangle + e^{-\mu_1-\mu_2-\mu_3} \langle\omega^{1,6}\rangle + e^{-\mu_1-2\mu_3} \langle\omega^{3,5}\rangle + e^{-\mu_2-2\mu_3} \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\mu_1-2\mu_2}\ \langle\omega^{1,2,4}\rangle + e^{-2\mu_1-2\mu_3}\ \langle\omega^{1,3,5}\rangle + e^{-3\mu_1-\mu_2-\mu_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\mu_2-2\mu_3}\ \langle\omega^{2,3,6}\rangle + e^{-2\mu_1-2\mu_2-\mu_3}\ \langle\omega^{1,2,7} + \omega^{2,4,5}\rangle + e^{-\mu_1-3\mu_2-\mu_3}\ \langle\omega^{2,4,6}\rangle + e^{-2\mu_1-\mu_2-2\mu_3}\ \langle\omega^{1,3,7}-\omega^{3,4,5}\rangle + e^{-\mu_1-2\mu_2-2\mu_3}\ \langle\omega^{2,3,7}+\omega^{3,4,6}\rangle + e^{-\mu_1-\mu_2-3\mu_3}\ \langle\omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\mu_1-3\mu_2-\mu_3}\ \langle\omega^{1,2,4,7}\rangle + e^{-3\mu_1-\mu_2-3\mu_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-3\mu_1-2\mu_2-2\mu_3}\ \langle\omega^{1,2,5,7} - \omega^{1,3,4,7}\rangle + e^{-4\mu_1-2\mu_2-2\mu_3}\ \langle\omega^{1,4,5,7}\rangle + e^{-2\mu_1-3\mu_2-2\mu_3}\ \langle\omega^{2,3,4,7} + \omega^{2,4,5,6}\rangle + e^{-2\mu_1-2\mu_2-3\mu_3}\ \langle\omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-\mu_1-3\mu_2-3\mu_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-2\mu_1-4\mu_2-2\mu_3}\ \langle\omega^{2,4,6,7}\rangle + e^{-2\mu_1-2\mu_2-4\mu_3}\ \langle\omega^{3,5,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\mu_1-3\mu_2-2\mu_3}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-3\mu_1-4\mu_2-2\mu_3}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-4\mu_1-2\mu_2-3\mu_3}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-3\mu_1-3\mu_2-3\mu_3}\ \langle\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-3\mu_1-2\mu_2-4\mu_3}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-4\mu_1-3\mu_2-3\mu_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-2\mu_1-4\mu_2-3\mu_3}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-2\mu_1-3\mu_2-4\mu_3}\ \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-4\mu_1-3\mu_2-3\mu_3}\ \langle\omega^{1,2,3,4,5,7}\rangle + e^{-4\mu_1-4\mu_2-3\mu_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\mu_1-3\mu_2-4\mu_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\mu_1-4\mu_2-4\mu_3}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-4\mu_1-4\mu_2-4\mu_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + 5Y^2 + 4Y^3 + 5Y^4 + 3Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 19, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 9, 9, 8, 4, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 40.

Riemann Hyp.: no

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} - \omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} =$

$\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,5} = \omega^{1,6} + 2\omega^{2,5}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,6} + 2\omega^{3,4}$; $\Delta\omega^{3,7} = 2\omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 3\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 3\omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + 3\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 3\omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + 3\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 3\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^6\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,4}\rangle+ e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle 2\omega^{1,6} - \omega^{2,5} - \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,4}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle+e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,7}-2\omega^{1,4,6}+\omega^{2,4,5}\rangle+e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,4,6}\rangle+ e^{-2\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,7}-2\omega^{1,5,6}-\omega^{3,4,5}\rangle+e^{-\beta_1-2\beta_2-2\beta_3}\ \langle 2\omega^{2,3,7}-\omega^{2,5,6}+\omega^{3,4,6}\rangle+ e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-2\beta_1-2\beta_2-4\beta_3}\ \langle-\omega^{3,5,6,7}\rangle + e^{-2\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{2,4,6,7}\rangle + e^{-\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}\ \langle-\omega^{1,4,5,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3}\ \langle(-(\omega^{1,3,6,7} - 2\omega^{2,3,5,7} - \omega^{3,4,5,6}))/6\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle(-(\omega^{1,2,6,7} + 2\omega^{2,3,4,7} + \omega^{2,4,5,6}))/6\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle(-(\omega^{1,2,5,7} - \omega^{1,3,4,7} - 2\omega^{1,4,5,6}))/6\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,4,7}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-2\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-2\beta_1-4\beta_2-3\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-4\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/6\rangle + e^{-3\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-3\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-4\beta_3}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-3\beta_3}\ \langle-\omega^{1,2,3,4,5,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-4\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 3Y + 5Y^2 + 4Y^3 + 5Y^4 + 3Y^5 + Y^6)(1 + Y).$

5.5.3 Algebra $\mathcal{G}_{7,3.2}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_1, x_6] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & \xi_3^6 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4+\delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_1^6, ad\ x_5 = 0, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_3}\ \langle\delta_1^5\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle + 2e^{\beta_1}\ \langle\delta_3^5, \delta_2^4\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle + e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{3\beta_1}\ \langle\delta_2^7\rangle + e^{3\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle + e^{2\beta_1}\ \langle\delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^4, \delta_2^6] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^5, \delta_3^6] = -\delta_1^7$; $[\delta_2^5, \delta_3^6] = -\delta_2^7$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_3^5, \delta_3^6] = -\delta_3^7$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_2^4] = \delta_2^5$; $[\delta_1^6, \delta_2^4] = -\delta_1^7$; $[\delta_3^6, \delta_2^4] = -\delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{3\beta_1}\ \langle\delta_2^7\rangle + e^{3\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle + e^{2\beta_1}\ \langle\delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_2^5, \delta_3^6] = -\delta_2^7$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_3^5, \delta_3^6] = -\delta_3^7$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_2^4] = \delta_2^5$; $[\delta_3^6, \delta_2^4] = -\delta_3^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,5}\rangle + e^{-4\beta_1}\ \langle x_2\omega^{1,7}\rangle + 2e^{-2\beta_1}\ \langle x_3\omega^{1,5}, x_2\omega^{1,4}\rangle + e^{-4\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,7}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,6}\rangle + 2e^{-\beta_1}\ \langle x_3\omega^{1,3}, x_2\omega^{1,2}\rangle + e^{-3\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,6}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + 4e^{-\beta_2}\ \langle x_3\omega^{2,3} + x_5\omega^{2,5}, x_1\omega^{1,2} + x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} -$

$x_3\omega^{2,3} - x_4\omega^{2,4} + x_7\omega^{2,7}, -x_3\omega^{2,3} + x_5\omega^{3,4}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,4} + x_5\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_3(\omega^{2,5} + \omega^{3,4}) + x_5(\omega^{4,5} + 2\omega^{3,6}), -x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_4\omega^{2,6} - x_5\omega^{3,6} + x_6\omega^{2,7} + x_7\omega^{4,7}\rangle + e^{-2\beta_1-2\beta_2+\beta_3}\ \langle x_5(\omega^{4,6} - \omega^{2,7})\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 3x_2\omega^{2,3} + 2x_4\omega^{3,4} + x_6\omega^{3,6}, -x_1\omega^{1,3} + 2x_2\omega^{2,3} - x_4\omega^{3,4} + x_7\omega^{3,7}, x_5\omega^{3,5}\rangle + e^{3\beta_1-\beta_3}\ \langle x_7\omega^{2,3}\rangle + e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle + 2e^{2\beta_1-\beta_3}\ \langle x_6\omega^{2,3} + x_7\omega^{2,5}, -x_6\omega^{2,3} + x_7\omega^{3,4}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + e^{2\beta_1+\beta_2-2\beta_3}\ \langle x_7\omega^{3,5}\rangle + e^{\beta_1-\beta_3}\ \langle 2x_4\omega^{2,3} + x_6(\omega^{2,5} - \omega^{3,4}) + x_7\omega^{4,5}\rangle + e^{-\beta_1-\beta_3}\ \langle -2/3x_1\omega^{1,5} + x_2(\omega^{2,5} + \omega^{3,4}) + 2/3x_3\omega^{3,5} + 1/3x_4(\omega^{4,5} + 4\omega^{3,6}) + x_6(\omega^{3,7} + 1/3\omega^{5,6}) + x_7\omega^{5,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{2\beta_1-\beta_2-\beta_3} + e^{2\beta_1-2\beta_3} + e^{\beta_1-\beta_2-\beta_3} + e^{\beta_1-2\beta_3} + 4e^{-\beta_1-\beta_2-\beta_3} + 3e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + 3e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2} + 3e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2+\beta_3} + 2e^{-3\beta_1-\beta_3} + 4e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + 2e^{-4\beta_1-\beta_3} + 2e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2+\beta_3} + e^{-5\beta_1-\beta_3} + e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2+\beta_3} + 3e^{-\beta_2-\beta_3} + e^{-2\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + 3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-2\beta_3} + 3e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 2e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-2\beta_3} + 4e^{-5\beta_1-\beta_2-\beta_3} + 2e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-4\beta_2+\beta_3} + e^{-5\beta_1-2\beta_3} + 2e^{-6\beta_1-\beta_2-\beta_3} + 3e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2+\beta_3} + e^{-7\beta_1-\beta_2-\beta_3} + 2e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2+\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + 2e^{-5\beta_1-\beta_2-2\beta_3} + 2e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-4\beta_2-\beta_3} + e^{-6\beta_1-\beta_2-2\beta_3} + 2e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + 2e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-\beta_2-2\beta_3} + 3e^{-7\beta_1-2\beta_2-\beta_3} + 3e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2+\beta_3} + e^{-8\beta_1-2\beta_2-\beta_3} + e^{-8\beta_1-3\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-6\beta_1-4\beta_2-\beta_3} + e^{-6\beta_1-4\beta_2-2\beta_3} + e^{-7\beta_1-2\beta_2-2\beta_3} + e^{-7\beta_1-3\beta_2-\beta_3} + e^{-7\beta_1-3\beta_2-2\beta_3} + e^{-7\beta_1-4\beta_2-\beta_3} + e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-2\beta_2-2\beta_3} + 2e^{-8\beta_1-3\beta_2-\beta_3} + e^{-8\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-7\beta_1-4\beta_2-2\beta_3} + e^{-8\beta_1-3\beta_2-2\beta_3} + e^{-8\beta_1-4\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 17, 62, 130, 158, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 30, 45, 43, 28, 13, 3)$

S=$2(4\beta_1 + 2\beta_2 + \beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 20, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 10, 10, 7, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,6}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,6}$;

$d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} - \omega^{1,5,6}$; $d\omega^{6,7} = -\omega^{1,4,7}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,5,7} = -\omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7}-\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6}+\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7}+\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-7\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,4,6}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,4,7}+\omega^{2,4,5,6}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,7}+\omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,5,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-7\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2-\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-8\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+3Y^2+Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1,3,11,20,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1,3,7,10,10,7,3,1)$

Total dimension of trivial cohomology: 42.

Length associated polynomial: 42.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{4}$; $d^{\star}\omega^{1,3} = -\omega^{5}$; $d^{\star}\omega^{1,4} = -\omega^{6}$; $d^{\star}\omega^{1,6} = -\omega^{7}$; $d^{\star}\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^{\star}\omega^{1,2,4} = -\omega^{2,6}$; $d^{\star}\omega^{1,2,5} = -\omega^{4,5}$; $d^{\star}\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^{\star}\omega^{1,2,7} = -\omega^{4,7}$; $d^{\star}\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^{\star}\omega^{1,3,6} = -\omega^{3,7} - \omega^{5,6}$; $d^{\star}\omega^{1,3,7} = -\omega^{5,7}$; $d^{\star}\omega^{1,4,5} = \omega^{5,6}$; $d^{\star}\omega^{1,4,6} = -\omega^{4,7}$; $d^{\star}\omega^{1,4,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,5,6} = -\omega^{5,7}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{2,3,6} + \omega^{2,4,5}$; $d^{\star}\omega^{1,2,3,5} = \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $d^{\star}\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^{\star}\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^{\star}\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^{\star}\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^{\star}\omega^{1,3,4,6} = -\omega^{3,4,7} + \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^{\star}\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^{\star}\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{2,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^{\star}\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{2,3,4,5,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7} + \omega^{5,6}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = \omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,3,7} - \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + 2\omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$;

$\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7} - \omega^{1,5,6}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2} \langle-\omega^{2,4,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle(\omega^{2,3,5,7} + \omega^{3,4,5,6})/2\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle(-(\omega^{2,3,4,7} + \omega^{2,4,5,6}))/2\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-7\beta_1-2\beta_2-\beta_3} \langle-\omega^{1,5,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{1,4,5,7}))/2\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,5,7} - \omega^{1,4,5,6}))/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-4\beta_2-\beta_3} \langle-\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle-\omega^{2,3,4,5,6}\rangle + e^{-8\beta_1-3\beta_2-\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-2\beta_2-2\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-7\beta_1-4\beta_2} \langle-\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-7\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-3\beta_2-2\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-8\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y + Y^2)(1 + Y).$

5.5.4 Algebra $\mathcal{G}_{7,3.3}$ †

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_4]=x_6$; $[x_1,x_6]=x_7$; $[x_2,x_3]=x_5$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^3 & \xi_2^2+\xi_3^3 & 0 & 0\\ \xi_1^6 & \xi_2^6 & 0 & \xi_2^4 & 0 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^6 & 0 & \xi_2^4 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4+\delta_3^5, ad\ x_3=-\delta_2^5, ad\ x_4=-\delta_1^6, ad\ x_5=0, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})=\ e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_2+\beta_3}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=\ e^{\beta_2+\beta_3}\ \langle x_5\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=\ e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle+2e^{\beta_2}\ \langle\delta_1^4,\delta_3^5\rangle+e^{-\beta_1+\beta_2+\beta_3}\ \langle\delta_1^5\rangle+e^{\beta_3}\ \langle\delta_2^5\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle+3e^0\ \langle t_3,t_1,t_2\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{3\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^5]=-\delta_2^5$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^4,\delta_1^3]=\delta_1^5$; $[\delta_1^4,\delta_2^6]=-\delta_1^7$; $[\delta_1^4,\delta_2^4]=-\delta_1^6$; $[\delta_3^5,\delta_1^3]=\delta_1^5$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_2^4]=-\delta_2^5$; $[\delta_1^6,\delta_2^4]=-\delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=\ e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle+e^{-\beta_1+\beta_2+\beta_3}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle+3e^0\ \langle t_3,t_1,t_2\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{3\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_1^4,\delta_1^3]=\delta_1^5$; $[\delta_3^5,\delta_1^3]=\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})=\ e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}+x_3\omega^{1,3},x_2\omega^{1,2}+x_5\omega^{1,5},x_4\omega^{1,4}\rangle+e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,4}\rangle+e^{-2\beta_2}\ \langle x_3\omega^{2,5}\rangle+e^{-4\beta_1+\beta_3}\ \langle x_5\omega^{1,7}\rangle+e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle+3e^{-\beta_2}\ \langle x_3\omega^{2,3},x_1\omega^{1,2}+2x_4\omega^{2,4}+x_6\omega^{2,6},-x_1\omega^{1,2}-x_4\omega^{2,4}+x_7\omega^{2,7}\rangle+3e^{-\beta_3}\ \langle -x_2\omega^{2,3}+x_4\omega^{1,5},x_1\omega^{1,3}-3x_2\omega^{2,3}+2x_4\omega^{3,4}+x_6\omega^{3,6},-x_1\omega^{1,3}+2x_2\omega^{2,3}-x_4\omega^{3,4}+x_7\omega^{3,7}\rangle+e^{-3\beta_1}\ \langle x_4\omega^{1,7}\rangle+e^{-3\beta_1-\beta_2+\beta_3}\ \langle -x_3\omega^{1,6}+x_5\omega^{4,6}\rangle+e^{-2\beta_1}\ \langle x_4\omega^{1,6}\rangle+e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,4}\rangle+e^{3\beta_1-\beta_2-\beta_3}\ \langle x_7\omega^{2,5}\rangle+e^{2\beta_1-\beta_3}\ \langle -x_6\omega^{2,3}+x_7\omega^{3,4}\rangle+e^{3\beta_1-2\beta_3}\ \langle x_7\omega^{3,5}\rangle+e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,4}+2x_2\omega^{2,4}+x_3(\omega^{1,5}-\omega^{3,4})+x_4\omega^{2,6}-x_5\omega^{4,5}+x_6\omega^{2,7}+x_7\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{3\beta_1-\beta_2-2\beta_3}+e^{2\beta_1-\beta_2-2\beta_3}+e^{2\beta_1-2\beta_2-\beta_3}+3e^{-\beta_1-\beta_2-\beta_3}+3e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2}+e^{-\beta_1-\beta_3}+e^{-\beta_1-2\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-\beta_3}+e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-\beta_2}+3e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2+\beta_3}+e^{-3\beta_1-\beta_3}+3e^{-4\beta_1-\beta_2}+e^{-4\beta_1-2\beta_2+\beta_3}+e^{-4\beta_1-\beta_3}+e^{-5\beta_1-\beta_2}+e^{-6\beta_1-\beta_2+\beta_3}+e^{-6\beta_1-2\beta_2+\beta_3}+2e^{-\beta_2-2\beta_3}+3e^{-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{2\beta_1-2\beta_2-2\beta_3}+e^{-\beta_1-\beta_2-2\beta_3}+e^{-\beta_1-2\beta_2-\beta_3}+e^{-\beta_1-2\beta_2-2\beta_3}+2e^{-\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-2\beta_3}+e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2-\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+3e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2}+2e^{-4\beta_1-\beta_2-\beta_3}+e^{-4\beta_1-\beta_2-2\beta_3}+e^{-4\beta_1-2\beta_2-\beta_3}+2e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-3\beta_2}+e^{-5\beta_1-\beta_2-\beta_3}+e^{-5\beta_1-2\beta_2-\beta_3}+e^{-5\beta_1-4\beta_2}+e^{-6\beta_1-\beta_2-\beta_3}+3e^{-6\beta_1-2\beta_2}+e^{-6\beta_1-3\beta_2+\beta_3}+e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-4\beta_2+\beta_3}+e^{-7\beta_1-2\beta_2+\beta_3}+2e^{-2\beta_2-2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-\beta_1-3\beta_2-2\beta_3}+e^{-2\beta_1-4\beta_2-2\beta_3}+2e^{-3\beta_1-3\beta_2-2\beta_3}+2e^{-3\beta_1-4\beta_2-\beta_3}+2e^{-4\beta_1-2\beta_2-2\beta_3}+2e^{-4\beta_1-3\beta_2-\beta_3}+e^{-5\beta_1-2\beta_2-2\beta_3}+e^{-5\beta_1-4\beta_2-\beta_3}+e^{-5\beta_1-5\beta_2-\beta_3}+e^{-6\beta_1-2\beta_2-\beta_3}+e^{-6\beta_1-2\beta_2-2\beta_3}+3e^{-6\beta_1-3\beta_2-\beta_3}+3e^{-6\beta_1-4\beta_2}+e^{-6\beta_1-5\beta_2}+e^{-7\beta_1-2\beta_2-\beta_3}+2e^{-7\beta_1-3\beta_2}+e^{-7\beta_1-4\beta_2+\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-4\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-2\beta_3}+e^{-5\beta_1-5\beta_2-2\beta_3}+e^{-6\beta_1-3\beta_2-2\beta_3}+e^{-6\beta_1-4\beta_2-\beta_3}+e^{-6\beta_1-4\beta_2-2\beta_3}+2e^{-6\beta_1-5\beta_2-\beta_3}+e^{-7\beta_1-3\beta_2-\beta_3}+e^{-7\beta_1-3\beta_2-2\beta_3}+e^{-7\beta_1-4\beta_2-\beta_3}+e^{-7\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-6\beta_1-5\beta_2-2\beta_3}+e^{-7\beta_1-4\beta_2-2\beta_3}+e^{-7\beta_1-5\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,15,56,126,156,115,45,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,10,22,35,37,26,13,3)$

S=$7\beta_1+5\beta_2+2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,11,19,25,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,7,9,9,7,3,1)$

$d\omega^4=-\omega^{1,2}$; $d\omega^5=-\omega^{2,3}$; $d\omega^6=-\omega^{1,4}$; $d\omega^7=-\omega^{1,6}$; $d\omega^{1,5}=\omega^{1,2,3}$; $d\omega^{2,6}=-\omega^{1,2,4}$; $d\omega^{2,7}=-\omega^{1,2,6}$; $d\omega^{3,4}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{1,3,4}$; $d\omega^{3,7}=-\omega^{1,3,6}$; $d\omega^{4,5}=-\omega^{1,2,5}+\omega^{2,3,4}$; $d\omega^{4,6}=-\omega^{1,2,6}$; $d\omega^{4,7}=-\omega^{1,2,7}-\omega^{1,4,6}$; $d\omega^{5,6}=\omega^{1,4,5}-\omega^{2,3,6}$; $d\omega^{5,7}=-\omega^{1,5,6}-\omega^{2,3,7}$; $d\omega^{6,7}=-\omega^{1,4,7}$; $d\omega^{1,4,5}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=\omega^{1,2,3,6}$; $d\omega^{1,5,7}=\omega^{1,2,3,7}$; $d\omega^{2,3,6}=-\omega^{1,2,3,4}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{1,2,4,5}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}$; $d\omega^{2,6,7}=-\omega^{1,2,4,7}$; $d\omega^{3,4,5}=\omega^{1,2,3,5}$; $d\omega^{3,4,6}=\omega^{1,2,3,6}$; $d\omega^{3,4,7}=\omega^{1,2,3,7}-\omega^{1,3,4,6}$; $d\omega^{3,5,6}=\omega^{1,3,4,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=-\omega^{1,3,4,7}$; $d\omega^{4,5,6}=-\omega^{1,2,5,6}+\omega^{2,3,4,6}$; $d\omega^{4,5,7}=-\omega^{1,2,5,7}-\omega^{1,4,5,6}+\omega^{2,3,4,7}$; $d\omega^{4,6,7}=-\omega^{1,2,6,7}$; $d\omega^{5,6,7}=\omega^{1,4,5,7}-\omega^{2,3,6,7}$; $d\omega^{1,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,7}=$

$-\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{2,3,4,6,7}$; $d\omega^{1,4,5,6,7} = -\omega^{1,2,3,4,6,7}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,5} - \omega^{3,4}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6} - \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,5,7}\rangle + e^{-3\beta_1-4\beta_2-\beta_3} \langle\omega^{2,4,5,6}\rangle + e^{-6\beta_1-4\beta_2} \langle\omega^{2,4,6,7}\rangle + e^{-3\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,7} + \omega^{3,4,5,6}\rangle + e^{-6\beta_1-3\beta_2-\beta_3} \langle\omega^{1,5,6,7} - \omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,5,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-5\beta_2-\beta_3} \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_1-5\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-5\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-7\beta_1-5\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 11, 19, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 7, 9, 9, 7, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^5$; $d^\star\omega^{1,2,3} = \omega^{1,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{4,5}$; $d^\star\omega^{2,3,6} = -\omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,5} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,5} = \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} =$

$\omega^{2,5,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{2,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,5,7} - \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,5} + \omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = \omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = 2\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,7} + \omega^{1,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} + \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,5} + \omega^{2,3,4}$; $\Delta\omega^{2,3,6} = 2\omega^{2,3,6}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{4,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 3\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 3\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = 2\omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = \omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,2,5,6,7} +$

$\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,5} - \omega^{3,4}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{2,7} - \omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{1,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5} + \omega^{2,3,4}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6} - \omega^{2,3,7} - 2\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2} \langle-\omega^{2,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-\beta_3} \langle-\omega^{2,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-2\beta_3} \langle(\omega^{2,3,5,7} + \omega^{3,4,5,6})/2\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-6\beta_1-3\beta_2-\beta_3} \langle(-(\omega^{1,5,6,7} - \omega^{3,4,6,7}))/2\rangle + e^{-7\beta_1-3\beta_2} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle-\omega^{1,3,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle(2\omega^{1,2,5,7} - \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-5\beta_2-\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3} \langle-\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-3\beta_2-2\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle(-\omega^{1,2,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-7\beta_1-4\beta_2} \langle-\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle(\omega^{1,2,3,5,7} + \omega^{1,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-6\beta_1-5\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2-2\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-5\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-7\beta_1-5\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 2Y + 4Y^2 + 2Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.5.5 Algebra $\mathcal{G}_{7,3,4}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_2, x_4] = x_6$; $[x_3, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & 0 & -\xi_1^5 & 0 & \xi_1^1+2\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_2^6, ad\ x_5 = -\delta_3^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : T=$\langle t_1, t_2, t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+2\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $2e^{\beta_1}\ \langle \delta_2^4, \delta_3^5\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + e^{\beta_1+\beta_2}\ \langle \delta_2^6\rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^6\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{2\beta_3}\ \langle \delta_1^7\rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle \delta_2^7\rangle + e^{\beta_1+\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_3}\ \langle \delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4, \delta_1^4] = \delta_2^6$; $[\delta_3^5, \delta_1^5] = \delta_3^7$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1}\ \langle \delta_2^4\rangle + e^{2\beta_2}\ \langle \delta_1^6\rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^6\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{2\beta_3}\ \langle \delta_1^7\rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle \delta_2^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

ch($H^2_{\mathcal{G}}$)= $e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle + 2e^{-2\beta_1}\ \langle x_2\omega^{1,4}, x_3\omega^{1,5}\rangle + 3e^{-\beta_3}\ \langle -2x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} + x_6\omega^{3,6}, x_1\omega^{1,3} + x_4\omega^{2,5}\rangle + e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) - 2x_3\omega^{3,5} + x_4\omega^{4,5}\rangle + 3e^{-\beta_2}\ \langle -2x_1\omega^{1,2} - x_3\omega^{2,3} + x_5\omega^{2,5}, x_4\omega^{2,4}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_7\omega^{2,7}\rangle + e^{-2\beta_2}\ \langle x_4\omega^{2,6}\rangle + e^{-2\beta_3}\ \langle x_1\omega^{3,5} + x_4\omega^{2,7}\rangle + e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5}\rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle x_6\omega^{1,5}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_6\omega^{1,6}, x_3\omega^{1,3} + x_7\omega^{1,7}\rangle + e^{2\beta_2-3\beta_3}\ \langle x_6\omega^{3,7}\rangle + e^{-\beta_1-\beta_2+2\beta_3}\ \langle x_7\omega^{1,4}\rangle + e^{-3\beta_2+2\beta_3}\ \langle x_7\omega^{2,6}\rangle$.

ch($H^3_{\mathcal{G}}$)=$e^{-\beta_1+2\beta_2-4\beta_3} + 3e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2+2\beta_3} + 3e^{-\beta_1-3\beta_3} + e^{-2\beta_1+2\beta_2-3\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-3\beta_2+2\beta_3} + e^{-2\beta_1-3\beta_2} + 3e^{-2\beta_1-\beta_3} + e^{-2\beta_1-3\beta_3} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-\beta_2-3\beta_3} + e^{-2\beta_2-\beta_3} + e^{-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-3\beta_3} + e^{-\beta_1-\beta_2-4\beta_3} + e^{-\beta_1-2\beta_2-3\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-3\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1+2\beta_2-4\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-3\beta_3} + 3e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-2\beta_1-4\beta_2+2\beta_3} + 2e^{-2\beta_1-4\beta_2} + 2e^{-2\beta_1-4\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + 3e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-3\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-3\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-\beta_2-4\beta_3} + e^{-2\beta_1-2\beta_2-4\beta_3} + e^{-2\beta_1-3\beta_2-3\beta_3} + e^{-2\beta_1-3\beta_2-4\beta_3} + e^{-2\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1-4\beta_2-3\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-\beta_2-4\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + 3e^{-3\beta_1-3\beta_2-3\beta_3} + e^{-3\beta_1-4\beta_2-\beta_3} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-4\beta_3} + 2e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-4\beta_2} + e^{-4\beta_1-4\beta_3} + e^{-5\beta_1-2\beta_2-3\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-3\beta_2-4\beta_3} + e^{-3\beta_1-4\beta_2-3\beta_3} + e^{-3\beta_1-4\beta_2-4\beta_3} + e^{-4\beta_1-\beta_2-4\beta_3} + e^{-4\beta_1-3\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-4\beta_3} + e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2-3\beta_3} + e^{-5\beta_1-2\beta_2-4\beta_3} + 2e^{-5\beta_1-3\beta_2-3\beta_3} + e^{-5\beta_1-4\beta_2-2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-4\beta_1-4\beta_2-4\beta_3} + e^{-5\beta_1-3\beta_2-4\beta_3} + e^{-5\beta_1-4\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(2, 13, 55, 124, 157, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(2, 8, 19, 32, 36, 26, 12, 3)$

S$=5\beta_1 + 4\beta_2 + 4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,4}$; $d\omega^7 = -\omega^{3,5}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = -\omega^{2,4,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,3,5,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7}$; $d\omega^{3,5,6} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{2,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,4,5,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2}\ \langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_3}\ \langle\omega^{1,5}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle+ e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,5}+\omega^{3,4}\rangle + e^{-\beta_1-3\beta_3}\ \langle\omega^{3,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-3\beta_3}\ \langle\omega^{1,5,7}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3}\ \langle\omega^{2,5,7}+\omega^{3,4,7}\rangle + e^{-2\beta_1-4\beta_3}\ \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-4\beta_2}\ \langle\omega^{1,2,4,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,2,5,7}\rangle + e^{-3\beta_1-4\beta_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3}\ \langle\omega^{2,3,4,6}\rangle + e^{-2\beta_1-\beta_2-4\beta_3}\ \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,5,6,7}+\omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\beta_1-4\beta_2-\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle + e^{-4\beta_1-\beta_2-4\beta_3}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-4\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-4\beta_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-5\beta_1-4\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+3Y^2+Y^4)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,10,21,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,3,6,10,10,6,3,1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$;

$d^{\star}\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,3,5,6}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,6} = \omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^{4} = \omega^{4}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-3\beta_3} \langle\omega^{3,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,5} + \omega^{1,3,4}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle +$

$e^{-3\beta_1-3\beta_2}$ $\langle\omega^{1,4,6}\rangle$ + $e^{-3\beta_1-3\beta_3}$ $\langle\omega^{1,5,7}\rangle$ + $e^{-\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,3,6}\rangle$ + $e^{-\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{2,3,7}\rangle$ + $e^{-2\beta_1-4\beta_2}$ $\langle\omega^{2,4,6}\rangle$ + $e^{-2\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,5,6}+\omega^{3,4,6}\rangle$ + $e^{-2\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{2,5,7}+\omega^{3,4,7}\rangle + e^{-2\beta_1-4\beta_3}$ $\langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=$ $e^{-3\beta_1-3\beta_2-3\beta_3}$ $\langle(\omega^{2,5,6,7}+\omega^{3,4,6,7})/2\rangle$ + $e^{-2\beta_1-3\beta_2-3\beta_3}$ $\langle\omega^{2,3,6,7}\rangle$ + $e^{-2\beta_1-\beta_2-4\beta_3}$ $\langle-\omega^{2,3,5,7}\rangle$ + $e^{-2\beta_1-4\beta_2-\beta_3}$ $\langle-\omega^{2,3,4,6}\rangle$ + $e^{-4\beta_1-\beta_2-3\beta_3}$ $\langle-\omega^{1,4,5,7}\rangle$ + $e^{-4\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,4,5,6}\rangle$ + $e^{-3\beta_1-4\beta_3}$ $\langle\omega^{1,3,5,7}\rangle$ + $e^{-3\beta_1-\beta_2-3\beta_3}$ $\langle(-(\omega^{1,2,5,7}+\omega^{1,3,4,7}))/2\rangle$ + $e^{-3\beta_1-3\beta_2-\beta_3}$ $\langle(\omega^{1,2,5,6}+\omega^{1,3,4,6})/2\rangle$ + $e^{-3\beta_1-4\beta_2}$ $\langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=$ $e^{-3\beta_1-3\beta_2-4\beta_3}$ $\langle\omega^{2,3,5,6,7}\rangle$ + $e^{-3\beta_1-4\beta_2-3\beta_3}$ $\langle-\omega^{2,3,4,6,7}\rangle$ + $e^{-5\beta_1-3\beta_2-3\beta_3}$ $\langle\omega^{1,4,5,6,7}\rangle$ + $e^{-4\beta_1-\beta_2-4\beta_3}$ $\langle-\omega^{1,3,4,5,7}\rangle$ + $e^{-4\beta_1-3\beta_2-3\beta_3}$ $\langle(\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7})/2\rangle$ + $e^{-4\beta_1-4\beta_2-\beta_3}$ $\langle-\omega^{1,2,4,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=$ $e^{-4\beta_1-4\beta_2-4\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-5\beta_1-3\beta_2-4\beta_3}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-5\beta_1-4\beta_2-3\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})=$ $e^{-5\beta_1-4\beta_2-4\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y^2+Y^4)(1+Y)^3.$

5.5.6 Algebra $\mathcal{G}_{7,3.5}$ †

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_2,x_4]=x_6$; $[x_2,x_5]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & \xi_2^3 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^4 & 0 & \xi_1^1+2\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & -\xi_1^5 & -\xi_1^4 & 2\xi_2^3 & \xi_1^1+\xi_2^2+\xi_3^3 \end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-(\delta_2^6+\delta_3^7), ad\ x_5=-\delta_2^7, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+\beta_3}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_2^6,\delta_3^7\rangle+e^{\beta_1+2\beta_2-\beta_3}\ \langle\delta_3^6\rangle+e^{\beta_2+\beta_3}\ \langle\delta_1^7\rangle+e^{\beta_1+\beta_3}\ \langle\delta_2^7\rangle+e^{\beta_3}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle+3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^5,\delta_1^4]=\delta_2^7$; $[\delta_2^4,\delta_1^5]=\delta_2^7$; $[\delta_2^4,\delta_1^4]=\delta_3^7+\delta_2^6$; $[\delta_1^6,\delta_2^3]=-2\delta_1^7$; $[\delta_2^6,\delta_2^3]=-2\delta_2^7$; $[\delta_3^6,\delta_2^3]=-2\delta_3^7+\delta_2^6$; $[\delta_3^7,\delta_2^3]=\delta_2^7$; $[\delta_1^4,\delta_2^3]=-\delta_1^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+2\beta_2-\beta_3}\ \langle\delta_3^6\rangle+e^{\beta_2+\beta_3}\ \langle\delta_1^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_2^6\rangle+e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle+3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^6,\delta_2^3]=-2\delta_1^7$; $[\delta_3^6,\delta_2^3]=3\delta_2^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle+e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,4}\rangle+e^{-2\beta_1}\ \langle x_2\omega^{1,4}+x_3\omega^{1,5}\rangle+e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})-2x_3\omega^{3,5}+x_4\omega^{4,5}\rangle+e^{-3\beta_2+\beta_3}\ \langle x_5\omega^{2,6}\rangle+e^{-2\beta_2}\ \langle -x_1\omega^{2,4}+x_5\omega^{2,7}\rangle+e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4}+2x_2\omega^{2,4}+x_3(\omega^{2,5}+\omega^{3,4})+x_5\omega^{4,5}\rangle+e^{-\beta_1+2\beta_2-\beta_3}\ \langle x_6\omega^{1,5}\rangle+2e^{-\beta_1}\ \langle -x_3\omega^{1,3}+x_4\omega^{1,4}+x_6\omega^{1,6}, x_2\omega^{1,2}+2x_3\omega^{1,3}-x_4\omega^{1,4}+x_7\omega^{1,7}\rangle+e^{2\beta_2-2\beta_3}\ \langle x_6\omega^{3,5}\rangle+3e^{-\beta_3}\ \langle 2x_1\omega^{1,3}-x_2\omega^{2,3}+x_4\omega^{2,5}+x_6\omega^{3,6}, -3x_1\omega^{1,3}+x_2\omega^{2,3}-x_4(\omega^{2,5}-\omega^{3,4}), -2x_1\omega^{1,3}+x_2\omega^{2,3}-x_4\omega^{2,5}+x_6\omega^{2,7}\rangle+e^{\beta_2-2\beta_3}\ \langle x_4\omega^{3,5}+x_6\omega^{3,7}\rangle+e^{-\beta_1+\beta_2}\ \langle x_6\omega^{1,4}\rangle+e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}+x_7\omega^{1,6}\rangle+$

$e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + 3e^{-\beta_2}\ \langle -x_1\omega^{1,2}+x_5\omega^{2,5}, -x_1\omega^{1,2}-x_3\omega^{2,3}+x_4\omega^{2,4}, x_6\omega^{2,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{-\beta_1+2\beta_2-2\beta_3} + e^{-\beta_1+\beta_2-3\beta_3} + e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + 3e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2+\beta_3} + e^{-\beta_1-4\beta_2} + 3e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2} + 3e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-\beta_3} + e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3} + e^{-2\beta_2-2\beta_3} + e^{-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-3\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + 2e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + e^{-\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1+\beta_2-3\beta_3} + 2e^{-2\beta_1-\beta_2-3\beta_3} + 3e^{-2\beta_1-2\beta_2-2\beta_3} + 3e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-4\beta_2-\beta_3} + 2e^{-2\beta_1-4\beta_2} + e^{-2\beta_1-5\beta_2} + 2e^{-2\beta_1-2\beta_3} + e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + 3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2+\beta_3} + e^{-3\beta_1-2\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + 2e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-3\beta_3} + 2e^{-2\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1-4\beta_2-3\beta_3} + e^{-2\beta_1-5\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2} + 2e^{-3\beta_1-5\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-3\beta_3} + 3e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-5\beta_2} + e^{-5\beta_1-3\beta_2-2\beta_3} + e^{-5\beta_1-4\beta_2-\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-3\beta_1-4\beta_2-3\beta_3} + e^{-3\beta_1-5\beta_2-2\beta_3} + e^{-3\beta_1-5\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-4\beta_2-3\beta_3} + e^{-4\beta_1-5\beta_2-\beta_3} + e^{-4\beta_1-5\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-3\beta_3} + 2e^{-5\beta_1-4\beta_2-2\beta_3} + e^{-5\beta_1-5\beta_2-\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-4\beta_1-5\beta_2-3\beta_3} + e^{-5\beta_1-4\beta_2-3\beta_3} + e^{-5\beta_1-5\beta_2-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 56, 125, 157, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 9, 21, 34, 37, 26, 12, 3)$

S=$5\beta_1 + 5\beta_2 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,4}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{2,3,4,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} =$

$\omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,7}+\omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,7}+\omega^{1,5,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,5,7}+\omega^{3,4,7}+2\omega^{3,5,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,5,7}+\omega^{1,3,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1-5\beta_2-\beta_3} \langle\omega^{2,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\beta_1-5\beta_2-\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,3,4,5,7}\rangle + e^{-5\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-5\beta_2-2\beta_3} \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-3\beta_3} \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-5\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_2-3\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-5\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-5\beta_1-5\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+3Y^2+Y^4)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1,3,10,21,24,17,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1,3,6,10,10,6,3,1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = \omega^{1,6}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7} - \omega^{3,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7} + \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{1,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{1,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} + \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{2,7} - \omega^{3,6}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7} - \omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = -\omega^{1,2,7} + 2\omega^{1,3,6}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{1,5,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7} + \omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 3\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = 3\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7} - \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{2,4,7} + 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7} - \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7} - \omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{2,5,7} - \omega^{3,4,7} + \omega^{3,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,4,7} + 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = 3\omega^{1,3,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 3\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} =$

$\omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = 2\omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2} \langle\omega^{2,6}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,7} + \omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,7} + \omega^{1,5,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle 2\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7} + 2\omega^{3,5,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-3\beta_1-5\beta_2-\beta_3} \langle -\omega^{2,4,6,7}\rangle + e^{-2\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle -\omega^{2,3,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle(\omega^{2,3,4,7} + \omega^{2,3,5,6})/2\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle -\omega^{1,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle(-(\omega^{1,2,5,7} + \omega^{1,3,4,7} + 2\omega^{1,3,5,6}))/6\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle(2\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6})/6\rangle + e^{-3\beta_1-4\beta_2} \langle -\omega^{1,2,4,6}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-3\beta_1-4\beta_2-3\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-5\beta_2-2\beta_3} \langle -\omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle -\omega^{1,3,4,5,7}\rangle + e^{-4\beta_1-5\beta_2-\beta_3} \langle -\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle(\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-4\beta_1-5\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_2-3\beta_3} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-5\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-5\beta_1-5\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3.$

5.5.7 Algebra $\mathcal{G}_{7,3.6}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_5] = x_7$; $[x_2, x_3] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 & \xi_3^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1 + \xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^3 & \xi_1^2 & \xi_2^2 + \xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_3^5 & 0 & 2\xi_1^1 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4 + \delta_3^6, ad\ x_3 = -(\delta_1^5 + \delta_2^6), ad\ x_4 = 0, ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1} \langle x_1 \rangle + e^{\beta_2} \langle x_2 \rangle + e^{\beta_3} \langle x_3 \rangle + e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{\beta_1+\beta_3} \langle x_5 \rangle + e^{\beta_2+\beta_3} \langle x_6 \rangle + e^{2\beta_1+\beta_3} \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_2} \langle x_4 \rangle + e^{\beta_2+\beta_3} \langle x_6 \rangle + e^{2\beta_1+\beta_3} \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = 2e^{\beta_2} \langle \delta_1^4, \delta_3^6 \rangle + 2e^{\beta_1} \langle \delta_2^4, \delta_3^5 \rangle + e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + e^{\beta_2-\beta_3} \langle \delta_3^2 \rangle + 2e^{\beta_3} \langle \delta_1^5, \delta_2^6 \rangle + e^{-\beta_1+\beta_2+\beta_3} \langle \delta_1^6 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + 3e^0 \langle t_2, t_1, t_3 \rangle + e^{\beta_1+\beta_3} \langle \delta_1^7 \rangle + e^{2\beta_1-\beta_2+\beta_3} \langle \delta_2^7 \rangle + e^{2\beta_1} \langle \delta_3^7 \rangle + e^{\beta_1-\beta_2+\beta_3} \langle \delta_2^5 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4, \delta_1^3] = \delta_1^6$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_2^4, \delta_3^2] = \delta_3^4$; $[\delta_2^4, \delta_1^3] = \delta_2^6$; $[\delta_2^4, \delta_1^2] = \delta_1^4$; $[\delta_2^4, \delta_2^5] = -\delta_2^7$; $[\delta_3^4, \delta_1^3] = \delta_3^6 + \delta_1^4$; $[\delta_3^4, \delta_2^5] = -\delta_3^7$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^2, \delta_2^6] = -\delta_3^6$; $[\delta_3^2, \delta_1^3] = \delta_1^2$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_3^2, \delta_2^5] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_3^5] = \delta_3^4$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = \delta_1^6$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^5] = \delta_2^6 - \delta_1^5$; $[\delta_1^2, \delta_3^5] = \delta_3^6$; $[\delta_2^5, \delta_3^5] = -\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{\beta_1+\beta_2-\beta_3} \langle \delta_3^4 \rangle + e^{\beta_2-\beta_3} \langle \delta_3^2 \rangle + e^{-\beta_1+\beta_2+\beta_3} \langle \delta_1^6 \rangle + e^{\beta_3} \langle \delta_1^5 \rangle + e^{\beta_2} \langle \delta_1^4 \rangle + e^{-\beta_1+\beta_3} \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2} \langle \delta_1^2 \rangle + 3e^0 \langle t_2, t_1, t_3 \rangle + e^{2\beta_1-\beta_2+\beta_3} \langle \delta_2^7 \rangle + e^{2\beta_1} \langle \delta_3^7 \rangle + e^{\beta_1-\beta_2+\beta_3} \langle \delta_2^5 \rangle + e^{\beta_1} \langle \delta_2^4 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4, \delta_1^3] = 2\delta_1^4$; $[\delta_3^4, \delta_2^5] = -\delta_3^7$; $[\delta_3^2, \delta_1^5] = \delta_1^4$; $[\delta_3^2, \delta_1^3] = \delta_1^2$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_3^2, \delta_2^5] = 2\delta_2^4$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_1^5, \delta_1^2] = -\delta_1^6$; $[\delta_1^4, \delta_1^3] = \delta_1^6$; $[\delta_1^3, \delta_2^4] = \delta_1^5$; $[\delta_1^2, \delta_2^5] = -2\delta_1^5$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_2^5, \delta_2^4] = \delta_2^7$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = 4e^{-\beta_3} \langle -x_2\omega^{2,3} + x_4\omega^{3,4}, x_2\omega^{2,3} + x_4\omega^{2,5}, -x_2\omega^{2,3} + x_4\omega^{1,6}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_7\omega^{3,7} \rangle + e^{\beta_2-2\beta_3} \langle x_4\omega^{3,5} \rangle + e^{\beta_1-2\beta_3} \langle x_4\omega^{3,6} \rangle +$

$e^{-\beta_1+\beta_2-2\beta_3}\ \langle x_2\omega^{3,5} + x_4\omega^{3,7}\rangle + 2e^{-\beta_1-\beta_3}\ \langle -x_1\omega^{1,5} - x_2(\omega^{2,5} + \omega^{3,4}) + x_3\omega^{3,5} - 3x_4(\omega^{2,7} - 1/3\omega^{4,5}), 2x_1\omega^{1,5} + x_2(\omega^{1,6} + \omega^{2,5}) - 2x_3\omega^{3,5} + 3x_4\omega^{2,7} + x_6\omega^{5,6}\rangle + e^{-2\beta_3}\ \langle x_1\omega^{3,5} + x_2\omega^{3,6} + x_4\omega^{5,6}\rangle + 4e^{-\beta_2}\ \langle x_1\omega^{1,2} + 2x_3\omega^{2,3} + x_5\omega^{2,5}, -x_3\omega^{2,3} + x_5\omega^{1,6}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_7\omega^{2,7}, x_4\omega^{2,4}\rangle + e^{-2\beta_1+\beta_3}\ \langle x_6\omega^{1,4}\rangle + 4e^{-\beta_1}\ \langle x_2\omega^{1,2} + x_6\omega^{1,6}, -x_2\omega^{1,2} + x_3\omega^{1,3}, x_4\omega^{1,4}, x_5\omega^{1,5}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_6\omega^{1,7}\rangle + 2e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}, x_5\omega^{1,4}\rangle + 2e^{-2\beta_1}\ \langle x_5\omega^{1,7}, -x_2\omega^{1,4} - x_3\omega^{1,5} + x_6\omega^{4,5}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_4\omega^{1,7}\rangle + e^{-3\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,7} + x_6\omega^{5,7}\rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle x_7\omega^{2,4}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_7\omega^{2,6}\rangle + e^{\beta_1-\beta_2}\ \langle -x_5\omega^{2,3} + x_7\omega^{3,4}\rangle + e^{2\beta_1-\beta_2-\beta_3}\ \langle x_7\omega^{3,6}\rangle + e^{-\beta_1-\beta_2}\ \langle -2/3x_1\omega^{1,4} + 2/3x_2\omega^{2,4} + x_3(\omega^{1,6} + 4/3\omega^{2,5} + 1/3\omega^{3,4}) + x_5(\omega^{2,7} + 1/3\omega^{4,5}) - x_6\omega^{4,6} + x_7\omega^{4,7}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle x_4\omega^{2,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{2\beta_1-2\beta_2-\beta_3} + e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + e^{\beta_1-3\beta_2} + 6e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + 4e^{-\beta_1-2\beta_2} + 4e^{-\beta_1-2\beta_3} + e^{-\beta_1-3\beta_3} + e^{-2\beta_1+\beta_2-3\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + 4e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + 2e^{-2\beta_1-\beta_3} + 2e^{-2\beta_1-2\beta_3} + 2e^{-3\beta_1+\beta_2-2\beta_3} + 2e^{-3\beta_1-\beta_2} + 4e^{-3\beta_1-\beta_3} + e^{-4\beta_1+\beta_2-\beta_3} + e^{-4\beta_1-\beta_3} + e^{-4\beta_1} + 4e^{-\beta_2-2\beta_3} + 3e^{-2\beta_2-\beta_3} + e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-3\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2-2\beta_3} + 2e^{-\beta_1-3\beta_2-\beta_3} + 4e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-\beta_2-3\beta_3} + 4e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + 4e^{-3\beta_1-\beta_2-2\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2} + 4e^{-3\beta_1-3\beta_3} + e^{-4\beta_1+\beta_2-3\beta_3} + 4e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2} + 4e^{-4\beta_1-2\beta_3} + e^{-5\beta_1-\beta_3} + 2e^{-2\beta_2-2\beta_3} + e^{-2\beta_2-3\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-\beta_1-2\beta_2-3\beta_3} + e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-4\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + 2e^{-3\beta_1-\beta_2-3\beta_3} + 2e^{-3\beta_1-\beta_2-4\beta_3} + 2e^{-3\beta_1-2\beta_2-2\beta_3} + 2e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + 4e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 4e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_3} + e^{-4\beta_1-4\beta_3} + 2e^{-5\beta_1-\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-3\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-4\beta_3} + e^{-3\beta_1-3\beta_2-4\beta_3} + e^{-4\beta_1-\beta_2-4\beta_3} + 3e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-4\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-3\beta_3} + e^{-5\beta_1-\beta_2-3\beta_3} + e^{-5\beta_1-\beta_2-4\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-3\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2-4\beta_3} + e^{-5\beta_1-2\beta_2-4\beta_3} + e^{-5\beta_1-3\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(3, 18, 64, 134, 163, 117, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \leq j \leq 7$: $(3, 14, 33, 51, 52, 35, 15, 3)$

S=$5\beta_1 + 3\beta_2 + 4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 12, 23, 26, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 8, 14, 14, 8, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,5}$;

$d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = \omega^{1,5,6} - \omega^{2,3,7}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,6}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,6,7}+\omega^{2,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{1,5,6}+\omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,5,6}+\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,6}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,5,7}\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,4,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,6,7}\rangle + e^{-\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,6,7} - \omega^{3,4,5,6}\rangle + e^{-4\beta_1-\beta_2-3\beta_3} \langle\omega^{1,5,6,7} + \omega^{3,4,5,7}\rangle + e^{-3\beta_1-\beta_2-4\beta_3} \langle\omega^{3,5,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,4,6,7}\rangle + 2e^{-4\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,2,5,6,7}, \omega^{1,3,4,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,4,5,7}\rangle + e^{-4\beta_1-\beta_2-4\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-2\beta_1-3\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-4\beta_3} \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-4\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-4\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-5\beta_1-3\beta_2-4\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) \ = \ (1 + 2Y + 6Y^2 + 8Y^3 + 6Y^4 + 2Y^5 + Y^6)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{g}}$'s $0 \le j \le 7$: $(1, 3, 12, 23, 26, 17, 7, 1)$

Dimensions of $H^j_{\mathfrak{g}}$'s $0 \le j \le 7$: $(1, 3, 8, 14, 14, 8, 3, 1)$

Total dimension of trivial cohomology: 52.

Length associated polynomial: 52.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = -\omega^{2,7} - \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,4,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5} - \omega^{3,4}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = \omega^{1,6} - \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 3\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} - \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + 3\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} =$

$3\omega^{1,2,3,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} - \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{4,5,6,7} = 3\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{A}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{A}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,7}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6}\rangle + 2e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{A}}) = e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7} - \omega^{1,4,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6} + \omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6} + \omega^{2,3,5}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,6,7} + \omega^{2,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{-\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{A}}) = e^{-3\beta_1-\beta_2-4\beta_3} \langle-\omega^{3,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle(-(\omega^{2,3,6,7} - \omega^{3,4,5,6}))/2\rangle + e^{-\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-4\beta_1-\beta_2-3\beta_3} \langle(-(\omega^{1,5,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,4,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{2,3,5,7}))/2\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,3,4,6} + \omega^{2,3,4,5})/2\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle(2\omega^{1,2,6,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,2,5,7} + \omega^{1,3,4,7}))/2\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{A}}) = e^{-3\beta_1-2\beta_2-4\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-2\beta_1-3\beta_2-3\beta_3} \langle-\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-\beta_2-4\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3} \langle-\omega^{1,3,4,5,7}\rangle + 2e^{-4\beta_1-2\beta_2-3\beta_3} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} +$

$\omega^{2,3,4,5,7})/3\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle -\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3}\ \langle \omega^{1,2,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{T}})= \quad e^{-4\beta_1-3\beta_2-4\beta_3} \quad \langle \omega^{2,3,4,5,6,7}\rangle \ + \ e^{-5\beta_1-2\beta_2-4\beta_3} \quad \langle -\omega^{1,3,4,5,6,7}\rangle \ + \ e^{-5\beta_1-3\beta_2-3\beta_3}\ \langle \omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{T}})= e^{-5\beta_1-3\beta_2-4\beta_3}\ \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1+2Y+6Y^2+8Y^3+6Y^4+2Y^5+Y^6)(1+Y)$.

5.5.8 Algebra $\mathcal{G}_{7,3.7}$

Adjoint cohomology

$[x_1,x_2]=x_5$; $[x_1,x_5]=x_6$; $[x_2,x_4]=x_6$; $[x_3,x_4]=-x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0\\ 0 & 0 & 0 & 2\xi_1^1 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^5 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & 0 & 0 & 2\xi_1^1+\xi_3^3 \end{pmatrix}$$

$ad\ x_1=\delta_2^5, ad\ x_2=-\delta_1^5+\delta_4^6, ad\ x_3=-\delta_4^7, ad\ x_4=-\delta_2^6+\delta_3^7, ad\ x_5=-\delta_1^6, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{2\beta_1}\ \langle x_4\rangle+e^{\beta_1+\beta_2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{-2\beta_1+\beta_3}\ \langle\delta_4^3\rangle+2e^{\beta_2}\ \langle\delta_1^5,\delta_4^6\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+2e^{2\beta_1}\ \langle\delta_2^6,\delta_3^7\rangle+e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle+e^{\beta_1}\ \langle\delta_2^5\rangle+3e^0\ \langle t_2,t_1,t_3\rangle+e^{\beta_1+\beta_3}\ \langle\delta_1^7\rangle+e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle+e^{\beta_3}\ \langle\delta_4^7\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_4^3,\delta_3^6]=-\delta_4^6$; $[\delta_4^3,\delta_3^7]=-\delta_4^7$; $[\delta_1^5,\delta_2^5]=-\delta_1^6$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^5]=\delta_4^6-\delta_1^5$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{-2\beta_1+\beta_3}\ \langle\delta_4^3\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle+e^{\beta_2}\ \langle\delta_1^5\rangle+3e^0\ \langle t_2,t_1,t_3\rangle+e^{\beta_1+\beta_3}\ \langle\delta_1^7\rangle+e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3,\delta_3^6]=-\delta_1^5$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{-2\beta_2}\ \langle x_1\omega^{2,5}\rangle+e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,4}\rangle+e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,5}-\omega^{2,4})+x_2\omega^{2,5}\rangle+e^{-3\beta_1+\beta_3}\ \langle x_3\omega^{1,4}\rangle+e^{-4\beta_1}\ \langle x_3\omega^{4,7}\rangle+3e^{-\beta_1}\ \langle -x_2\omega^{1,2}-x_3\omega^{1,3}+x_4\omega^{1,4}, x_2\omega^{1,2}+x_5\omega^{2,4}, x_3\omega^{1,3}+x_7\omega^{1,7}\rangle+e^{2\beta_1-\beta_2-\beta_3}\ \langle x_4\omega^{2,3}\rangle+3e^{-\beta_3}\ \langle x_1\omega^{1,3}-2x_2\omega^{2,3}-2x_4\omega^{3,4}+x_5\omega^{3,5}, -x_1\omega^{1,3}+x_2\omega^{2,3}+2x_4\omega^{3,4}+x_6\omega^{3,6}, x_4\omega^{3,4}+x_6\omega^{2,7}\rangle+4e^{-2\beta_1}\ \langle -x_1\omega^{1,4}-2x_2\omega^{1,5}+x_5\omega^{4,5}, -x_1\omega^{1,4}-x_2(\omega^{1,5}-\omega^{2,4}), -x_1\omega^{1,4}-x_2\omega^{1,5}+x_5\omega^{1,6}, x_3\omega^{3,4}\rangle+e^{-3\beta_1}\ \langle -x_2(\omega^{1,6}-\omega^{4,5})+x_5\omega^{4,6}\rangle+e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}+x_5\omega^{3,4}\rangle+e^{\beta_1-\beta_3}\ \langle x_4\omega^{1,3}+x_5\omega^{2,3}\rangle+e^{\beta_2-2\beta_3}\ \langle x_6\omega^{3,7}\rangle+e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{3,4}+x_5\omega^{1,7}\rangle+e^{-\beta_1-\beta_2+\beta_3}\ \langle -x_3\omega^{1,2}+x_7\omega^{1,6}\rangle+e^{\beta_1-2\beta_2+\beta_3}\ \langle x_7\omega^{2,5}\rangle+3e^{-\beta_2}\ \langle x_3\omega^{2,3}+x_7\omega^{2,7}, -x_3\omega^{2,3}+x_4\omega^{2,4}, -x_1\omega^{1,2}+x_6\omega^{2,6}\rangle+e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2-\beta_3} + 2e^{-\beta_1-\beta_2-\beta_3} + 3e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2+\beta_3} + 2e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-\beta_3} + 2e^{-2\beta_1-2\beta_3} + e^{-3\beta_1+\beta_2-2\beta_3} + 4e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-\beta_3} + 2e^{-3\beta_1} + 2e^{-4\beta_1-\beta_2} + 3e^{-4\beta_1-\beta_3} + e^{-4\beta_1} + e^{-5\beta_1+\beta_2-\beta_3} + e^{-5\beta_1-\beta_2+\beta_3} + 2e^{-5\beta_1} + 2e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-\beta_2-2\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2} + 3e^{-3\beta_1-3\beta_2} + 2e^{-3\beta_1-2\beta_3} + 3e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + 3e^{-4\beta_1-2\beta_3} + e^{-5\beta_1+\beta_2-2\beta_3} + 2e^{-5\beta_1-\beta_2-\beta_3} + 2e^{-5\beta_1-\beta_2} + 2e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2+\beta_3} + 2e^{-5\beta_1-\beta_3} + e^{-5\beta_1-2\beta_3} + e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2+\beta_3} + e^{-6\beta_1-\beta_3} + e^{-7\beta_1-\beta_2} + e^{-7\beta_1-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-3\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2-2\beta_3} + 2e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + 2e^{-4\beta_1-2\beta_2-\beta_3} + 2e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2} + 3e^{-5\beta_1-\beta_2-2\beta_3} + 2e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-\beta_3} + 2e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-2\beta_3} + e^{-6\beta_1-\beta_2-2\beta_3} + e^{-6\beta_1-2\beta_2-\beta_3} + 2e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2+\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-2\beta_3} + 2e^{-7\beta_1-\beta_2-\beta_3} + e^{-7\beta_1-2\beta_2-\beta_3} + e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-2\beta_3} + e^{-8\beta_1-\beta_2-\beta_3} + e^{-8\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + 2e^{-5\beta_1-3\beta_2-2\beta_3} + 2e^{-6\beta_1-2\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-\beta_2-2\beta_3} + e^{-7\beta_1-2\beta_2-2\beta_3} + 2e^{-7\beta_1-3\beta_2-\beta_3} + e^{-8\beta_1-\beta_2-2\beta_3} + 2e^{-8\beta_1-2\beta_2-\beta_3} + e^{-8\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-6\beta_1-3\beta_2-2\beta_3} + e^{-7\beta_1-3\beta_2-2\beta_3} + e^{-8\beta_1-2\beta_2-2\beta_3} + e^{-8\beta_1-3\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 61, 131, 164, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 27, 45, 50, 38, 18, 4)$

S=$8\beta_1 + 3\beta_2 + 2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^7 = \omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = -\omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = -\omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} - \omega^{2,4,7} - \omega^{3,4,6}$; $d\omega^{1,2,7} = \omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} + \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,5,7} = \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} - \omega^{2,3,4,7}$;

$d\omega^{4,5,6} = \omega^{1,2,4,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{3,7}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{4,5}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-2\beta_3} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,6}\rangle + e^{-5\beta_1-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,6} - \omega^{2,4,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6} + \omega^{3,4,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,7} + \omega^{3,4,6}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle + e^{-5\beta_1-2\beta_3} \langle\omega^{1,3,4,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,5,6} - \omega^{2,3,4,6}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-7\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6,7} - \omega^{3,4,5,6}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,3,5,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,4,5,6}\rangle + e^{-7\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-8\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-6\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-6\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-8\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 8, 11, 11, 8, 4, 1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,4} = \omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,6} + \omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$;

$d^{\star}\omega^{1,3,4} = -\omega^{1,7}$; $d^{\star}\omega^{1,3,5} = -\omega^{3,6}$; $d^{\star}\omega^{1,4,5} = -\omega^{4,6}$; $d^{\star}\omega^{1,5,7} = -\omega^{6,7}$; $d^{\star}\omega^{2,3,4} = -\omega^{2,7} - \omega^{3,6}$; $d^{\star}\omega^{2,4,5} = \omega^{5,6}$; $d^{\star}\omega^{2,4,7} = -\omega^{6,7}$; $d^{\star}\omega^{3,4,5} = -\omega^{5,7}$; $d^{\star}\omega^{3,4,6} = -\omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{3,4,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{2,3,6}$; $d^{\star}\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^{\star}\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^{\star}\omega^{1,2,4,5} = -\omega^{1,5,6} - \omega^{2,4,6}$; $d^{\star}\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^{\star}\omega^{1,2,4,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{1,5,7} - \omega^{3,4,6}$; $d^{\star}\omega^{1,3,4,6} = \omega^{1,6,7}$; $d^{\star}\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^{\star}\omega^{2,3,4,5} = \omega^{2,5,7} + \omega^{3,5,6}$; $d^{\star}\omega^{2,3,4,6} = \omega^{2,6,7}$; $d^{\star}\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{3,4,5,6} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = -\omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $d^{\star}\omega^{1,2,3,4,6} = -\omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = \omega^{1,3,6,7} - \omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^{\star}\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^{\star}\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{1,5,6,7}$; $d^{\star}\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{2,3,4,5,6} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = \omega^{1,2,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = 2\omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{4,5}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,4} = \omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{2,7} + 2\omega^{3,6}$; $\Delta\omega^{4,5} = \omega^{1,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,3,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} + \omega^{2,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,2,7} + \omega^{1,3,6} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{2,3,4} = \omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,5,6} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,5,7} + \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,4,7} + 2\omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,5,6} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,3,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$;

$\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,7} = 2\omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,4,5,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,3,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{3,7}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{4,5}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-2\beta_3} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,6}\rangle + e^{-5\beta_1-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,6} + \omega^{2,4,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,6} - \omega^{2,4,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7} - 2\omega^{1,3,6} - \omega^{3,4,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,7} - 2\omega^{2,4,7} + \omega^{3,4,6}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-5\beta_1-3\beta_2} \langle-\omega^{2,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-7\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,3,5,7} - \omega^{2,3,4,7}))/2\rangle + e^{-5\beta_1-2\beta_3} \langle-\omega^{1,3,4,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,6,7} + 2\omega^{2,4,5,7} - \omega^{3,4,5,6}))/6\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,5,7} - 2\omega^{1,3,5,6} + \omega^{2,3,4,6}))/6\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-7\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-8\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-7\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle-\omega^{1,2,4,5,6}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle(-\omega^{1,2,3,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-2\beta_2-2\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-2\beta_3} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

5.5.9 Algebra $\mathcal{G}_{7,3.8}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_5] = x_7$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^3 & \xi_1^1+\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^4+\xi_2^5 & \xi_3^5 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5 + \delta_4^7, ad\ x_3 = -\delta_1^6, ad\ x_4 = -\delta_2^7, ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{2\beta_1}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= 2e^{-\beta_1+\beta_3}\ \langle \delta_1^3, \delta_4^6\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_4^7\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{\beta_3}\ \langle \delta_1^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6\rangle + 3e^{\beta_1}\ \langle \delta_3^6, \delta_1^4, \delta_2^5\rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{2\beta_1}\ \langle \delta_2^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_2^4, \delta_1^2] = \delta_2^5 + \delta_1^4$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^5, \delta_1^4] = \delta_1^7$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^3] = \delta_4^6 - \delta_1^3$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^4] = -(\delta_4^7 + \delta_1^5)$; $[\delta_1^2, \delta_2^5] = \delta_4^7 - \delta_1^5$; $[\delta_1^6, \delta_3^5] = -\delta_1^7$; $[\delta_2^6, \delta_3^5] = -\delta_2^7$; $[\delta_3^6, \delta_2^3] = \delta_2^6$; $[\delta_3^6, \delta_3^5] = -\delta_3^7$; $[\delta_4^6, \delta_1^4] = \delta_1^6$; $[\delta_4^6, \delta_3^5] = -\delta_4^7$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_2^5] = \delta_2^6$; $[\delta_2^3, \delta_3^5] = \delta_3^6 - \delta_2^5$; $[\delta_4^7, \delta_1^4] = \delta_1^7$; $[\delta_1^4, \delta_2^5] = -\delta_2^7$; $[\delta_1^4, \delta_3^5] = -\delta_3^7$; $[\delta_2^5, \delta_3^5] = \delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= 2e^{-\beta_1+\beta_3}\ \langle \delta_1^3, \delta_4^6\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6\rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_2}\ \langle \delta_1^5\rangle + 2e^{\beta_1}\ \langle \delta_1^4, \delta_2^5\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5\rangle$.

Comm. relations among the δ_i^j's $\ i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^4, \delta_1^2] = 2\delta_1^5$; $[\delta_1^4, \delta_3^5] = -\delta_3^7$; $[\delta_2^4, \delta_1^2] = \delta_2^5 + \delta_1^4$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_1^2, \delta_2^3] = \delta_4^6 - \delta_1^3$; $[\delta_2^5, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_3^5] = \delta_3^7$; $[\delta_4^6, \delta_3^5] = -\delta_1^5$; $[\delta_2^3, \delta_3^5] = -2\delta_2^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,4}\rangle + e^{-3\beta_1+\beta_3}\ \langle x_3\omega^{1,4}\rangle + 5e^{-2\beta_1}\ \langle x_3\omega^{1,6}, -x_1\omega^{1,4} - x_2(\omega^{1,5} - \omega^{2,4}) + x_3\omega^{3,4}, -x_2(\omega^{1,5} + \omega^{2,4}) + x_5\omega^{4,5}, -x_1\omega^{1,4} - x_2(\omega^{1,5} -$

$\omega^{2,4}) + x_6\omega^{4,6}, -x_2\omega^{2,4} + x_5\omega^{1,7}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_4\omega^{1,4}, x_3\omega^{1,3}, x_2\omega^{1,2} + x_5\omega^{2,4}, x_5\omega^{1,5}\rangle + e^{2\beta_1-\beta_2-\beta_3}\ \langle x_4\omega^{2,3}\rangle + e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3} + x_4(\omega^{2,6} + \omega^{3,5})\rangle + 2e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3} + x_5\omega^{3,4}, x_5\omega^{1,6}\rangle + 4e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} - 2x_4\omega^{3,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + 2x_4\omega^{3,4} + x_7\omega^{3,7}, x_2\omega^{2,3} + x_4(\omega^{1,6} + \omega^{3,4}) + x_5\omega^{2,6}, x_6\omega^{3,6}\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle -x_2(\omega^{1,6} + \omega^{3,4}) + x_5\omega^{4,6}\rangle + e^{-3\beta_1}\ \langle -x_2(\omega^{1,7} - \omega^{4,5}) + x_5\omega^{4,7}\rangle + 2e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5}, -x_3\omega^{2,4} + x_6\omega^{4,5}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_6\omega^{2,5}\rangle + 4e^{-\beta_2}\ \langle x_3\omega^{2,3} + x_6\omega^{2,6}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_5\omega^{2,5}, x_4\omega^{2,4}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,5} + x_6\omega^{2,7}\rangle + e^{-3\beta_1-\beta_2+\beta_3}\ \langle -x_3(\omega^{1,7} - \omega^{4,5}) + x_6\omega^{4,7}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,5} - \omega^{2,4}) + x_2\omega^{2,5} + x_3(\omega^{2,6} + \omega^{3,5}) + x_6(\omega^{3,7} + \omega^{5,6}), x_2\omega^{2,5} + x_4(\omega^{1,7} - \omega^{4,5}) + x_5\omega^{2,7}\rangle + e^{\beta_1-\beta_2}\ \langle x_4\omega^{1,2}\rangle + e^{\beta_2-\beta_3}\ \langle x_5\omega^{1,3}\rangle + e^{\beta_1-\beta_3}\ \langle x_4\omega^{1,3} + x_5\omega^{2,3}\rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle x_7\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + e^{-\beta_1+\beta_2-2\beta_3} + 5e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 4e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + 2e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + 7e^{-2\beta_1-\beta_3} + e^{-3\beta_1+\beta_2-\beta_3} + 6e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2+\beta_3} + 3e^{-3\beta_1-\beta_3} + 2e^{-3\beta_1} + e^{-4\beta_1+\beta_2-\beta_3} + e^{-4\beta_1-\beta_2+\beta_3} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + 2e^{-4\beta_1} + e^{-5\beta_1-\beta_2+\beta_3} + e^{-5\beta_1} + 3e^{-\beta_2-\beta_3} + 2e^{-2\beta_2-\beta_3} + e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = 2e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2-2\beta_3} + 5e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2} + 2e^{-2\beta_1-2\beta_3} + 6e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + 3e^{-3\beta_1-2\beta_2} + 3e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-2\beta_3} + e^{-4\beta_1+\beta_2-2\beta_3} + 5e^{-4\beta_1-\beta_2-\beta_3} + 5e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2+\beta_3} + 2e^{-4\beta_1-\beta_3} + e^{-4\beta_1-2\beta_3} + 3e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2+\beta_3} + e^{-5\beta_1-2\beta_2} + 2e^{-5\beta_1-\beta_3} + 2e^{-6\beta_1-\beta_2} + e^{-6\beta_1-2\beta_2+\beta_3} + e^{-6\beta_1-\beta_3} + e^{-2\beta_2-2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = 2e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2-2\beta_3} + 3e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + 3e^{-4\beta_1-\beta_2-2\beta_3} + 4e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + 2e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-2\beta_3} + e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + 4e^{-5\beta_1-2\beta_2-\beta_3} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-2\beta_3} + 3e^{-6\beta_1-\beta_2-\beta_3} + 3e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2+\beta_3} + e^{-6\beta_1-2\beta_3} + e^{-7\beta_1-\beta_2-\beta_3} + e^{-7\beta_1-2\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-5\beta_1-2\beta_2-2\beta_3} + 3e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3} + e^{-6\beta_1-\beta_2-2\beta_3} + e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2} + e^{-7\beta_1-\beta_2-2\beta_3} + 2e^{-7\beta_1-2\beta_2-\beta_3} + e^{-7\beta_1-3\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-5\beta_1-3\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3} + e^{-7\beta_1-2\beta_2-2\beta_3} + e^{-7\beta_1-3\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 19, 68, 139, 169, 120, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 38, 60, 63, 44, 19, 4)$

S=$7\beta_1 + 3\beta_2 + 2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,12,23,26,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,9,14,14,9,4,1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -\omega^{1,5} - \omega^{2,4}$; $d\omega^{1,7} = \omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,4,6}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1} \langle\omega^{1,4}\rangle + 2e^{-2\beta_1-\beta_3} \langle\omega^{1,6}, \omega^{3,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{1,4,6}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,7}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}, \omega^{1,3,7} + \omega^{3,4,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} - \omega^{2,4,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,7}\rangle + e^{-3\beta_1-2\beta_3} \langle\omega^{3,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,3,4,6}\rangle + 2e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6,7}, \omega^{2,3,4,7} - \omega^{2,4,5,6}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,4,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{-6\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,3,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6,7} - \omega^{3,4,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,4,5,7}\rangle + 2e^{-5\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}, \omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-6\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-5\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle +$

$e^{-7\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

ch($H^7_{\mathfrak{T}}$)= $e^{-7\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+Y+3Y^2+Y^3+Y^4)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{T}}$'s $0\le j\le 7$: $(1,4,12,23,26,18,7,1)$

Dimensions of $H^j_{\mathfrak{T}}$'s $0\le j\le 7$: $(1,4,9,14,14,9,4,1)$

Total dimension of trivial cohomology: 56.

Length associated polynomial: 56.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^5$; $d^\star\omega^{1,3}=-\omega^6$; $d^\star\omega^{1,5}=-\omega^7$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{1,2,3}=-\omega^{2,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4}=\omega^{1,7}+\omega^{4,5}$; $d^\star\omega^{1,2,5}=-\omega^{2,7}$; $d^\star\omega^{1,2,6}=-\omega^{5,6}$; $d^\star\omega^{1,2,7}=-\omega^{5,7}$; $d^\star\omega^{1,3,4}=\omega^{4,6}$; $d^\star\omega^{1,3,5}=-\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{1,3,7}=-\omega^{6,7}$; $d^\star\omega^{1,4,5}=-\omega^{4,7}$; $d^\star\omega^{1,5,6}=\omega^{6,7}$; $d^\star\omega^{2,3,4}=-\omega^{3,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=\omega^{1,3,7}+\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{2,3,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,3,6}=\omega^{3,5,6}$; $d^\star\omega^{1,2,3,7}=-\omega^{2,6,7}+\omega^{3,5,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{4,5,7}$; $d^\star\omega^{1,2,5,6}=\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{3,4,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7}=\omega^{4,6,7}$; $d^\star\omega^{1,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{1,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,7}$; $d^\star\omega^{2,3,4,6}=\omega^{3,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,3,5,7}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,3,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6}=\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,7}=\omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7}=\omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6}=\omega^{1,5,6,7}+\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7}=\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=\omega^{1,3,5,6,7}+\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7}=-\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7}=-\omega^{3,4,5,6,7}$;

$\Delta\omega^5=\omega^5$; $\Delta\omega^6=\omega^6$; $\Delta\omega^7=2\omega^7$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,3}=\omega^{1,3}$; $\Delta\omega^{1,5}=\omega^{1,5}+\omega^{2,4}$; $\Delta\omega^{1,7}=\omega^{1,7}+\omega^{4,5}$; $\Delta\omega^{2,4}=\omega^{1,5}+\omega^{2,4}$; $\Delta\omega^{2,6}=\omega^{2,6}-\omega^{3,5}$; $\Delta\omega^{2,7}=\omega^{2,7}$; $\Delta\omega^{3,5}=-\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,7}=2\omega^{3,7}-\omega^{5,6}$; $\Delta\omega^{4,5}=\omega^{1,7}+\omega^{4,5}$; $\Delta\omega^{4,6}=\omega^{4,6}$; $\Delta\omega^{4,7}=\omega^{4,7}$; $\Delta\omega^{5,6}=-\omega^{3,7}+2\omega^{5,6}$; $\Delta\omega^{5,7}=2\omega^{5,7}$; $\Delta\omega^{6,7}=3\omega^{6,7}$; $\Delta\omega^{1,2,3}=2\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=2\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=\omega^{1,2,5}$; $\Delta\omega^{1,2,6}=\omega^{1,2,6}-\omega^{1,3,5}$; $\Delta\omega^{1,2,7}=\omega^{1,2,7}-\omega^{2,4,5}$; $\Delta\omega^{1,3,4}=\omega^{1,3,4}$; $\Delta\omega^{1,3,5}=-\omega^{1,2,6}+2\omega^{1,3,5}+\omega^{2,3,4}$; $\Delta\omega^{1,3,7}=2\omega^{1,3,7}-\omega^{1,5,6}-\omega^{3,4,5}$; $\Delta\omega^{1,4,5}=\omega^{1,4,5}$; $\Delta\omega^{1,5,6}=-\omega^{1,3,7}+\omega^{1,5,6}+\omega^{2,4,6}$; $\Delta\omega^{1,5,7}=\omega^{1,5,7}+\omega^{2,4,7}$; $\Delta\omega^{1,6,7}=\omega^{1,6,7}-\omega^{4,5,6}$; $\Delta\omega^{2,3,4}=\omega^{1,3,5}+\omega^{2,3,4}$; $\Delta\omega^{2,3,7}=\omega^{2,3,7}-\omega^{2,5,6}$; $\Delta\omega^{2,4,5}=-\omega^{1,2,7}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6}=\omega^{1,5,6}+2\omega^{2,4,6}-\omega^{3,4,5}$; $\Delta\omega^{2,4,7}=\omega^{1,5,7}+\omega^{2,4,7}$; $\Delta\omega^{2,5,6}=-\omega^{2,3,7}+\omega^{2,5,6}$; $\Delta\omega^{2,6,7}=2\omega^{2,6,7}-\omega^{3,5,7}$; $\Delta\omega^{3,4,5}=$

$-\omega^{1,3,7} - \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} + \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,5,7} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} + \omega^{2,3,4,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,5,6,7} + 2\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = \omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1} \langle\omega^{1,4}\rangle + 2e^{-2\beta_1-\beta_3} \langle\omega^{1,6}, \omega^{3,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{1,4,6}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6} + \omega^{1,3,5} - \omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,7} + \omega^{2,4,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} - \omega^{2,4,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,7}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7} + \omega^{2,4,6} + 2\omega^{3,4,5}, \omega^{1,5,6} - \omega^{2,4,6} - \omega^{3,4,5}\rangle + e^{-3\beta_1-2\beta_3} \langle\omega^{3,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,6,7} - \omega^{3,4,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,5,6,7} - \omega^{2,4,6,7} - \omega^{3,4,5,7})/3\rangle + e^{-6\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_2} \langle-\omega^{1,4,5,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,3,6,7} - \omega^{3,4,5,6}))/2\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle(\omega^{1,3,5,6} - \omega^{2,3,4,6})/2\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle(-(\omega^{1,3,4,7} - \omega^{1,4,5,6}))/2\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,3,4,6}\rangle + 2e^{-4\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,2,6,7} + \omega^{2,3,4,7} - \omega^{2,4,5,6})/3, (-(\omega^{1,3,5,7} - 2\omega^{2,3,4,7} + \omega^{2,4,5,6}))/3\rangle + e^{-4\beta_1-3\beta_2} \langle-\omega^{1,2,5,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{2,3,4,5}))/3\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + 2e^{-5\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,4,5,7}, \omega^{1,2,5,6,7}\rangle +$

$e^{-7\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,4,5,6,7}\rangle$ + $e^{-5\beta_1-2\beta_2-2\beta_3}$ $\langle(\omega^{1,3,5,6,7}$ $-$ $\omega^{2,3,4,6,7})/2\rangle$ + $e^{-6\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,3,4,6,7}\rangle$ + $e^{-6\beta_1-2\beta_2-\beta_3}$ $\langle(-\omega^{1,2,4,6,7}$ $-$ $\omega^{1,3,4,5,7})/2\rangle$ + $e^{-6\beta_1-3\beta_2}$ $\langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}$ $\langle(-\omega^{1,2,3,6,7} - \omega^{2,3,4,5,6})/2\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=$ $e^{-6\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-7\beta_1-2\beta_2-2\beta_3}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-7\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-2\beta_3}$ $\langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})=$ $e^{-7\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \; = \; (1 + Y + 3Y^2 + Y^3 + Y^4)(1 + Y)^3.$

5.5.10 Algebra $\mathcal{G}_{7,3.9}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = -x_7$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & 2\xi_1^1 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & -\xi_1^3 & \xi_2^2+\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -(\xi_1^4+\xi_2^5) & \xi_3^4 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5 + \delta_3^6 + \delta_4^7, ad\ x_3 = -\delta_2^6, ad\ x_4 = -\delta_2^7, ad\ x_5 = \delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0$.

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{2\beta_1}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_2+\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_2+\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle + 3e^{\beta_2}\ \langle \delta_1^5, \delta_3^6, \delta_4^7\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6\rangle + e^{\beta_3}\ \langle \delta_2^6\rangle + e^{-2\beta_1+\beta_2+\beta_3}\ \langle \delta_4^6\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^7\rangle + e^{2\beta_1}\ \langle \delta_2^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7\rangle + 2e^{\beta_1}\ \langle \delta_1^4, \delta_2^5\rangle + e^{2\beta_1-\beta_3}\ \langle \delta_3^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^4] = -\delta_2^4$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_1^5, \delta_1^3] = \delta_1^6$; $[\delta_1^5, \delta_1^4] = \delta_1^7$; $[\delta_1^5, \delta_2^5] = \delta_1^7$; $[\delta_1^6, \delta_3^4] = -\delta_1^7$; $[\delta_2^6, \delta_3^4] = -\delta_2^7$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_4^6, \delta_1^4] = \delta_1^6$; $[\delta_4^6, \delta_3^4] = -\delta_4^7 + \delta_3^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_2^5] = -\delta_2^6$; $[\delta_1^3, \delta_3^4] = -\delta_1^4$; $[\delta_4^7, \delta_1^4] = \delta_1^7$; $[\delta_4^7, \delta_3^4] = \delta_3^7$; $[\delta_1^4, \delta_2^5] = -\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + e^{2\beta_1-\beta_2}\ \langle \delta_2^4\rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6\rangle + e^{-2\beta_1+\beta_2+\beta_3}\ \langle \delta_4^6\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7\rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_3^6\rangle + e^{\beta_1}\ \langle \delta_1^4\rangle + e^{2\beta_1-\beta_3}\ \langle \delta_3^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^3, \delta_3^4] = -\delta_2^4$; $[\delta_1^5, \delta_1^3] = \delta_1^6$; $[\delta_3^6, \delta_1^3] = \delta_1^6$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_4^6, \delta_1^4] = \delta_1^6$; $[\delta_4^6, \delta_3^4] = 2\delta_3^6 - \delta_1^5$; $[\delta_1^3, \delta_3^4] = -\delta_1^4$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 2e^{-2\beta_2}\ \langle x_1\omega^{2,5}, x_3\omega^{2,6}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,5}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_3\omega^{1,3} + x_4\omega^{1,4}, x_3\omega^{1,3} + x_6\omega^{1,6}, x_5\omega^{1,5}, -x_2\omega^{1,2} + x_3\omega^{1,3} + x_5\omega^{2,4}\rangle + e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,5}\rangle + e^{2\beta_1-2\beta_2-\beta_3}\ \langle x_4\omega^{2,6}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{1,3} + x_5\omega^{3,4}\rangle + 4e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} - 2x_4\omega^{3,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + 2x_4\omega^{3,4} + x_7\omega^{3,7}, -x_2\omega^{2,3} - x_4\omega^{3,4} + x_5\omega^{1,6}, x_6\omega^{3,6}\rangle + 4e^{-2\beta_1}\ \langle x_1\omega^{1,4} - 2x_2\omega^{2,4} + 2x_3\omega^{3,4} +$

$x_5\omega^{4,5}, -x_3\omega^{3,4} + x_6\omega^{4,6}, -x_2\omega^{2,4} + x_3\omega^{3,4} + x_5\omega^{1,7}, x_1\omega^{1,4} - x_2(\omega^{1,5} + \omega^{2,4}) + 2x_3\omega^{3,4} + x_6\omega^{3,7}\rangle + e^{-3\beta_1+\beta_2+\beta_3}\ \langle x_6\omega^{1,4}\rangle + 2e^{-3\beta_1+\beta_3}\ \langle x_3\omega^{1,4} + x_6\omega^{1,7}, x_3\omega^{1,4} + x_6\omega^{4,5}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle + 3e^{-\beta_2}\ \langle x_3\omega^{2,3}, x_4\omega^{2,4}, x_1\omega^{1,2} + x_4\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{2,4}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_5\omega^{1,4}\rangle + 2e^{-\beta_1-\beta_2}\ \langle 2x_1(\omega^{1,5} + \omega^{2,4}) - 2x_2\omega^{2,5} - x_3(\omega^{1,6} - \omega^{3,5}) + x_6\omega^{5,6}, -x_1(\omega^{1,5} + \omega^{2,4}) - x_4(\omega^{1,7} - \omega^{4,5}) + x_5\omega^{2,7}\rangle + e^{2\beta_1-\beta_2-\beta_3}\ \langle x_4\omega^{2,3}\rangle + e^{\beta_2-\beta_3}\ \langle x_5\omega^{1,3}\rangle + e^{\beta_1-\beta_3}\ \langle -x_4\omega^{1,3} + x_5\omega^{2,3}\rangle + e^{2\beta_1-2\beta_3}\ \langle x_7\omega^{3,6}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3} - x_4(\omega^{1,6} - \omega^{3,5}) + x_5\omega^{2,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{2\beta_1-2\beta_2-2\beta_3} + e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + e^{\beta_1-3\beta_2-\beta_3} + 5e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2} + 4e^{-2\beta_1-\beta_3} + e^{-3\beta_1+\beta_2-\beta_3} + 5e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-\beta_3} + 3e^{-3\beta_1} + e^{-4\beta_1-\beta_2+\beta_3} + e^{-4\beta_1} + e^{-5\beta_1-\beta_2+\beta_3} + e^{-5\beta_1+\beta_3} + 2e^{-\beta_2-2\beta_3} + 4e^{-2\beta_2-\beta_3} + e^{-3\beta_2-\beta_3} + e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{\beta_1-3\beta_2-2\beta_3} + 2e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2-2\beta_3} + 4e^{-\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2-2\beta_3} + 4e^{-2\beta_1-2\beta_2-\beta_3} + 2e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2-\beta_3} + 5e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2} + 5e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + 2e^{-4\beta_1-\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2+\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-\beta_3} + 3e^{-5\beta_1-\beta_2} + e^{-5\beta_1-2\beta_2+\beta_3} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2+\beta_3} + e^{-5\beta_1-\beta_3} + e^{-6\beta_1-\beta_2+\beta_3} + 2e^{-2\beta_2-2\beta_3} + e^{-3\beta_2-2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3} + 2e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-2\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1-4\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-2\beta_3} + 4e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + 3e^{-3\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + 2e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + 2e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2} + e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + 3e^{-5\beta_1-2\beta_2-\beta_3} + 3e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2} + e^{-6\beta_1-\beta_2-\beta_3} + 2e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-3\beta_2+\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-2\beta_1-4\beta_2-2\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + 2e^{-3\beta_1-4\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3} + 2e^{-5\beta_1-4\beta_2-\beta_3} + e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-4\beta_1-4\beta_2-2\beta_3} + e^{-5\beta_1-4\beta_2-2\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3} + e^{-6\beta_1-4\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 18, 65, 137, 168, 120, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 13, 34, 55, 60, 43, 19, 4)$

S=$2(3\beta_1 + 2\beta_2 + \beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 12, 22, 26, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 13, 13, 9, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = \omega^{1,5} - \omega^{2,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{1,7} = \omega^{1,2,4}$; $d\omega^{2,7} = \omega^{1,2,5}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = \omega^{1,3,5} - \omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{4,7} = \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6}+\omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7}+\omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7} - \omega^{1,2,4,6}$; $d\omega^{2,3,7} = \omega^{1,2,3,5}$; $d\omega^{2,4,7} = \omega^{1,2,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = \omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = -\omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-3\beta_1-\beta_3} \langle\omega^{1,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6} + \omega^{2,3,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,7}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7} + \omega^{3,4,5}, \omega^{1,4,6} - \omega^{3,4,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,7}\rangle + 2e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,6,7}, \omega^{2,3,5,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,6}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,4,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,5,7} + \omega^{2,3,4,7}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,7}\rangle + e^{-3\beta_1-4\beta_2-\beta_3} \langle\omega^{2,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,6,7} - \omega^{3,4,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6,7} - \omega^{3,4,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,4,5,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-3\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-5\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-4\beta_2-\beta_3} \langle\omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-4\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-6\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2)^3(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 12, 22, 26, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 9, 13, 13, 9, 4, 1)$

Total dimension of trivial cohomology: 54.

Length associated polynomial: 54.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = \omega^7$; $d^\star\omega^{2,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{1,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,7} + \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{2,7}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,5} = \omega^{3,7}$; $d^\star\omega^{1,4,5} = \omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7} + \omega^{4,6}$; $d^\star\omega^{2,3,5} = \omega^{5,6}$; $d^\star\omega^{2,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,5,6} + \omega^{2,3,7}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{1,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7} + \omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,4,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7} - \omega^{4,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5} - \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = -\omega^{1,5} + \omega^{2,4}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} - \omega^{2,4,5}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5} - \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} - \omega^{2,4,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = -\omega^{1,3,5} + 2\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,5,6} - \omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = -\omega^{1,5,7} + \omega^{2,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + 2\omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} - \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{2,3,4,5}$;

$\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,7} + 2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7} - \omega^{2,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,5,6} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,5,7} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = -\omega^{1,5,6,7} + \omega^{2,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5} + \omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_2-\beta_3} \langle\omega^{2,6}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-\beta_2-2\beta_3} \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-3\beta_1-\beta_3} \langle\omega^{1,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-5\beta_1-\beta_2} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6} + \omega^{2,3,5}\rangle + e^{-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,2,7} + \omega^{2,4,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,5,6} + \omega^{2,3,7} + 2\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} + \omega^{2,4,7}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,7}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7} + \omega^{3,4,5}, \omega^{1,4,6} - \omega^{3,4,5}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-3\beta_1-4\beta_2-\beta_3} \langle\omega^{2,5,6,7}\rangle + e^{-5\beta_1-3\beta_2} \langle\omega^{2,4,5,7}\rangle + e^{-\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,4,6,7} - \omega^{3,4,5,7})/2\rangle + e^{-6\beta_1-2\beta_2} \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle(-(\omega^{1,3,6,7} - \omega^{3,4,5,6}))/2\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(2\omega^{1,3,5,7} + \omega^{1,4,5,6} + \omega^{2,3,4,7})/6\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle(-(\omega^{1,3,5,6} + \omega^{2,3,4,6}))/2\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle-\omega^{1,3,4,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,4,6}\rangle + 2e^{-3\beta_1-3\beta_2-\beta_3} \langle(\omega^{1,2,6,7} - \omega^{2,3,5,7} - 2\omega^{2,4,5,6})/3, (-(\omega^{1,2,6,7} + 2\omega^{2,3,5,7} + \omega^{2,4,5,6}))/3\rangle + e^{-4\beta_1-3\beta_2} \langle-\omega^{1,2,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-5\beta_1-4\beta_2-\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle(-\omega^{1,3,5,6,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle-\omega^{1,3,4,5,7}\rangle + e^{-4\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-\beta_3} \langle(\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,2,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-2\beta_3} \langle(-\omega^{1,2,3,6,7} - \omega^{2,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{a}})= \quad e^{-5\beta_1-4\beta_2-2\beta_3} \quad \langle\omega^{2,3,4,5,6,7}\rangle \;+\; e^{-6\beta_1-3\beta_2-2\beta_3} \quad \langle-\omega^{1,3,4,5,6,7}\rangle \;+$ $e^{-6\beta_1-4\beta_2-\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathfrak{a}})= e^{-6\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) \ = \ (1+Y+Y^2)^3(1+Y).$

5.5.11 Algebra $\mathcal{G}_{7,3.10}$

Adjoint cohomology

$[x_1,x_2]=x_5$; $[x_1,x_3]=x_6$; $[x_2,x_4]=x_6$; $[x_2,x_6]=x_7$; $[x_3,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0\\ 2\xi_2^3 & \xi_2^4 & -\xi_2^1 & \xi_1^1-\xi_2^2+\xi_3^3 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_2^5 & -\xi_1^5 & -\xi_2^3 & \xi_1^1+\xi_3^3 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^6 & -\xi_1^5 & \xi_1^1+\xi_2^2+\xi_3^3 \end{pmatrix}$$

$ad\ x_1=\delta_2^5, ad\ x_2=-\delta_1^5, ad\ x_3=-\delta_1^6, ad\ x_4=-\delta_2^6, ad\ x_5=-\delta_3^7, ad\ x_6=-\delta_2^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+\beta_2+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{\beta_1-2\beta_2+\beta_3}\ \langle\delta_2^4\rangle + e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^6\rangle + e^{\beta_1}\ \langle\delta_2^5\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_2+\beta_3}\ \langle\delta_1^7\rangle + e^{\beta_1+\beta_3}\ \langle\delta_2^7\rangle + e^{\beta_1+\beta_2}\ \langle\delta_3^7\rangle + e^{2\beta_2}\ \langle\delta_4^7\rangle + e^{\beta_3}\ \langle\delta_1^6\rangle + e^{\beta_2}\ \langle\delta_1^5\rangle + 3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^4,\delta_4^7]=-\delta_2^7$; $[\delta_2^4,\delta_1^5]=\delta_2^6$; $[\delta_2^1,\delta_2^3]=-3\delta_2^4$; $[\delta_2^1,\delta_1^7]=-\delta_2^7$; $[\delta_2^1,\delta_4^7]=\delta_3^7$; $[\delta_2^1,\delta_1^6]=-\delta_2^6$; $[\delta_2^1,\delta_1^5]=-\delta_2^5$; $[\delta_3^5,\delta_2^3]=\delta_2^5$; $[\delta_3^5,\delta_1^6]=\delta_3^7$; $[\delta_2^6,\delta_1^5]=\delta_2^7$; $[\delta_2^5,\delta_2^3]=2\delta_2^6$; $[\delta_2^5,\delta_1^6]=\delta_2^7$; $[\delta_2^5,\delta_1^5]=\delta_3^7$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_2^3,\delta_4^7]=-2\delta_1^7$; $[\delta_2^3,\delta_1^5]=\delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1-2\beta_2+\beta_3}\ \langle\delta_2^4\rangle + e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_2+\beta_3}\ \langle\delta_1^7\rangle + e^{2\beta_2}\ \langle\delta_4^7\rangle + 3e^0\ \langle t_1,t_2,t_3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^1,\delta_2^3]=-3\delta_2^4$; $[\delta_2^3,\delta_4^7]=-2\delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5}\rangle + e^{-2\beta_1}\ \langle -x_2\omega^{1,5} - x_3(\omega^{1,6}-\omega^{4,5})\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,5}+x_4\omega^{1,5}\rangle + e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle + e^{-3\beta_2+\beta_3}\ \langle x_4\omega^{2,5}\rangle + 3e^{-\beta_3}\ \langle -x_1\omega^{2,4}+x_4\omega^{3,4}, -x_1\omega^{1,3}+x_5\omega^{3,5}, 2x_1(\omega^{1,3}-1/2\omega^{2,4})-x_2\omega^{2,3}+x_5\omega^{2,6}\rangle + e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,5}-x_2\omega^{2,5}-x_3(\omega^{2,6}+\omega^{3,5})-x_4(\omega^{1,6}-\omega^{4,5})\rangle + e^{2\beta_2-2\beta_3}\ \langle x_5\omega^{3,4}\rangle + e^{\beta_2-2\beta_3}\ \langle -x_1\omega^{3,4}+x_5\omega^{3,6}\rangle + 3e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_1\omega^{1,4}+x_3\omega^{3,4}+x_5\omega^{4,5}, -x_1\omega^{1,4}-x_2\omega^{1,3}+x_5\omega^{1,6}, x_1\omega^{1,4}-x_2\omega^{2,4}-x_3\omega^{3,4}+x_7\omega^{4,7}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_4\omega^{1,2}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,5}+x_4(\omega^{2,6}+\omega^{3,5})\rangle + e^{-\beta_1+2\beta_2}\ \langle x_7\omega^{1,4}\rangle +$

$3e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_3\omega^{1,3}-x_4\omega^{1,4}+x_7\omega^{1,7},-x_3\omega^{1,3}+x_5\omega^{1,5},x_3\omega^{2,4}+2x_4\omega^{1,4}+x_6\omega^{4,5}\rangle+3e^{-\beta_2}\ \langle -x_1\omega^{1,2}+x_4\omega^{1,3},x_4\omega^{2,4},-x_1\omega^{1,2}-x_3\omega^{2,3}+x_5\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1+2\beta_2-3\beta_3}+2e^{-\beta_1+\beta_2-2\beta_3}+e^{-\beta_1+\beta_2-3\beta_3}+2e^{-\beta_1-\beta_2-\beta_3}+e^{-\beta_1-2\beta_2-\beta_3}+3e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2+\beta_3}+e^{-\beta_1-3\beta_2}+e^{-\beta_1-4\beta_2}+e^{-\beta_1-\beta_3}+2e^{-\beta_1-2\beta_3}+e^{-2\beta_1+\beta_2-\beta_3}+2e^{-2\beta_1+\beta_2-2\beta_3}+3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+\beta_3}+4e^{-2\beta_1-\beta_3}+e^{-3\beta_1+2\beta_2-2\beta_3}+e^{-3\beta_1+\beta_2-\beta_3}+e^{-3\beta_1}+e^{-\beta_2-\beta_3}+e^{-\beta_2-2\beta_3}+e^{-2\beta_2-\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-3\beta_3}+2e^{-\beta_1-2\beta_2-2\beta_3}+e^{-\beta_1-2\beta_2-3\beta_3}+2e^{-\beta_1-3\beta_2-2\beta_3}+e^{-\beta_1-4\beta_2-\beta_3}+2e^{-2\beta_1+\beta_2-3\beta_3}+e^{-2\beta_1+\beta_2-4\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+e^{-2\beta_1-\beta_2-2\beta_3}+e^{-2\beta_1-\beta_2-3\beta_3}+e^{-2\beta_1-2\beta_2-\beta_3}+2e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-2\beta_2}+3e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-2\beta_3}+e^{-2\beta_1-3\beta_3}+e^{-3\beta_1+2\beta_2-3\beta_3}+2e^{-3\beta_1+\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-\beta_3}+e^{-3\beta_1-2\beta_3}+e^{-3\beta_1-3\beta_3}+e^{-4\beta_1+\beta_2-2\beta_3}+e^{-4\beta_1-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-\beta_2-4\beta_3}+3e^{-2\beta_1-2\beta_2-3\beta_3}+2e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-2\beta_1-3\beta_2-3\beta_3}+e^{-2\beta_1-4\beta_2-2\beta_3}+e^{-3\beta_1+\beta_2-3\beta_3}+3e^{-3\beta_1-\beta_2-3\beta_3}+2e^{-3\beta_1-2\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-4\beta_2-\beta_3}+2e^{-3\beta_1-4\beta_3}+e^{-4\beta_1+\beta_2-4\beta_3}+e^{-4\beta_1-\beta_2-3\beta_3}+2e^{-4\beta_1-2\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2-\beta_3}+e^{-4\beta_1-2\beta_3}+e^{-4\beta_1-3\beta_3}+e^{-5\beta_1-\beta_2-2\beta_3}+e^{-5\beta_1-3\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-\beta_2-4\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-2\beta_2-4\beta_3}+2e^{-3\beta_1-3\beta_2-3\beta_3}+e^{-3\beta_1-4\beta_2-2\beta_3}+e^{-4\beta_1-\beta_2-4\beta_3}+3e^{-4\beta_1-2\beta_2-3\beta_3}+2e^{-4\beta_1-3\beta_2-2\beta_3}+e^{-4\beta_1-4\beta_3}+2e^{-5\beta_1-\beta_2-3\beta_3}+e^{-5\beta_1-2\beta_2-2\beta_3}+e^{-5\beta_1-4\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-4\beta_1-2\beta_2-4\beta_3}+e^{-4\beta_1-3\beta_2-3\beta_3}+e^{-5\beta_1-\beta_2-4\beta_3}+e^{-5\beta_1-2\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,15,58,124,157,118,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,9,24,35,36,30,17,4)$

S=$5\beta_1+2\beta_2+4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,10,19,24,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,7,8,8,7,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,3}-\omega^{2,4}$; $d\omega^7=-\omega^{2,6}-\omega^{3,5}$; $d\omega^{1,6}=\omega^{1,2,4}$; $d\omega^{1,7}=\omega^{1,2,6}+\omega^{1,3,5}$; $d\omega^{2,6}=-\omega^{1,2,3}$; $d\omega^{2,7}=\omega^{2,3,5}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,6}=-\omega^{2,3,4}$; $d\omega^{3,7}=-\omega^{2,3,6}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=\omega^{1,3,4}$; $d\omega^{4,7}=-\omega^{2,4,6}-\omega^{3,4,5}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{1,3,5}+\omega^{2,4,5}$; $d\omega^{5,7}=-\omega^{1,2,7}-\omega^{2,5,6}$; $d\omega^{6,7}=-\omega^{1,3,7}-\omega^{2,4,7}+\omega^{3,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,5}$; $d\omega^{1,3,6}=\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,6}$; $d\omega^{1,4,7}=\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $d\omega^{1,5,6}=-\omega^{1,2,4,5}$; $d\omega^{1,5,7}=\omega^{1,2,5,6}$; $d\omega^{1,6,7}=\omega^{1,2,4,7}-\omega^{1,3,5,6}$; $d\omega^{2,4,6}=\omega^{1,2,3,4}$; $d\omega^{2,4,7}=\omega^{2,3,4,5}$; $d\omega^{2,5,6}=$

$\omega^{1,2,3,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{2,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} - \omega^{2,3,5,6}$; $d\omega^{3,6,7} = -\omega^{2,3,4,7}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{2,4,5,6}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,5,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7} = \omega^{1,2,4,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,5,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-3\beta_1-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,5,7}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,5,7}\rangle + e^{-3\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,3,4,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle(2\omega^{1,3,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7})/2\rangle + e^{-4\beta_1-2\beta_3}\ \langle\omega^{1,4,5,6}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,6,7} - \omega^{1,3,5,7}\rangle + e^{-3\beta_1-4\beta_3}\ \langle\omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-4\beta_1-4\beta_3}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-3\beta_1-\beta_2-4\beta_3}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-4\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-\beta_2-4\beta_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-5\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 10, 19, 24, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 7, 8, 8, 7, 4, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{2,6} = -\omega^7$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,6} + \omega^{4,5}$; $d^\star\omega^{1,2,6} = \omega^{1,7} - \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{1,7} + \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{2,3,6} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,6} = -\omega^{4,7}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,5,6} = -\omega^{5,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{1,3,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,4,6} = \omega^{1,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{1,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7} + \omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7} - \omega^{3,5,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} - \omega^{1,3,5,6} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} + \omega^{1,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{4,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,4} = \omega^{1,3} + \omega^{2,4}$; $\Delta\omega^{2,6} = 2\omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,5} = 2\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{1,6} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = 2\omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} + \omega^{2,4,7}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6} + 2\omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} + 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 3\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{1,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 3\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = 2\omega^{1,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$;

$\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{1,3,5,6} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} + 2\omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,3,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = 2\omega^{1,2,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = 2\omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}-\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6}-\omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-3\beta_1-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6} - \omega^{1,3,5} + 2\omega^{2,4,5}\rangle + e^{-2\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,5,7}\rangle + e^{-2\beta_1-2\beta_3}\ \langle 2\omega^{1,3,6}-\omega^{2,4,6}+\omega^{3,4,5}\rangle + e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-4\beta_3}\ \langle\omega^{3,4,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle-\omega^{2,3,5,7}\rangle + e^{-4\beta_1-2\beta_3}\ \langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle(-(2\omega^{1,3,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}))/6\rangle + e^{-3\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,3,4,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{2,4,5,7})/6\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle-\omega^{1,2,5,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-3\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-\beta_2-4\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle(\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-4\beta_1-4\beta_3}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3}\ \langle(-\omega^{1,2,3,6,7} + \omega^{2,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-4\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-5\beta_1-\beta_2-4\beta_3}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-3\beta_3}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-5\beta_1-2\beta_2-4\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1+Y+Y^2+Y^3+Y^4)(1+Y)^3.$

5.5.12 Algebra $\mathcal{G}_{7,3.11}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_5] = x_7$; $[x_2, x_4] = x_6$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & -\xi_1^2 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & \xi_1^1 - \xi_2^2 + \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^3 - \xi_1^4 & \xi_1^1 + \xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^5 & 0 & 2\xi_1^1 + \xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5 + \delta_4^6, ad\ x_3 = -\delta_1^6, ad\ x_4 = -\delta_2^6, ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{2\beta_1+\beta_2}\ \langle x_7 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{2\beta_1+\beta_2}\ \langle x_7 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4 \rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_4^6 \rangle + e^{\beta_3}\ \langle \delta_1^6 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + 2e^{\beta_1}\ \langle \delta_3^6, \delta_2^5 \rangle + 2e^{-\beta_2+\beta_3}\ \langle \delta_2^3, \delta_1^4 \rangle + 3e^0\ \langle t_3, t_1, t_2 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^7 \rangle + e^{2\beta_1}\ \langle \delta_2^7 \rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^7 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^4] = -(\delta_1^4 + \delta_2^3)$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_3^6] = \delta_4^6$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^2, \delta_1^4] = -\delta_1^3$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_3^7] = \delta_4^7$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^5, \delta_1^4] = \delta_1^6$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_3^6, \delta_2^3] = \delta_2^6$; $[\delta_4^6, \delta_1^4] = \delta_1^6$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_2^5] = \delta_2^6$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^4, \delta_2^5] = -\delta_2^6$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_3}\ \langle \delta_1^3 \rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4 \rangle + e^{\beta_2}\ \langle \delta_1^5 \rangle + 2e^{-\beta_2+\beta_3}\ \langle \delta_2^3, \delta_1^4 \rangle + 3e^0\ \langle t_3, t_1, t_2 \rangle + e^{2\beta_1}\ \langle \delta_2^7 \rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^7 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^7 \rangle + e^{\beta_1}\ \langle \delta_2^5 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^4] = -(\delta_1^4 + \delta_2^3)$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^2, \delta_1^4] = -\delta_1^3$; $[\delta_1^2, \delta_3^7] = \delta_4^7$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_1+2\beta_2-\beta_3}\ \langle x_2\omega^{1,4} \rangle + e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,4} \rangle + e^{-3\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,7} \rangle + 4e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3} + x_3\omega^{3,4}, x_1\omega^{1,4} + x_2(\omega^{1,3} - 2\omega^{2,4}) + x_5\omega^{4,5}, -x_2\omega^{2,4} + x_5\omega^{1,6}, -x_1\omega^{1,4} - x_2(\omega^{1,3} - \omega^{2,4}) + x_7\omega^{4,7} \rangle + 2e^{-2\beta_1}\ \langle -x_2\omega^{1,5} - x_3(\omega^{1,6} - \omega^{4,5}), x_5\omega^{1,7} \rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_4\omega^{1,4}, x_3\omega^{1,3}, x_2\omega^{1,2} + x_3\omega^{2,4}, x_5\omega^{1,5} \rangle +$

$e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle + e^{-3\beta_2+\beta_3}\ \langle x_4\omega^{2,5}\rangle + e^{-2\beta_2}\ \langle -x_1\omega^{2,5} + x_4(\omega^{2,6} + \omega^{3,5})\rangle + 2e^{-\beta_1-\beta_2}\ \langle x_1\omega^{1,5} - x_2\omega^{2,5} - x_3(\omega^{2,6} + \omega^{3,5}) - x_4(\omega^{1,6} - \omega^{4,5}), x_1\omega^{1,5} - x_2\omega^{2,5} + x_3(\omega^{2,6} + \omega^{3,5}) + 3x_6(\omega^{3,7} + 1/3\omega^{5,6})\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} - 2x_4\omega^{3,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{3,4} + x_7\omega^{3,7}, x_4\omega^{3,4} + x_6\omega^{3,6}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_4\omega^{1,2}\rangle + 4e^{-\beta_2}\ \langle x_3\omega^{2,3} + x_4\omega^{1,3}, x_4\omega^{2,4}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_5\omega^{2,5}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_7\omega^{2,7}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_3\omega^{2,5} + x_4\omega^{1,5}\rangle + e^{-2\beta_1-2\beta_2+\beta_3}\ \langle -x_4\omega^{1,7} + x_6\omega^{5,7}\rangle + e^{2\beta_1-\beta_3}\ \langle x_7\omega^{2,3}\rangle + e^{\beta_1-\beta_3}\ \langle x_5\omega^{2,3} + x_7\omega^{2,6}\rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle x_7\omega^{3,4}\rangle + e^{\beta_2-\beta_3}\ \langle x_5\omega^{1,3}\rangle + e^{2\beta_2-2\beta_3}\ \langle -x_5\omega^{3,4} + x_7\omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_3} + 3e^{-\beta_1+\beta_2-2\beta_3} + 5e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + e^{-2\beta_1+2\beta_2-2\beta_3} + e^{-2\beta_1+\beta_2-\beta_3} + 2e^{-2\beta_1+\beta_2-2\beta_3} + 2e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-4\beta_2+\beta_3} + 6e^{-2\beta_1-\beta_3} + e^{-3\beta_1+2\beta_2-2\beta_3} + 2e^{-3\beta_1+\beta_2-\beta_3} + 4e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-\beta_3} + e^{-4\beta_1+\beta_2-\beta_3} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + e^{-4\beta_1} + e^{2\beta_2-3\beta_3} + e^{\beta_2-2\beta_3} + 2e^{-\beta_2-\beta_3} + 2e^{-2\beta_2-\beta_3} + e^{-3\beta_2}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=3e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + 2e^{-2\beta_1+\beta_2-3\beta_3} + 2e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-4\beta_2} + 5e^{-2\beta_1-2\beta_3} + e^{-3\beta_1+2\beta_2-3\beta_3} + 3e^{-3\beta_1+\beta_2-2\beta_3} + 4e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + 4e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2+\beta_3} + 2e^{-3\beta_1-2\beta_3} + 2e^{-4\beta_1+\beta_2-2\beta_3} + 4e^{-4\beta_1-\beta_2-\beta_3} + 4e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2+\beta_3} + 2e^{-4\beta_1-\beta_3} + e^{-5\beta_1+\beta_2-2\beta_3} + e^{-5\beta_1-\beta_2} + 2e^{-5\beta_1-\beta_3} + e^{\beta_2-3\beta_3}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-3\beta_3} + 2e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_3} + e^{-3\beta_1+\beta_2-3\beta_3} + 2e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + 2e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2-\beta_3} + e^{-3\beta_1-3\beta_3} + e^{-4\beta_1+\beta_2-3\beta_3} + 3e^{-4\beta_1-\beta_2-2\beta_3} + 4e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2} + e^{-4\beta_1-2\beta_3} + e^{-5\beta_1+\beta_2-3\beta_3} + 2e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + 2e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-3\beta_2} + 3e^{-5\beta_1-2\beta_3} + e^{-6\beta_1-\beta_2-\beta_3} + e^{-6\beta_1-2\beta_3}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + 2e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-4\beta_2-\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + e^{-5\beta_1-\beta_2-3\beta_3} + 3e^{-5\beta_1-2\beta_2-2\beta_3} + 2e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-3\beta_3} + 2e^{-6\beta_1-\beta_2-2\beta_3} + e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-3\beta_3}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-5\beta_1-2\beta_2-3\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3} + e^{-6\beta_1-\beta_2-3\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 18, 65, 136, 166, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 13, 34, 54, 57, 40, 18, 4)$

S$=6\beta_1 + 2\beta_2 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathfrak{a}}$'s $0 \le j \le 7$: $(1, 4, 11, 23, 25, 18, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1,4,8,13,13,8,4,1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,2,7}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = \omega^{1,3,5,6} - \omega^{2,3,4,7}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}})= e^{-2\beta_1+\beta_2-\beta_3} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-2\beta_3} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6}+\omega^{3,5}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}})= e^{-3\beta_1+\beta_2-2\beta_3} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,7}\rangle + 2e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6} + \omega^{3,4,5}, \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1+\beta_2-3\beta_3} \langle\omega^{3,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,6,7} - \omega^{4,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,7}\rangle + 2e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6,7}, \omega^{1,3,5,7}\rangle + e^{-3\beta_1+\beta_2-3\beta_3} \langle\omega^{1,3,4,6}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,3,4,7}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-5\beta_1-2\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,3,6}\rangle + e^{-2\beta_1-3\beta_3} \langle\omega^{2,3,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6} - \omega^{2,4,5,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-5\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-3\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-\beta_2-2\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-5\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

ch($H^7_{\mathbb{C}}$)= $e^{-6\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y+5Y^2+8Y^3+5Y^4+3Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,11,23,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,8,13,13,8,4,1)$

Total dimension of trivial cohomology: 52.

Length associated polynomial: 52.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,6}+\omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{4,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,6}+\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7}+\omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,6}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,6}-\omega^{2,3,4,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7}+\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{4,5}$; $\Delta\omega^{2,4} = \omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6}-\omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,5} = -\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}-\omega^{5,6}$; $\Delta\omega^{4,5} = \omega^{1,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = -\omega^{3,7}+3\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}-\omega^{1,3,5}-\omega^{2,4,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6}+2\omega^{1,3,5}+\omega^{2,4,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}+\omega^{2,4,6}-\omega^{3,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}-\omega^{1,5,6}+\omega^{2,4,7}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,5,6} = -\omega^{1,3,7}+2\omega^{1,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}+\omega^{4,5,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}-\omega^{2,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,6}+\omega^{1,3,5}+\omega^{2,4,5}$; $\Delta\omega^{2,4,6} = \omega^{1,3,6}+\omega^{2,4,6}-\omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7}+2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{2,3,7}+\omega^{2,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}-\omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,6}-\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}+\omega^{4,5,6}$;

$\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,5,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,3,7} + \omega^{1,2,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,3,5,6} + \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} + 2\omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,3,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{a}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{a}}) = e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3} - \omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{a}}) = e^{-3\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_3}\ \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle 2\omega^{1,3,7} + \omega^{1,5,6} - \omega^{2,4,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,7} + \omega^{2,5,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{2,5,7}\rangle + 2e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6} + \omega^{3,4,5}, \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6,7} - \omega^{4,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{a}}) = e^{-3\beta_1-3\beta_2-\beta_3}\ \langle -\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-3\beta_3}\ \langle -\omega^{2,3,4,6}\rangle + e^{-5\beta_1-2\beta_3}\ \langle\omega^{1,4,6,7}\rangle + e^{-5\beta_1-\beta_2-\beta_3}\ \langle -\omega^{1,4,5,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3}\ \langle(-(\omega^{1,3,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle(\omega^{1,3,5,6} + \omega^{2,3,4,7} - 2\omega^{2,4,5,6})/6\rangle + e^{-4\beta_1-2\beta_3}\ \langle(-(\omega^{1,3,4,7} - \omega^{1,4,5,6}))/2\rangle + e^{-3\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,3,4,6}\rangle + 2e^{-4\beta_1-2\beta_2-\beta_3}\ \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + 2\omega^{2,4,5,7})/3, (\omega^{1,2,6,7} + 2\omega^{1,3,5,7} - \omega^{2,4,5,7})/3\rangle + e^{-4\beta_1-3\beta_2}\ \langle -\omega^{1,2,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle(-(\omega^{1,2,3,6} - \omega^{2,3,4,5}))/2\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{a}}) = e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle -\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3}\ \langle(\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle +$

$e^{-5\beta_1-3\beta_3}$ $\langle\omega^{1,3,4,6,7}\rangle$ + $e^{-5\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,5,6,7}\rangle$ + $e^{-5\beta_1-\beta_2-2\beta_3}$ $\langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}$ $\langle(-\omega^{1,2,3,6,7} + \omega^{2,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})=$ $e^{-5\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-6\beta_1-\beta_2-3\beta_3}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-6\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-2\beta_3}$ $\langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-6\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1 + 3Y + 5Y^2 + 8Y^3 + 5Y^4 + 3Y^5 + Y^6)(1 + Y)$.

5.5.13 Algebra $\mathcal{G}_{7,3.12}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_2, x_4] = x_6$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & \xi_4^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & -\xi_4^1 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & \xi_3^3 & -\xi_1^2 & 0 & 0 & 0 \\ -\xi_2^3 & 0 & -\xi_2^1 & \xi_1^1 - \xi_2^2 + \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1 + \xi_2^2 & -\xi_4^1 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & 2\xi_2^3 & \xi_1^1 + \xi_3^3 & -2\xi_4^1 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & 0 & \xi_2^3 & \xi_1^1 - \xi_2^2 + 2\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5 + \delta_4^6, ad\ x_3 = -\delta_1^6 + \delta_4^7, ad\ x_4 = -(\delta_2^6 + \delta_3^7), ad\ x_5 = 0, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle x_7 \rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle x_7 \rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_4^6 \rangle + 2e^{\beta_1}\ \langle \delta_2^5, \delta_3^6 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_4^5 \rangle + 2e^{\beta_3}\ \langle \delta_1^6, \delta_4^7 \rangle + 2e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6, \delta_3^7 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_4^1 \rangle + e^{-\beta_2+2\beta_3}\ \langle \delta_1^7 \rangle + e^{\beta_1-2\beta_2+2\beta_3}\ \langle \delta_2^7 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + 3e^0\ \langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^1] = t_3 + t_2 - t_1$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_3^5] = \delta_4^5$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_3^6] = \delta_4^6$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_3^7] = \delta_4^7$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_4^5] = \delta_3^5$; $[\delta_2^1, \delta_1^6] = -\delta_2^6$; $[\delta_2^1, \delta_4^6] = \delta_3^6$; $[\delta_2^1, \delta_1^7] = -\delta_2^7$; $[\delta_2^1, \delta_4^7] = \delta_3^7$; $[\delta_1^5, \delta_4^1] = \delta_4^5$; $[\delta_1^5, \delta_2^3] = -2\delta_1^6$; $[\delta_2^5, \delta_4^1] = -\delta_3^5$; $[\delta_2^5, \delta_2^3] = -2\delta_2^6$; $[\delta_3^5, \delta_2^3] = -2\delta_3^6 + \delta_2^5$; $[\delta_4^5, \delta_2^3] = -2\delta_4^6 - \delta_1^5$; $[\delta_1^6, \delta_4^1] = \delta_4^6 + \delta_1^5$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_2^6, \delta_4^1] = -\delta_3^6 + \delta_2^5$; $[\delta_2^6, \delta_2^3] = -\delta_2^7$; $[\delta_3^6, \delta_4^1] = \delta_3^5$; $[\delta_3^6, \delta_2^3] = -\delta_3^7 + \delta_2^6$; $[\delta_4^6, \delta_4^1] = \delta_4^5$; $[\delta_4^6, \delta_2^3] = -(\delta_4^7 + \delta_1^6)$; $[\delta_4^1, \delta_1^7] = -\delta_4^7 - 2\delta_1^6$; $[\delta_4^1, \delta_2^7] = \delta_3^7 - 2\delta_2^6$; $[\delta_4^1, \delta_3^7] = -2\delta_3^6$; $[\delta_4^1, \delta_4^7] = -2\delta_4^6$; $[\delta_4^1, \delta_2^3] = t_3 - t_2 - t_1$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_4^7, \delta_2^3] = -\delta_1^7$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_4^5 \rangle + e^{\beta_1}\ \langle \delta_2^5 \rangle + e^{\beta_2}\ \langle \delta_1^5 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_4^1 \rangle + e^{-\beta_2+2\beta_3}\ \langle \delta_1^7 \rangle + e^{\beta_1-2\beta_2+2\beta_3}\ \langle \delta_2^7 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{\beta_3}\ \langle \delta_1^6 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + 3e^0\ \langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^1] = t_3 + t_2 - t_1$; $[\delta_1^2, \delta_3^5] = \delta_4^5$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_2^1, \delta_4^5] = \delta_3^5$; $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^7] = -\delta_2^7$; $[\delta_2^1, \delta_1^6] = -\delta_2^6$; $[\delta_3^5, \delta_2^3] = 3\delta_2^5$; $[\delta_4^5, \delta_2^3] = -3\delta_1^5$; $[\delta_2^5, \delta_4^1] = -\delta_3^5$; $[\delta_2^5, \delta_2^3] = -2\delta_2^6$; $[\delta_1^5, \delta_4^1] = \delta_4^5$; $[\delta_1^5, \delta_2^3] = -2\delta_1^6$; $[\delta_4^1, \delta_1^7] = -3\delta_1^6$;

$[\delta_4^1,\delta_2^7] = -3\delta_2^6$; $[\delta_4^1,\delta_2^6] = -2\delta_2^5$; $[\delta_4^1,\delta_1^6] = -2\delta_1^5$; $[\delta_4^1,\delta_2^3] = t_3 - t_2 - t_1$; $[\delta_2^6,\delta_2^3] = -\delta_2^7$; $[\delta_1^6,\delta_2^3] = -\delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle + e^{-2\beta_1+2\beta_2-\beta_3}\ \langle x_2\omega^{1,4}\rangle + 5e^{-\beta_3}\ \langle -x_1(\omega^{1,3} - \omega^{2,4}) + x_2\omega^{2,3}, -x_1\omega^{1,3} + x_5\omega^{3,5}, x_1\omega^{1,3} + x_5\omega^{2,6}, -x_1\omega^{2,4} + x_4\omega^{3,4}, -x_1\omega^{2,4} + x_6\omega^{2,7}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_3\omega^{1,4}\rangle + 5e^{-\beta_1}\ \langle -x_2\omega^{1,2} - x_3\omega^{1,3} + x_4\omega^{1,4}, x_5\omega^{1,5}, x_3\omega^{1,3} + x_7\omega^{1,7}, x_2\omega^{1,2} + x_3\omega^{2,4}, x_2\omega^{1,2} + x_6\omega^{1,6}\rangle + e^{\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle + 2e^{\beta_2-2\beta_3}\ \langle -x_1\omega^{3,4} + x_5\omega^{3,6}, x_1\omega^{3,4} + x_5\omega^{2,7}\rangle + e^{2\beta_2-3\beta_3}\ \langle x_5\omega^{3,7}\rangle + 5e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{2,4} + x_5\omega^{4,5}, x_1\omega^{1,4} + x_2(\omega^{1,3} - \omega^{2,4}), -x_2\omega^{2,4} + x_5\omega^{1,6}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3\omega^{3,4}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_6\omega^{1,7}\rangle + 2e^{-\beta_1+2\beta_2-2\beta_3}\ \langle -x_2\omega^{3,4} + x_5\omega^{4,6}, -x_2\omega^{3,4} + x_5\omega^{1,7}\rangle + e^{-\beta_1+3\beta_2-3\beta_3}\ \langle x_5\omega^{4,7}\rangle + 5e^{-\beta_2}\ \langle x_5\omega^{2,5}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_7\omega^{2,7}, -x_1\omega^{1,2} + x_4\omega^{1,3}, -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_6\omega^{2,6}\rangle + e^{-\beta_1-2\beta_2+2\beta_3}\ \langle x_7\omega^{1,5}\rangle + 2e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2} + x_7\omega^{1,6}, x_6\omega^{1,5}\rangle + e^{-3\beta_2+2\beta_3}\ \langle x_7\omega^{2,5}\rangle + 2e^{-2\beta_2+\beta_3}\ \langle x_4\omega^{1,2} + x_7\omega^{2,6}, x_6\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{-\beta_1+3\beta_2-4\beta_3} + e^{-\beta_1+2\beta_2-3\beta_3} + e^{-\beta_1+2\beta_2-4\beta_3} + e^{-\beta_1+\beta_2-2\beta_3} + 4e^{-\beta_1+\beta_2-3\beta_3} + 6e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2} + e^{-\beta_1-2\beta_2+\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2+2\beta_3} + e^{-\beta_1-3\beta_2+\beta_3} + 2e^{-\beta_1-\beta_3} + 6e^{-\beta_1-2\beta_3} + e^{-2\beta_1+3\beta_2-4\beta_3} + 4e^{-2\beta_1+2\beta_2-3\beta_3} + e^{-2\beta_1+\beta_2-\beta_3} + 6e^{-2\beta_1+\beta_2-2\beta_3} + 4e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + 6e^{-2\beta_1-\beta_3} + e^{-3\beta_1+3\beta_2-3\beta_3} + 2e^{-3\beta_1+2\beta_2-2\beta_3} + 2e^{-3\beta_1+\beta_2-\beta_3} + e^{-3\beta_1} + e^{-\beta_2-\beta_3} + 2e^{-\beta_2-2\beta_3} + 2e^{-2\beta_2-\beta_3} + e^{-3\beta_2} + e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1+\beta_2-4\beta_3} + 3e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-\beta_2-3\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-3\beta_3} + e^{-\beta_1-4\beta_3} + e^{-2\beta_1+3\beta_2-5\beta_3} + 3e^{-2\beta_1+2\beta_2-4\beta_3} + 5e^{-2\beta_1+\beta_2-3\beta_3} + 2e^{-2\beta_1+\beta_2-4\beta_3} + 5e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_3} + 3e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + 6e^{-2\beta_1-2\beta_3} + 3e^{-2\beta_1-3\beta_3} + e^{-3\beta_1+3\beta_2-4\beta_3} + 2e^{-3\beta_1+2\beta_2-3\beta_3} + 2e^{-3\beta_1+2\beta_2-4\beta_3} + 3e^{-3\beta_1+\beta_2-2\beta_3} + 3e^{-3\beta_1+\beta_2-3\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-\beta_3} + 3e^{-3\beta_1-2\beta_3} + e^{-4\beta_1+3\beta_2-4\beta_3} + 2e^{-4\beta_1+2\beta_2-3\beta_3} + 2e^{-4\beta_1+\beta_2-2\beta_3} + e^{-4\beta_1-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1+\beta_2-5\beta_3} + 3e^{-2\beta_1-\beta_2-3\beta_3} + e^{-2\beta_1-\beta_2-4\beta_3} + 3e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + 3e^{-2\beta_1-4\beta_3} + 2e^{-3\beta_1+2\beta_2-5\beta_3} + 5e^{-3\beta_1+\beta_2-4\beta_3} + 5e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + 5e^{-3\beta_1-3\beta_3} + e^{-3\beta_1-4\beta_3} + e^{-4\beta_1+3\beta_2-5\beta_3} + 3e^{-4\beta_1+2\beta_2-4\beta_3} + 3e^{-4\beta_1+\beta_2-3\beta_3} + e^{-4\beta_1+\beta_2-4\beta_3} + e^{-4\beta_1-\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_3} + e^{-4\beta_1-3\beta_3} + e^{-5\beta_1+2\beta_2-4\beta_3} + e^{-5\beta_1+\beta_2-3\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1+\beta_2-5\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + 2e^{-3\beta_1-\beta_2-4\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-4\beta_3} + e^{-3\beta_1-5\beta_3} + e^{-4\beta_1+2\beta_2-5\beta_3} + e^{-4\beta_1+\beta_2-4\beta_3} + e^{-4\beta_1+\beta_2-5\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_3} + 3e^{-4\beta_1-4\beta_3} + e^{-5\beta_1+2\beta_2-5\beta_3} + 2e^{-5\beta_1+\beta_2-4\beta_3} + e^{-5\beta_1-3\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-4\beta_1-\beta_2-4\beta_3} + e^{-4\beta_1-5\beta_3} + e^{-5\beta_1+\beta_2-5\beta_3} + e^{-5\beta_1-4\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 19, 66, 147, 172, 122, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 15, 36, 66, 74, 49, 21, 4)$

S=$5(\beta_1 + \beta_3)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 14, 21, 28, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 14, 14, 11, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3} - \omega^{2,4}$; $d\omega^7 = -\omega^{3,4}$; $d\omega^{1,6} = \omega^{1,2,4}$; $d\omega^{1,7} = \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,4,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} - \omega^{2,4,7} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,5,7} = -\omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,5,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = -\omega^{2,3,4,7}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = \omega^{1,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3} \langle\omega^4\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-2\beta_1+\beta_2-\beta_3} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{2,7} + \omega^{3,6}\rangle + e^{-\beta_1+\beta_2-3\beta_3} \langle\omega^{3,7}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,6} - \omega^{4,5}\rangle + e^{-2\beta_1+\beta_2-2\beta_3} \langle\omega^{1,7} - \omega^{4,6}\rangle + e^{-2\beta_1+2\beta_2-3\beta_3} \langle\omega^{4,7}\rangle$.

ch($H^3_{\mathbb{C}}$)= $e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + 2e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6}, \omega^{1,3,5}\rangle + 2e^{-2\beta_1+\beta_2-3\beta_3} \langle\omega^{1,3,7}, \omega^{2,4,7}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-3\beta_1+\beta_2-2\beta_3} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1+2\beta_2-3\beta_3} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_3} \langle\omega^{2,3,7}\rangle + 2e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6} + \omega^{3,4,5}, \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1+2\beta_2-4\beta_3} \langle\omega^{3,4,7}\rangle$.

ch($H^4_{\mathbb{C}}$)= $e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + 2e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,5,7} + \omega^{2,4,5,6}, \omega^{1,3,5,6} - \omega^{2,4,5,6}\rangle + 2e^{-3\beta_1-3\beta_3} \langle\omega^{1,3,5,7}, \omega^{1,2,6,7} + \omega^{3,4,5,6}\rangle + 2e^{-3\beta_1+\beta_2-4\beta_3} \langle\omega^{1,3,6,7} + \omega^{3,4,5,7}, \omega^{2,4,6,7} + \omega^{3,4,5,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1+\beta_2-3\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-4\beta_1+2\beta_2-4\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-4\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1+2\beta_2-5\beta_3} \langle\omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}})= e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,3,5,6}\rangle + e^{-4\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,6}\rangle + e^{-4\beta_1-3\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1+\beta_2-4\beta_3}\ \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1+2\beta_2-5\beta_3}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-5\beta_1+\beta_2-4\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{1,2,3,5,7}\rangle + e^{-3\beta_1-4\beta_3}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-3\beta_1+\beta_2-5\beta_3}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-4\beta_1-4\beta_3}\ \langle\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}})= e^{-4\beta_1-\beta_2-4\beta_3}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-5\beta_1-4\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1+\beta_2-5\beta_3}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-5\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})= e^{-5\beta_1-5\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y+7Y^2+3Y^3+Y^4)(1+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,4,14,21,28,18,7,1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0\le j\le 7$: $(1,4,11,14,14,11,4,1)$

Total dimension of trivial cohomology: 60.

Length associated polynomial: 60.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{2,4} = -\omega^6$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,6}+\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}+\omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}-\omega^{3,6}$; $d^\star\omega^{2,4,5} = \omega^{5,6}$; $d^\star\omega^{2,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7}+\omega^{1,3,6}+\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7}+\omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,6}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{1,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}+\omega^{3,5,6}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,4,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7}-\omega^{1,3,5,6}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{1,3,6,7}+\omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,3,5,6,7}-\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{4,5}$; $\Delta\omega^{1,7} = \omega^{1,7}+\omega^{4,6}$; $\Delta\omega^{2,4} = \omega^{1,3}+\omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6}-\omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}-\omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,6} = -\omega^{2,7}+\omega^{3,6}$; $\Delta\omega^{4,5} = \omega^{1,6}+\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{1,7}+\omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,6} =$

$\omega^{1,2,6} - \omega^{1,3,5} - \omega^{2,4,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,6} - \omega^{2,4,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{1,3,6} = -\omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{2,4,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{4,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{2,3,4} = 2\omega^{2,3,4}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,6} + \omega^{1,3,5} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,2,7} + \omega^{1,3,6} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{1,3,7} + \omega^{2,4,7} - \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,6} - \omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,3,7} - \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = -\omega^{2,5,7} + 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,5,6} - \omega^{2,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,5,7} - \omega^{2,4,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = 2\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,5,7} + \omega^{1,3,5,6} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{1,3,6,7} + \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,5,7} - \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,3,6,7} - \omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,3,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{1,3,5,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}-\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,6}+\omega^{3,5}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{2,7}+\omega^{3,6}\rangle + e^{-\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,7}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6}-\omega^{4,5}\rangle + e^{-2\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,7}-\omega^{4,6}\rangle + e^{-2\beta_1+2\beta_2-3\beta_3}\ \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-3\beta_1-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1+2\beta_2-3\beta_3}\ \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_3}\ \langle\omega^{2,3,7}\rangle + 2e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6} + \omega^{2,4,5}, \omega^{1,3,5} - \omega^{2,4,5}\rangle + 2e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6} - \omega^{2,4,6}, \omega^{1,2,7} + 2\omega^{2,4,6} + \omega^{3,4,5}\rangle + 2e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,3,7} + \omega^{3,4,6}, \omega^{2,4,7} + \omega^{3,4,6}\rangle + e^{-2\beta_1+2\beta_2-4\beta_3}\ \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1+2\beta_2-5\beta_3} \langle\omega^{3,4,6,7}\rangle + e^{-2\beta_1-4\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-4\beta_1+2\beta_2-4\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1+\beta_2-3\beta_3} \langle-\omega^{1,4,5,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,4,5,6}\rangle + 2e^{-3\beta_1+\beta_2-4\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{2,4,6,7} + 2\omega^{3,4,5,7}))/3, (\omega^{1,3,6,7} - 2\omega^{2,4,6,7} - \omega^{3,4,5,7})/3\rangle + 2e^{-3\beta_1-3\beta_3} \langle(\omega^{1,2,6,7} - \omega^{1,3,5,7} + 3\omega^{2,4,5,7} + \omega^{3,4,5,6})/4, (\omega^{1,2,6,7} + \omega^{1,3,5,7} + \omega^{2,4,5,7} + \omega^{3,4,5,6})/4\rangle + 2e^{-3\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,2,5,7} - \omega^{1,3,5,6} + 2\omega^{2,4,5,6}))/3, (-(\omega^{1,2,5,7} + 2\omega^{1,3,5,6} - \omega^{2,4,5,6}))/3\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-\beta_2-4\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1+\beta_2-5\beta_3} \langle-\omega^{2,3,4,6,7}\rangle + e^{-5\beta_1+\beta_2-4\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_3} \langle(\omega^{1,3,5,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-4\beta_1+2\beta_2-5\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1+\beta_2-4\beta_3} \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-4\beta_1-3\beta_3} \langle(\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6})/2\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle-\omega^{1,2,4,5,6}\rangle + e^{-3\beta_1-4\beta_3} \langle(-\omega^{1,2,3,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle(\omega^{1,2,3,5,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-4\beta_1-5\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-5\beta_1+\beta_2-5\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-4\beta_3} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-5\beta_1-5\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + 7Y^2 + 3Y^3 + Y^4)(1 + Y^2)(1 + Y).$

5.5.14 Algebra $\mathcal{G}_{7,3.13}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & \xi_1^1 + 2\xi_2^2 - \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^5 & 2\xi_1^1 + \xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^5 & \xi_1^2 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5, ad\ x_3 = \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -(\delta_1^6 + \delta_2^7), ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1-2\beta_2+2\beta_3}\ \langle \delta_4^3\rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^4\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + 2e^{\beta_1+\beta_2}\ \langle \delta_1^6, \delta_2^7\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_4^6\rangle + e^{\beta_1}\ \langle \delta_2^5\rangle + e^{2\beta_2}\ \langle \delta_1^7\rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_3}\ \langle \delta_4^7\rangle + e^{\beta_2}\ \langle \delta_1^5\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_3^6] = -\delta_4^6$; $[\delta_4^3, \delta_3^7] = -\delta_4^7$; $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = -\delta_2^7 + \delta_1^6$; $[\delta_3^6, \delta_1^2] = -\delta_3^7$; $[\delta_4^6, \delta_1^2] = -\delta_4^7$; $[\delta_2^5, \delta_1^5] = \delta_2^7 + \delta_1^6$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^7, \delta_1^2] = \delta_1^7$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1-2\beta_2+2\beta_3}\ \langle \delta_4^3\rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^4\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_4^6\rangle + e^{2\beta_2}\ \langle \delta_1^7\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^6\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_3^6] = -\delta_4^6$; $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_1^6, \delta_1^2] = -\delta_1^7$; $[\delta_2^6, \delta_1^2] = 2\delta_1^6$.

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_1-2\beta_2+2\beta_3}\ \langle x_3\omega^{1,4}\rangle + e^{-\beta_1-3\beta_2+2\beta_3}\ \langle x_3\omega^{2,4}\rangle + e^{2\beta_2-2\beta_3}\ \langle x_4\omega^{1,3}\rangle + 3e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2} + x_3\omega^{1,3} + x_7\omega^{1,7}, x_6\omega^{1,6}\rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle x_4\omega^{2,3}\rangle + 3e^{-\beta_2}\ \langle -x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_6\omega^{2,6}, x_1\omega^{1,2} - x_3\omega^{2,3} - x_5\omega^{3,4} + x_6\omega^{1,7}\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} + 3x_4\omega^{2,5} + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_2\omega^{2,3} - x_4\omega^{2,5} + x_6\omega^{3,6}, x_4\omega^{3,4}\rangle + 3e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_1\omega^{1,4} - 2x_2\omega^{2,4} - 3x_3\omega^{2,5} + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3\omega^{2,5} + x_6\omega^{4,6}, x_3\omega^{3,4}\rangle + e^{2\beta_1-\beta_3}\ \langle x_6\omega^{2,3}\rangle + e^{\beta_1-2\beta_2+\beta_3}\ \langle x_6\omega^{2,4}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3} + x_6\omega^{3,7}\rangle + e^{-3\beta_2+\beta_3}\ \langle -x_1\omega^{2,4} + x_6\omega^{4,7}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{2,5}\rangle +$

$e^{-2\beta_1+\beta_2}\ \langle x_7\omega^{1,6}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{1,3} + x_4\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,5} + x_2\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-3\beta_2+\beta_3} + e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2+\beta_3} + 2e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2+\beta_3} + e^{-2\beta_1-5\beta_2+2\beta_3} + e^{-2\beta_1-\beta_3} + 2e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-4\beta_2+2\beta_3} + e^{-3\beta_1-\beta_3} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + e^{-4\beta_1-3\beta_2+2\beta_3} + 2e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3} + e^{-3\beta_2} + e^{-4\beta_2+\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$e^{-\beta_1-2\beta_2-2\beta_3} + 2e^{-\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_3} + 2e^{-2\beta_1-3\beta_2-\beta_3} + 3e^{-2\beta_1-4\beta_2} + 2e^{-2\beta_1-5\beta_2+\beta_3} + e^{-2\beta_1-6\beta_2+\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2-\beta_3} + 4e^{-3\beta_1-3\beta_2} + 2e^{-3\beta_1-4\beta_2+\beta_3} + 2e^{-3\beta_1-5\beta_2+\beta_3} + e^{-3\beta_1-6\beta_2+2\beta_3} + e^{-3\beta_1-2\beta_3} + e^{-4\beta_1-\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2+\beta_3} + 2e^{-4\beta_1-4\beta_2+\beta_3} + e^{-4\beta_1-5\beta_2+2\beta_3} + e^{-5\beta_1-3\beta_2+\beta_3} + e^{-5\beta_1-4\beta_2+2\beta_3} + e^{-4\beta_2}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$2e^{-2\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1-5\beta_2} + e^{-2\beta_1-6\beta_2} + 2e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2-\beta_3} + e^{-3\beta_1-5\beta_2} + 2e^{-3\beta_1-6\beta_2+\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2} + 2e^{-4\beta_1-5\beta_2+\beta_3} + e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-7\beta_2+\beta_3} + e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-4\beta_2+\beta_3} + 2e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2+\beta_3} + e^{-5\beta_1-6\beta_2+\beta_3} + e^{-5\beta_1-7\beta_2+2\beta_3} + e^{-6\beta_1-5\beta_2+\beta_3} + e^{-6\beta_1-6\beta_2+2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-3\beta_1-6\beta_2} + e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-5\beta_2-\beta_3} + e^{-4\beta_1-7\beta_2} + 2e^{-5\beta_1-4\beta_2-\beta_3} + 2e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2} + 2e^{-5\beta_1-7\beta_2+\beta_3} + e^{-6\beta_1-5\beta_2} + 2e^{-6\beta_1-6\beta_2+\beta_3} + e^{-6\beta_1-7\beta_2+2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-5\beta_1-5\beta_2-\beta_3} + e^{-5\beta_1-7\beta_2} + e^{-6\beta_1-6\beta_2} + e^{-6\beta_1-7\beta_2+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 59, 127, 159, 117, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 11, 26, 39, 41, 31, 16, 4)$

S=$6\beta_1 + 7\beta_2$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 21, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 9, 9, 6, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = -\omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{2,3,4,5}$;

$d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-2\beta_1-2\beta_2+\beta_3} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-3\beta_2+\beta_3} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,7}+\omega^{2,4,6}\rangle + e^{-2\beta_1-5\beta_2+\beta_3} \langle\omega^{2,4,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,5,7} - \omega^{3,4,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,5,7} - \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-4\beta_2} \langle\omega^{1,3,4,7} + \omega^{2,3,4,6}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-5\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-4\beta_1-5\beta_2+\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-3\beta_1-6\beta_2+\beta_3} \langle\omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{C}}) = e^{-5\beta_1-5\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}\rangle + e^{-5\beta_1-4\beta_2-\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-6\beta_1-6\beta_2+\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1-5\beta_2-\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-7\beta_2+\beta_3} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{C}}) = e^{-5\beta_1-5\beta_2-\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-7\beta_2+\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-6\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-6\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)^2(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 21, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 9, 9, 6, 4, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7} - \omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} =$

$-\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} - \omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} - \omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,7} = 2\omega^{1,7} - \omega^{2,6}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = -\omega^{1,7} + \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{2,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{2,4,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7} - \omega^{2,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,3,7} + \omega^{2,3,6}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,4,7} + \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = -\omega^{1,5,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,3,4,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{2,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,3,4,7} + 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{1,3,5,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,4,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} =$

$2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,3,4,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-2\beta_1-2\beta_2+\beta_3} \langle\omega^{1,4}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5} - \omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-3\beta_2+\beta_3} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,7}+\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,7}\rangle + e^{-3\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,7}+\omega^{2,4,6}\rangle + e^{-2\beta_1-5\beta_2+\beta_3} \langle\omega^{2,4,7}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{1,5,7} + 2\omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,5,7} - \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-3\beta_1-6\beta_2+\beta_3} \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-4\beta_1-5\beta_2+\beta_3} \langle(-(\omega^{1,4,5,7} + \omega^{2,4,5,6}))/2\rangle + e^{-5\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle(\omega^{2,3,5,6} + \omega^{1,3,5,7})/2\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle-\omega^{1,3,5,6}\rangle + e^{-3\beta_1-4\beta_2} \langle(-(\omega^{2,3,4,6} - \omega^{1,2,5,7} + 2\omega^{1,3,4,7}))/6\rangle + e^{-4\beta_1-3\beta_2} \langle(-(\omega^{1,2,5,6} - \omega^{1,3,4,6}))/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-5\beta_1-7\beta_2+\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-4\beta_1-5\beta_2-\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-6\beta_2+\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_2-\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-5\beta_1-7\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-6\beta_1-6\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-7\beta_2+\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-5\beta_2-\beta_3} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-6\beta_1-7\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)^2(1 + Y)$.

5.5.15 Algebra $\mathcal{G}_{7,3.14}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & 0 & 2\xi_1^1 - \xi_2^2 + \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^2 & 0 & \xi_1^1 + \xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^4 + \xi_2^6 & \xi_3^6 & 2\xi_1^1 + \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5 + \delta_4^7, ad\ x_3 = -\delta_1^6, ad\ x_4 = -\delta_2^7, ad\ x_5 = 0, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{2\beta_1-\beta_2+\beta_3}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{2\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4\rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_4^7\rangle + 2e^{\beta_1}\ \langle \delta_2^5, \delta_3^6\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5\rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^5\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{\beta_3}\ \langle \delta_1^6\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{\beta_1+\beta_3}\ \langle \delta_1^7\rangle + e^{2\beta_1-\beta_2+\beta_3}\ \langle \delta_2^7\rangle + e^{2\beta_1}\ \langle \delta_3^7\rangle + 2e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_1^4, \delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_2^4, \delta_4^5] = -\delta_2^5$; $[\delta_2^4, \delta_1^2] = \delta_2^6 + \delta_1^4$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_1^5, \delta_1^4] = \delta_1^7$; $[\delta_1^5, \delta_2^6] = -\delta_1^7$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^5, \delta_1^4] = \delta_2^7$; $[\delta_2^5, \delta_2^6] = -\delta_2^7$; $[\delta_3^5, \delta_1^4] = \delta_3^7$; $[\delta_3^5, \delta_2^6] = -\delta_3^7$; $[\delta_4^5, \delta_1^4] = \delta_4^7 + \delta_1^5$; $[\delta_4^5, \delta_2^6] = -\delta_4^7$; $[\delta_1^6, \delta_3^6] = -\delta_1^7$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^4] = -\delta_1^6$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_3^6] = \delta_4^7$; $[\delta_4^7, \delta_1^4] = \delta_1^7$; $[\delta_2^6, \delta_3^6] = -\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{2\beta_1-2\beta_2+\beta_3}\ \langle \delta_2^4\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5\rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle \delta_4^5\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{2\beta_1}\ \langle \delta_3^7\rangle + e^{\beta_2}\ \langle \delta_1^5\rangle + 2e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_1^4, \delta_2^6\rangle + e^{\beta_1}\ \langle \delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_2^4, \delta_4^5] = -\delta_2^5$; $[\delta_2^4, \delta_1^2] = \delta_2^6 + \delta_1^4$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_3^5, \delta_1^4] = \delta_3^7$; $[\delta_3^5, \delta_2^6] = -\delta_3^7$; $[\delta_4^5, \delta_1^4] = 2\delta_1^5$; $[\delta_4^5, \delta_2^6] = -\delta_1^5$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-3\beta_1+2\beta_2-\beta_3}\ \langle x_2\omega^{1,4}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_3\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5}\rangle + 4e^{-2\beta_1+\beta_2-\beta_3}\ \langle -x_1\omega^{1,4} - x_2(\omega^{1,6} - \omega^{2,4}) + x_3\omega^{3,4}, -x_2\omega^{2,4} + x_5\omega^{4,5}, -x_1\omega^{1,4} - 2x_2(\omega^{1,6} - 1/2\omega^{2,4}) + x_6\omega^{4,6}, -x_2\omega^{2,4} + x_5\omega^{1,7}\rangle + 4e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_4\omega^{1,4}, x_3\omega^{1,3}, x_2\omega^{1,2} + x_6\omega^{2,4}, x_5\omega^{1,5}\rangle + e^{2\beta_1-2\beta_2}\ \langle x_4\omega^{2,3}\rangle + e^{\beta_1-3\beta_2+\beta_3}\ \langle x_4\omega^{2,5}\rangle +$

$e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}+x_4\omega^{3,6}\rangle+e^{-\beta_1+2\beta_2-2\beta_3}\ \langle x_5\omega^{3,4}\rangle+3e^{-\beta_3}\ \langle -x_2\omega^{2,3}-x_4\omega^{3,4}+x_5\omega^{3,5}, x_2\omega^{2,3}+x_4\omega^{3,4}+x_5\omega^{2,6}, -x_1\omega^{1,3}+x_2\omega^{2,3}+2x_4\omega^{3,4}+x_7\omega^{3,7}\rangle+e^{\beta_2-2\beta_3}\ \langle x_5\omega^{3,6}\rangle+e^{-2\beta_1+2\beta_2-2\beta_3}\ \langle -x_2\omega^{3,4}+x_5\omega^{4,6}\rangle+e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle+2e^{-2\beta_1}\ \langle x_3\omega^{1,6}, -x_2\omega^{1,5}-x_3\omega^{2,4}+x_6\omega^{4,5}\rangle+e^{-2\beta_2+\beta_3}\ \langle x_4\omega^{1,5}+x_6\omega^{2,5}\rangle+4e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_3\omega^{2,3}+x_6\omega^{2,6}, x_4\omega^{2,4}, -x_1\omega^{1,2}-x_3\omega^{2,3}+x_4\omega^{1,6}, x_5\omega^{2,5}\rangle+e^{\beta_1-2\beta_2+\beta_3}\ \langle x_4\omega^{1,2}\rangle+e^{\beta_1-\beta_2}\ \langle x_4\omega^{1,3}+x_7\omega^{3,5}\rangle+e^{\beta_1-\beta_3}\ \langle x_5\omega^{2,3}\rangle+e^{-\beta_1+\beta_2-\beta_3}\ \langle x_5\omega^{2,4}\rangle+e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,5}+2x_2\omega^{2,5}+3x_3(\omega^{2,6}+\omega^{3,5})+3x_4(\omega^{1,7}-\omega^{4,5})+4x_6(\omega^{2,7}-1/4\omega^{5,6})+x_7\omega^{5,7}\rangle+e^{-\beta_1-\beta_3}\ \langle x_1\omega^{1,6}-x_3\omega^{3,6}+x_5\omega^{2,7}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2-\beta_3}+e^{\beta_1-3\beta_2}+3e^{-\beta_1-\beta_2-\beta_3}+4e^{-\beta_1-2\beta_2}+4e^{-\beta_1-2\beta_3}+e^{-2\beta_1+2\beta_2-3\beta_3}+3e^{-2\beta_1+\beta_2-2\beta_3}+e^{-2\beta_1+\beta_2-3\beta_3}+2e^{-2\beta_1-\beta_2-\beta_3}+4e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+\beta_3}+4e^{-2\beta_1-\beta_3}+e^{-3\beta_1+2\beta_2-3\beta_3}+e^{-3\beta_1+\beta_2-\beta_3}+3e^{-3\beta_1+\beta_2-2\beta_3}+2e^{-3\beta_1-\beta_2}+5e^{-3\beta_1-\beta_3}+e^{-4\beta_1+2\beta_2-2\beta_3}+2e^{-4\beta_1+\beta_2-\beta_3}+e^{-4\beta_1}+e^{-5\beta_1+2\beta_2-2\beta_3}+e^{-5\beta_1+\beta_2-\beta_3}+3e^{-\beta_2-\beta_3}+e^{-\beta_2-2\beta_3}+e^{-2\beta_2-\beta_3}+e^{-3\beta_2+\beta_3}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-2\beta_3}+2e^{-\beta_1-2\beta_2-\beta_3}+2e^{-\beta_1-2\beta_2-2\beta_3}+4e^{-2\beta_1-\beta_2-2\beta_3}+e^{-2\beta_1-\beta_2-3\beta_3}+2e^{-2\beta_1-2\beta_2-\beta_3}+e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-2\beta_3}+e^{-2\beta_1-3\beta_3}+3e^{-3\beta_1+\beta_2-3\beta_3}+5e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2}+3e^{-3\beta_1-2\beta_3}+2e^{-3\beta_1-3\beta_3}+e^{-4\beta_1+2\beta_2-4\beta_3}+2e^{-4\beta_1+\beta_2-3\beta_3}+3e^{-4\beta_1-\beta_2-\beta_3}+e^{-4\beta_1-2\beta_2}+e^{-4\beta_1-\beta_3}+5e^{-4\beta_1-2\beta_3}+e^{-5\beta_1+2\beta_2-3\beta_3}+3e^{-5\beta_1+\beta_2-2\beta_3}+2e^{-5\beta_1-\beta_3}+e^{-6\beta_1+2\beta_2-3\beta_3}+2e^{-6\beta_1+\beta_2-2\beta_3}+e^{-6\beta_1-\beta_3}+e^{-3\beta_2-\beta_3}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+3e^{-3\beta_1-\beta_2-3\beta_3}+e^{-3\beta_1-2\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_2-\beta_3}+3e^{-4\beta_1-\beta_2-2\beta_3}+e^{-4\beta_1-\beta_2-3\beta_3}+2e^{-4\beta_1-2\beta_2-\beta_3}+e^{-4\beta_1-2\beta_2-2\beta_3}+4e^{-4\beta_1-3\beta_3}+e^{-4\beta_1-4\beta_3}+2e^{-5\beta_1+\beta_2-4\beta_3}+3e^{-5\beta_1-\beta_2-2\beta_3}+e^{-5\beta_1-2\beta_2-\beta_3}+e^{-5\beta_1-2\beta_3}+2e^{-5\beta_1-3\beta_3}+e^{-6\beta_1+2\beta_2-4\beta_3}+3e^{-6\beta_1+\beta_2-3\beta_3}+e^{-6\beta_1-\beta_2-\beta_3}+3e^{-6\beta_1-2\beta_3}+e^{-7\beta_1+\beta_2-3\beta_3}+e^{-7\beta_1-2\beta_3}.$

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-3\beta_2-2\beta_3}+e^{-4\beta_1-\beta_2-3\beta_3}+2e^{-4\beta_1-2\beta_2-3\beta_3}+2e^{-5\beta_1-\beta_2-3\beta_3}+e^{-5\beta_1-\beta_2-4\beta_3}+2e^{-5\beta_1-2\beta_2-2\beta_3}+e^{-5\beta_1-4\beta_3}+e^{-6\beta_1+\beta_2-4\beta_3}+e^{-6\beta_1-\beta_2-2\beta_3}+e^{-6\beta_1-\beta_2-3\beta_3}+e^{-6\beta_1-3\beta_3}+e^{-6\beta_1-4\beta_3}+e^{-7\beta_1+\beta_2-4\beta_3}+e^{-7\beta_1-\beta_2-2\beta_3}+2e^{-7\beta_1-3\beta_3}.$

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-5\beta_1-2\beta_2-3\beta_3}+e^{-6\beta_1-\beta_2-4\beta_3}+e^{-7\beta_1-\beta_2-3\beta_3}+e^{-7\beta_1-4\beta_3}.$

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,18,65,135,166,120,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(2,13,34,53,56,41,19,4)$

$\mathrm{S}=7\beta_1+\beta_2+4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,12,21,26,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,9,12,12,9,4,1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -\omega^{1,6} - \omega^{2,4}$; $d\omega^{1,7} = \omega^{1,2,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,6}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,6} - \omega^{2,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{1,4,6}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} - \omega^{1,5,6} + \omega^{2,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{2,4,6}$; $d\omega^{1,3,7} = \omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,2,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,4,6}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $d\omega^{3,6,7} = \omega^{2,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = \omega^{1,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{1,3,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,3,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1+\beta_2-\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-3\beta_1+\beta_2-\beta_3} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1+\beta_2-2\beta_3} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7}, \omega^{1,5,6}\rangle + e^{-4\beta_1-\beta_3} \langle\omega^{1,4,5}\rangle + e^{-5\beta_1+\beta_2-2\beta_3} \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,6,7} - \omega^{2,4,7}\rangle + e^{-3\beta_1-2\beta_3} \langle\omega^{1,3,7} + \omega^{3,4,5}\rangle + e^{-3\beta_1+\beta_2-3\beta_3} \langle\omega^{3,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,7} + \omega^{3,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,7}\rangle + 2e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,6,7} - \omega^{3,4,5,6}, \omega^{2,3,4,7} - \omega^{3,4,5,6}\rangle + e^{-5\beta_1-2\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-6\beta_1-2\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-6\beta_1+\beta_2-3\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,6,7}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,5,6,7} + \omega^{2,4,5,7}\rangle + e^{-5\beta_1+\beta_2-4\beta_3} \langle\omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-6\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,4,5,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-3\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-6\beta_1+\beta_2-4\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-3\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}\rangle + e^{-5\beta_1-4\beta_3} \langle\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-5\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-7\beta_1-\beta_2-3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-\beta_2-4\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-7\beta_1-\beta_2-4\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \;=\; (1 + 3Y + 5Y^2 + 3Y^3 + Y^4)(1 + Y^2)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 12, 21, 26, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 4, 9, 12, 12, 9, 4, 1)$

Total dimension of trivial cohomology: 52.

Length associated polynomial: 52.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{2,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{1,7}+\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{2,7}-\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{2,3,4} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = \omega^{5,7}$; $d^\star\omega^{2,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = \omega^{1,3,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,5,7}$; $d^\star\omega^{1,2,4,6} = -\omega^{1,6,7} - \omega^{2,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{3,4,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{2,3,4,5} = \omega^{3,5,7}$; $d^\star\omega^{2,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,3,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,3,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{2,4}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{2,4} = \omega^{1,6} + \omega^{2,4}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{3,5} = -\omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{2,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{2,4,7} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = \omega^{1,3,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{3,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} - \omega^{1,5,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = 2\omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,4,7} = \omega^{1,6,7} + \omega^{2,4,7} - \omega^{4,5,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,3,7} - \omega^{2,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + 3\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} - \omega^{2,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,5,6} +$

$\omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 3\omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,3,5,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + 2\omega^{1,3,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} - \omega^{1,3,5,6} + \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = \omega^{1,3,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,5,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,4,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1+\beta_2-\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6} - \omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-2\beta_1+\beta_2-2\beta_3}\ \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,6}\rangle + e^{-3\beta_1-\beta_3}\ \langle\omega^{1,7} - \omega^{4,5}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-4\beta_1-\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-5\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,4,7}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6} - \omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,7} + \omega^{2,4,5}, \omega^{1,5,6} + \omega^{2,4,5}\rangle + e^{-4\beta_1-2\beta_3}\ \langle\omega^{1,6,7} - \omega^{2,4,7}\rangle + e^{-3\beta_1-2\beta_3}\ \langle\omega^{1,3,7} + \omega^{2,4,6} + 2\omega^{3,4,5}\rangle + e^{-3\beta_1+\beta_2-3\beta_3}\ \langle\omega^{3,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,7} + \omega^{3,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-5\beta_1+\beta_2-4\beta_3}\ \langle\omega^{3,4,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-5\beta_1-\beta_2-2\beta_3}\ \langle(\omega^{1,5,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-6\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_3}\ \langle-\omega^{1,4,5,7}\rangle + 2e^{-4\beta_1-3\beta_3}\ \langle(-(\omega^{1,3,6,7} + \omega^{2,3,4,7} - 2\omega^{3,4,5,6}))/3, (-(\omega^{1,3,6,7} - 2\omega^{2,3,4,7} + \omega^{3,4,5,6}))/3\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle(\omega^{1,3,5,6} + \omega^{2,3,4,5})/2\rangle + e^{-4\beta_1-\beta_2-2\beta_3}\ \langle(2\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{2,4,5,6})/6\rangle + e^{-4\beta_1-2\beta_2-\beta_3}\ \langle-\omega^{1,2,5,7}\rangle + e^{-5\beta_1-2\beta_3}\ \langle(\omega^{1,2,4,7} + \omega^{1,4,5,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-4\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-7\beta_1-3\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3}\ \langle(\omega^{1,3,5,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-6\beta_1+\beta_2-4\beta_3}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-3\beta_3}\ \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-6\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-\beta_2-3\beta_3}\ \langle(-\omega^{1,2,3,6,7} - \omega^{2,3,4,5,6})/2\rangle.$

$\mathrm{ch}(H^6_{\mathfrak{C}})=$ $e^{-6\beta_1-\beta_2-4\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$ + $e^{-7\beta_1-4\beta_3}$ $\langle-\omega^{1,3,4,5,6,7}\rangle$ + $e^{-7\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3}$ $\langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{C}})=$ $e^{-7\beta_1-\beta_2-4\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) \ = \ (1+3Y+5Y^2+3Y^3+Y^4)(1+Y^2)(1+Y).$

5.5.16 Algebra $\mathcal{G}_{7,3.15}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1 + 2\xi_2^2 - \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & -\xi_2^3 & \xi_1^1 + \xi_2^2 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^3 & \xi_1^1 + \xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & -\xi_1^5 & -\xi_1^4 & \xi_1^1 + 2\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5, ad\ x_3 = -\delta_1^6 + \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -\delta_2^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_1+2\beta_2}\ \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_1+2\beta_2}\ \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^4 \rangle + 2e^{\beta_1}\ \langle \delta_2^5, \delta_3^6 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_2^4 \rangle + 2e^{\beta_3}\ \langle \delta_1^6, \delta_4^7 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{-2\beta_2+2\beta_3}\ \langle \delta_4^6 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + 3e^0\ \langle t_3, t_1, t_2 \rangle + e^{2\beta_2}\ \langle \delta_1^7 \rangle + e^{\beta_1+\beta_2}\ \langle \delta_2^7 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle \delta_3^7 \rangle + e^{\beta_2}\ \langle \delta_1^5 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_1^4 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_3^4, \delta_2^3] = \delta_2^4$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_2^5, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_1^5] = \delta_2^7$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_2^4, \delta_2^3] = -\delta_3^6 + 2\delta_2^5$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_2^4, \delta_1^5] = \delta_3^7$; $[\delta_1^6, \delta_1^4] = \delta_1^7$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_3^6, \delta_2^3] = \delta_2^6$; $[\delta_3^6, \delta_1^4] = \delta_3^7$; $[\delta_4^6, \delta_1^4] = \delta_4^7 + \delta_1^6$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_1^5] = -\delta_4^7 + \delta_1^6$; $[\delta_2^3, \delta_1^4] = -\delta_1^5$; $[\delta_4^7, \delta_1^4] = \delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^4 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_2^4 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{\beta_1}\ \langle \delta_2^5 \rangle + e^{-2\beta_2+2\beta_3}\ \langle \delta_4^6 \rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3 \rangle + 3e^0\ \langle t_3, t_1, t_2 \rangle + e^{2\beta_2}\ \langle \delta_1^7 \rangle + e^{\beta_3}\ \langle \delta_1^6 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_1^4 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4, \delta_4^6] = \delta_2^5$; $[\delta_3^4, \delta_2^3] = \delta_2^4$; $[\delta_2^4, \delta_4^6] = -\delta_2^6$; $[\delta_2^4, \delta_2^3] = 3\delta_2^5$; $[\delta_2^5, \delta_2^3] = -\delta_2^6$; $[\delta_4^6, \delta_1^4] = 2\delta_1^6$; $[\delta_1^6, \delta_1^4] = \delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3} \rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,5} \rangle + e^{-2\beta_1-2\beta_2+2\beta_3}\ \langle x_3\omega^{1,4} \rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_2\omega^{1,4} + x_3\omega^{1,5} \rangle + 3e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2} + x_3\omega^{1,3} + x_7\omega^{1,7}, x_6\omega^{1,6} \rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{1,3} + x_4\omega^{1,5} \rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle x_4\omega^{2,3} \rangle + 4e^{-\beta_2}\ \langle -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2} + x_6\omega^{2,6}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_5\omega^{3,4}, x_5\omega^{2,5} \rangle + e^{2\beta_2-3\beta_3}\ \langle x_4\omega^{3,6} \rangle + 4e^{-\beta_3}\ \langle -2x_1\omega^{1,3} + x_2\omega^{2,3} + x_5\omega^{3,5}, x_1\omega^{1,3} +$

$x_5\omega^{2,6}, x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} + x_4\omega^{2,5}\rangle + e^{\beta_2-2\beta_3}\ \langle x_4(\omega^{2,6}+\omega^{3,5}) + x_5\omega^{3,6}\rangle + 3e^{-\beta_1-2\beta_2+\beta_3}\ \langle -2x_1\omega^{1,4} + x_2\omega^{2,4} + 2x_3\omega^{3,4} + x_5\omega^{4,5}, -x_3\omega^{3,4} + x_6\omega^{4,6}, x_1\omega^{1,4} - x_2\omega^{2,4} - x_3(\omega^{2,5}+2\omega^{3,4}) + x_6\omega^{1,7}\rangle + e^{-3\beta_2+2\beta_3}\ \langle x_6\omega^{2,4}\rangle + e^{-3\beta_2+\beta_3}\ \langle -x_1\omega^{2,4} + x_6\omega^{2,7}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + e^{-2\beta_2}\ \langle x_1\omega^{3,4} + x_5\omega^{2,7}\rangle + e^{-\beta_1-3\beta_2+2\beta_3}\ \langle -x_3\omega^{2,4} + x_6\omega^{4,5}\rangle + e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,5} + 2x_2(\omega^{2,5}+1/2\omega^{3,4}) + x_3(\omega^{2,6}+\omega^{3,5}) - x_5\omega^{4,6} + x_6\omega^{5,6}\rangle + e^{-\beta_1+2\beta_2-\beta_3}\ \langle x_7\omega^{1,6}\rangle + e^{2\beta_2-2\beta_3}\ \langle x_4\omega^{1,3}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle -x_3\omega^{1,2} + x_5\omega^{1,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{-\beta_1+2\beta_2-3\beta_3} + e^{-\beta_1+\beta_2-2\beta_3} + 4e^{-\beta_1-\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-2\beta_2} + 2e^{-\beta_1-3\beta_2+\beta_3} + 3e^{-\beta_1-3\beta_2} + 2e^{-\beta_1-4\beta_2+\beta_3} + e^{-\beta_1-5\beta_2+2\beta_3} + 3e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + 4e^{-2\beta_1-\beta_2} + 2e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+2\beta_3} + 2e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-5\beta_2+2\beta_3} + 3e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-3\beta_2+2\beta_3} + e^{-3\beta_1-\beta_3} + e^{\beta_2-3\beta_3} + e^{-\beta_2-\beta_3} + 2e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3} + e^{-3\beta_2-\beta_3} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-\beta_2-3\beta_3} + 2e^{-\beta_1-2\beta_2-2\beta_3} + 3e^{-\beta_1-3\beta_2-\beta_3} + e^{-\beta_1-3\beta_2-2\beta_3} + e^{-\beta_1-4\beta_2-\beta_3} + e^{-\beta_1-4\beta_2} + e^{-\beta_1-5\beta_2} + e^{-2\beta_1+\beta_2-3\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-2\beta_2-\beta_3} + 3e^{-2\beta_1-3\beta_2-\beta_3} + 2e^{-2\beta_1-3\beta_2} + 4e^{-2\beta_1-4\beta_2} + 3e^{-2\beta_1-5\beta_2+\beta_3} + e^{-2\beta_1-6\beta_2+2\beta_3} + 2e^{-2\beta_1-2\beta_3} + 3e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + 2e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-3\beta_2+\beta_3} + 3e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2+\beta_3} + e^{-3\beta_1-5\beta_2+2\beta_3} + e^{-3\beta_1-5\beta_2+\beta_3} + e^{-3\beta_1-2\beta_3} + e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2+\beta_3} + e^{-4\beta_1-4\beta_2+\beta_3} + e^{-4\beta_1-5\beta_2+2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-2\beta_1-2\beta_2-3\beta_3} + 2e^{-2\beta_1-3\beta_2-2\beta_3} + 3e^{-2\beta_1-4\beta_2-\beta_3} + e^{-2\beta_1-4\beta_2-2\beta_3} + e^{-2\beta_1-5\beta_2-\beta_3} + e^{-2\beta_1-5\beta_2} + e^{-2\beta_1-6\beta_2} + e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + 3e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-4\beta_2-\beta_3} + 2e^{-3\beta_1-4\beta_2} + 3e^{-3\beta_1-5\beta_2} + 2e^{-3\beta_1-6\beta_2+\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + 2e^{-4\beta_1-4\beta_2} + 3e^{-4\beta_1-5\beta_2+\beta_3} + e^{-4\beta_1-6\beta_2+2\beta_3} + e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2+\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-3\beta_1-2\beta_2-3\beta_3} + 2e^{-3\beta_1-4\beta_2-2\beta_3} + e^{-3\beta_1-5\beta_2-\beta_3} + e^{-3\beta_1-6\beta_2-\beta_3} + e^{-3\beta_1-6\beta_2} + 2e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-4\beta_1-4\beta_2-\beta_3} + e^{-4\beta_1-5\beta_2-\beta_3} + e^{-4\beta_1-5\beta_2} + e^{-4\beta_1-6\beta_2+\beta_3} + e^{-4\beta_1-6\beta_2} + e^{-5\beta_1-4\beta_2-\beta_3} + 2e^{-5\beta_1-5\beta_2} + e^{-5\beta_1-6\beta_2+\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-4\beta_1-4\beta_2-2\beta_3} + e^{-4\beta_1-6\beta_2-\beta_3} + e^{-5\beta_1-5\beta_2-\beta_3} + e^{-5\beta_1-6\beta_2}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 17, 63, 133, 164, 119, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 12, 31, 49, 52, 38, 18, 4)$

S$=5\beta_1 + 6\beta_2 + \beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 11, 21, 25, 18, 7, 1)$

Dimensions of $H_{\mathfrak{L}}^j$'s $0 \le j \le 7$: $(1,4,8,11,11,8,4,1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{2,3,5}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$; $d\omega^{4,7} = -\omega^{2,4,5}$; $d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,3,7} = \omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,2,4,5}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,5,7} = -\omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,7} = -\omega^{2,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}$; $d\omega^{3,6,7} = \omega^{2,3,5,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = \omega^{1,3,4,7} + \omega^{2,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H_{\mathfrak{L}}^1) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^2) = e^{-2\beta_1-2\beta_2+\beta_3} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6}+\omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^3) = e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2+\beta_3} \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-5\beta_2+\beta_3} \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-3\beta_2} \langle\omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,7} - \omega^{2,5,6}\rangle + e^{-2\beta_1-4\beta_2} \langle\omega^{2,5,7} - \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^4) = e^{-3\beta_1-4\beta_2} \langle\omega^{1,2,5,7} + \omega^{2,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-5\beta_2+\beta_3} \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-5\beta_2} \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,6,7}\rangle + e^{-3\beta_1-6\beta_2+\beta_3} \langle\omega^{2,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,6,7} + \omega^{3,4,5,6}\rangle + e^{-3\beta_1-5\beta_2} \langle\omega^{2,4,6,7} + \omega^{3,4,5,7}\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^5) = e^{-4\beta_1-6\beta_2+\beta_3} \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-5\beta_2} \langle\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-5\beta_2} \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-6\beta_2} \langle\omega^{2,3,4,5,7}\rangle + e^{-3\beta_1-5\beta_2-\beta_3} \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_3} \langle\omega^{2,3,5,6,7}\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^6) = e^{-4\beta_1-4\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-5\beta_1-6\beta_2} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-5\beta_1-5\beta_2-\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-6\beta_2-\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H_{\mathfrak{L}}^7) = e^{-5\beta_1-6\beta_2-\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,11,21,25,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,8,11,11,8,4,1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = \omega^{1,7}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7} + \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7} - \omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,3,6,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,7} = 2\omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{2,5} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{2,4,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = 2\omega^{1,3,7} - \omega^{3,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{4,5,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,3,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$;

$\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,5,7} + 2\omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,5,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5} - \omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6} + \omega^{1,3,5}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,5,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-5\beta_2+\beta_3}\ \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{1,2,7} + 2\omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,3,7} - \omega^{2,5,6} + 2\omega^{3,4,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,5,7} - \omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-3\beta_1-5\beta_2}\ \langle(-(\omega^{2,4,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-3\beta_1-6\beta_2+\beta_3}\ \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3}\ \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-5\beta_2}\ \langle\omega^{2,3,4,7}\rangle + e^{-4\beta_1-5\beta_2+\beta_3}\ \langle-\omega^{1,4,5,7}\rangle + e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,4,5,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle-\omega^{1,3,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle(\omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{3,4,5,6})/6\rangle + e^{-3\beta_1-4\beta_2}\ \langle(-(2\omega^{1,2,5,7} - \omega^{1,3,4,7} + \omega^{2,4,5,6}))/6\rangle + e^{-3\beta_1-2\beta_2-\beta_3}\ \langle(-(\omega^{1,2,5,6} - \omega^{1,3,4,6}))/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-3\beta_1-4\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-5\beta_2-\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-\beta_3}\ \langle(-\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-4\beta_1-5\beta_2}\ \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-4\beta_1-6\beta_2+\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-4\beta_1-6\beta_2-\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-5\beta_1-5\beta_2-\beta_3}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-6\beta_2}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2-2\beta_3}\ \langle-\omega^{1,2,3,5,6,7}\rangle$.

$$\mathrm{ch}(H^7_{\mathfrak{a}}) = e^{-5\beta_1-6\beta_2-\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$$
$$P(Y) = (1+Y+Y^2)(1+Y^2)(1+Y)^3.$$

5.5.17 Algebra $\mathcal{G}_{7,3.16}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_5] = x_7$; $[x_3, x_4] = x_6$; $[x_3, x_6] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^4 & 2\xi_1^1+\xi_2^2-2\xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & 0 & \xi_1^1+\xi_2^2 & 0 & 0 \\ \xi_3^5 & 0 & \xi_3^6 & \xi_4^6 & 0 & 2\xi_1^1+\xi_2^2-\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^5 & \xi_4^6 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5, ad\ x_3 = \delta_4^6, ad\ x_4 = -\delta_3^6, ad\ x_5 = -\delta_1^7, ad\ x_6 = -\delta_3^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{2\beta_1+\beta_2-2\beta_3}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{2\beta_1+\beta_2-3\beta_3}\ \langle\delta_3^4\rangle + e^{\beta_2}\ \langle\delta_1^5\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{2\beta_1+\beta_2-2\beta_3}\ \langle\delta_3^6\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{2\beta_1}\ \langle\delta_2^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle + e^{2\beta_3}\ \langle\delta_4^7\rangle + e^{\beta_1}\ \langle\delta_2^5\rangle + e^{\beta_3}\ \langle\delta_4^6\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_1^5, \delta_2^5] = -\delta_1^7$; $[\delta_3^5, \delta_2^5] = -\delta_3^7$; $[\delta_3^5, \delta_4^6] = -\delta_1^7$; $[\delta_3^6, \delta_4^6] = -\delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{2\beta_1+\beta_2-3\beta_3}\ \langle\delta_3^4\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1}\ \langle\delta_2^7\rangle + e^{2\beta_3}\ \langle\delta_4^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: .

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_2}\ \langle x_1\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + e^{-3\beta_1+2\beta_3}\ \langle x_2\omega^{1,4}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,5} + x_2\omega^{2,5}\rangle + e^{-4\beta_1-2\beta_2+4\beta_3}\ \langle x_3\omega^{4,6}\rangle + e^{\beta_1+\beta_2-3\beta_3}\ \langle x_4\omega^{1,3}\rangle + e^{2\beta_1-3\beta_3}\ \langle x_4\omega^{2,3}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle -x_3\omega^{3,6} + x_4\omega^{4,6}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,5}\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_5\omega^{3,5}, x_4\omega^{3,4}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_6\omega^{1,5}\rangle + 2e^{-2\beta_1-\beta_2+2\beta_3}\ \langle x_1\omega^{1,4} - 2x_2\omega^{2,4} + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} - x_3\omega^{3,4} + x_7\omega^{4,7}\rangle + e^{-3\beta_1-\beta_2+3\beta_3}\ \langle x_3\omega^{1,4} + x_5\omega^{4,6}\rangle + 3e^{-\beta_1}\ \langle -x_3\omega^{1,3} + 2x_4\omega^{1,4} + x_6\omega^{1,6}, x_2\omega^{1,2}, x_3\omega^{1,3} - x_4\omega^{1,4} + x_5\omega^{3,6}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3} + x_6\omega^{2,5}\rangle + 2e^{-\beta_2}\ \langle -x_3\omega^{2,3} + 2x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} + x_3\omega^{2,3} - x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + e^{-2\beta_3}\ \langle x_4\omega^{3,6}\rangle + e^{-\beta_2+2\beta_3}\ \langle x_7\omega^{2,4}\rangle$.

ch$(H^3_{\mathcal{G}})$=$e^{\beta_1-\beta_2-3\beta_3}+e^{-\beta_1-\beta_2-\beta_3}+3e^{-\beta_1-2\beta_2}+e^{-\beta_1-\beta_3}+e^{-\beta_1-2\beta_3}+e^{-\beta_1-3\beta_3}+e^{-2\beta_1-\beta_2-\beta_3}+3e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+2\beta_3}+e^{-2\beta_1-2\beta_2+\beta_3}+e^{-2\beta_1-3\beta_2+2\beta_3}+e^{-2\beta_1-\beta_3}+2e^{-3\beta_1-\beta_2+2\beta_3}+e^{-3\beta_1-\beta_2+\beta_3}+e^{-3\beta_1-\beta_2}+e^{-3\beta_1-2\beta_2+2\beta_3}+e^{-3\beta_1-3\beta_2+3\beta_3}+e^{-3\beta_1-\beta_3}+e^{-4\beta_1-\beta_2+2\beta_3}+3e^{-4\beta_1-2\beta_2+3\beta_3}+e^{-4\beta_1-3\beta_2+4\beta_3}+e^{-5\beta_1-\beta_2+3\beta_3}+e^{-5\beta_1-2\beta_2+4\beta_3}+2e^{-\beta_2-\beta_3}+e^{-\beta_2-2\beta_3}+e^{-2\beta_2-\beta_3}$.

ch$(H^4_{\mathcal{G}})$=$e^{-\beta_1-2\beta_2-\beta_3}+e^{-\beta_1-2\beta_2-2\beta_3}+e^{-\beta_1-2\beta_2-3\beta_3}+e^{-2\beta_1-\beta_2-3\beta_3}+e^{-2\beta_1-2\beta_2-\beta_3}+2e^{-2\beta_1-2\beta_2}+e^{-2\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-\beta_2-\beta_3}+e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+2e^{-3\beta_1-3\beta_2+2\beta_3}+e^{-3\beta_1-3\beta_2+\beta_3}+e^{-3\beta_1-3\beta_2}+e^{-4\beta_1-\beta_2-\beta_3}+2e^{-4\beta_1-2\beta_2+2\beta_3}+3e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-3\beta_2+3\beta_3}+e^{-4\beta_1-4\beta_2+3\beta_3}+e^{-5\beta_1-\beta_2}+e^{-5\beta_1-2\beta_2+3\beta_3}+e^{-5\beta_1-2\beta_2+2\beta_3}+e^{-5\beta_1-2\beta_2+\beta_3}+e^{-5\beta_1-3\beta_2+3\beta_3}+e^{-5\beta_1-4\beta_2+4\beta_3}+e^{-6\beta_1-2\beta_2+3\beta_3}+e^{-6\beta_1-3\beta_2+3\beta_3}+e^{-7\beta_1-2\beta_2+3\beta_3}+e^{-7\beta_1-3\beta_2+4\beta_3}$.

ch$(H^5_{\mathcal{G}})$=$e^{-2\beta_1-2\beta_2-3\beta_3}+2e^{-3\beta_1-3\beta_2-\beta_3}+e^{-3\beta_1-3\beta_2-2\beta_3}+2e^{-4\beta_1-2\beta_2-\beta_3}+e^{-4\beta_1-2\beta_2-2\beta_3}+e^{-4\beta_1-3\beta_2+2\beta_3}+2e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2}+2e^{-5\beta_1-2\beta_2}+e^{-5\beta_1-3\beta_2}+e^{-5\beta_1-4\beta_2+3\beta_3}+e^{-5\beta_1-4\beta_2+2\beta_3}+e^{-5\beta_1-4\beta_2+\beta_3}+e^{-6\beta_1-2\beta_2}+2e^{-6\beta_1-3\beta_2+2\beta_3}+e^{-6\beta_1-3\beta_2+\beta_3}+e^{-6\beta_1-4\beta_2+3\beta_3}+e^{-6\beta_1-5\beta_2+3\beta_3}+e^{-7\beta_1-2\beta_2+2\beta_3}+2e^{-7\beta_1-3\beta_2+3\beta_3}+e^{-7\beta_1-4\beta_2+3\beta_3}+e^{-7\beta_1-5\beta_2+4\beta_3}+e^{-8\beta_1-3\beta_2+3\beta_3}+e^{-8\beta_1-4\beta_2+4\beta_3}$.

ch$(H^6_{\mathcal{G}})$=$e^{-4\beta_1-3\beta_2-\beta_3}+e^{-4\beta_1-3\beta_2-2\beta_3}+2e^{-5\beta_1-4\beta_2}+2e^{-6\beta_1-3\beta_2}+e^{-6\beta_1-4\beta_2+2\beta_3}+e^{-6\beta_1-4\beta_2+\beta_3}+e^{-6\beta_1-5\beta_2+2\beta_3}+e^{-7\beta_1-3\beta_2+2\beta_3}+e^{-7\beta_1-4\beta_2+2\beta_3}+2e^{-7\beta_1-5\beta_2+3\beta_3}+e^{-8\beta_1-3\beta_2+2\beta_3}+2e^{-8\beta_1-4\beta_2+3\beta_3}+e^{-8\beta_1-5\beta_2+4\beta_3}$.

ch$(H^7_{\mathcal{G}})$=$e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-5\beta_2+2\beta_3}+e^{-8\beta_1-4\beta_2+2\beta_3}+e^{-8\beta_1-5\beta_2+3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,14,58,123,157,118,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,8,23,34,35,30,17,4)$

S=$8\beta_1+5\beta_2-2\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,10,19,24,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,7,8,8,7,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{3,4}$; $d\omega^7=-\omega^{1,5}-\omega^{3,6}$; $d\omega^{1,6}=\omega^{1,3,4}$; $d\omega^{1,7}=\omega^{1,3,6}$; $d\omega^{2,6}=\omega^{2,3,4}$; $d\omega^{2,7}=-\omega^{1,2,5}+\omega^{2,3,6}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,7}=-\omega^{1,3,5}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,7}=-\omega^{1,4,5}-\omega^{3,4,6}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{3,4,5}$; $d\omega^{5,7}=-\omega^{1,2,7}-\omega^{3,5,6}$; $d\omega^{6,7}=\omega^{1,5,6}-\omega^{3,4,7}$; $d\omega^{1,2,6}=-\omega^{1,2,3,4}$; $d\omega^{1,2,7}=-\omega^{1,2,3,6}$; $d\omega^{1,4,7}=\omega^{1,3,4,6}$; $d\omega^{1,5,6}=-\omega^{1,3,4,5}$; $d\omega^{1,5,7}=\omega^{1,3,5,6}$; $d\omega^{1,6,7}=\omega^{1,3,4,7}$; $d\omega^{2,3,7}=-\omega^{1,2,3,5}$; $d\omega^{2,4,7}=-\omega^{1,2,4,5}+\omega^{2,3,4,6}$; $d\omega^{2,5,6}=-\omega^{2,3,4,5}$; $d\omega^{2,5,7}=\omega^{2,3,5,6}$; $d\omega^{2,6,7}=\omega^{1,2,5,6}+\omega^{2,3,4,7}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,4,7}=-\omega^{1,3,4,5}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,7}=\omega^{1,2,3,7}$; $d\omega^{3,6,7}=\omega^{1,3,5,6}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}$; $d\omega^{4,5,7}=\omega^{1,2,4,7}-\omega^{3,4,5,6}$; $d\omega^{4,6,7}=\omega^{1,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,2,6,7}+\omega^{3,4,5,7}$;

$d\omega^{1,2,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{1,2,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,4,5,7} = \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7} = \omega^{2,3,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,4,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7}$; $d\omega^{1,2,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1-\beta_2+2\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2+2\beta_3} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-4\beta_1-2\beta_2+3\beta_3} \langle\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-2\beta_2+3\beta_3} \langle\omega^{1,4,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + e^{-3\beta_1-3\beta_2+2\beta_3} \langle\omega^{2,4,5}\rangle + e^{-4\beta_1-3\beta_2+3\beta_3} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2+2\beta_3} \langle\omega^{1,4,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} - \omega^{3,6,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2} \langle\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2+3\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-3\beta_2+2\beta_3} \langle\omega^{1,2,4,5}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,7} + \omega^{2,3,6,7}\rangle + e^{-5\beta_1-4\beta_2+3\beta_3} \langle\omega^{2,4,5,6}\rangle + e^{-6\beta_1-3\beta_2+2\beta_3} \langle\omega^{1,4,5,7} - \omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,3,5,7}\rangle + e^{-7\beta_1-3\beta_2+2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{1,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2+3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-4\beta_2+2\beta_3} \langle\omega^{1,2,4,5,7} + \omega^{2,3,4,6,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-7\beta_1-5\beta_2+3\beta_3} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-6\beta_1-4\beta_2} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-8\beta_1-5\beta_2+3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2+2\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-5\beta_2+2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-8\beta_1-5\beta_2+2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+Y+Y^2+Y^3+Y^4)(1+Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,10,19,24,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,7,8,8,7,4,1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^6$; $d^\star\omega^{3,6} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,7}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,6}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = \omega^{1,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,6}$; $d^\star\omega^{2,3,6} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = \omega^{5,6}$; $d^\star\omega^{3,4,6} = -\omega^{4,7}$; $d^\star\omega^{3,4,7} = -\omega^{6,7}$; $d^\star\omega^{3,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} =$

$-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,2,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,7}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,6} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = \omega^{1,4,7}$; $d^\star\omega^{1,3,4,7} = \omega^{1,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,5,7} + \omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,6}$; $d^\star\omega^{2,3,4,6} = \omega^{2,4,7}$; $d^\star\omega^{2,3,4,7} = \omega^{2,6,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,5,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{3,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,2,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,5,7} + \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,5,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,5,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,2,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = 2\omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5} + \omega^{3,6}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{1,5} + \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{2,3,6}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} + \omega^{3,4,6}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{3,6,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,5} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}$; $\Delta\omega^{2,4,7} = 2\omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,4,5} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{1,5,7} + \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5} - \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{2,3,6,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = 2\omega^{1,3,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = -\omega^{1,2,4,5} + \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = -\omega^{1,2,5,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{2,3,4,6,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = -\omega^{1,2,4,5,7} + \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} =$

$\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{L}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-2\beta_1-\beta_2+2\beta_3} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{L}}) = e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2+2\beta_3} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-2\beta_1-\beta_2} \langle\omega^{1,5} - \omega^{3,6}\rangle + e^{-4\beta_1-2\beta_2+3\beta_3} \langle\omega^{4,6}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{L}}) = e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-5\beta_1-2\beta_2+3\beta_3} \langle\omega^{1,4,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5} + \omega^{2,3,6}\rangle + e^{-3\beta_1-3\beta_2+2\beta_3} \langle\omega^{2,4,5}\rangle + e^{-4\beta_1-3\beta_2+3\beta_3} \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-2\beta_2+2\beta_3} \langle\omega^{1,4,5} - \omega^{3,4,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,5,7} - \omega^{3,6,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{L}}) = e^{-5\beta_1-4\beta_2+3\beta_3} \langle-\omega^{2,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-7\beta_1-3\beta_2+3\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-3\beta_2+2\beta_3} \langle(-(\omega^{1,4,5,7} - \omega^{3,4,6,7}))/2\rangle + e^{-5\beta_1-2\beta_2} \langle-\omega^{1,3,6,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-3\beta_2} \langle(\omega^{1,2,5,7} + \omega^{2,3,6,7})/2\rangle + e^{-4\beta_1-3\beta_2+2\beta_3} \langle(-(\omega^{1,2,4,5} + \omega^{2,3,4,6}))/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{L}}) = e^{-7\beta_1-5\beta_2+3\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-5\beta_1-4\beta_2} \langle\omega^{2,3,5,6,7}\rangle + e^{-8\beta_1-4\beta_2+3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2} \langle-\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-3\beta_2+2\beta_3} \langle\omega^{1,3,4,6,7}\rangle + e^{-6\beta_1-4\beta_2+2\beta_3} \langle(-\omega^{1,2,4,5,7} - \omega^{2,3,4,6,7})/2\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,3,5,7}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{L}}) = e^{-7\beta_1-5\beta_2+2\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2+2\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-5\beta_2+3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-4\beta_2} \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{L}}) = e^{-8\beta_1-5\beta_2+2\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + Y + Y^2 + Y^3 + Y^4)(1 + Y)^3.$

5.5.18 Algebra $\mathcal{G}_{7,3.17}$

Adjoint cohomology

$[x_1,x_2]=x_5$; $[x_1,x_5]=x_6$; $[x_1,x_6]=x_7$; $[x_3,x_4]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & 0 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0\\ \xi_1^4 & 0 & \xi_3^4 & 3\xi_1^1+\xi_2^2-\xi_3^3 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & 0 & 0 & \xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_1^4 & -\xi_1^3 & \xi_2^5 & 2\xi_1^1+\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^6 & \xi_2^5 & 3\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1=\delta_2^5, ad\ x_2=-\delta_1^5, ad\ x_3=\delta_4^7, ad\ x_4=-\delta_3^7, ad\ x_5=-\delta_1^6, ad\ x_6=-\delta_1^7, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{3\beta_1+\beta_2-\beta_3}\ \langle x_4\rangle+e^{\beta_1+\beta_2}\ \langle x_5\rangle+e^{2\beta_1+\beta_2}\ \langle x_6\rangle+e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{3\beta_1+\beta_2}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{-3\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle+e^{3\beta_1+\beta_2-2\beta_3}\ \langle\delta_3^4\rangle+3e^0\ \langle t_3,t_1,t_2\rangle+e^{\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle+e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_1^4\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle+e^{2\beta_1+\beta_2}\ \langle\delta_1^7\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{3\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle+e^{\beta_3}\ \langle\delta_4^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle+e^{\beta_1}\ \langle\delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_4^3,\delta_3^4]=t_3$; $[\delta_4^3,\delta_1^4]=\delta_1^3$; $[\delta_4^3,\delta_3^7]=-\delta_4^7$; $[\delta_3^4,\delta_1^3]=\delta_1^4$; $[\delta_3^4,\delta_4^7]=-\delta_3^7$; $[\delta_1^5,\delta_2^6]=-\delta_1^7$; $[\delta_1^5,\delta_2^5]=-\delta_1^6$; $[\delta_1^6,\delta_2^5]=-\delta_1^7$; $[\delta_1^4,\delta_1^3]=2\delta_1^6$; $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_1^4,\delta_2^5]=-\delta_3^7$; $[\delta_1^3,\delta_3^7]=-\delta_1^7$; $[\delta_1^3,\delta_2^5]=\delta_4^7$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{-3\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle+e^{3\beta_1+\beta_2-2\beta_3}\ \langle\delta_3^4\rangle+3e^0\ \langle t_3,t_1,t_2\rangle+e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_1^4\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle+e^{3\beta_1}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3,\delta_3^4]=t_3$; $[\delta_4^3,\delta_1^4]=\delta_1^3$; $[\delta_3^4,\delta_1^3]=\delta_1^4$.

ch($H^2_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle+e^{-4\beta_1+\beta_3}\ \langle x_2\omega^{1,4}\rangle+e^{-4\beta_1-\beta_2+2\beta_3}\ \langle x_3\omega^{1,4}\rangle+e^{-3\beta_1-2\beta_2+2\beta_3}\ \langle x_3\omega^{2,4}\rangle+e^{2\beta_1+\beta_2-2\beta_3}\ \langle x_4\omega^{1,3}\rangle+3e^{-\beta_1}\ \langle -x_3\omega^{1,3}+x_4\omega^{1,4},x_2\omega^{1,2},x_3\omega^{1,3}+x_6\omega^{3,4}\rangle+e^{3\beta_1-2\beta_3}\ \langle x_4\omega^{2,3}\rangle+3e^{-\beta_2}\ \langle -x_3\omega^{2,3}+x_4\omega^{2,4},x_1\omega^{1,2}+2x_5\omega^{2,5}+x_6\omega^{2,6},-x_1\omega^{1,2}+x_3\omega^{2,3}-x_5\omega^{2,5}+x_7\omega^{2,7}\rangle+e^{-3\beta_1}\ \langle x_2\omega^{1,6}\rangle+3e^{-\beta_3}\ \langle x_1\omega^{1,3}-2x_2\omega^{2,3}-x_4\omega^{1,6}+x_5\omega^{3,5},-x_1\omega^{1,3}+x_2\omega^{2,3}+2x_4\omega^{1,6}+x_6\omega^{3,6},x_4\omega^{3,4}\rangle+3e^{-3\beta_1-\beta_2+\beta_3}\ \langle x_1\omega^{1,4}-2x_2\omega^{2,4}+x_3\omega^{1,6}+$

$x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} - 2x_3\omega^{1,6} + x_6\omega^{4,6}, x_3\omega^{3,4}\rangle + e^{-2\beta_1} \langle x_2\omega^{1,5}\rangle + e^{2\beta_1-\beta_3} \langle -x_4\omega^{1,2} + x_6\omega^{2,3}\rangle + e^{-\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,2} + x_6\omega^{2,4}\rangle + e^{2\beta_1-\beta_2} \langle x_7\omega^{2,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{2\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2} + e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-\beta_3} + e^{-3\beta_1-\beta_2-\beta_3} + 2e^{-3\beta_1-\beta_2} + 2e^{-3\beta_1-2\beta_2+\beta_3} + 2e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-\beta_3} + e^{-4\beta_1-\beta_2+\beta_3} + 3e^{-4\beta_1-\beta_2} + 2e^{-4\beta_1-2\beta_2+\beta_3} + e^{-4\beta_1-3\beta_2+2\beta_3} + e^{-4\beta_1-\beta_3} + e^{-5\beta_1-\beta_2+\beta_3} + e^{-6\beta_1-\beta_2+\beta_3} + e^{-6\beta_1-2\beta_2+\beta_3} + e^{-6\beta_1-3\beta_2+2\beta_3} + e^{-7\beta_1-\beta_2+\beta_3} + e^{-7\beta_1-2\beta_2+2\beta_3} + 2e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + 3e^{-3\beta_1-3\beta_2} + 3e^{-4\beta_1-\beta_2-\beta_3} + 4e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_2+\beta_3} + e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-4\beta_2+\beta_3} + e^{-6\beta_1-\beta_2-\beta_3} + 3e^{-6\beta_1-2\beta_2} + 3e^{-6\beta_1-3\beta_2+\beta_3} + e^{-6\beta_1-4\beta_2+2\beta_3} + e^{-7\beta_1-\beta_2} + 3e^{-7\beta_1-2\beta_2+\beta_3} + e^{-7\beta_1-3\beta_2+2\beta_3} + e^{-8\beta_1-2\beta_2+\beta_3} + e^{-9\beta_1-2\beta_2+\beta_3} + e^{-9\beta_1-3\beta_2+2\beta_3} + e^{-2\beta_2-2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2-\beta_3} + 2e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-4\beta_2+\beta_3} + 2e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2-\beta_3} + e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2+\beta_3} + 2e^{-7\beta_1-3\beta_2} + e^{-7\beta_1-4\beta_2+\beta_3} + e^{-8\beta_1-2\beta_2} + e^{-8\beta_1-5\beta_2+\beta_3} + e^{-9\beta_1-2\beta_2} + 2e^{-9\beta_1-3\beta_2+\beta_3} + e^{-9\beta_1-4\beta_2+\beta_3} + e^{-9\beta_1-5\beta_2+2\beta_3} + e^{-10\beta_1-3\beta_2+\beta_3} + e^{-10\beta_1-4\beta_2+2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-4\beta_1-3\beta_2-2\beta_3} + 2e^{-6\beta_1-4\beta_2-\beta_3} + 2e^{-7\beta_1-3\beta_2-\beta_3} + 2e^{-7\beta_1-4\beta_2} + e^{-8\beta_1-5\beta_2} + e^{-9\beta_1-3\beta_2} + e^{-9\beta_1-4\beta_2} + 2e^{-9\beta_1-5\beta_2+\beta_3} + e^{-10\beta_1-3\beta_2} + 2e^{-10\beta_1-4\beta_2+\beta_3} + e^{-10\beta_1-5\beta_2+2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-7\beta_1-4\beta_2-\beta_3} + e^{-9\beta_1-5\beta_2} + e^{-10\beta_1-4\beta_2} + e^{-10\beta_1-5\beta_2+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 16, 56, 127, 157, 117, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(1, 10, 23, 36, 39, 29, 16, 4)$

S=$5(2\beta_1 + \beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 9, 21, 23, 18, 7, 1)$

Dimensions of $H^j_{\mathfrak{C}}$'s $0 \le j \le 7$: $(1, 4, 6, 9, 9, 6, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,5}$; $d\omega^7 = -\omega^{1,6} - \omega^{3,4}$; $d\omega^{1,7} = \omega^{1,3,4}$; $d\omega^{2,6} = -\omega^{1,2,5}$; $d\omega^{2,7} = -\omega^{1,2,6} + \omega^{2,3,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,5}$; $d\omega^{3,7} = -\omega^{1,3,6}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = -\omega^{1,4,5}$; $d\omega^{4,7} = -\omega^{1,4,6}$; $d\omega^{5,6} = -\omega^{1,2,6}$; $d\omega^{5,7} = -\omega^{1,2,7} - \omega^{1,5,6} + \omega^{3,4,5}$; $d\omega^{6,7} = -\omega^{1,5,7} + \omega^{3,4,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,4}$; $d\omega^{1,5,7} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,3,4,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,5}$; $d\omega^{2,3,7} = -\omega^{1,2,3,6}$; $d\omega^{2,4,6} = -\omega^{1,2,4,5}$; $d\omega^{2,4,7} = -\omega^{1,2,4,6}$; $d\omega^{2,5,7} = -\omega^{1,2,5,6} - \omega^{2,3,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,4,6} = -\omega^{1,3,4,5}$; $d\omega^{3,4,7} = -\omega^{1,3,4,6}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6}$; $d\omega^{3,6,7} = -\omega^{1,3,5,7}$;

$d\omega^{4,5,6} = \omega^{1,2,4,6}$; $d\omega^{4,5,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $d\omega^{4,6,7} = -\omega^{1,4,5,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d\omega^{1,2,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{2,3,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7} = -\omega^{1,2,3,5,6}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,4,5,7} = -\omega^{1,2,4,5,6}$; $d\omega^{2,4,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{2,5,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7} - \omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7}$; $d\omega^{1,2,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-3\beta_1-\beta_2+\beta_3} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2+\beta_3} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2+\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-7\beta_1-2\beta_2+\beta_3} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-4\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - \omega^{3,4,5}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7} - \omega^{3,4,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{3,5,6}\rangle + e^{-6\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-7\beta_1-2\beta_2} \langle\omega^{1,3,4,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,3,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-7\beta_1-3\beta_2+\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-9\beta_1-3\beta_2+\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,5,6}\rangle + e^{-6\beta_1-3\beta_2} \langle\omega^{2,3,4,7} - \omega^{3,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-6\beta_1-4\beta_2+\beta_3} \langle\omega^{2,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-9\beta_1-5\beta_2+\beta_3} \langle\omega^{2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-7\beta_1-4\beta_2-\beta_3} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-10\beta_1-5\beta_2+\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+3Y+Y^2)(1+Y^2)^2(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,9,21,23,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,6,9,9,6,4,1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^6$; $d^\star\omega^{1,6} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,5} = -\omega^{2,6}$; $d^\star\omega^{1,2,6} = -\omega^{2,7} - \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7}$; $d^\star\omega^{1,3,5} = -\omega^{3,6}$; $d^\star\omega^{1,3,6} = -\omega^{3,7}$;

$d^\star\omega^{1,4,5} = -\omega^{4,6}$; $d^\star\omega^{1,4,6} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,5,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,6}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,3,7} + \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,6}$; $d^\star\omega^{1,2,4,6} = -\omega^{2,4,7} + \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{2,5,7}$; $d^\star\omega^{1,2,5,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{3,4,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7} - \omega^{3,4,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{3,5,7}$; $d^\star\omega^{1,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} - \omega^{2,3,4,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{2,3,4,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7} - \omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,4}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{3,4} = \omega^{1,6}+\omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{2,7} + \omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6} - \omega^{2,3,4}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{1,5,6}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6}$; $\Delta\omega^{1,5,6} = \omega^{1,2,7} + \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6}+\omega^{2,3,4}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}-\omega^{3,5,6}$; $\Delta\omega^{2,4,6} = \omega^{2,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}-\omega^{4,5,6}$; $\Delta\omega^{2,5,7} = 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{2,3,7}+\omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,4,7}+\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}-\omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,3,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}-\omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,4,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6}$;

$\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,4,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-3\beta_1-\beta_2+\beta_3} \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-4\beta_1-\beta_2+\beta_3} \langle\omega^{1,4}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-3\beta_1-2\beta_2+\beta_3} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6} - \omega^{3,4}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,7}\rangle + e^{-7\beta_1-2\beta_2+\beta_3} \langle\omega^{1,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-4\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,5,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,2,7} - 2\omega^{1,5,6} - \omega^{3,4,5}\rangle + e^{-6\beta_1-2\beta_2} \langle\omega^{1,6,7} - \omega^{3,4,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,7} + \omega^{3,5,6}\rangle + e^{-6\beta_1-3\beta_2+\beta_3} \langle\omega^{2,4,7} + \omega^{4,5,6}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-6\beta_1-4\beta_2+\beta_3} \langle-\omega^{2,4,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-9\beta_1-3\beta_2+\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle-\omega^{1,3,6,7}\rangle + e^{-7\beta_1-2\beta_2} \langle-\omega^{1,3,4,7}\rangle + e^{-6\beta_1-3\beta_2} \langle(-(\omega^{1,2,6,7} + 2\omega^{2,3,4,7} - \omega^{3,4,5,6}))/6\rangle + e^{-4\beta_1-3\beta_2} \langle(-(\omega^{1,2,5,6} - \omega^{2,3,4,5}))/2\rangle + e^{-7\beta_1-3\beta_2+\beta_3} \langle(\omega^{1,2,4,7} + \omega^{1,4,5,6})/2\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,3,7} + \omega^{1,3,5,6}))/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-9\beta_1-5\beta_2+\beta_3} \langle-\omega^{2,4,5,6,7}\rangle + e^{-6\beta_1-4\beta_2-\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-10\beta_1-4\beta_2+\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-9\beta_1-3\beta_2} \langle\omega^{1,3,4,6,7}\rangle + e^{-7\beta_1-4\beta_2} \langle(-\omega^{1,2,5,6,7} + \omega^{2,3,4,5,7})/2\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-9\beta_1-5\beta_2} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-10\beta_1-4\beta_2} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-10\beta_1-5\beta_2+\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-7\beta_1-4\beta_2-\beta_3} \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-10\beta_1-5\beta_2} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 3Y + Y^2)(1 + Y^2)^2(1 + Y).$

5.5.19 Algebra $\mathcal{G}_{7,3.18}$

Adjoint cohomology

$[x_1, x_2] = x_6$; $[x_1, x_6] = x_7$; $[x_2, x_5] = x_7$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & 2\xi_1^1+\xi_2^2-\xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_2^4 & -\xi_2^3 & 2\xi_1^1 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_1^4 & -\xi_1^3 & -\xi_1^2 & \xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_5^7 & -\xi_1^5+\xi_2^6 & 2\xi_1^1+\xi_2^2 \end{pmatrix}$$

$ad\ x_1 = \delta_2^6, ad\ x_2 = -\delta_1^6 + \delta_5^7, ad\ x_3 = \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_1^7, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle x_4\rangle + e^{2\beta_1}\ \langle x_5\rangle + e^{\beta_1+\beta_2}\ \langle x_6\rangle + e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{2\beta_1+\beta_2}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-2\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle + e^{2\beta_1+\beta_2-2\beta_3}\ \langle\delta_3^4\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1-\beta_2}\ \langle\delta_2^5\rangle + e^{2\beta_1-\beta_3}\ \langle\delta_2^4\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + 2e^{\beta_2}\ \langle\delta_1^6, \delta_5^7\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_1^4\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^7\rangle + e^{2\beta_1}\ \langle\delta_2^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^7\rangle + e^{\beta_3}\ \langle\delta_4^7\rangle + 2e^{\beta_1}\ \langle\delta_1^5, \delta_2^6\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_2^4] = \delta_2^3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_4^3, \delta_3^7] = -\delta_4^7$; $[\delta_3^4, \delta_2^3] = \delta_2^4$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_2^5, \delta_1^2] = \delta_2^6 + \delta_1^5$; $[\delta_2^5, \delta_5^7] = -\delta_2^7$; $[\delta_2^4, \delta_2^3] = 2\delta_2^5$; $[\delta_2^4, \delta_1^3] = \delta_2^6 + \delta_1^5$; $[\delta_2^4, \delta_1^2] = \delta_1^4$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_2^4, \delta_5^7] = -\delta_3^7$; $[\delta_2^3, \delta_1^4] = -(\delta_2^6+\delta_1^5)$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_5^7] = \delta_4^7$; $[\delta_1^6, \delta_1^5] = \delta_1^7$; $[\delta_1^6, \delta_2^6] = -\delta_1^7$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^4, \delta_1^5] = \delta_3^7$; $[\delta_1^4, \delta_2^6] = -\delta_3^7$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_1^5] = -\delta_4^7$; $[\delta_1^3, \delta_2^6] = \delta_4^7$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_1^5] = -(\delta_5^7 + \delta_1^6)$; $[\delta_1^2, \delta_2^6] = \delta_5^7 - \delta_1^6$; $[\delta_5^7, \delta_1^5] = \delta_1^7$; $[\delta_1^5, \delta_2^6] = -\delta_2^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-2\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle + e^{2\beta_1+\beta_2-2\beta_3}\ \langle\delta_3^4\rangle + 3e^0\ \langle t_3, t_1, t_2\rangle + e^{2\beta_1-\beta_2}\ \langle\delta_2^5\rangle + e^{2\beta_1-\beta_3}\ \langle\delta_2^4\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_1^4\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{\beta_2}\ \langle\delta_1^6\rangle + e^{\beta_1}\ \langle\delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^3, \delta_2^4] = \delta_2^3$; $[\delta_4^3, \delta_1^4] = \delta_1^3$; $[\delta_3^4, \delta_2^3] = \delta_2^4$; $[\delta_3^4, \delta_1^3] = \delta_1^4$; $[\delta_2^5, \delta_1^2] = \delta_1^5$; $[\delta_2^4, \delta_2^3] = 2\delta_2^5$; $[\delta_2^4, \delta_1^3] = \delta_1^5$; $[\delta_2^4, \delta_1^2] = \delta_1^4$; $[\delta_2^3, \delta_1^4] = -\delta_1^5$; $[\delta_2^3, \delta_1^2] = \delta_1^3$; $[\delta_1^4, \delta_1^3] = 2\delta_1^6$; $[\delta_1^2, \delta_1^5] = -2\delta_1^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{-2\beta_2}\ \langle x_1\omega^{2,6}\rangle + e^{-3\beta_1+\beta_2}\ \langle x_2\omega^{1,5}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1(\omega^{1,6}-\omega^{2,5}) + x_2\omega^{2,6}\rangle + e^{-3\beta_1-\beta_2+2\beta_3}\ \langle x_3\omega^{1,4}\rangle + 2e^{-3\beta_1+\beta_3}\ \langle x_2\omega^{1,4}+x_3\omega^{1,5}, x_2\omega^{1,4}+x_6\omega^{4,5}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3}\ \langle x_3\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle x_1\omega^{2,4}+x_3\omega^{2,6}\rangle + e^{-4\beta_1-\beta_2+2\beta_3}\ \langle x_3\omega^{4,5}\rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle x_4\omega^{1,3}\rangle + 4e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, -x_2\omega^{1,2} + x_5\omega^{1,5}, x_3\omega^{1,3} + x_6\omega^{3,4}, x_2\omega^{1,2} + x_6\omega^{2,5}\rangle + 2e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{1,3} + x_4\omega^{1,5}, x_2\omega^{1,3} + x_6\omega^{3,5}\rangle + e^{2\beta_1-2\beta_3}\ \langle x_4\omega^{2,3}\rangle + 4e^{-\beta_2}\ \langle -x_3\omega^{2,3}+x_4\omega^{2,4}, x_3\omega^{2,3}+x_5\omega^{3,4}, x_5\omega^{2,5}, -x_1\omega^{1,2} + x_5\omega^{1,6}\rangle + e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3}+x_4\omega^{2,6}\rangle + e^{\beta_2-2\beta_3}\ \langle x_4\omega^{3,5}\rangle + 4e^{-2\beta_1}\ \langle -x_3\omega^{3,5} + x_4\omega^{4,5}, -x_1\omega^{1,5}-2x_2\omega^{1,6}+x_6\omega^{5,6}, -x_1\omega^{1,5}-x_2(\omega^{1,6}-\omega^{2,5}), -x_1\omega^{1,5}-x_2(\omega^{1,6}+\omega^{3,4}) - x_3\omega^{3,5} + x_6\omega^{1,7}\rangle + e^{\beta_1-\beta_3}\ \langle x_4\omega^{1,2} + x_5\omega^{1,3}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle -x_3\omega^{1,2} + x_5\omega^{1,4}\rangle + e^{2\beta_1-\beta_2-\beta_3}\ \langle x_5\omega^{2,3}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,4}\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle + 4e^{-\beta_3}\ \langle -x_1\omega^{1,3} + x_2\omega^{2,3} + x_4\omega^{1,6} + x_5\omega^{3,5}, -x_1\omega^{1,3} + 2x_4\omega^{1,6} + x_6\omega^{3,6}, x_4\omega^{3,4}, x_1\omega^{1,3} - x_2\omega^{2,3} - x_4(\omega^{1,6} - \omega^{2,5})\rangle + 4e^{-2\beta_1-\beta_2+\beta_3}\ \langle -x_1\omega^{1,4} + x_2\omega^{2,4} - x_3\omega^{1,6} + x_5\omega^{4,5}, -x_1\omega^{1,4} - 2x_3\omega^{1,6} + x_6\omega^{4,6}, x_3\omega^{3,4}, -x_1\omega^{1,4} + x_2\omega^{2,4} - x_3(\omega^{1,6}-\omega^{2,5})\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + e^{-\beta_1+\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2+\beta_3} + 2e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3} + 5e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_3} + 4e^{-2\beta_1-\beta_3} + e^{-3\beta_1+\beta_2-\beta_3} + 2e^{-3\beta_1-\beta_2+\beta_3} + 3e^{-3\beta_1-\beta_2} + 3e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-3\beta_2+2\beta_3} + e^{-3\beta_1-\beta_3} + 3e^{-3\beta_1} + 4e^{-4\beta_1-\beta_2+\beta_3} + e^{-4\beta_1-2\beta_2+2\beta_3} + e^{-4\beta_1} + e^{-5\beta_1-\beta_2+2\beta_3} + e^{-5\beta_1-\beta_2+\beta_3} + e^{-5\beta_1-2\beta_2+2\beta_3} + e^{-5\beta_1+\beta_3} + 3e^{-\beta_2-\beta_3} + e^{-2\beta_2-\beta_3} + e^{-2\beta_2} + e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-2\beta_1-3\beta_2} + 4e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + 4e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-3\beta_2+\beta_3} + 2e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2+\beta_3} + e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + 2e^{-4\beta_1-\beta_2-\beta_3} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + 3e^{-4\beta_1-2\beta_2} + 3e^{-4\beta_1-3\beta_2+\beta_3} + e^{-4\beta_1-4\beta_2+\beta_3} + e^{-4\beta_1-\beta_3} + e^{-5\beta_1-\beta_2+\beta_3} + 3e^{-5\beta_1-\beta_2} + 4e^{-5\beta_1-2\beta_2+\beta_3} + e^{-5\beta_1-3\beta_2+2\beta_3} + e^{-5\beta_1-3\beta_2+\beta_3} + e^{-5\beta_1-4\beta_2+2\beta_3} + e^{-5\beta_1-\beta_3} + e^{-6\beta_1-\beta_2+\beta_3} + 2e^{-6\beta_1-2\beta_2+\beta_3} + e^{-6\beta_1-3\beta_2+2\beta_3} + e^{-7\beta_1-\beta_2+\beta_3} + e^{-7\beta_1-2\beta_2+2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + 3e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-4\beta_2} + e^{-4\beta_1-\beta_2-2\beta_3} + 4e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + 4e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2+\beta_3} + e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-2\beta_2-\beta_3} + 2e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2+\beta_3} + 3e^{-5\beta_1-3\beta_2} + 3e^{-5\beta_1-4\beta_2+\beta_3} + e^{-6\beta_1-\beta_2-\beta_3} + 4e^{-6\beta_1-2\beta_2} + 4e^{-6\beta_1-3\beta_2+\beta_3} + e^{-6\beta_1-4\beta_2+2\beta_3} + e^{-6\beta_1-4\beta_2+\beta_3} + e^{-7\beta_1-\beta_2} + 2e^{-7\beta_1-2\beta_2+\beta_3} + e^{-7\beta_1-3\beta_2+\beta_3} + e^{-7\beta_1-4\beta_2+2\beta_3} + e^{-8\beta_1-2\beta_2+\beta_3} + e^{-8\beta_1-3\beta_2+2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-4\beta_1-2\beta_2-2\beta_3} + 2e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-4\beta_2} + 2e^{-5\beta_1-3\beta_2-\beta_3} + 2e^{-5\beta_1-4\beta_2} + 2e^{-6\beta_1-2\beta_2-\beta_3} + 4e^{-6\beta_1-3\beta_2} + 2e^{-6\beta_1-4\beta_2+\beta_3} + e^{-6\beta_1-4\beta_2} + e^{-7\beta_1-2\beta_2} + e^{-7\beta_1-3\beta_2} + 2e^{-7\beta_1-4\beta_2+\beta_3} + e^{-8\beta_1-2\beta_2} + 2e^{-8\beta_1-3\beta_2+\beta_3} + e^{-8\beta_1-4\beta_2+2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-6\beta_1-3\beta_2-\beta_3}+e^{-6\beta_1-4\beta_2}+e^{-7\beta_1-4\beta_2}+e^{-8\beta_1-3\beta_2}+e^{-8\beta_1-4\beta_2+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,19,70,135,170,125,47,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,13,40,58,60,50,25,5)$

S=$4(2\beta_1+\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,5,12,20,26,19,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,5,10,11,11,10,5,1)$

$d\omega^6=-\omega^{1,2}$; $d\omega^7=-\omega^{1,6}-\omega^{2,5}-\omega^{3,4}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,4}$; $d\omega^{2,7}=-\omega^{1,2,6}+\omega^{2,3,4}$; $d\omega^{3,6}=\omega^{1,2,3}$; $d\omega^{3,7}=-\omega^{1,3,6}-\omega^{2,3,5}$; $d\omega^{4,6}=\omega^{1,2,4}$; $d\omega^{4,7}=-\omega^{1,4,6}-\omega^{2,4,5}$; $d\omega^{5,6}=\omega^{1,2,5}$; $d\omega^{5,7}=-\omega^{1,5,6}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,2,7}+\omega^{2,5,6}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}-\omega^{1,3,4,6}$; $d\omega^{2,3,7}=-\omega^{1,2,3,6}$; $d\omega^{2,4,7}=-\omega^{1,2,4,6}$; $d\omega^{2,5,7}=-\omega^{1,2,5,6}-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{2,3,4,6}$; $d\omega^{3,4,6}=-\omega^{1,2,3,4}$; $d\omega^{3,4,7}=-\omega^{1,3,4,6}-\omega^{2,3,4,5}$; $d\omega^{3,5,6}=-\omega^{1,2,3,5}$; $d\omega^{3,5,7}=-\omega^{1,3,5,6}$; $d\omega^{3,6,7}=\omega^{1,2,3,7}+\omega^{2,3,5,6}$; $d\omega^{4,5,6}=-\omega^{1,2,4,5}$; $d\omega^{4,5,7}=-\omega^{1,4,5,6}$; $d\omega^{4,6,7}=\omega^{1,2,4,7}+\omega^{2,4,5,6}$; $d\omega^{5,6,7}=\omega^{1,2,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{1,5,6,7}=\omega^{1,3,4,5,6}$; $d\omega^{2,3,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{2,3,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{2,4,5,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7}=\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{3,4,5,7}=-\omega^{1,3,4,5,6}$; $d\omega^{3,4,6,7}=-\omega^{1,2,3,4,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,2,3,5,7}$; $d\omega^{4,5,6,7}=-\omega^{1,2,4,5,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7}=\omega^{1,2,3,4,5,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle+e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle+e^{-2\beta_1}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+e^{-3\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle+e^{-3\beta_1}\ \langle\omega^{1,5}\rangle+2e^{-2\beta_1-\beta_2}\ \langle\omega^{1,6},\omega^{2,5}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\ \langle\omega^{2,6}\rangle+e^{-2\beta_1-\beta_3}\ \langle\omega^{3,5}\rangle+e^{-4\beta_1-\beta_2+\beta_3}\ \langle\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})$= $e^{-3\beta_1-\beta_3}\ \langle\omega^{1,3,5}\rangle+e^{-5\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4,5}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,6}\rangle+e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6}\rangle+e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,6}\rangle+e^{-4\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,6}\rangle+e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4,6}\rangle+e^{-4\beta_1-\beta_2}\ \langle\omega^{1,5,6}\rangle+e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,7}-\omega^{3,4,6}\rangle+e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,7}+\omega^{3,5,6}\rangle+e^{-5\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,7}+\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})$= $e^{-5\beta_1-2\beta_2}\ \langle\omega^{1,3,4,7}-\omega^{3,4,5,6}\rangle+e^{-5\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,5,7}\rangle+e^{-7\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,5,7}\rangle+e^{-4\beta_1-3\beta_2}\ \langle\omega^{1,2,6,7}+\omega^{2,3,4,7}\rangle+e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,3,7}\rangle+e^{-4\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,3,6,7}-\omega^{2,3,5,7}\rangle+e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,6,7}\rangle+e^{-5\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,2,4,7}\rangle+e^{-6\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,4,6,7}-\omega^{2,4,5,7}\rangle+e^{-5\beta_1-4\beta_2+\beta_3}\ \langle\omega^{2,4,6,7}\rangle+e^{-6\beta_1-2\beta_2}\ \langle\omega^{1,5,6,7}+\omega^{3,4,5,7}\rangle$.

ch$(H^5_{\mathcal{C}})$= $e^{-4\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,3,6,7}\rangle$ + $e^{-6\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{1,2,4,6,7}\rangle$ + $2e^{-6\beta_1-3\beta_2}$ $\langle\omega^{1,2,5,6,7}-\omega^{2,3,4,5,7},\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}\rangle$ + $e^{-7\beta_1-2\beta_2}$ $\langle\omega^{1,3,4,5,7}\rangle$ + $e^{-6\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,3,5,6,7}\rangle$ + $e^{-8\beta_1-3\beta_2+\beta_3}$ $\langle\omega^{1,4,5,6,7}\rangle$ + $e^{-5\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,6,7}\rangle$ + $e^{-5\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,3,5,6,7}\rangle$ + $e^{-7\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{2,4,5,6,7}\rangle$.

ch$(H^6_{\mathcal{C}})$= $e^{-6\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,6,7}\rangle$ + $e^{-6\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,3,5,6,7}\rangle$ + $e^{-8\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle$ + $e^{-8\beta_1-3\beta_2}$ $\langle\omega^{1,3,4,5,6,7}\rangle$ + $e^{-7\beta_1-4\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$.

ch$(H^7_{\mathcal{C}})$= $e^{-8\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y+6Y^2+5Y^3+6Y^4+4Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,5,12,20,26,19,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0\le j\le 7$: $(1,5,10,11,11,10,5,1)$

Total dimension of trivial cohomology: 54.

Length associated polynomial: 54.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^6$; $d^\star\omega^{1,6}=-\omega^7$; $d^\star\omega^{2,5}=-\omega^7$; $d^\star\omega^{3,4}=-\omega^7$; $d^\star\omega^{1,2,3}=\omega^{3,6}$; $d^\star\omega^{1,2,4}=\omega^{4,6}$; $d^\star\omega^{1,2,5}=\omega^{1,7}+\omega^{5,6}$; $d^\star\omega^{1,2,6}=-\omega^{2,7}$; $d^\star\omega^{1,2,7}=-\omega^{6,7}$; $d^\star\omega^{1,3,4}=\omega^{1,7}$; $d^\star\omega^{1,3,6}=-\omega^{3,7}$; $d^\star\omega^{1,4,6}=-\omega^{4,7}$; $d^\star\omega^{1,5,6}=-\omega^{5,7}$; $d^\star\omega^{2,3,4}=\omega^{2,7}$; $d^\star\omega^{2,3,5}=-\omega^{3,7}$; $d^\star\omega^{2,4,5}=-\omega^{4,7}$; $d^\star\omega^{2,5,6}=\omega^{6,7}$; $d^\star\omega^{3,4,5}=\omega^{5,7}$; $d^\star\omega^{3,4,6}=\omega^{6,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,3,5}=\omega^{1,3,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,3,6}=-\omega^{2,3,7}$; $d^\star\omega^{1,2,3,7}=\omega^{3,6,7}$; $d^\star\omega^{1,2,4,5}=\omega^{1,4,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,4,6}=-\omega^{2,4,7}$; $d^\star\omega^{1,2,4,7}=\omega^{4,6,7}$; $d^\star\omega^{1,2,5,6}=-\omega^{1,6,7}-\omega^{2,5,7}$; $d^\star\omega^{1,2,5,7}=\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5}=-\omega^{1,5,7}$; $d^\star\omega^{1,3,4,6}=-\omega^{1,6,7}-\omega^{3,4,7}$; $d^\star\omega^{1,3,5,6}=-\omega^{3,5,7}$; $d^\star\omega^{1,4,5,6}=-\omega^{4,5,7}$; $d^\star\omega^{2,3,4,5}=-\omega^{2,5,7}-\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6}=-\omega^{2,6,7}$; $d^\star\omega^{2,3,5,6}=\omega^{3,6,7}$; $d^\star\omega^{2,4,5,6}=\omega^{4,6,7}$; $d^\star\omega^{3,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=\omega^{1,2,5,7}+\omega^{1,3,4,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=\omega^{1,2,6,7}-\omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,5,6}=-\omega^{1,3,6,7}-\omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,5,7}=-\omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6}=-\omega^{1,4,6,7}-\omega^{2,4,5,7}$; $d^\star\omega^{1,2,4,5,7}=-\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6}=\omega^{1,5,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{2,3,4,5,6}=\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6}=-\omega^{1,2,5,6,7}-\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $d^\star\omega^{1,2,3,4,5,7}=\omega^{3,4,5,6,7}$;

$\Delta\omega^6=\omega^6$; $\Delta\omega^7=3\omega^7$; $\Delta\omega^{1,2}=\omega^{1,2}$; $\Delta\omega^{1,6}=\omega^{1,6}+\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{1,7}=2\omega^{1,7}+\omega^{5,6}$; $\Delta\omega^{2,5}=\omega^{1,6}+\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{2,7}=2\omega^{2,7}$; $\Delta\omega^{3,4}=\omega^{1,6}+\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,6}=\omega^{3,6}$; $\Delta\omega^{3,7}=2\omega^{3,7}$; $\Delta\omega^{4,6}=\omega^{4,6}$; $\Delta\omega^{4,7}=2\omega^{4,7}$; $\Delta\omega^{5,6}=\omega^{1,7}+\omega^{5,6}$; $\Delta\omega^{5,7}=2\omega^{5,7}$; $\Delta\omega^{6,7}=3\omega^{6,7}$; $\Delta\omega^{1,2,3}=\omega^{1,2,3}$; $\Delta\omega^{1,2,4}=\omega^{1,2,4}$; $\Delta\omega^{1,2,5}=2\omega^{1,2,5}+\omega^{1,3,4}$; $\Delta\omega^{1,2,6}=\omega^{1,2,6}-\omega^{2,3,4}$;

$\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{2,5,6}$; $\Delta\omega^{1,3,4} = \omega^{1,2,5} + \omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{3,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7} - \omega^{4,5,6}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6} - \omega^{3,4,5}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,3,4} = -\omega^{1,2,6} + \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}$; $\Delta\omega^{2,4,5} = \omega^{1,4,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = -\omega^{1,2,7} + \omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{1,6,7} + 2\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,5,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{2,5,6} + 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,6,7} + \omega^{2,5,7} + 2\omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,3,7} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,4,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = \omega^{1,2,3,7} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{1,2,5,6} = 2\omega^{1,2,5,6} + \omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,5,6} + 2\omega^{1,3,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,3,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,6,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{1,4,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,3,4,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,5,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle + e^{-2\beta_1}\ \langle\omega^5\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle + e^{-3\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle + e^{-3\beta_1}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,6}\rangle + 2e^{-2\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{3,4}, \omega^{2,5} - \omega^{3,4}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{3,5}\rangle + e^{-4\beta_1-\beta_2+\beta_3}\ \langle\omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-3\beta_1-\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-5\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,6} + \omega^{2,3,4}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6} - \omega^{2,3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-4\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,6} - \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-4\beta_1-\beta_2}\ \langle\omega^{1,5,6} + \omega^{3,4,5}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,2,7} + 2\omega^{2,5,6} - \omega^{3,4,6}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,7} + \omega^{3,5,6}\rangle + e^{-5\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,7} + \omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-5\beta_1-4\beta_2+\beta_3}\ \langle -\omega^{2,4,6,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-6\beta_1-2\beta_2}\ \langle(-(\omega^{1,5,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-6\beta_1-3\beta_2+\beta_3}\ \langle(-(\omega^{1,4,6,7} -$

$\omega^{2,4,5,7}))/2\rangle + e^{-7\beta_1-2\beta_2+\beta_3} \langle -\omega^{1,4,5,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,3,6,7} - \omega^{2,3,5,7})/2\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle \omega^{1,3,5,7}\rangle + e^{-4\beta_1-3\beta_2} \langle(\omega^{1,2,6,7} + \omega^{2,3,4,7})/2\rangle + e^{-5\beta_1-2\beta_2} \langle(\omega^{1,2,5,7} - 2\omega^{1,3,4,7} + \omega^{3,4,5,6})/6\rangle + e^{-5\beta_1-3\beta_2+\beta_3} \langle(\omega^{1,2,4,7} - \omega^{2,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle(-(\omega^{1,2,3,7} - \omega^{2,3,5,6}))/2\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-7\beta_1-4\beta_2+\beta_3} \langle -\omega^{2,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-\beta_3} \langle \omega^{2,3,5,6,7}\rangle + e^{-5\beta_1-4\beta_2} \langle -\omega^{2,3,4,6,7}\rangle + e^{-8\beta_1-3\beta_2+\beta_3} \langle \omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle -\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-2\beta_2} \langle -\omega^{1,3,4,5,7}\rangle + 2e^{-6\beta_1-3\beta_2} \langle(-\omega^{1,2,5,6,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,4,5,7})/3, (-\omega^{1,2,5,6,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,4,5,7})/3\rangle + e^{-6\beta_1-4\beta_2+\beta_3} \langle -\omega^{1,2,4,6,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3} \langle \omega^{1,2,3,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-7\beta_1-4\beta_2} \langle \omega^{2,3,4,5,6,7}\rangle + e^{-8\beta_1-3\beta_2} \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-8\beta_1-4\beta_2+\beta_3} \langle \omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-\beta_3} \langle -\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-4\beta_2} \langle \omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-8\beta_1-4\beta_2} \langle \omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 4Y + 6Y^2 + 5Y^3 + 6Y^4 + 4Y^5 + Y^6)(1 + Y)$.

5.5.20 Algebra $\mathcal{G}_{7,3.19}$

Adjoint cohomology

$[x_1, x_2] = x_6$; $[x_1, x_3] = x_7$; $[x_3, x_4] = x_6$; $[x_4, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & \xi_4^1 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & \xi_4^2 & 0 & 0 & 0 \\ -\xi_4^2 & \xi_4^1 & \xi_3^3 & \xi_4^3 & (-\xi_3^2)/2 & 0 & 0 \\ \xi_3^2/2 & 0 & 0 & \xi_1^1+\xi_2^2-\xi_3^3 & 0 & 0 & 0 \\ \xi_4^3 & 0 & -2\xi_4^1 & \xi_4^5 & -\xi_2^2+2\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_5^6 & \xi_1^1+\xi_2^2 & \xi_3^2/2 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_5^7 & \xi_4^1 & \xi_1^1+\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^6 + \delta_3^7, ad\ x_2 = -\delta_1^6, ad\ x_3 = \delta_4^6 - \delta_1^7, ad\ x_4 = -\delta_3^6 + \delta_5^7, ad\ x_5 = -\delta_4^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle x_4\rangle + e^{-\beta_2+2\beta_3}\ \langle x_5\rangle + e^{\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+\beta_3}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+\beta_3}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_4^2\rangle + e^{-\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle + e^{-\beta_1-2\beta_2+3\beta_3}\ \langle\delta_4^5\rangle + e^{\beta_2}\ \langle\delta_1^6\rangle + 2e^{\beta_1}\ \langle\delta_2^6,\delta_3^7\rangle + 2e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6,\delta_5^7\rangle + 2e^{\beta_3}\ \langle\delta_4^6,\delta_1^7\rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle\delta_5^6\rangle + 3e^0\ \langle t_2,t_1,t_3\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle + e^{-\beta_2+2\beta_3}\ \langle\delta_4^7\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_4^1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^6] = -\delta_1^6$; $[\delta_1^2,\delta_2^7] = -\delta_1^7$; $[\delta_1^2,\delta_4^1] = \delta_4^2$; $[\delta_4^2,\delta_2^6] = -\delta_4^6$; $[\delta_4^2,\delta_3^6] = \delta_1^6$; $[\delta_4^2,\delta_3^2] = 3/2\delta_1^2$; $[\delta_4^2,\delta_2^7] = -\delta_4^7$; $[\delta_4^2,\delta_3^7] = \delta_1^7$; $[\delta_4^2,\delta_4^1] = -2\delta_4^3$; $[\delta_4^3,\delta_3^6] = -\delta_4^6$; $[\delta_4^3,\delta_5^6] = -\delta_1^6$; $[\delta_4^3,\delta_3^2] = -\delta_4^2$; $[\delta_4^3,\delta_3^7] = -\delta_4^7$; $[\delta_4^3,\delta_5^7] = -\delta_1^7$; $[\delta_4^3,\delta_4^1] = 3\delta_4^5$; $[\delta_4^5,\delta_5^6] = -\delta_4^6$; $[\delta_4^5,\delta_3^2] = 1/2\delta_4^3$; $[\delta_4^5,\delta_5^7] = -\delta_4^7$; $[\delta_1^6,\delta_4^1] = -\delta_1^7+\delta_4^6$; $[\delta_2^6,\delta_3^2] = \delta_3^6$; $[\delta_2^6,\delta_4^1] = -\delta_2^7$; $[\delta_3^6,\delta_3^2] = -1/2\delta_5^6$; $[\delta_3^6,\delta_4^1] = -\delta_3^7+\delta_2^6$; $[\delta_4^6,\delta_3^2] = 1/2\delta_1^6$; $[\delta_4^6,\delta_4^1] = -\delta_4^7$; $[\delta_5^6,\delta_4^1] = -\delta_5^7 - 2\delta_3^6$; $[\delta_3^2,\delta_1^7] = 1/2\delta_1^6$; $[\delta_3^2,\delta_2^7] = -\delta_3^7+1/2\delta_2^6$; $[\delta_3^2,\delta_3^7] = 1/2(\delta_5^7+\delta_3^6)$; $[\delta_3^2,\delta_4^7] = -1/2\delta_1^7+1/2\delta_4^6$; $[\delta_3^2,\delta_5^7] = 1/2\delta_5^6$; $[\delta_3^2,\delta_4^1] = t_2 - 1/2t_1$; $[\delta_1^7,\delta_4^1] = \delta_4^7$; $[\delta_3^7,\delta_4^1] = \delta_2^7$; $[\delta_5^7,\delta_4^1] = -2\delta_3^7$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_4^2\rangle + e^{-\beta_1-\beta_2+2\beta_3}\ \langle\delta_4^3\rangle + e^{-\beta_1-2\beta_2+3\beta_3}\ \langle\delta_4^5\rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle\delta_5^6\rangle + 3e^0\ \langle t_2,t_1,t_3\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{\beta_3}\ \langle\delta_4^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle + e^{\beta_1}\ \langle\delta_2^6\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_4^1\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^7] = -\delta_4^6$; $[\delta_1^2,\delta_4^1] = \delta_4^2$; $[\delta_3^2,\delta_4^2] = (-3\delta_1^2)/2$; $[\delta_3^2,\delta_4^3] = \delta_4^2$; $[\delta_3^2,\delta_4^5] = (-\delta_4^3)/2$; $[\delta_3^2,\delta_2^7] = (3\delta_2^6)/2$; $[\delta_3^2,\delta_2^6] = -\delta_3^6$; $[\delta_3^2,\delta_3^6] = \delta_5^6/2$; $[\delta_3^2,\delta_4^1] = (2t_2-t_1)/2$; $[\delta_4^2,\delta_2^6] = -\delta_4^6$; $[\delta_4^2,\delta_4^1] =$

$-2\delta_4^3$; $[\delta_4^3,\delta_3^6]=-\delta_4^6$; $[\delta_4^3,\delta_4^1]=3\delta_4^5$; $[\delta_4^5,\delta_5^6]=-\delta_4^6$; $[\delta_5^6,\delta_4^1]=-3\delta_3^6$; $[\delta_2^6,\delta_4^1]=-\delta_2^7$; $[\delta_3^6,\delta_4^1]=2\delta_2^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle + e^{-2\beta_1+\beta_3}\ \langle x_2\omega^{1,4}\rangle + e^{-\beta_1+2\beta_2-2\beta_3}\ \langle x_2\omega^{1,5}\rangle + 5e^{-\beta_3}\ \langle -x_1\omega^{1,3}+x_2\omega^{2,3}, -x_4\omega^{3,4}+x_5\omega^{3,5}, x_1(\omega^{1,3}+\omega^{4,5})-x_4\omega^{3,4}+x_6\omega^{2,7}, -x_1(\omega^{1,3}-\omega^{4,5})+x_3\omega^{2,5}+x_4(\omega^{1,2}-\omega^{3,4}), -x_1\omega^{1,3}+x_6\omega^{3,6}\rangle + e^{-2\beta_1-\beta_2+2\beta_3}\ \langle x_3\omega^{1,4}\rangle + 2e^{-\beta_1+\beta_2-\beta_3}\ \langle -x_2\omega^{4,5}+x_3\omega^{1,5}, x_2\omega^{1,3}\rangle + 4e^{-\beta_2}\ \langle x_1(\omega^{1,2}-\omega^{3,4})+x_3\omega^{2,3}, x_1\omega^{3,4}-x_4\omega^{2,4}+x_5\omega^{2,5}, -x_1(\omega^{1,2}+\omega^{3,4})+x_4\omega^{2,4}+x_7\omega^{2,7}, x_1\omega^{3,4}+x_6\omega^{2,6}\rangle + 5e^{-\beta_1}\ \langle x_2\omega^{3,4}+x_3\omega^{4,5}+x_4\omega^{1,4}, x_3\omega^{4,5}+x_5\omega^{1,5}, x_2\omega^{1,2}, x_2\omega^{3,4}+x_3\omega^{1,3}, -x_2\omega^{3,4}+x_6\omega^{4,7}\rangle + 2e^{2\beta_2-3\beta_3}\ \langle x_2\omega^{3,5}+x_4\omega^{1,5}, -x_2\omega^{3,5}+x_6\omega^{5,7}\rangle + e^{\beta_1-2\beta_3}\ \langle x_1\omega^{2,5}+x_4\omega^{2,3}\rangle + e^{\beta_1+\beta_2-3\beta_3}\ \langle -x_1\omega^{3,5}+x_4\omega^{2,5}\rangle + e^{\beta_1+2\beta_2-4\beta_3}\ \langle x_4\omega^{3,5}\rangle + e^{-2\beta_1-2\beta_2+3\beta_3}\ \langle x_5\omega^{1,4}\rangle + 2e^{-2\beta_2+\beta_3}\ \langle -x_1\omega^{2,4}+x_5\omega^{2,3}, -x_1\omega^{2,4}+x_7\omega^{2,6}\rangle + e^{-\beta_1-3\beta_2+3\beta_3}\ \langle x_5\omega^{2,4}\rangle + 4e^{\beta_2-2\beta_3}\ \langle x_1\omega^{1,5}-2x_2\omega^{2,5}-x_3\omega^{3,5}+x_6\omega^{5,6}, x_1\omega^{1,5}-x_2\omega^{2,5}+x_4\omega^{1,3}, x_1\omega^{1,5}-x_2\omega^{2,5}-x_3\omega^{3,5}+x_4\omega^{4,5}, x_1\omega^{1,5}-x_2\omega^{2,5}-x_3\omega^{3,5}+x_6\omega^{3,7}\rangle + 5e^{-\beta_1-\beta_2+\beta_3}\ \langle x_2\omega^{2,4}+x_3\omega^{1,2}, -x_1\omega^{1,4}+x_2\omega^{2,4}+x_3\omega^{3,4}, -x_2\omega^{2,4}+x_6\omega^{4,6}, -x_1\omega^{1,4}+x_2\omega^{2,4}+x_5\omega^{1,3}, x_5\omega^{4,5}\rangle + 2e^{-\beta_1-2\beta_2+2\beta_3}\ \langle -x_3\omega^{2,4}+x_5\omega^{1,2}, x_5\omega^{3,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1+\beta_2-4\beta_3}+e^{\beta_1-3\beta_3}+e^{-\beta_1+2\beta_2-3\beta_3}+e^{-\beta_1+2\beta_2-4\beta_3}+3e^{-\beta_1+\beta_2-2\beta_3}+2e^{-\beta_1+\beta_2-3\beta_3}+3e^{-\beta_1-\beta_2-\beta_3}+3e^{-\beta_1-\beta_2}+3e^{-\beta_1-2\beta_2+\beta_3}+2e^{-\beta_1-2\beta_2}+e^{-\beta_1-3\beta_2+2\beta_3}+e^{-\beta_1-3\beta_2+\beta_3}+3e^{-\beta_1-\beta_3}+3e^{-\beta_1-2\beta_3}+e^{-2\beta_1+2\beta_2-3\beta_3}+2e^{-2\beta_1+\beta_2-2\beta_3}+e^{-2\beta_1-\beta_2+\beta_3}+6e^{-2\beta_1-\beta_2}+4e^{-2\beta_1-2\beta_2+\beta_3}+2e^{-2\beta_1-3\beta_2+2\beta_3}+e^{-2\beta_1-4\beta_2+3\beta_3}+4e^{-2\beta_1-\beta_3}+2e^{-3\beta_1-\beta_2+\beta_3}+2e^{-3\beta_1-2\beta_2+2\beta_3}+e^{-3\beta_1-3\beta_2+3\beta_3}+e^{-3\beta_1}+e^{2\beta_2-4\beta_3}+e^{2\beta_2-5\beta_3}+3e^{\beta_2-3\beta_3}+e^{\beta_2-4\beta_3}+3e^{-\beta_2-\beta_3}+e^{-\beta_2-2\beta_3}+e^{-2\beta_2-\beta_3}+e^{-2\beta_2}+3e^{-2\beta_3}+e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{-\beta_1+2\beta_2-5\beta_3}+4e^{-\beta_1+\beta_2-4\beta_3}+5e^{-\beta_1-\beta_2-2\beta_3}+4e^{-\beta_1-2\beta_2-\beta_3}+e^{-\beta_1-3\beta_2}+5e^{-\beta_1-3\beta_3}+e^{-2\beta_1+2\beta_2-4\beta_3}+4e^{-2\beta_1+\beta_2-3\beta_3}+8e^{-2\beta_1-\beta_2-\beta_3}+7e^{-2\beta_1-2\beta_2}+4e^{-2\beta_1-3\beta_2+\beta_3}+e^{-2\beta_1-4\beta_2+2\beta_3}+7e^{-2\beta_1-2\beta_3}+e^{-3\beta_1+\beta_2-2\beta_3}+4e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2+\beta_3}+2e^{-3\beta_1-3\beta_2+2\beta_3}+e^{-3\beta_1-4\beta_2+3\beta_3}+2e^{-3\beta_1-\beta_3}+e^{-4\beta_1-\beta_2}+e^{-4\beta_1-2\beta_2+\beta_3}+e^{-4\beta_1-3\beta_2+2\beta_3}+e^{\beta_2-5\beta_3}+e^{-\beta_2-3\beta_3}+e^{-3\beta_3}+2e^{-4\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=2e^{-\beta_1-\beta_2-3\beta_3}+e^{-\beta_1-\beta_2-4\beta_3}+2e^{-\beta_1-4\beta_3}+e^{-\beta_1-5\beta_3}+e^{-2\beta_1+\beta_2-4\beta_3}+e^{-2\beta_1+\beta_2-5\beta_3}+3e^{-2\beta_1-\beta_2-2\beta_3}+2e^{-2\beta_1-\beta_2-3\beta_3}+3e^{-2\beta_1-2\beta_2-\beta_3}+2e^{-2\beta_1-2\beta_2-2\beta_3}+e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-3\beta_2}+3e^{-2\beta_1-3\beta_3}+2e^{-2\beta_1-4\beta_3}+e^{-3\beta_1+\beta_2-4\beta_3}+e^{-3\beta_1-\beta_2-\beta_3}+3e^{-3\beta_1-\beta_2-2\beta_3}+3e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2+\beta_3}+2e^{-3\beta_1-3\beta_2}+e^{-3\beta_1-4\beta_2+\beta_3}+e^{-3\beta_1-2\beta_3}+2e^{-3\beta_1-3\beta_3}+2e^{-4\beta_1-\beta_2-\beta_3}+3e^{-4\beta_1-2\beta_2}+2e^{-4\beta_1-3\beta_2+\beta_3}+e^{-4\beta_1-4\beta_2+2\beta_3}+e^{-4\beta_1-2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-2\beta_1-\beta_2-3\beta_3}+2e^{-2\beta_1-\beta_2-4\beta_3}+e^{-2\beta_1-2\beta_2-3\beta_3}+e^{-2\beta_1-5\beta_3}+3e^{-3\beta_1-\beta_2-3\beta_3}+3e^{-3\beta_1-2\beta_2-2\beta_3}+2e^{-3\beta_1-3\beta_2-\beta_3}+2e^{-3\beta_1-4\beta_3}+2e^{-4\beta_1-\beta_2-2\beta_3}+3e^{-4\beta_1-2\beta_2-\beta_3}+2e^{-4\beta_1-3\beta_2}+e^{-4\beta_1-4\beta_2+\beta_3}+e^{-4\beta_1-3\beta_3}$.

ch($H^7_{\mathcal{G}}$)=$e^{-3\beta_1-\beta_2-4\beta_3}$ + $e^{-3\beta_1-2\beta_2-3\beta_3}$ + $e^{-4\beta_1-\beta_2-3\beta_3}$ + $e^{-4\beta_1-2\beta_2-2\beta_3}$ + $e^{-4\beta_1-3\beta_2-\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 19, 70, 148, 171, 124, 47, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 40, 71, 74, 50, 24, 5)$

S=$4\beta_1 + 2\beta_2 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 5, 11, 25, 25, 19, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 5, 9, 15, 15, 9, 5, 1)$

$d\omega^6 = -\omega^{1,2} - \omega^{3,4}$; $d\omega^7 = -\omega^{1,3} - \omega^{4,5}$; $d\omega^{1,6} = \omega^{1,3,4}$; $d\omega^{1,7} = \omega^{1,4,5}$; $d\omega^{2,6} = \omega^{2,3,4}$; $d\omega^{2,7} = -\omega^{1,2,3} + \omega^{2,4,5}$; $d\omega^{3,6} = \omega^{1,2,3}$; $d\omega^{3,7} = \omega^{3,4,5}$; $d\omega^{4,6} = \omega^{1,2,4}$; $d\omega^{4,7} = \omega^{1,3,4}$; $d\omega^{5,6} = \omega^{1,2,5} + \omega^{3,4,5}$; $d\omega^{5,7} = \omega^{1,3,5}$; $d\omega^{6,7} = -\omega^{1,2,7} + \omega^{1,3,6} - \omega^{3,4,7} + \omega^{4,5,6}$; $d\omega^{1,2,6} = -\omega^{1,2,3,4}$; $d\omega^{1,2,7} = -\omega^{1,2,4,5}$; $d\omega^{1,3,7} = -\omega^{1,3,4,5}$; $d\omega^{1,5,6} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $d\omega^{2,3,7} = -\omega^{2,3,4,5}$; $d\omega^{2,4,7} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{2,3,4,5}$; $d\omega^{2,5,7} = \omega^{1,2,3,5}$; $d\omega^{2,6,7} = \omega^{1,2,3,6} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $d\omega^{3,4,6} = -\omega^{1,2,3,4}$; $d\omega^{3,5,6} = -\omega^{1,2,3,5}$; $d\omega^{3,6,7} = \omega^{1,2,3,7} - \omega^{3,4,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,4,5}$; $d\omega^{4,5,7} = -\omega^{1,3,4,5}$; $d\omega^{4,6,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6}$; $d\omega^{5,6,7} = \omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{3,4,5,7}$; $d\omega^{1,2,3,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{1,5,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{2,3,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{2,4,5,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = -\omega^{1,2,3,4,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{3,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $d\omega^{1,2,3,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,4,5,7}$

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1}$ $\langle\omega^1\rangle + e^{-\beta_2}$ $\langle\omega^2\rangle + e^{-\beta_3}$ $\langle\omega^3\rangle + e^{-\beta_1-\beta_2+\beta_3}$ $\langle\omega^4\rangle + e^{\beta_2-2\beta_3}$ $\langle\omega^5\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-2\beta_1-\beta_2+\beta_3}$ $\langle\omega^{1,4}\rangle$ + $e^{-\beta_1+\beta_2-2\beta_3}$ $\langle\omega^{1,5}\rangle$ + $e^{-\beta_2-\beta_3}$ $\langle\omega^{2,3}\rangle$ + $e^{-\beta_1-2\beta_2+\beta_3}$ $\langle\omega^{2,4}\rangle$ + $e^{-2\beta_3}$ $\langle\omega^{2,5}\rangle$ + $e^{-\beta_1-\beta_2}$ $\langle\omega^{1,2}\rangle$ + $e^{\beta_2-3\beta_3}$ $\langle\omega^{3,5}\rangle$ + $e^{-\beta_1-\beta_3}$ $\langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2}$ $\langle\omega^{1,6} - \omega^{4,7}\rangle$.

ch($H^3_{\mathbb{C}}$)= $2e^{-2\beta_1-\beta_2-\beta_3}$ $\langle\omega^{1,3,6}, \omega^{1,2,7} - \omega^{4,5,6}\rangle$ + $e^{-3\beta_1-2\beta_2+\beta_3}$ $\langle\omega^{1,4,6}\rangle$ + $e^{-3\beta_1-\beta_2}$ $\langle\omega^{1,4,7}\rangle + e^{-2\beta_1+\beta_2-3\beta_3}$ $\langle\omega^{1,5,7}\rangle + e^{-3\beta_3}$ $\langle\omega^{2,3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{2,3,6}\rangle$ + $e^{-2\beta_1-3\beta_2+\beta_3}$ $\langle\omega^{2,4,6}\rangle$ + $e^{-\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{2,3,7} - \omega^{2,5,6}\rangle$ + $2e^{-2\beta_1-2\beta_2}$ $\langle\omega^{1,2,6} - \omega^{3,4,6}, \omega^{2,4,7} + \omega^{3,4,6}\rangle$ + $e^{-\beta_1-3\beta_3}$ $\langle\omega^{2,5,7} + \omega^{3,5,6}\rangle$ + $e^{-\beta_1+\beta_2-4\beta_3}$ $\langle\omega^{3,5,7}\rangle$ + $2e^{-2\beta_1-2\beta_3}$ $\langle\omega^{1,3,7} - \omega^{4,5,7}, \omega^{1,5,6} - \omega^{4,5,7}\rangle$.

ch($H^4_{\mathbb{C}}$)= $2e^{-2\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,2,3,6}, \omega^{2,3,4,7}\rangle$ + $e^{-3\beta_1-3\beta_2+\beta_3}$ $\langle\omega^{1,2,4,6}\rangle$ + $2e^{-2\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,2,5,6} - \omega^{3,4,5,6}, \omega^{2,4,5,7} + \omega^{3,4,5,6}\rangle + 2e^{-2\beta_1-3\beta_3}$ $\langle\omega^{1,2,5,7}, \omega^{1,3,5,6}\rangle$+

$e^{-3\beta_1-2\beta_2}$ $\langle\omega^{1,2,4,7}\rangle$ + $e^{-2\beta_1+\beta_2-4\beta_3}$ $\langle\omega^{1,3,5,7}\rangle$ + $e^{-3\beta_1-\beta_2-\beta_3}$ $\langle\omega^{1,3,4,7}\rangle$ + $e^{-3\beta_1-2\beta_3}$ $\langle\omega^{1,4,5,7}\rangle$ + $e^{-4\beta_1-2\beta_2}$ $\langle\omega^{1,4,6,7}\rangle$ + $e^{-2\beta_1-3\beta_2}$ $\langle\omega^{2,3,4,6}\rangle$ + $e^{-\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-4\beta_3}$ $\langle\omega^{2,3,5,7}\rangle$.

ch$(H^5_{\mathbb{C}})$= $e^{-4\beta_1-3\beta_2}$ $\langle\omega^{1,2,4,6,7}\rangle$ + $e^{-4\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,3,4,6,7}\rangle$ + $e^{-3\beta_1-4\beta_3}$ $\langle\omega^{1,3,5,6,7}\rangle$ + $e^{-4\beta_1-\beta_2-2\beta_3}$ $\langle\omega^{1,4,5,6,7}\rangle$ + $e^{-2\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{1,2,3,5,6}\rangle$ + $e^{-3\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,3,4,6,7}\rangle$ + $e^{-2\beta_1-\beta_2-4\beta_3}$ $\langle\omega^{2,3,5,6,7}\rangle$ + $e^{-3\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,2,3,6,7}$ + $\omega^{2,4,5,6,7}\rangle$ + $e^{-3\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{1,2,5,6,7} - \omega^{3,4,5,6,7}\rangle$.

ch$(H^6_{\mathbb{C}})$= $e^{-4\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,3,4,6,7}\rangle$ + $e^{-3\beta_1-\beta_2-4\beta_3}$ $\langle\omega^{1,2,3,5,6,7}\rangle$ + $e^{-4\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle$+$e^{-4\beta_1-\beta_2-3\beta_3}$ $\langle\omega^{1,3,4,5,6,7}\rangle$+$e^{-3\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{2,3,4,5,6,7}\rangle$.

ch$(H^7_{\mathbb{C}})$= $e^{-4\beta_1-2\beta_2-3\beta_3}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y+5Y^2+10Y^3+5Y^4+4Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,5,11,25,25,19,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,5,9,15,15,9,5,1)$

Total dimension of trivial cohomology: 60.

Length associated polynomial: 60.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^6$; $d^\star\omega^{1,3} = -\omega^7$; $d^\star\omega^{3,4} = -\omega^6$; $d^\star\omega^{4,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,7}+\omega^{3,6}$; $d^\star\omega^{1,2,4} = \omega^{4,6}$; $d^\star\omega^{1,2,5} = \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{6,7}$; $d^\star\omega^{1,3,4} = \omega^{1,6}+\omega^{4,7}$; $d^\star\omega^{1,3,5} = \omega^{5,7}$; $d^\star\omega^{1,3,6} = \omega^{6,7}$; $d^\star\omega^{1,4,5} = \omega^{1,7}$; $d^\star\omega^{2,3,4} = \omega^{2,6}$; $d^\star\omega^{2,4,5} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = \omega^{3,7}+\omega^{5,6}$; $d^\star\omega^{3,4,7} = -\omega^{6,7}$; $d^\star\omega^{4,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,6}+\omega^{2,4,7}-\omega^{3,4,6}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,7}-\omega^{3,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{2,6,7}$; $d^\star\omega^{1,2,3,7} = \omega^{3,6,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,2,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,2,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,3,7}-\omega^{1,5,6}-\omega^{4,5,7}$; $d^\star\omega^{1,3,4,6} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{1,6,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,3,7}-\omega^{2,5,6}$; $d^\star\omega^{2,3,4,7} = \omega^{2,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{3,6,7}$; $d^\star\omega^{3,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,3,7}+\omega^{1,2,5,6}-\omega^{2,4,5,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,2,6,7}-\omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,2,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,3,6,7}+\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,3,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,3,6,7}+\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,2,5,6,7}+\omega^{3,4,5,6,7}$;

$\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = 2\omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}+\omega^{3,4}$; $\Delta\omega^{1,3} = \omega^{1,3}+\omega^{4,5}$; $\Delta\omega^{1,6} = \omega^{1,6}+\omega^{4,7}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}-\omega^{3,6}$; $\Delta\omega^{3,4} = \omega^{1,2}+\omega^{3,4}$; $\Delta\omega^{3,6} = -\omega^{2,7}+\omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}+\omega^{5,6}$;

$\Delta\omega^{4,5} = \omega^{1,3} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = \omega^{1,6} + \omega^{4,7}$; $\Delta\omega^{5,6} = \omega^{3,7} + 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 4\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3} - \omega^{2,4,5}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{3,4,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{1,3,6} + \omega^{3,4,7}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = -\omega^{1,2,7} + \omega^{1,3,6} - \omega^{3,4,7} + \omega^{4,5,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} + \omega^{1,5,6} + \omega^{4,5,7}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,3,7} + \omega^{1,5,6} + \omega^{4,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,3} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,2,6} + \omega^{2,4,7} - \omega^{3,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,3,7} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,5,6}$; $\Delta\omega^{2,6,7} = 3\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,2,5} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,2,6} - \omega^{2,4,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,2,7} - \omega^{1,3,6} + \omega^{3,4,7} - \omega^{4,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{2,5,7} + \omega^{3,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{1,3,6} - \omega^{3,4,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{1,3,7} + \omega^{1,5,6} + \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,3,7} + \omega^{1,2,5,6} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{3,4,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{1,3,4,5} = 3\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,4,7} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,2,5,7} + \omega^{1,3,5,6} - \omega^{3,4,5,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{4,5,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,4,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = \omega^{1,2,3,6} + \omega^{2,3,4,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,3,6} - \omega^{2,3,4,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,2,3,7} - \omega^{1,2,5,6} + \omega^{2,4,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,2,5,6} - \omega^{2,4,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,2,5,7} - \omega^{1,3,5,6} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,2,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{1,3,6,7} + 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 4\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,5,6} = \omega^{1,2,3,5,6} + \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,4,5,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,4,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{3,4,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,4,5,7} + 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{1,2,3,5,6} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,4,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{1,2,5,6,7} + \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 2\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$;

$\mathrm{ch}(H^1_{\mathfrak{T}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle + e^{\beta_2-2\beta_3}\ \langle\omega^5\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{T}})= e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle + e^{-\beta_1+\beta_2-2\beta_3}\ \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2+\beta_3}\ \langle\omega^{2,4}\rangle + e^{-2\beta_3}\ \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2}\ \langle\omega^{1,2} - \omega^{3,4}\rangle + e^{\beta_2-3\beta_3}\ \langle\omega^{3,5}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3} - \omega^{4,5}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,6} - \omega^{4,7}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{T}})= e^{-3\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,6}\rangle + e^{-3\beta_1-\beta_2}\ \langle\omega^{1,4,7}\rangle + e^{-2\beta_1+\beta_2-3\beta_3}\ \langle\omega^{1,5,7}\rangle + e^{-3\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,4,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,7} -$

$\omega^{2,5,6}\rangle + 2e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,6} - \omega^{3,4,6}, \omega^{2,4,7} + \omega^{3,4,6}\rangle + 2e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6} + \omega^{3,4,7}, \omega^{1,2,7} - 2\omega^{3,4,7} - \omega^{4,5,6}\rangle + e^{-\beta_1-3\beta_3}\ \langle\omega^{2,5,7} + \omega^{3,5,6}\rangle + e^{-\beta_1+\beta_2-4\beta_3}\ \langle\omega^{3,5,7}\rangle + 2e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,7} - \omega^{4,5,7}, \omega^{1,5,6} - \omega^{4,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-\beta_1-4\beta_3}\ \langle-\omega^{2,3,5,7}\rangle + e^{-\beta_1-\beta_2-3\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-3\beta_2}\ \langle-\omega^{2,3,4,6}\rangle + e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,4,6,7}\rangle + e^{-3\beta_1-2\beta_3}\ \langle-\omega^{1,4,5,7}\rangle + e^{-2\beta_1+\beta_2-4\beta_3}\ \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle(\omega^{1,3,4,7} + \omega^{1,4,5,6})/2\rangle + 2e^{-2\beta_1-3\beta_3}\ \langle(-(2\omega^{3,4,5,7} - \omega^{1,2,5,7} + \omega^{1,3,5,6}))/3, (-(\omega^{3,4,5,7} + \omega^{1,2,5,7} + 2\omega^{1,3,5,6}))/3\rangle + 2e^{-2\beta_1-\beta_2-2\beta_3}\ \langle(3\omega^{2,4,5,7} + \omega^{3,4,5,6} + \omega^{1,2,3,7} + \omega^{1,2,5,6})/4, (\omega^{2,4,5,7} + \omega^{3,4,5,6} + \omega^{1,2,3,7} - \omega^{1,2,5,6})/4\rangle + e^{-3\beta_1-2\beta_2}\ \langle(\omega^{1,2,4,7} + \omega^{1,3,4,6})/2\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle-\omega^{1,2,4,6}\rangle + 2e^{-2\beta_1-2\beta_2-\beta_3}\ \langle(-(\omega^{2,3,4,7} + 2\omega^{2,4,5,6} + \omega^{1,2,3,6}))/3, (2\omega^{2,3,4,7} + \omega^{2,4,5,6} - \omega^{1,2,3,6})/3\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-2\beta_1-\beta_2-4\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_3}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3}\ \langle(-\omega^{1,2,5,6,7} + \omega^{3,4,5,6,7})/2\rangle + e^{-4\beta_1-3\beta_2}\ \langle-\omega^{1,2,4,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle(-\omega^{1,2,3,6,7} - \omega^{2,4,5,6,7})/2\rangle + e^{-2\beta_1-\beta_2-3\beta_3}\ \langle(-\omega^{1,2,3,5,6} + \omega^{2,3,4,5,7})/2\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-3\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-3\beta_3}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-4\beta_3}\ \langle-\omega^{1,2,3,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-4\beta_1-2\beta_2-3\beta_3}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1 + 4Y + 5Y^2 + 10Y^3 + 5Y^4 + 4Y^5 + Y^6)(1 + Y)$.

5.5.21 Algebra $\mathcal{G}_{7,3.20}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_1, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & \xi_3^2 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_2^3 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & \xi_3^4 & 2\xi_1^1+\xi_2^2 & \xi_3^2 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_3^5 & \xi_2^3 & 2\xi_1^1+\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_1^6, ad\ x_5 = -\delta_1^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{\beta_2}\ \langle\delta_1^4\rangle + e^{\beta_3}\ \langle\delta_1^5\rangle + e^{\beta_1+\beta_2}\ \langle\delta_1^6\rangle + 2e^{2\beta_1}\ \langle\delta_2^6, \delta_3^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle + 2e^{\beta_1}\ \langle\delta_2^4, \delta_3^5\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^4\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{\beta_1+\beta_3}\ \langle\delta_1^7\rangle + e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^4] = -\delta_1^4$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_1^4, \delta_2^5] = -\delta_1^7$; $[\delta_1^4, \delta_2^3] = -\delta_1^5$; $[\delta_1^5, \delta_3^4] = -\delta_1^6$; $[\delta_1^5, \delta_3^2] = -\delta_1^4$; $[\delta_1^5, \delta_3^5] = -\delta_1^7$; $[\delta_1^6, \delta_2^3] = -\delta_1^7$; $[\delta_2^6, \delta_3^2] = \delta_3^6$; $[\delta_2^6, \delta_2^3] = -\delta_2^7$; $[\delta_3^6, \delta_2^3] = -\delta_3^7+\delta_2^6$; $[\delta_2^4, \delta_3^4] = \delta_3^6$; $[\delta_2^4, \delta_3^2] = \delta_3^4$; $[\delta_2^4, \delta_2^5] = -\delta_2^7$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$; $[\delta_3^4, \delta_2^5] = -\delta_3^7 + \delta_2^6$; $[\delta_3^4, \delta_3^5] = \delta_3^6$; $[\delta_3^4, \delta_2^3] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_1^7] = \delta_1^6$; $[\delta_3^2, \delta_2^7] = -\delta_3^7 + \delta_2^6$; $[\delta_3^2, \delta_3^7] = \delta_3^6$; $[\delta_3^2, \delta_2^5] = -\delta_3^5 + \delta_2^4$; $[\delta_3^2, \delta_3^5] = \delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_2^5, \delta_3^5] = -\delta_2^7$; $[\delta_3^5, \delta_2^3] = \delta_2^5$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + 2e^{2\beta_1}\ \langle\delta_2^6, \delta_3^7\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^4\rangle + 3e^0\ \langle t_2, t_1, t_3\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle + e^{\beta_1}\ \langle\delta_2^4\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_2^6, \delta_3^2] = \delta_3^6$; $[\delta_2^6, \delta_2^3] = -\delta_2^7$; $[\delta_3^6, \delta_2^3] = -\delta_3^7 + \delta_2^6$; $[\delta_3^4, \delta_2^5] = -\delta_3^7 + \delta_2^6$; $[\delta_3^4, \delta_2^4] = -\delta_3^6$; $[\delta_3^4, \delta_2^3] = 2\delta_2^4$; $[\delta_3^2, \delta_2^7] = -\delta_3^7 + \delta_2^6$; $[\delta_3^2, \delta_3^7] = \delta_3^6$; $[\delta_3^2, \delta_2^5] = 2\delta_2^4$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^7, \delta_2^3] = \delta_2^7$; $[\delta_2^5, \delta_2^4] = \delta_2^7$; $[\delta_2^4, \delta_2^3] = -\delta_2^5$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 2e^{-3\beta_1} \langle x_2\omega^{1,6}, x_3\omega^{1,7}\rangle + e^{-3\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,7}\rangle + e^{-3\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,6}\rangle + 2e^{-2\beta_1} \langle x_2\omega^{1,4}, x_3\omega^{1,5}\rangle + e^{-2\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,5}\rangle + e^{-2\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,4}\rangle + 4e^{-\beta_2} \langle x_1\omega^{1,2} + 2x_3\omega^{2,3} + x_4\omega^{2,4} + x_5\omega^{2,5}, x_4\omega^{2,4} + x_6\omega^{2,6}, -x_1\omega^{1,2} - x_3\omega^{2,3} - x_4\omega^{2,4} + x_7\omega^{2,7}, -x_3\omega^{2,3} + x_5\omega^{3,4} + x_7\omega^{3,6}\rangle + 2e^{-\beta_1} \langle x_2\omega^{1,2}, x_3\omega^{1,3}\rangle + e^{-\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,3}\rangle + e^{2\beta_1-\beta_3} \langle x_6\omega^{2,3}\rangle + 2e^{\beta_1-\beta_3} \langle x_4\omega^{2,3} + x_6\omega^{2,5}, -x_4\omega^{2,3} + x_6\omega^{3,4}\rangle + 4e^{-\beta_3} \langle x_2\omega^{2,3} + x_4\omega^{2,5} + x_6\omega^{2,7}, -x_2\omega^{2,3} + x_4\omega^{3,4} + x_6\omega^{3,6}, x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_4\omega^{3,4} + x_5\omega^{3,5}, -x_1\omega^{1,3} + 2x_2\omega^{2,3} - x_4\omega^{3,4} + x_7\omega^{3,7}\rangle + e^{\beta_1+\beta_2-2\beta_3} \langle x_6\omega^{3,5}\rangle + e^{\beta_2-2\beta_3} \langle x_4\omega^{3,5} + x_6\omega^{3,7}\rangle + 2e^{-\beta_1-\beta_3} \langle -x_1\omega^{1,5} + x_2(\omega^{2,5} + \omega^{3,4}) + x_3\omega^{3,5} + x_4\omega^{3,6} + x_6\omega^{5,6}, x_1\omega^{1,5} - x_3\omega^{3,5} + x_4\omega^{2,7} + x_6\omega^{4,7}\rangle + e^{-\beta_1+\beta_2-2\beta_3} \langle x_2\omega^{3,5} + x_4\omega^{3,7} + x_6\omega^{5,7}\rangle + e^{-\beta_1-\beta_2+\beta_3} \langle x_3\omega^{1,2}\rangle + e^{2\beta_1-\beta_2} \langle x_7\omega^{2,3}\rangle + e^{\beta_1-2\beta_2+\beta_3} \langle x_7\omega^{2,4}\rangle + 2e^{\beta_1-\beta_2} \langle x_6\omega^{2,4}, -x_5\omega^{2,3} + x_7\omega^{3,4}\rangle + e^{-2\beta_2+\beta_3} \langle x_5\omega^{2,4} + x_7\omega^{2,6}\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle x_3\omega^{2,4} + x_5\omega^{2,6} + x_7\omega^{4,6}\rangle + 2e^{-\beta_1-\beta_2} \langle -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{2,7} + x_7\omega^{4,7}, x_2\omega^{2,4} + x_4\omega^{2,6} + x_6\omega^{4,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-\beta_3} + e^{\beta_1-2\beta_2} + e^{\beta_1-2\beta_3} + e^{-\beta_1+\beta_2-3\beta_3} + 5e^{-\beta_1-\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2+\beta_3} + 4e^{-\beta_1-2\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + 3e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-\beta_3} + e^{-2\beta_1-2\beta_3} + e^{-3\beta_1+\beta_2-2\beta_3} + 4e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+\beta_3} + 4e^{-3\beta_1-\beta_3} + e^{-4\beta_1+\beta_2-2\beta_3} + 3e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_3} + 3e^{-4\beta_1-\beta_3} + e^{-5\beta_1-\beta_2} + e^{-5\beta_1-\beta_3} + 3e^{-\beta_2-\beta_3} + e^{-2\beta_2} + e^{-2\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-\beta_1-\beta_2-2\beta_3} + 2e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-3\beta_2} + e^{-\beta_1-3\beta_3} + 3e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-\beta_3} + 3e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + 2e^{-3\beta_1-3\beta_2} + 2e^{-3\beta_1-3\beta_3} + e^{-4\beta_1+\beta_2-3\beta_3} + 5e^{-4\beta_1-\beta_2-\beta_3} + 4e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2+\beta_3} + 4e^{-4\beta_1-2\beta_3} + 4e^{-5\beta_1-\beta_2-\beta_3} + 2e^{-5\beta_1-2\beta_2} + 2e^{-5\beta_1-2\beta_3} + 2e^{-6\beta_1-\beta_2-\beta_3} + e^{-6\beta_1-2\beta_2} + e^{-6\beta_1-2\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-3\beta_3} + 3e^{-5\beta_1-\beta_2-2\beta_3} + 3e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-3\beta_3} + 3e^{-6\beta_1-\beta_2-2\beta_3} + 3e^{-6\beta_1-2\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2} + e^{-6\beta_1-3\beta_3} + e^{-7\beta_1-\beta_2-2\beta_3} + e^{-7\beta_1-2\beta_2-\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-3\beta_3} + e^{-6\beta_1-\beta_2-3\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3} + e^{-6\beta_1-2\beta_2-3\beta_3} + e^{-6\beta_1-3\beta_2-\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3} + e^{-7\beta_1-\beta_2-3\beta_3} + 2e^{-7\beta_1-2\beta_2-2\beta_3} + e^{-7\beta_1-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-6\beta_1-3\beta_2-3\beta_3} + e^{-7\beta_1-2\beta_2-3\beta_3} + e^{-7\beta_1-3\beta_2-2\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 19, 66, 133, 163, 115, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 14, 36, 52, 51, 33, 13, 3)$

S=$7\beta_1 + 3\beta_2 + 3\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1,3,11,23,25,17,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1,3,7,13,13,7,3,1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{1,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} + \omega^{1,2,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7} + \omega^{1,3,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7} + \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,7}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle\omega^{1,2,5,7}\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,6,7}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-6\beta_1-\beta_2-2\beta_3} \langle\omega^{1,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,4,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,7} + \omega^{2,3,5,6}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}}) = e^{-6\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,6,7}\rangle + e^{-6\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-6\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-7\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,4,5,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}}) = e^{-7\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-7\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}}) = e^{-7\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y^2+Y^4)(1+Y)^3.$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z_{\mathbb{C}}^j$'s $0 \le j \le 7$: $(1,3,11,23,25,17,7,1)$

Dimensions of $H_{\mathbb{C}}^j$'s $0 \le j \le 7$: $(1,3,7,13,13,7,3,1)$

Total dimension of trivial cohomology: 48.

Length associated polynomial: 48.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5}+\omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{2,7}-\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6}+\omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}+\omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{2,3,7}+\omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7}+\omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{2,4,7}+\omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{3,4,7}+\omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7}+\omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{2,3,4,7}+\omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7}+\omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7}+\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}+\omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{2,5} = \omega^{2,5}-\omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}+\omega^{4,5}$; $\Delta\omega^{3,4} = -\omega^{2,5}+\omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}-\omega^{4,5}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7}-\omega^{3,6}+2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}-\omega^{5,6}$; $\Delta\omega^{5,6} = -\omega^{4,7}+2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = 2\omega^{1,2,5}-\omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}+\omega^{1,4,5}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5}+2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6}-\omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7}-\omega^{1,3,6}+2\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = \omega^{1,4,7}-\omega^{1,5,6}$; $\Delta\omega^{1,5,6} = -\omega^{1,4,7}+\omega^{1,5,6}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6}-\omega^{2,4,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}-\omega^{3,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6}+\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}-\omega^{2,5,6}$; $\Delta\omega^{2,5,6} = -\omega^{2,4,7}+2\omega^{2,5,6}-\omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}-\omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}-\omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,3,7}+\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6}+\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7}+2\omega^{3,4,7}-\omega^{3,5,6}$; $\Delta\omega^{3,5,6} = -\omega^{3,4,7}+\omega^{3,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}-\omega^{4,5,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7}+2\omega^{4,5,7}$;

$\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,6} = -\omega^{1,2,4,7} + 2\omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7} - \omega^{1,3,5,6}$; $\Delta\omega^{1,3,5,6} = -\omega^{1,3,4,7} + \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} - \omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $\Delta\omega^{2,3,4,7} = \omega^{2,3,4,7} - \omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,6} = -\omega^{2,3,4,7} + \omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = 2\omega^{2,3,6,7} - \omega^{2,4,5,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + 2\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7} - \omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,6} = -\omega^{1,2,3,4,7} + 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-3\beta_1-\beta_3} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{2,7} - \omega^{3,6} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7} - \omega^{1,3,6} - \omega^{1,4,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,7} + \omega^{1,5,6}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,7}\rangle + e^{-5\beta_1-\beta_2-\beta_3} \langle\omega^{1,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,5}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{2,4,7} + \omega^{2,5,6} + \omega^{3,4,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7} + \omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2-2\beta_3} \langle(-(\omega^{2,3,6,7} + \omega^{2,4,5,7} - \omega^{3,4,5,6}))/3\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle(\omega^{2,3,4,7} + \omega^{2,3,5,6})/2\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,4,5}\rangle + e^{-6\beta_1-\beta_2-2\beta_3} \langle-\omega^{1,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6,7}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{1,4,5,7}))/2\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,2,6,7} + \omega^{1,4,5,6})/2\rangle + e^{-4\beta_1-\beta_2-2\beta_3} \langle(-(\omega^{1,2,5,7} + \omega^{1,3,4,7} + \omega^{1,3,5,6}))/3\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,2,4,7} + \omega^{1,2,5,6} + \omega^{1,3,4,6})/3\rangle + e^{-4\beta_1-3\beta_2} \langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_3} \langle-\omega^{2,3,4,5,6}\rangle + e^{-7\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-6\beta_1-\beta_2-3\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-6\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,4,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_3} \langle(-\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7} +

$\omega^{1,3,4,5,6})/3\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-6\beta_1-3\beta_2-3\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-7\beta_1-2\beta_2-3\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-7\beta_1-3\beta_2-2\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-7\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y^2+Y^4)(1+Y)^3$.

5.5.22 Algebra $\mathcal{G}_{7,3.21}$ †

Adjoint cohomology

$[x_1, x_2] = x_4$; $[x_1, x_3] = x_5$; $[x_1, x_4] = x_6$; $[x_3, x_5] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 \\ \xi_1^5 & 0 & \xi_3^5 & 0 & \xi_1^1+\xi_3^3 & 0 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_2^4 & \xi_3^4 & 2\xi_1^1+\xi_2^2 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & 0 & -\xi_1^5 & 0 & \xi_1^1+2\xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^4 + \delta_3^5, ad\ x_2 = -\delta_1^4, ad\ x_3 = -\delta_1^5, ad\ x_4 = -\delta_1^6, ad\ x_5 = -\delta_3^7, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_1+\beta_2}\ \langle x_4\rangle + e^{\beta_1+\beta_3}\ \langle x_5\rangle + e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{2\beta_1+\beta_2}\ \langle x_6\rangle + e^{\beta_1+2\beta_3}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{\beta_2}\ \langle \delta_1^4\rangle + 2e^{\beta_1}\ \langle \delta_3^5, \delta_2^4\rangle + e^{\beta_1+\beta_2}\ \langle \delta_1^6\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^6\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^4\rangle + 3e^{0}\ \langle t_2, t_1, t_3\rangle + e^{2\beta_3}\ \langle \delta_1^7\rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle \delta_2^7\rangle + e^{\beta_1+\beta_3}\ \langle \delta_3^7\rangle + e^{\beta_3}\ \langle \delta_1^5\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^4, \delta_2^4] = -\delta_1^6$; $[\delta_3^5, \delta_3^4] = -\delta_3^6$; $[\delta_3^5, \delta_1^5] = \delta_3^7$; $[\delta_2^4, \delta_3^4] = \delta_3^6$; $[\delta_3^4, \delta_1^5] = \delta_1^6$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{2\beta_1}\ \langle \delta_2^6\rangle + e^{2\beta_1+\beta_2-\beta_3}\ \langle \delta_3^6\rangle + e^{\beta_1}\ \langle \delta_2^4\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^4\rangle + 3e^{0}\ \langle t_2, t_1, t_3\rangle + e^{2\beta_3}\ \langle \delta_1^7\rangle + e^{\beta_1-\beta_2+2\beta_3}\ \langle \delta_2^7\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_2^4, \delta_3^4] = \delta_3^6$.

$\mathrm{ch}(H^2_{\mathcal{G}}) = e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,5}\rangle + e^{-3\beta_1}\ \langle x_2\omega^{1,6}\rangle + 2e^{-2\beta_1}\ \langle x_3\omega^{1,5}, x_2\omega^{1,4}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + 3e^{-\beta_3}\ \langle x_1\omega^{1,3} - 2x_2\omega^{2,3} + x_4\omega^{3,4}, -x_1\omega^{1,3} + x_2\omega^{2,3} + x_6\omega^{3,6}, x_5\omega^{3,5}\rangle + 3e^{-\beta_2}\ \langle -2x_1\omega^{1,2} - x_3\omega^{2,3} - 2x_4\omega^{2,4} + x_5\omega^{2,5}, x_4\omega^{2,4} + x_6\omega^{2,6}, x_1\omega^{1,2} + x_3\omega^{2,3} + x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + 2e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,4} + 2x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{4,5}, x_1\omega^{1,4} - x_2\omega^{2,4} + x_5\omega^{3,6}\rangle + 2e^{-\beta_1}\ \langle x_2\omega^{1,2}, x_3\omega^{1,3} + x_7\omega^{1,7}\rangle + e^{2\beta_1-\beta_3}\ \langle x_6\omega^{2,3}\rangle + e^{\beta_1-\beta_2}\ \langle x_5\omega^{2,3}\rangle + e^{\beta_1-\beta_3}\ \langle x_4\omega^{2,3} + x_6\omega^{2,5}\rangle + e^{\beta_1+\beta_2-3\beta_3}\ \langle x_6\omega^{3,7}\rangle + e^{-2\beta_3}\ \langle x_5\omega^{3,7}\rangle + e^{-\beta_1-\beta_3}\ \langle -x_1\omega^{1,5} + x_2(\omega^{2,5} + \omega^{3,4}) +$

$x_3\omega^{3,5} + x_4\omega^{3,6} + x_6\omega^{5,6}\rangle + e^{\beta_2-3\beta_3}\ \langle x_4\omega^{3,7} + x_6\omega^{5,7}\rangle + e^{-2\beta_1-\beta_2+2\beta_3}\ \langle x_7\omega^{1,6}\rangle + e^{-2\beta_2+2\beta_3}\ \langle x_7\omega^{2,4}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-3\beta_3} + 3e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + 3e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-3\beta_3} + e^{-2\beta_1+\beta_2-3\beta_3} + e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+2\beta_3} + 3e^{-2\beta_1-\beta_3} + e^{-2\beta_1-3\beta_3} + e^{-3\beta_1+\beta_2-3\beta_3} + 3e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+2\beta_3} + 2e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-\beta_3} + e^{\beta_2-4\beta_3} + 2e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3} + e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$2e^{-\beta_1-\beta_2-3\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-3\beta_3} + 3e^{-2\beta_1-\beta_2-3\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-\beta_3} + 2e^{-2\beta_1-4\beta_3} + e^{-3\beta_1+\beta_2-4\beta_3} + 2e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-3\beta_2+2\beta_3} + 2e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-3\beta_3} + 3e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-2\beta_3} + 3e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-3\beta_3} + e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_3} + e^{-4\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-\beta_2-4\beta_3} + e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-4\beta_3} + e^{-3\beta_1-\beta_2-4\beta_3} + 2e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-3\beta_1-3\beta_2-3\beta_3} + e^{-3\beta_1-4\beta_3} + 2e^{-4\beta_1-\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_3} + 3e^{-5\beta_1-\beta_2-3\beta_3} + 2e^{-5\beta_1-2\beta_2-\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_3} + e^{-6\beta_1-\beta_2-3\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-3\beta_1-2\beta_2-4\beta_3} + e^{-4\beta_1-3\beta_2-3\beta_3} + e^{-4\beta_1-3\beta_2-4\beta_3} + e^{-5\beta_1-\beta_2-4\beta_3} + e^{-5\beta_1-2\beta_2-3\beta_3} + e^{-5\beta_1-2\beta_2-4\beta_3} + e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-3\beta_2-3\beta_3} + e^{-6\beta_1-\beta_2-4\beta_3} + 2e^{-6\beta_1-2\beta_2-3\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-5\beta_1-3\beta_2-4\beta_3} + e^{-6\beta_1-2\beta_2-4\beta_3} + e^{-6\beta_1-3\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 58, 127, 159, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 24, 38, 41, 28, 12, 3)$

S=$6\beta_1 + 3\beta_2 + 4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{1,4}$; $d\omega^7 = -\omega^{3,5}$; $d\omega^{1,7} = \omega^{1,3,5}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = -\omega^{1,2,4}$; $d\omega^{2,7} = \omega^{2,3,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{1,3,4}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{3,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{1,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = -\omega^{1,4,7} + \omega^{3,5,6}$; $d\omega^{1,2,7} = -\omega^{1,2,3,5}$; $d\omega^{1,4,7} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = -\omega^{1,3,5,6}$; $d\omega^{2,3,6} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = \omega^{2,3,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = -\omega^{1,2,4,7} - \omega^{2,3,5,6}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6}$; $d\omega^{3,4,7} =$

$\omega^{1,2,3,7}$; $d\omega^{3,5,6} = \omega^{1,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,3,4,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{1,4,5,7}$; $d\omega^{1,2,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{1,4,6,7} = -\omega^{1,3,4,5,6}$; $d\omega^{2,3,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = -\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7}$; $d\omega^{3,5,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7}+\omega^{1,3,4,6,7}$; $d\omega^{1,2,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-3\beta_3} \langle\omega^{3,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7}\rangle + e^{-2\beta_1-4\beta_3} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-4\beta_1-3\beta_2} \langle\omega^{1,2,4,6}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-4\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-\beta_2-3\beta_3} \langle\omega^{1,3,6,7}\rangle + e^{-5\beta_1-\beta_2-3\beta_3} \langle\omega^{1,5,6,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle\omega^{2,3,4,6}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-\beta_2-4\beta_3} \langle\omega^{2,3,5,7}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,6,7} + \omega^{2,4,5,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-5\beta_1-3\beta_2-\beta_3} \langle\omega^{1,2,4,5,6}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-\beta_2-4\beta_3} \langle\omega^{1,3,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-4\beta_3} \langle\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-3\beta_3} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-6\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-4\beta_3} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-4\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-3\beta_2-4\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \leq j \leq 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,4} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,4} = -\omega^{2,6}$; $d^\star\omega^{1,2,5} = -\omega^{4,5}$; $d^\star\omega^{1,2,6} =$

$-\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,6} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = \omega^{1,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = \omega^{5,6}$; $d^\star\omega^{1,4,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,5} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,6}+\omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,4,7} = -\omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{1,4,7} + \omega^{3,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = -\omega^{3,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = \omega^{2,4,7}$; $d^\star\omega^{2,3,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = -\omega^{1,2,4,7} + \omega^{2,3,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{1,2,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{1,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,5} = \omega^{2,5}-\omega^{3,4}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = -\omega^{2,5} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,6} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7}$; $\Delta\omega^{1,3,4} = -\omega^{1,2,5} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{1,4,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,6}+\omega^{1,4,5}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = \omega^{2,3,6} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{2,3,6} + \omega^{2,4,5}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6} - \omega^{3,4,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} - \omega^{3,4,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = 2\omega^{3,5,6}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7} - \omega^{4,5,7}$; $\Delta\omega^{4,5,6} = 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = -\omega^{3,6,7} + 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,6} + \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,7} = 2\omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,5,7} + 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}-\omega^{1,4,5,7}$; $\Delta\omega^{1,4,5,7} = -\omega^{1,3,6,7}+\omega^{1,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,6} = 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{2,4,5,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{2,3,6,7} + \omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $\Delta\omega^{3,4,5,6} = 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$;

$\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7} - \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,5,7} = -\omega^{1,2,3,6,7} + \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_3} \langle\omega^{1,5}\rangle + e^{-3\beta_1-\beta_2} \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,5} + \omega^{3,4}\rangle + e^{-\beta_1-3\beta_3} \langle\omega^{3,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6} + \omega^{1,4,5}\rangle + e^{-4\beta_1-2\beta_2} \langle\omega^{1,4,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_2-3\beta_3} \langle\omega^{2,3,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6} + \omega^{2,4,5}\rangle + e^{-3\beta_1-3\beta_2} \langle\omega^{2,4,6}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{2,5,7} + \omega^{3,4,7}\rangle + e^{-2\beta_1-4\beta_3} \langle\omega^{3,5,7}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-3\beta_3} \langle(\omega^{2,3,6,7} + \omega^{2,4,5,7})/2\rangle + e^{-2\beta_1-\beta_2-4\beta_3} \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,4,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-5\beta_1-\beta_2-3\beta_3} \langle-\omega^{1,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,5,6}\rangle + e^{-4\beta_1-\beta_2-3\beta_3} \langle(-(\omega^{1,3,6,7} + \omega^{1,4,5,7}))/2\rangle + e^{-3\beta_1-4\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle(\omega^{1,2,5,6} + \omega^{1,3,4,6})/2\rangle + e^{-4\beta_1-3\beta_2} \langle-\omega^{1,2,4,6}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-4\beta_1-3\beta_2-3\beta_3} \langle-\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-2\beta_2-4\beta_3} \langle\omega^{2,3,4,5,7}\rangle + e^{-6\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-\beta_2-4\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-3\beta_3} \langle(\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7})/2\rangle + e^{-5\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,4,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-3\beta_2-4\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-4\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-3\beta_2-4\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3.$

5.5.23 Algebra $\mathcal{G}_{7,3.22}$ †

Adjoint cohomology

$[x_1,x_2]=x_4$; $[x_1,x_3]=x_5$; $[x_1,x_5]=x_7$; $[x_2,x_5]=x_6$; $[x_3,x_4]=x_6$.

$$\delta=\begin{pmatrix}\xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_2^2 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^4 & 0 & \xi_1^1+\xi_3^3 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & -\xi_1^5 & -\xi_1^4 & \xi_1^1+\xi_2^2+\xi_3^3 & \xi_1^2\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_2^5 & \xi_2^4 & 0 & 2\xi_1^1+\xi_3^3\end{pmatrix}$$

$ad\ x_1=\delta_2^4, ad\ x_2=-\delta_1^4, ad\ x_3=-\delta_1^5, ad\ x_4=-\delta_3^6, ad\ x_5=-(\delta_2^6+\delta_1^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : T=$\langle t_1,t_2,t_3\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{\beta_1+\beta_2}\ \langle x_4\rangle+e^{\beta_1+\beta_3}\ \langle x_5\rangle+e^{\beta_1+\beta_2+\beta_3}\ \langle x_6\rangle+e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2+\beta_3}\ \langle x_6\rangle+e^{2\beta_1+\beta_3}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^4\rangle+e^{\beta_2+\beta_3}\ \langle\delta_1^6\rangle+2e^{\beta_1+\beta_3}\ \langle\delta_2^6,\delta_1^7\rangle+e^{\beta_1+\beta_2}\ \langle\delta_3^6\rangle+e^{\beta_3}\ \langle\delta_1^5\rangle+e^{\beta_2}\ \langle\delta_1^4\rangle+3e^0\ \langle t_2,t_1,t_3\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_3^7\rangle+e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle+e^{\beta_1}\ \langle\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_3^4,\delta_1^5]=\delta_3^6$; $[\delta_3^4,\delta_2^5]=-\delta_3^7$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^5,\delta_2^4]=-(\delta_1^7+\delta_2^6)$; $[\delta_1^4,\delta_2^5]=-(\delta_1^7+\delta_2^6)$; $[\delta_1^4,\delta_2^4]=-\delta_3^6$; $[\delta_1^2,\delta_1^7]=\delta_1^6$; $[\delta_1^2,\delta_2^7]=-\delta_1^7+\delta_2^6$; $[\delta_1^2,\delta_3^7]=\delta_3^6$; $[\delta_1^2,\delta_2^5]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$.

ch($H^1_{\mathcal{G}}$)= $e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^4\rangle+e^{\beta_2+\beta_3}\ \langle\delta_1^6\rangle+3e^0\ \langle t_2,t_1,t_3\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{\beta_1+\beta_3}\ \langle\delta_2^6\rangle+e^{2\beta_1-\beta_2+\beta_3}\ \langle\delta_2^7\rangle+e^{2\beta_1}\ \langle\delta_3^7\rangle+e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^5\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_3^4,\delta_2^5]=-\delta_3^7$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^7]=2\delta_2^6$; .

ch($H^2_{\mathcal{G}}$)= $e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,4}\rangle+e^{\beta_2-2\beta_3}\ \langle x_4\omega^{3,5}\rangle+e^{-2\beta_3}\ \langle -x_1\omega^{3,5}+x_4\omega^{3,6}\rangle+e^{-\beta_1+\beta_2-2\beta_3}\ \langle x_2\omega^{3,5}+x_4\omega^{3,7}\rangle+e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,5}-x_2(\omega^{2,5}+\omega^{3,4})-2x_3\omega^{3,5}+x_4\omega^{4,5}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_3\omega^{1,3}+x_6\omega^{1,6},-x_3\omega^{1,3}+x_4\omega^{1,4},-x_2\omega^{1,2}+x_6\omega^{2,7}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,7}\rangle+2e^{-\beta_3}\ \langle -1/2x_2\omega^{2,3}+1/2x_4(\omega^{2,5}+2\omega^{3,4}),-x_1\omega^{1,3}+1/2x_2\omega^{2,3}-1/2x_4\omega^{2,5}+x_7\omega^{3,7}\rangle+e^{-2\beta_1}\ \langle x_2\omega^{1,4}+x_3\omega^{1,5}\rangle+e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,5}\rangle+e^{-\beta_2+\beta_3}\ \langle x_5\omega^{1,2}\rangle+3e^{-\beta_2}\ \langle x_1\omega^{1,2}+2x_3\omega^{2,3}-2x_4\omega^{2,4}+x_5\omega^{2,5},x_4\omega^{2,4}+x_6\omega^{2,6},-x_1\omega^{1,2}-x_3\omega^{2,3}+x_4\omega^{2,4}+x_7\omega^{2,7}\rangle+e^{2\beta_1-\beta_2}\ \langle x_7\omega^{2,3}\rangle+e^{\beta_1-2\beta_2+\beta_3}\ \langle x_7\omega^{2,4}\rangle+e^{\beta_1-\beta_3}\ \langle x_4\omega^{2,3}\rangle+e^{\beta_1-\beta_2-\beta_3}\ \langle -x_1\omega^{2,3}+x_7\omega^{3,6}\rangle+$

$e^{-2\beta_2}\ \langle -x_1\omega^{2,4} + x_5\omega^{2,6} + x_7\omega^{4,6}\rangle + e^{-\beta_1-\beta_2}\ \langle -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3(\omega^{2,5} + \omega^{3,4}) + x_5\omega^{2,7} + x_7\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})$=$e^{\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-\beta_3} + 3e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-2\beta_3} + e^{-\beta_1-3\beta_3} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1+\beta_2-3\beta_3} + 3e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-3\beta_3} + e^{-3\beta_1-\beta_2} + 3e^{-3\beta_1-\beta_3} + e^{-3\beta_1-2\beta_3} + e^{-4\beta_1+\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2} + e^{-4\beta_1-\beta_3} + 2e^{-\beta_2-\beta_3} + 2e^{-\beta_2-2\beta_3} + e^{-2\beta_2-\beta_3} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})$=$3e^{-\beta_1-2\beta_2-2\beta_3} + e^{-\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3} + 4e^{-2\beta_1-\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-3\beta_2} + 3e^{-3\beta_1-3\beta_3} + e^{-3\beta_1-4\beta_3} + e^{-4\beta_1+\beta_2-3\beta_3} + 2e^{-4\beta_1-\beta_2-\beta_3} + 2e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2} + 2e^{-4\beta_1-2\beta_3} + e^{-5\beta_1-\beta_2-\beta_3} + e^{-5\beta_1-2\beta_3} + e^{-3\beta_2-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})$=$e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-3\beta_2-2\beta_3} + e^{-2\beta_1-3\beta_2-3\beta_3} + e^{-3\beta_1-\beta_2-3\beta_3} + 3e^{-3\beta_1-2\beta_2-3\beta_3} + e^{-3\beta_1-2\beta_2-4\beta_3} + e^{-3\beta_1-3\beta_2-\beta_3} + e^{-3\beta_1-3\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-3\beta_3} + 2e^{-4\beta_1-\beta_2-4\beta_3} + e^{-4\beta_1-2\beta_2-\beta_3} + 3e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-3\beta_2-\beta_3} + e^{-4\beta_1-3\beta_3} + e^{-5\beta_1-\beta_2-2\beta_3} + e^{-5\beta_1-\beta_2-3\beta_3} + e^{-5\beta_1-2\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-\beta_3} + e^{-5\beta_1-4\beta_3} + e^{-6\beta_1-\beta_2-3\beta_3} + e^{-6\beta_1-2\beta_2-2\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})$=$e^{-3\beta_1-3\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-3\beta_3} + e^{-4\beta_1-2\beta_2-4\beta_3} + e^{-4\beta_1-3\beta_2-4\beta_3} + e^{-5\beta_1-\beta_2-4\beta_3} + e^{-5\beta_1-2\beta_2-4\beta_3} + e^{-5\beta_1-3\beta_2-2\beta_3} + e^{-5\beta_1-3\beta_2-3\beta_3} + e^{-6\beta_1-\beta_2-4\beta_3} + 2e^{-6\beta_1-2\beta_2-3\beta_3} + e^{-6\beta_1-3\beta_2-2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-5\beta_1-3\beta_2-4\beta_3} + e^{-6\beta_1-2\beta_2-4\beta_3} + e^{-6\beta_1-3\beta_2-3\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 15, 57, 124, 157, 114, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 10, 23, 34, 36, 26, 12, 3)$

S=$6\beta_1 + 3\beta_2 + 4\beta_3$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

$d\omega^4 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,5} - \omega^{3,4}$; $d\omega^7 = -\omega^{1,5}$; $d\omega^{1,6} = \omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,6} = \omega^{2,3,4}$; $d\omega^{2,7} = -\omega^{1,2,5}$; $d\omega^{3,4} = \omega^{1,2,3}$; $d\omega^{3,6} = -\omega^{2,3,5}$; $d\omega^{3,7} = -\omega^{1,3,5}$; $d\omega^{4,5} = -\omega^{1,2,5} + \omega^{1,3,4}$; $d\omega^{4,6} = -\omega^{1,2,6} - \omega^{2,4,5}$; $d\omega^{4,7} = -\omega^{1,2,7} - \omega^{1,4,5}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{3,4,5}$; $d\omega^{5,7} = -\omega^{1,3,7}$; $d\omega^{6,7} = \omega^{1,5,6} - \omega^{2,5,7} - \omega^{3,4,7}$; $d\omega^{1,2,6} = -\omega^{1,2,3,4}$; $d\omega^{1,3,6} = \omega^{1,2,3,5}$; $d\omega^{1,4,6} = \omega^{1,2,4,5}$; $d\omega^{1,5,6} = -\omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,5}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,4,7} = -\omega^{1,2,4,5}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6} - \omega^{2,3,4,5}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7}$; $d\omega^{2,6,7} = \omega^{1,2,5,6} + \omega^{2,3,4,7}$; $d\omega^{3,4,5} = \omega^{1,2,3,5}$; $d\omega^{3,4,6} = \omega^{1,2,3,6} - \omega^{2,3,4,5}$; $d\omega^{3,4,7} = \omega^{1,2,3,7} - \omega^{1,3,4,5}$; $d\omega^{3,6,7} =$

$\omega^{1,3,5,6} - \omega^{2,3,5,7}$; $d\omega^{4,5,6} = -\omega^{1,2,5,6} + \omega^{1,3,4,6}$; $d\omega^{4,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,7}$; $d\omega^{4,6,7} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,4,5,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} + \omega^{3,4,5,7}$; $d\omega^{1,2,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,3,4,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7} = \omega^{1,2,3,5,7}$; $d\omega^{1,4,6,7} = \omega^{1,2,4,5,7}$; $d\omega^{1,5,6,7} = -\omega^{1,3,4,5,7}$; $d\omega^{2,3,4,7} = -\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,5,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,4,6,7} = \omega^{1,2,4,5,6}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,5,6}$; $d\omega^{3,4,5,7} = \omega^{1,2,3,5,7}$; $d\omega^{3,4,6,7} = \omega^{1,2,3,6,7} + \omega^{1,3,4,5,6} - \omega^{2,3,4,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d\omega^{1,2,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{1,3,4,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,4,6,7} = \omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle \omega^1 \rangle + e^{-\beta_2} \langle \omega^2 \rangle + e^{-\beta_3} \langle \omega^3 \rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle \omega^{1,4} \rangle + e^{-3\beta_1-\beta_3} \langle \omega^{1,7} \rangle + e^{-\beta_2-\beta_3} \langle \omega^{2,3} \rangle + e^{-\beta_1-2\beta_2} \langle \omega^{2,4} \rangle + e^{-\beta_1-2\beta_3} \langle \omega^{3,5} \rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle \omega^{1,6} + 2\omega^{2,7} - \omega^{4,5} \rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2} \langle \omega^{1,2,4} \rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle \omega^{1,2,7} \rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle \omega^{1,4,7} \rangle + e^{-4\beta_1-2\beta_3} \langle \omega^{1,5,7} \rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle \omega^{2,3,6} \rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle \omega^{2,4,6} \rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle \omega^{1,4,6} + \omega^{2,4,7} \rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle \omega^{2,3,7} + \omega^{3,4,5} \rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle \omega^{3,5,6} \rangle + e^{-3\beta_1-3\beta_3} \langle \omega^{3,5,7} \rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-3\beta_1-3\beta_2-\beta_3} \langle \omega^{1,2,4,6} \rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle \omega^{1,2,4,7} \rangle + e^{-4\beta_1-3\beta_3} \langle \omega^{1,3,5,7} \rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle \omega^{1,4,5,7} \rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle \omega^{2,3,4,6} \rangle + e^{-2\beta_1-2\beta_2-3\beta_3} \langle \omega^{2,3,5,6} \rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle \omega^{1,3,5,6} \rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle \omega^{1,2,6,7} + \omega^{2,4,5,7} \rangle + e^{-3\beta_1-2\beta_2-3\beta_3} \langle \omega^{2,3,6,7} - \omega^{3,4,5,6} \rangle + e^{-4\beta_1-\beta_2-4\beta_3} \langle \omega^{3,5,6,7} \rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-5\beta_1-3\beta_2-2\beta_3} \langle \omega^{1,2,4,6,7} \rangle + e^{-5\beta_1-\beta_2-4\beta_3} \langle \omega^{1,3,5,6,7} \rangle + e^{-6\beta_1-2\beta_2-3\beta_3} \langle \omega^{1,4,5,6,7} \rangle + e^{-3\beta_1-3\beta_2-3\beta_3} \langle \omega^{2,3,4,5,6} \rangle + e^{-4\beta_1-2\beta_2-3\beta_3} \langle \omega^{1,2,3,6,7} \rangle + e^{-4\beta_1-2\beta_2-4\beta_3} \langle \omega^{2,3,5,6,7} \rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-6\beta_1-3\beta_2-3\beta_3} \langle \omega^{1,2,4,5,6,7} \rangle + e^{-6\beta_1-2\beta_2-4\beta_3} \langle \omega^{1,3,4,5,6,7} \rangle + e^{-5\beta_1-3\beta_2-4\beta_3} \langle \omega^{2,3,4,5,6,7} \rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-6\beta_1-3\beta_2-4\beta_3} \langle \omega^{1,2,3,4,5,6,7} \rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 10, 21, 24, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 6, 10, 10, 6, 3, 1)$

Total dimension of trivial cohomology: 40.

Length associated polynomial: 40.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^4$; $d^\star\omega^{1,3} = -\omega^5$; $d^\star\omega^{1,5} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^6$; $d^\star\omega^{3,4} = -\omega^6$; $d^\star\omega^{1,2,3} = -\omega^{2,5} + \omega^{3,4}$; $d^\star\omega^{1,2,5} = \omega^{1,6} - \omega^{2,7} - \omega^{4,5}$;

$d^\star\omega^{1,2,6} = -\omega^{4,6}$; $d^\star\omega^{1,2,7} = -\omega^{4,7}$; $d^\star\omega^{1,3,4} = \omega^{1,6} + \omega^{4,5}$; $d^\star\omega^{1,3,5} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = -\omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{5,7}$; $d^\star\omega^{1,4,5} = -\omega^{4,7}$; $d^\star\omega^{1,5,6} = \omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,6}$; $d^\star\omega^{2,3,5} = -\omega^{3,6}$; $d^\star\omega^{2,4,5} = -\omega^{4,6}$; $d^\star\omega^{2,5,7} = -\omega^{6,7}$; $d^\star\omega^{3,4,5} = \omega^{5,6}$; $d^\star\omega^{3,4,7} = -\omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,6} + \omega^{2,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,6} - \omega^{2,3,7} + \omega^{3,4,5}$; $d^\star\omega^{1,2,3,6} = -\omega^{2,5,6} + \omega^{3,4,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,5,7} + \omega^{3,4,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,6} - \omega^{2,4,7}$; $d^\star\omega^{1,2,5,6} = \omega^{2,6,7} - \omega^{4,5,6}$; $d^\star\omega^{1,2,5,7} = \omega^{1,6,7} - \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,6} - \omega^{3,4,7}$; $d^\star\omega^{1,3,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,3,4,7} = \omega^{1,6,7} + \omega^{4,5,7}$; $d^\star\omega^{1,3,5,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,6} - \omega^{3,4,6}$; $d^\star\omega^{2,3,4,7} = \omega^{2,6,7}$; $d^\star\omega^{2,3,5,7} = -\omega^{3,6,7}$; $d^\star\omega^{2,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{3,4,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,6} + \omega^{1,3,4,6} - \omega^{2,3,4,7}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{1,2,6,7} + \omega^{2,4,5,7}$; $d^\star\omega^{1,2,3,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,5,7} = \omega^{1,3,6,7} + \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,6,7} = -\omega^{2,5,6,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,4,5,7} = \omega^{1,4,6,7}$; $d^\star\omega^{1,2,5,6,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{3,4,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{2,5,6,7} - \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{2,3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,2,5,6,7} + \omega^{1,3,4,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,5,6,7} = \omega^{3,4,5,6,7}$;

$\Delta\omega^4 = \omega^4$; $\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = 2\omega^{1,6} - \omega^{2,7}$; $\Delta\omega^{2,5} = 2\omega^{2,5}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = -\omega^{1,6} + \omega^{2,7} + \omega^{4,5}$; $\Delta\omega^{3,4} = 2\omega^{3,4}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7}$; $\Delta\omega^{4,5} = \omega^{2,7} + 2\omega^{4,5}$; $\Delta\omega^{4,6} = 2\omega^{4,6}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = \omega^{5,7}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,5} = 3\omega^{1,2,5}$; $\Delta\omega^{1,2,6} = 2\omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,6} = 2\omega^{1,3,6} - \omega^{2,3,7}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,2,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} - \omega^{2,4,7}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6} - \omega^{2,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = -\omega^{1,3,6} + \omega^{2,3,7} - \omega^{3,4,5}$; $\Delta\omega^{2,4,5} = 2\omega^{2,4,5}$; $\Delta\omega^{2,4,7} = -\omega^{1,4,6} + \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = 2\omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,5,6} + 2\omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{4,5,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,3,7} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = 2\omega^{3,4,6}$; $\Delta\omega^{3,4,7} = 3\omega^{3,4,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{2,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = 2\omega^{4,5,7}$; $\Delta\omega^{4,6,7} = 3\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 2\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 2\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 3\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = 2\omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,5,6} = 3\omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = 2\omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = 2\omega^{1,3,4,6} - \omega^{2,3,4,7}$; $\Delta\omega^{1,3,4,7} = 2\omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,3,6,7} = 2\omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,2,6,7} + \omega^{1,4,5,6} - \omega^{2,4,5,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7} = -\omega^{1,3,4,6} + 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,3,5,6} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = -\omega^{1,4,5,6} + 2\omega^{2,4,5,7}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$;

$\Delta\omega^{3,4,5,6} = \omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = 2\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = 3\omega^{3,4,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = \omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7} = 2\omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = 2\omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,2,3,6,7}+\omega^{1,3,4,5,6}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,6,7} = 2\omega^{1,3,4,6,7}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,3,4,5,6} + 2\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,3,5,6,7} = \omega^{1,2,3,5,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{T}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle$.

$\mathrm{ch}(H^2_{\mathfrak{T}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,4}\rangle+e^{-3\beta_1-\beta_3} \langle\omega^{1,7}\rangle+e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle+e^{-\beta_1-2\beta_2} \langle\omega^{2,4}\rangle+ e^{-\beta_1-2\beta_3} \langle\omega^{3,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,6} + 2\omega^{2,7} - \omega^{4,5}\rangle$.

$\mathrm{ch}(H^3_{\mathfrak{T}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,4}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,7} - \omega^{1,4,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3} \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-2\beta_3} \langle\omega^{1,5,7}\rangle + e^{-\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-3\beta_2-\beta_3} \langle\omega^{2,4,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,4,6} + \omega^{2,4,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,6} + 2\omega^{2,3,7} + \omega^{3,4,5}\rangle + e^{-2\beta_1-\beta_2-3\beta_3} \langle\omega^{3,5,6}\rangle + e^{-3\beta_1-3\beta_3} \langle\omega^{3,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathfrak{T}})= e^{-4\beta_1-\beta_2-4\beta_3} \langle-\omega^{3,5,6,7}\rangle+e^{-3\beta_1-2\beta_2-3\beta_3} \langle(-(\omega^{2,3,6,7}-\omega^{3,4,5,6}))/2\rangle+ e^{-2\beta_1-2\beta_2-3\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-3\beta_2-2\beta_3} \langle-\omega^{2,3,4,6}\rangle + e^{-5\beta_1-\beta_2-2\beta_3} \langle-\omega^{1,4,5,7}\rangle + e^{-4\beta_1-3\beta_3} \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3} \langle(-(\omega^{1,3,5,6} + \omega^{2,3,5,7}))/2\rangle + e^{-4\beta_1-2\beta_2-2\beta_3} \langle(\omega^{1,2,6,7} + 2\omega^{1,4,5,6} + \omega^{2,4,5,7})/6\rangle + e^{-4\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,4,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3} \langle-\omega^{1,2,4,6}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{T}})= e^{-4\beta_1-2\beta_2-4\beta_3} \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-3\beta_2-3\beta_3} \langle-\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-2\beta_2-3\beta_3} \langle\omega^{1,4,5,6,7}\rangle + e^{-5\beta_1-\beta_2-4\beta_3} \langle-\omega^{1,3,5,6,7}\rangle + e^{-5\beta_1-3\beta_2-2\beta_3} \langle-\omega^{1,2,4,6,7}\rangle+e^{-4\beta_1-2\beta_2-3\beta_3} \langle(-\omega^{1,2,3,6,7}+2\omega^{1,3,4,5,6}+\omega^{2,3,4,5,7})/6\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{T}})= e^{-5\beta_1-3\beta_2-4\beta_3} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-6\beta_1-2\beta_2-4\beta_3} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-3\beta_2-3\beta_3} \langle\omega^{1,2,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{T}})= e^{-6\beta_1-3\beta_2-4\beta_3} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 3Y^2 + Y^4)(1 + Y)^3$.

5.5.24 Algebra $\mathcal{G}_{7,3.23}$ †

Adjoint cohomology

$[x_1,x_2]=x_3$; $[x_1,x_3]=x_5$; $[x_1,x_4]=x_7$; $[x_2,x_3]=x_6$.

$$\delta=\begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & 0 & 0 & 0\\ \xi_1^3 & \xi_2^3 & \xi_1^1+\xi_2^2 & 0 & 0 & 0 & 0\\ \xi_1^4 & \xi_2^4 & 0 & \xi_4^4 & 0 & 0 & 0\\ \xi_1^5 & \xi_2^5 & \xi_2^3 & \xi_4^5 & 2\xi_1^1+\xi_2^2 & 0 & 0\\ \xi_1^6 & \xi_2^6 & -\xi_1^3 & \xi_4^6 & \xi_1^2 & \xi_1^1+2\xi_2^2 & 0\\ \xi_1^7 & \xi_2^7 & \xi_2^4 & \xi_4^7 & 0 & 0 & \xi_1^1+\xi_4^4 \end{pmatrix}$$

$ad\ x_1=\delta_2^3+\delta_4^7, ad\ x_2=-\delta_1^3, ad\ x_3=-(\delta_1^5+\delta_2^6), ad\ x_4=-\delta_1^7, ad\ x_5=0, ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_4\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_1+\beta_2}\ \langle x_3\rangle+e^{\beta_4}\ \langle x_4\rangle+e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{2\beta_1+\beta_2}\ \langle x_5\rangle+e^{\beta_1+2\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_4}\ \langle\delta_1^4\rangle+2e^{\beta_1+\beta_2}\ \langle\delta_1^5,\delta_2^6\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+2e^{\beta_1}\ \langle\delta_2^3,\delta_4^7\rangle+e^{2\beta_1+\beta_2-\beta_4}\ \langle\delta_4^5\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_2}\ \langle\delta_1^3\rangle+e^{\beta_1+2\beta_2-\beta_4}\ \langle\delta_4^6\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+3e^0\ \langle t_2,t_1,t_4\rangle+e^{\beta_4}\ \langle\delta_1^7\rangle+e^{\beta_1-\beta_2+\beta_4}\ \langle\delta_2^7\rangle+e^{-\beta_2+\beta_4}\ \langle\delta_2^4\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^4,\delta_4^5]=-\delta_1^5$; $[\delta_1^4,\delta_4^6]=-\delta_1^6$; $[\delta_1^4,\delta_4^7]=-\delta_1^7$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_2^5,\delta_1^2]=-\delta_2^6+\delta_1^5$; $[\delta_2^3,\delta_1^3]=\delta_2^6+\delta_1^5$; $[\delta_2^3,\delta_1^2]=\delta_1^3$; $[\delta_2^3,\delta_2^4]=-\delta_2^7$; $[\delta_4^5,\delta_1^2]=-\delta_4^6$; $[\delta_4^5,\delta_2^4]=\delta_2^5$; $[\delta_2^6,\delta_1^2]=\delta_1^6$; $[\delta_1^3,\delta_2^4]=-\delta_1^7$; $[\delta_4^6,\delta_2^4]=\delta_2^6$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_2^4,\delta_4^7]=-\delta_2^7$.

ch($H^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_4}\ \langle\delta_1^4\rangle+e^{2\beta_1}\ \langle\delta_2^5\rangle+e^{2\beta_1+\beta_2-\beta_4}\ \langle\delta_4^5\rangle+e^{2\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1+\beta_2}\ \langle\delta_1^5\rangle+e^{\beta_1+2\beta_2-\beta_4}\ \langle\delta_4^6\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+3e^0\ \langle t_2,t_1,t_4\rangle+e^{\beta_1-\beta_2+\beta_4}\ \langle\delta_2^7\rangle+e^{-\beta_2+\beta_4}\ \langle\delta_2^4\rangle+e^{\beta_1}\ \langle\delta_2^3\rangle$.

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^4,\delta_4^5]=-\delta_1^5$; $[\delta_1^4,\delta_4^6]=-\delta_1^6$; $[\delta_2^5,\delta_1^2]=2\delta_1^5$; $[\delta_4^5,\delta_1^2]=-\delta_4^6$; $[\delta_4^5,\delta_2^4]=\delta_2^5$; $[\delta_1^5,\delta_1^2]=-\delta_1^6$; $[\delta_4^6,\delta_2^4]=-\delta_1^5$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_2^4,\delta_2^3]=\delta_2^7$.

ch($H^2_{\mathcal{G}}$)= $e^{-3\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,5}\rangle+2e^{-2\beta_1}\ \langle x_4\omega^{1,7},x_2\omega^{1,3}\rangle+4e^{-\beta_2}\ \langle x_1\omega^{1,2}+x_3\omega^{2,3}+x_4\omega^{2,4},x_3\omega^{2,3}+x_5\omega^{2,5},-x_3\omega^{2,3}+x_5\omega^{1,6},-x_1\omega^{1,2}-x_3\omega^{2,3}+x_7\omega^{2,7}\rangle+e^{2\beta_1-\beta_4}\ \langle x_5\omega^{2,4}\rangle+e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,6}\rangle+e^{\beta_1-\beta_4}\ \langle x_3\omega^{2,4}+x_5\omega^{2,7}\rangle+3e^{-\beta_4}\ \langle -2/3x_1\omega^{1,4}+1/3x_2\omega^{2,4}-1/3x_3\omega^{3,4}+x_5\omega^{4,5},1/3x_1\omega^{1,4}-2/3x_2\omega^{2,4}-1/3x_3\omega^{3,4}+x_6\omega^{4,6},x_1\omega^{1,4}+x_3\omega^{2,7}+x_5\omega^{3,7}\rangle+e^{\beta_1-\beta_2-\beta_4}\ \langle -x_1\omega^{2,4}+x_5\omega^{4,6}\rangle+$

$e^{\beta_1+\beta_2-2\beta_4}\ \langle x_5\omega^{4,7}\rangle+e^{-2\beta_1+\beta_2}\ \langle x_6\omega^{1,5}\rangle+3e^{-\beta_1}\ \langle x_2\omega^{1,2}+x_6\omega^{1,6}, x_5\omega^{1,5}, x_4\omega^{1,4}\rangle+e^{-\beta_1+2\beta_2-\beta_4}\ \langle x_6\omega^{1,7}\rangle + e^{\beta_2-\beta_4}\ \langle x_5\omega^{1,7}\rangle + e^{-\beta_1+\beta_2-\beta_4}\ \langle -x_2\omega^{1,4} + x_6\omega^{4,5}\rangle + e^{2\beta_2-2\beta_4}\ \langle x_6\omega^{4,7}\rangle + e^{-2\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,3}\rangle + e^{-\beta_1-2\beta_2+\beta_4}\ \langle x_4\omega^{2,3} + x_7\omega^{2,5}\rangle + e^{-3\beta_2+\beta_4}\ \langle x_7\omega^{2,6}\rangle+e^{-2\beta_1-2\beta_2+\beta_4}\ \langle x_4(\omega^{1,6}+\omega^{2,5})+x_7\omega^{3,5}\rangle+e^{-\beta_1-3\beta_2+\beta_4}\ \langle x_4\omega^{2,6}+x_7\omega^{3,6}\rangle+2e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,3}+2x_2\omega^{2,3}+x_4(\omega^{2,7}-\omega^{3,4})+x_7\omega^{3,7}, x_2\omega^{2,3}+x_3\omega^{2,5}+x_5\omega^{3,5}\rangle + e^{-2\beta_2}\ \langle x_3\omega^{2,6} + x_5\omega^{3,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-2\beta_2-\beta_4} + e^{\beta_1-2\beta_4} + e^{-\beta_1+2\beta_2-2\beta_4} + 4e^{-\beta_1-\beta_2-\beta_4} + e^{-\beta_1-2\beta_2-\beta_4} + e^{-\beta_1-2\beta_2} + 3e^{-\beta_1-3\beta_2} + e^{-\beta_1-4\beta_2+\beta_4} + 2e^{-\beta_1-2\beta_4} + e^{-2\beta_1+\beta_2-2\beta_4} + e^{-2\beta_1-\beta_2-\beta_4} + e^{-2\beta_1-\beta_2} + 3e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+\beta_4} + 2e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2+\beta_4} + 4e^{-2\beta_1-\beta_4} + e^{-3\beta_1+\beta_2-\beta_4} + 3e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+\beta_4} + 2e^{-3\beta_1-2\beta_2} + e^{-3\beta_1-3\beta_2+\beta_4} + e^{-3\beta_1-\beta_4} + 2e^{-4\beta_1-\beta_2} + e^{-4\beta_1-2\beta_2+\beta_4} + e^{\beta_2-2\beta_4} + 2e^{-\beta_2-\beta_4} + e^{-\beta_2-2\beta_4} + 2e^{-2\beta_2-\beta_4} + e^{-3\beta_2}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-\beta_1-\beta_2-2\beta_4} + e^{-\beta_1-2\beta_2-2\beta_4} + 2e^{-\beta_1-3\beta_2-\beta_4} + e^{-\beta_1-4\beta_2-\beta_4} + e^{-2\beta_1-\beta_2-2\beta_4} + 2e^{-2\beta_1-2\beta_2-\beta_4} + 4e^{-2\beta_1-3\beta_2-\beta_4} + 3e^{-2\beta_1-4\beta_2} + 2e^{-2\beta_1-2\beta_4} + e^{-3\beta_1+\beta_2-2\beta_4} + 2e^{-3\beta_1-\beta_2-\beta_4} + 4e^{-3\beta_1-2\beta_2-\beta_4} + 4e^{-3\beta_1-3\beta_2} + e^{-3\beta_1-4\beta_2} + e^{-3\beta_1-5\beta_2+\beta_4} + e^{-3\beta_1-2\beta_4} + 3e^{-4\beta_1-\beta_2-\beta_4} + e^{-4\beta_1-2\beta_2-\beta_4} + 3e^{-4\beta_1-2\beta_2} + e^{-4\beta_1-3\beta_2} + e^{-4\beta_1-4\beta_2+\beta_4} + e^{-5\beta_1-\beta_2-\beta_4} + e^{-5\beta_1-2\beta_2} + e^{-5\beta_1-3\beta_2} + e^{-5\beta_1-4\beta_2+\beta_4} + e^{-2\beta_2-2\beta_4}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-2\beta_1-3\beta_2-2\beta_4} + e^{-2\beta_1-4\beta_2-\beta_4} + e^{-2\beta_1-4\beta_2-2\beta_4} + e^{-3\beta_1-2\beta_2-2\beta_4} + e^{-3\beta_1-3\beta_2-\beta_4} + 2e^{-3\beta_1-3\beta_2-2\beta_4} + 2e^{-3\beta_1-4\beta_2-\beta_4} + e^{-3\beta_1-5\beta_2-\beta_4} + e^{-4\beta_1-\beta_2-2\beta_4} + e^{-4\beta_1-2\beta_2-\beta_4} + 2e^{-4\beta_1-2\beta_2-2\beta_4} + 3e^{-4\beta_1-3\beta_2-\beta_4} + e^{-4\beta_1-4\beta_2-\beta_4} + 2e^{-4\beta_1-5\beta_2} + e^{-5\beta_1-\beta_2-2\beta_4} + 2e^{-5\beta_1-2\beta_2-\beta_4} + e^{-5\beta_1-3\beta_2-\beta_4} + 3e^{-5\beta_1-4\beta_2} + e^{-5\beta_1-5\beta_2+\beta_4} + e^{-6\beta_1-3\beta_2-\beta_4} + e^{-6\beta_1-4\beta_2}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-3\beta_1-4\beta_2-2\beta_4} + 2e^{-4\beta_1-3\beta_2-2\beta_4} + e^{-4\beta_1-5\beta_2-\beta_4} + e^{-4\beta_1-5\beta_2-2\beta_4} + e^{-5\beta_1-2\beta_2-2\beta_4} + e^{-5\beta_1-4\beta_2-\beta_4} + e^{-5\beta_1-4\beta_2-2\beta_4} + e^{-5\beta_1-5\beta_2-\beta_4} + e^{-5\beta_1-5\beta_2} + e^{-6\beta_1-3\beta_2-2\beta_4} + 2e^{-6\beta_1-4\beta_2-\beta_4} + e^{-6\beta_1-5\beta_2}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-5\beta_1-5\beta_2-2\beta_4} + e^{-6\beta_1-4\beta_2-2\beta_4} + e^{-6\beta_1-5\beta_2-\beta_4}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 17, 63, 132, 159, 116, 45, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 13, 31, 48, 46, 30, 14, 3)$

S=$6\beta_1 + 5\beta_2 + 2\beta_4$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

$d\omega^3 = -\omega^{1,2}$; $d\omega^5 = -\omega^{1,3}$; $d\omega^6 = -\omega^{2,3}$; $d\omega^7 = -\omega^{1,4}$; $d\omega^{1,6} = \omega^{1,2,3}$; $d\omega^{2,5} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,4}$; $d\omega^{3,4} = -\omega^{1,2,4}$; $d\omega^{3,5} = -\omega^{1,2,5}$; $d\omega^{3,6} = -\omega^{1,2,6}$; $d\omega^{3,7} = -\omega^{1,2,7} - \omega^{1,3,4}$; $d\omega^{4,5} = \omega^{1,3,4}$; $d\omega^{4,6} = \omega^{2,3,4}$; $d\omega^{5,6} = -\omega^{1,3,6} + \omega^{2,3,5}$; $d\omega^{5,7} = -\omega^{1,3,7} + \omega^{1,4,5}$; $d\omega^{6,7} = \omega^{1,4,6} - \omega^{2,3,7}$;

$d\omega^{1,4,6} = -\omega^{1,2,3,4}$; $d\omega^{1,5,6} = -\omega^{1,2,3,5}$; $d\omega^{1,6,7} = \omega^{1,2,3,7}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,5} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = -\omega^{1,2,3,6}$; $d\omega^{2,5,7} = -\omega^{1,2,3,7} + \omega^{1,2,4,5}$; $d\omega^{2,6,7} = \omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,4,5}$; $d\omega^{3,4,6} = -\omega^{1,2,4,6}$; $d\omega^{3,4,7} = -\omega^{1,2,4,7}$; $d\omega^{3,5,6} = -\omega^{1,2,5,6}$; $d\omega^{3,5,7} = -\omega^{1,2,5,7} + \omega^{1,3,4,5}$; $d\omega^{3,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,3,4,6} - \omega^{2,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,3,4,7}$; $d\omega^{4,6,7} = \omega^{2,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,3,6,7} - \omega^{1,4,5,6} + \omega^{2,3,5,7}$; $d\omega^{1,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = -\omega^{1,2,3,6,7} - \omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,4,5,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,4,5,7}$; $d\omega^{3,4,6,7} = -\omega^{1,2,4,6,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,5,6,7} - \omega^{1,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,3,4,6,7} - \omega^{2,3,4,5,7}$; $d\omega^{1,4,5,6,7} = \omega^{1,2,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = \omega^{1,2,3,4,6,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,4,5,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_4}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_4}\ \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_4}\ \langle\omega^{2,4}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6} + \omega^{2,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_4}\ \langle\omega^{2,7} - \omega^{3,4}\rangle + e^{-\beta_1-2\beta_4}\ \langle\omega^{4,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,3,7}\rangle + e^{-2\beta_1-2\beta_4}\ \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,5,7}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,4,6} + \omega^{2,4,5}\rangle + e^{-\beta_1-3\beta_2-\beta_4}\ \langle\omega^{2,4,6}\rangle + e^{-\beta_1-\beta_2-2\beta_4}\ \langle\omega^{2,4,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,6,7} + \omega^{2,5,7} + \omega^{3,4,5}\rangle + e^{-2\beta_1-3\beta_2-\beta_4}\ \langle\omega^{2,6,7} + \omega^{3,4,6}\rangle.$

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-4\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,2,5,7}\rangle + e^{-5\beta_1-4\beta_2}\ \langle\omega^{1,3,5,6}\rangle + e^{-5\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,3,5,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_4}\ \langle\omega^{1,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_4}\ \langle\omega^{1,2,6,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_4}\ \langle\omega^{2,3,4,6}\rangle + e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-4\beta_1-3\beta_2-\beta_4}\ \langle\omega^{1,3,6,7}\rangle + e^{-3\beta_1-4\beta_2-\beta_4}\ \langle\omega^{2,3,6,7} - \omega^{2,4,5,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{1,4,6,7} + \omega^{2,4,5,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_4}\ \langle\omega^{2,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle + e^{-5\beta_1-4\beta_2-\beta_4}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-5\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{1,3,4,5,7}\rangle + e^{-6\beta_1-4\beta_2-\beta_4}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-5\beta_2-\beta_4}\ \langle\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-3\beta_2-2\beta_4}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-2\beta_4}\ \langle\omega^{2,3,4,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-6\beta_1-5\beta_2-\beta_4}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-6\beta_1-4\beta_2-2\beta_4}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-5\beta_1-5\beta_2-2\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}}) = e^{-6\beta_1-5\beta_2-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + 2Y + 5Y^2 + 6Y^3 + 5Y^4 + 2Y^5 + Y^6)(1 + Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 11, 21, 25, 17, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 3, 7, 11, 11, 7, 3, 1)$

Total dimension of trivial cohomology: 44.

Length associated polynomial: 44.

Riemann Hyp.: yes

$d^{\star}\omega^{1,2} = -\omega^{3}$; $d^{\star}\omega^{1,3} = -\omega^{5}$; $d^{\star}\omega^{1,4} = -\omega^{7}$; $d^{\star}\omega^{2,3} = -\omega^{6}$; $d^{\star}\omega^{1,2,3} = \omega^{1,6} - \omega^{2,5}$; $d^{\star}\omega^{1,2,4} = -\omega^{2,7} - \omega^{3,4}$; $d^{\star}\omega^{1,2,5} = -\omega^{3,5}$; $d^{\star}\omega^{1,2,6} = -\omega^{3,6}$; $d^{\star}\omega^{1,2,7} = -\omega^{3,7}$; $d^{\star}\omega^{1,3,4} = -\omega^{3,7} + \omega^{4,5}$; $d^{\star}\omega^{1,3,6} = -\omega^{5,6}$; $d^{\star}\omega^{1,3,7} = -\omega^{5,7}$; $d^{\star}\omega^{1,4,5} = \omega^{5,7}$; $d^{\star}\omega^{1,4,6} = \omega^{6,7}$; $d^{\star}\omega^{2,3,4} = \omega^{4,6}$; $d^{\star}\omega^{2,3,5} = \omega^{5,6}$; $d^{\star}\omega^{2,3,7} = -\omega^{6,7}$; $d^{\star}\omega^{1,2,3,4} = -\omega^{1,4,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $d^{\star}\omega^{1,2,3,5} = -\omega^{1,5,6}$; $d^{\star}\omega^{1,2,3,6} = -\omega^{2,5,6}$; $d^{\star}\omega^{1,2,3,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $d^{\star}\omega^{1,2,4,5} = \omega^{2,5,7} - \omega^{3,4,5}$; $d^{\star}\omega^{1,2,4,6} = \omega^{2,6,7} - \omega^{3,4,6}$; $d^{\star}\omega^{1,2,4,7} = -\omega^{3,4,7}$; $d^{\star}\omega^{1,2,5,6} = -\omega^{3,5,6}$; $d^{\star}\omega^{1,2,5,7} = -\omega^{3,5,7}$; $d^{\star}\omega^{1,2,6,7} = -\omega^{3,6,7}$; $d^{\star}\omega^{1,3,4,5} = \omega^{3,5,7}$; $d^{\star}\omega^{1,3,4,6} = \omega^{3,6,7} + \omega^{4,5,6}$; $d^{\star}\omega^{1,3,4,7} = \omega^{4,5,7}$; $d^{\star}\omega^{1,3,6,7} = -\omega^{5,6,7}$; $d^{\star}\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^{\star}\omega^{2,3,4,5} = -\omega^{4,5,6}$; $d^{\star}\omega^{2,3,4,7} = \omega^{4,6,7}$; $d^{\star}\omega^{2,3,5,7} = \omega^{5,6,7}$; $d^{\star}\omega^{1,2,3,4,5} = \omega^{1,4,5,6} + \omega^{2,3,5,7}$; $d^{\star}\omega^{1,2,3,4,6} = \omega^{2,3,6,7} + \omega^{2,4,5,6}$; $d^{\star}\omega^{1,2,3,4,7} = -\omega^{1,4,6,7} + \omega^{2,4,5,7}$; $d^{\star}\omega^{1,2,3,5,7} = -\omega^{1,5,6,7}$; $d^{\star}\omega^{1,2,3,6,7} = -\omega^{2,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6} = -\omega^{2,5,6,7} - \omega^{3,4,5,6}$; $d^{\star}\omega^{1,2,4,5,7} = -\omega^{3,4,5,7}$; $d^{\star}\omega^{1,2,4,6,7} = -\omega^{3,4,6,7}$; $d^{\star}\omega^{1,2,5,6,7} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,5,6} = -\omega^{3,5,6,7}$; $d^{\star}\omega^{1,3,4,6,7} = \omega^{4,5,6,7}$; $d^{\star}\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,6} = -\omega^{2,3,5,6,7}$; $d^{\star}\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^{\star}\omega^{1,2,3,4,6,7} = \omega^{2,4,5,6,7}$; $d^{\star}\omega^{1,2,4,5,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{3} = \omega^{3}$; $\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{1,6} = \omega^{1,6} - \omega^{2,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,5} = -\omega^{1,6} + \omega^{2,5}$; $\Delta\omega^{2,7} = \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,4} = \omega^{2,7} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7} - \omega^{4,5}$; $\Delta\omega^{4,5} = -\omega^{3,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} + \omega^{1,3,4}$; $\Delta\omega^{1,3,4} = \omega^{1,2,7} + 2\omega^{1,3,4}$; $\Delta\omega^{1,3,6} = \omega^{1,3,6} - \omega^{2,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,5}$; $\Delta\omega^{1,4,5} = -\omega^{1,3,7} + \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = 2\omega^{1,4,6} - \omega^{2,4,5}$; $\Delta\omega^{1,5,6} = \omega^{1,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7} - \omega^{2,5,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,3,6} + \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = 2\omega^{2,3,7} - \omega^{2,4,5}$; $\Delta\omega^{2,4,5} = -\omega^{1,4,6} - \omega^{2,3,7} + \omega^{2,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = -\omega^{1,6,7} + 2\omega^{2,5,7} - \omega^{3,4,5}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7} - \omega^{3,4,6}$; $\Delta\omega^{3,4,5} = -\omega^{2,5,7} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{2,6,7} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = 2\omega^{3,5,7}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7} + \omega^{4,5,6}$; $\Delta\omega^{4,5,6} = \omega^{3,6,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,5} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = 2\omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,5,7} + \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,6,7} + 2\omega^{1,3,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7} + \omega^{1,4,5,6} - \omega^{2,3,5,7}$; $\Delta\omega^{1,4,5,6} =$

$\omega^{1,3,6,7}+2\omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7}=\omega^{1,4,6,7}-\omega^{2,4,5,7}$; $\Delta\omega^{1,5,6,7}=\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5}=-\omega^{1,3,4,6}+\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,7}=\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7}=-\omega^{1,3,6,7}+2\omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7}=\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,6}=\omega^{2,3,6,7}+\omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7}=-\omega^{1,4,6,7}+\omega^{2,4,5,7}$; $\Delta\omega^{2,5,6,7}=2\omega^{2,5,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,6}=\omega^{2,5,6,7}+\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7}=\omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7}=\omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7}=2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7}=2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5}=2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6}=2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7}=2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7}=\omega^{1,2,3,5,7}$; $\Delta\omega^{1,2,3,6,7}=\omega^{1,2,3,6,7}+\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,6}=\omega^{1,2,3,6,7}+2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7}=\omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7}=\omega^{1,2,4,6,7}$; $\Delta\omega^{1,2,5,6,7}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,6}=\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7}=\omega^{1,3,4,6,7}-\omega^{2,3,4,5,7}$; $\Delta\omega^{1,4,5,6,7}=\omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,7}=-\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7}=\omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7}=\omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7}=\omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6}=\omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7}=\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7}=\omega^{1,2,3,4,6,7}$; $\Delta\omega^{1,2,4,5,6,7}=\omega^{1,2,4,5,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_4}\ \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-3\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle+e^{-2\beta_1-\beta_4}\ \langle\omega^{1,7}\rangle+e^{-\beta_2-\beta_4}\ \langle\omega^{2,4}\rangle+e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,6}+\omega^{2,5}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,6}\rangle + e^{-\beta_1-\beta_2-\beta_4}\ \langle\omega^{2,7}-\omega^{3,4}\rangle + e^{-\beta_1-2\beta_4}\ \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-4\beta_1-2\beta_2}\ \langle\omega^{1,3,5}\rangle + e^{-3\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,3,7}+\omega^{1,4,5}\rangle + e^{-2\beta_1-2\beta_4}\ \langle\omega^{1,4,7}\rangle + e^{-4\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,5,7}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,3,6}+\omega^{2,3,5}\rangle + e^{-2\beta_1-4\beta_2}\ \langle\omega^{2,3,6}\rangle+e^{-2\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,4,6}+\omega^{2,3,7}+2\omega^{2,4,5}\rangle+e^{-\beta_1-3\beta_2-\beta_4}\ \langle\omega^{2,4,6}\rangle+e^{-\beta_1-\beta_2-2\beta_4}\ \langle\omega^{2,4,7}\rangle+e^{-3\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,6,7}+\omega^{2,5,7}+\omega^{3,4,5}\rangle+e^{-2\beta_1-3\beta_2-\beta_4}\ \langle\omega^{2,6,7}+\omega^{3,4,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-3\beta_2-2\beta_4}\ \langle-\omega^{2,4,6,7}\rangle + e^{-3\beta_1-4\beta_2-\beta_4}\ \langle(\omega^{2,3,6,7}-\omega^{2,4,5,6})/2\rangle + e^{-4\beta_1-5\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-4\beta_2-\beta_4}\ \langle-\omega^{2,3,4,6}\rangle + e^{-3\beta_1-2\beta_2-2\beta_4}\ \langle(\omega^{1,4,6,7}+\omega^{2,4,5,7})/2\rangle + e^{-4\beta_1-\beta_2-2\beta_4}\ \langle-\omega^{1,4,5,7}\rangle + e^{-4\beta_1-3\beta_2-\beta_4}\ \langle(-(2\omega^{1,3,6,7}-\omega^{1,4,5,6}+\omega^{2,3,5,7}))/6\rangle + e^{-5\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,3,5,7}\rangle + e^{-5\beta_1-4\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + e^{-3\beta_1-3\beta_2-\beta_4}\ \langle(\omega^{1,2,6,7}+\omega^{1,3,4,6}+\omega^{2,3,4,5})/3\rangle + e^{-4\beta_1-2\beta_2-\beta_4}\ \langle(-(\omega^{1,2,5,7}+\omega^{1,3,4,5}))/2\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-4\beta_2-2\beta_4}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-5\beta_2-\beta_4}\ \langle-\omega^{2,3,4,5,6}\rangle + e^{-6\beta_1-4\beta_2-\beta_4}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-3\beta_2-2\beta_4}\ \langle(\omega^{1,3,4,6,7}+\omega^{2,3,4,5,7})/2\rangle + e^{-5\beta_1-2\beta_2-2\beta_4}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-5\beta_1-4\beta_2-\beta_4}\ \langle(-\omega^{1,2,5,6,7}+\omega^{1,3,4,5,6})/2\rangle + e^{-5\beta_1-5\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-5\beta_1-5\beta_2-2\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-6\beta_1-4\beta_2-2\beta_4}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-6\beta_1-5\beta_2-\beta_4}\ \langle-\omega^{1,2,3,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-6\beta_1-5\beta_2-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y)\ =\ (1+2Y+5Y^2+6Y^3+5Y^4+2Y^5+Y^6)(1+Y)$.

5.5.25 Algebra $\mathcal{G}_{7,3.24}$

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_2, x_3] = x_6$; $[x_2, x_4] = x_7$; $[x_3, x_4] = x_5$.

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & \xi_4^1 & 0 & 0 & 0 \\ 0 & \xi_2^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2^3 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0 \\ 0 & \xi_2^4 & \xi_3^4 & \xi_1^1 + \xi_2^2 - \xi_3^3 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1 + \xi_2^2 & -(\xi_3^1 + \xi_2^4) & -\xi_4^1 + \xi_2^3 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & 0 & \xi_2^2 + \xi_3^3 & \xi_4^3 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & 0 & \xi_3^4 & \xi_1^1 + 2\xi_2^2 - \xi_3^3 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5, ad\ x_2 = -\delta_1^5 + \delta_3^6 + \delta_4^7, ad\ x_3 = \delta_4^5 - \delta_2^6, ad\ x_4 = -(\delta_3^5 + \delta_2^7), ad\ x_5 = 0, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_2+\beta_3}\ \langle x_6 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_2+\beta_3}\ \langle x_6 \rangle + e^{\beta_1+2\beta_2-\beta_3}\ \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + 3e^{\beta_2}\ \langle \delta_1^5, \delta_3^6, \delta_4^7 \rangle + e^{\beta_1}\ \langle \delta_2^5 \rangle + 2e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5, \delta_2^7 \rangle + 2e^{\beta_3}\ \langle \delta_4^5, \delta_2^6 \rangle + 2e^{\beta_1-\beta_3}\ \langle \delta_3^1, \delta_2^4 \rangle + 2e^{-\beta_2+\beta_3}\ \langle \delta_4^1, \delta_2^3 \rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6 \rangle + e^{-\beta_1+2\beta_3}\ \langle \delta_4^6 \rangle + e^{-\beta_1-\beta_2+2\beta_3}\ \langle \delta_4^3 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_1^7 \rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^7 \rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle \delta_3^4 \rangle + 3e^0\ \langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^5] = -\delta_2^5$; $[\delta_2^1, \delta_1^6] = -\delta_2^6$; $[\delta_2^1, \delta_1^7] = -\delta_2^7$; $[\delta_1^5, \delta_3^1] = \delta_3^5$; $[\delta_1^5, \delta_4^1] = \delta_4^5$; $[\delta_3^5, \delta_2^3] = \delta_2^5$; $[\delta_3^5, \delta_4^3] = \delta_4^5$; $[\delta_4^5, \delta_2^4] = \delta_2^5$; $[\delta_4^5, \delta_3^4] = \delta_3^5$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_3^1, \delta_1^6] = -(\delta_3^6 + \delta_1^5)$; $[\delta_3^1, \delta_2^6] = -\delta_2^5$; $[\delta_3^1, \delta_3^6] = -\delta_3^5$; $[\delta_3^1, \delta_4^6] = -\delta_4^5$; $[\delta_3^1, \delta_4^3] = \delta_4^1$; $[\delta_3^1, \delta_1^7] = -\delta_3^7$; $[\delta_2^4, \delta_4^1] = -\delta_2^1$; $[\delta_2^4, \delta_1^6] = -\delta_1^5$; $[\delta_2^4, \delta_2^6] = -\delta_2^5$; $[\delta_2^4, \delta_3^6] = -\delta_3^5$; $[\delta_2^4, \delta_4^6] = -(\delta_2^6 + \delta_4^5)$; $[\delta_2^4, \delta_4^3] = -\delta_2^3$; $[\delta_2^4, \delta_4^7] = -\delta_2^7$; $[\delta_4^1, \delta_1^6] = -\delta_4^6$; $[\delta_4^1, \delta_1^7] = -(\delta_4^7 + \delta_1^5)$; $[\delta_4^1, \delta_2^7] = -\delta_2^5$; $[\delta_4^1, \delta_3^7] = -\delta_3^5$; $[\delta_4^1, \delta_4^7] = -\delta_4^5$; $[\delta_4^1, \delta_3^4] = \delta_3^1$; $[\delta_2^3, \delta_3^6] = -\delta_2^6$; $[\delta_2^3, \delta_1^7] = \delta_1^5$; $[\delta_2^3, \delta_2^7] = \delta_2^5$; $[\delta_2^3, \delta_3^7] = -\delta_2^7 + \delta_3^5$; $[\delta_2^3, \delta_4^7] = \delta_4^5$; $[\delta_2^3, \delta_3^4] = -\delta_2^4$; $[\delta_1^6, \delta_3^4] = -\delta_1^7$; $[\delta_2^6, \delta_3^4] = -\delta_2^7$; $[\delta_3^6, \delta_4^3] = \delta_4^6$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_4^6, \delta_3^4] = -\delta_4^7 + \delta_3^6$; $[\delta_4^3, \delta_1^7] = \delta_1^6$; $[\delta_4^3, \delta_2^7] = \delta_2^6$; $[\delta_4^3, \delta_3^7] = -\delta_4^7 + \delta_3^6$; $[\delta_4^3, \delta_4^7] = \delta_4^6$; $[\delta_4^3, \delta_3^4] = t_3$; $[\delta_4^7, \delta_3^4] = \delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{\beta_1-\beta_2}\ \langle \delta_2^1 \rangle + 2e^{\beta_1-\beta_3}\ \langle \delta_3^1, \delta_2^4 \rangle + 2e^{-\beta_2+\beta_3}\ \langle \delta_4^1, \delta_2^3 \rangle + e^{-\beta_1+\beta_2+\beta_3}\ \langle \delta_1^6 \rangle + e^{\beta_3}\ \langle \delta_4^5 \rangle + e^{-\beta_1+2\beta_3}\ \langle \delta_4^6 \rangle + e^{-\beta_1-\beta_2+2\beta_3}\ \langle \delta_4^3 \rangle + e^{2\beta_2-\beta_3}\ \langle \delta_1^7 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{\beta_1+2\beta_2-2\beta_3}\ \langle \delta_3^7 \rangle + 2e^{\beta_2}\ \langle \delta_1^5, \delta_3^6 \rangle + e^{\beta_1+\beta_2-2\beta_3}\ \langle \delta_3^4 \rangle + 3e^0\ \langle t_1, t_2, t_3 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^1, \delta_1^6] = -\delta_4^5$; $[\delta_2^1, \delta_1^7] = \delta_3^5$; $[\delta_3^1, \delta_2^3] = \delta_2^1$; $[\delta_3^1, \delta_1^5] = -\delta_3^5$; $[\delta_3^1, \delta_1^6] = -\delta_3^6 - \delta_1^5$; $[\delta_3^1, \delta_3^6] = -\delta_3^5$; $[\delta_3^1, \delta_4^6] = -\delta_4^5$;

$[\delta_3^1,\delta_4^3]=\delta_4^1$; $[\delta_3^1,\delta_1^7]=-\delta_3^7$; $[\delta_4^1,\delta_2^4]=\delta_2^1$; $[\delta_4^1,\delta_1^5]=-\delta_4^5$; $[\delta_4^1,\delta_1^6]=-\delta_4^6$; $[\delta_4^1,\delta_1^7]=\delta_3^6-2\delta_1^5$; $[\delta_4^1,\delta_3^7]=-\delta_3^5$; $[\delta_4^1,\delta_3^4]=\delta_3^1$; $[\delta_2^3,\delta_3^6]=-\delta_4^5$; $[\delta_2^3,\delta_1^7]=\delta_1^5$; $[\delta_2^3,\delta_3^7]=2\delta_3^5$; $[\delta_2^3,\delta_3^4]=-\delta_2^4$; $[\delta_2^4,\delta_1^6]=-\delta_1^5$; $[\delta_2^4,\delta_3^6]=-\delta_3^5$; $[\delta_2^4,\delta_4^6]=-2\delta_4^5$; $[\delta_2^4,\delta_4^3]=-\delta_2^3$; $[\delta_1^6,\delta_3^4]=-\delta_1^7$; $[\delta_4^5,\delta_3^4]=\delta_3^5$; $[\delta_3^6,\delta_4^3]=\delta_4^6$; $[\delta_3^6,\delta_3^4]=-\delta_3^7$; $[\delta_4^6,\delta_3^4]=2\delta_3^6-\delta_1^5$; $[\delta_4^3,\delta_1^7]=\delta_1^6$; $[\delta_4^3,\delta_3^5]=-\delta_4^5$; $[\delta_4^3,\delta_3^7]=2\delta_3^6-\delta_1^5$; $[\delta_4^3,\delta_3^4]=t_3$.

$\mathrm{ch}(H^2_{\mathcal{G}})=e^{\beta_1-2\beta_2-\beta_3}\ \langle x_1\omega^{2,6}\rangle+e^{-3\beta_2+\beta_3}\ \langle x_1\omega^{2,7}\rangle+3e^{-2\beta_2}\ \langle -x_1(\omega^{2,5}-\omega^{4,6})+x_3\omega^{2,6},-x_1(\omega^{2,5}+\omega^{3,7})+x_4\omega^{2,7},-2x_1(\omega^{2,5}+1/2\omega^{3,7}-1/2\omega^{4,6})+x_5\omega^{6,7}\rangle+e^{-\beta_1-3\beta_2+2\beta_3}\ \langle x_1\omega^{4,7}+x_3\omega^{2,7}\rangle+5e^{-\beta_1}\ \langle -x_2(\omega^{1,2}-2\omega^{3,4})+x_3\omega^{1,3}+x_4\omega^{1,4},x_2\omega^{3,4}+x_3\omega^{1,3}+x_6\omega^{1,6},x_2(\omega^{1,2}-\omega^{3,4})-x_3\omega^{1,3}+x_7\omega^{1,7},-x_2\omega^{3,4}+x_5\omega^{1,5},x_2\omega^{3,4}+x_3\omega^{1,3}+x_6\omega^{3,5}\rangle+e^{\beta_1-\beta_2-2\beta_3}\ \langle -x_1\omega^{3,6}+x_4\omega^{2,6}\rangle+5e^{-\beta_2}\ \langle x_1\omega^{1,2},-x_1\omega^{3,4}+x_3\omega^{2,3},x_1\omega^{3,4}+x_5\omega^{4,6},-x_1\omega^{3,4}+x_4\omega^{2,4},-x_1\omega^{3,4}+x_5\omega^{3,7}\rangle+e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle+e^{-2\beta_2+\beta_3}\ \langle x_1\omega^{2,4}\rangle+e^{-2\beta_1+2\beta_3}\ \langle x_6\omega^{1,4}\rangle+e^{-2\beta_1+\beta_3}\ \langle -x_2\omega^{1,4}+x_6\omega^{1,5}\rangle+2e^{-2\beta_1-\beta_2+2\beta_3}\ \langle x_3\omega^{1,4}+x_6\omega^{1,7},x_3\omega^{1,4}+x_6\omega^{4,5}\rangle+6e^{-\beta_1-\beta_2+\beta_3}\ \langle x_1\omega^{1,4}+x_3\omega^{1,2},x_1\omega^{1,4}-x_2\omega^{2,4}+x_3\omega^{3,4},-x_1\omega^{1,4}+x_5\omega^{4,5},x_1\omega^{1,4}-x_2\omega^{2,4}+x_6\omega^{4,6},-x_1\omega^{1,4}+x_5\omega^{1,7},-x_1\omega^{1,4}+x_2\omega^{2,4}+x_6\omega^{3,7}\rangle+2e^{-\beta_1-2\beta_2+2\beta_3}\ \langle x_3\omega^{2,4},x_5\omega^{4,7}\rangle+e^{-2\beta_1-2\beta_2+3\beta_3}\ \langle x_6\omega^{4,7}\rangle+e^{-\beta_1-2\beta_2+\beta_3}\ \langle -x_1(\omega^{1,7}-\omega^{4,5})+2/3x_2\omega^{2,7}+4/3x_3(\omega^{2,5}+1/4\omega^{3,7}-3/4\omega^{4,6})-2/3x_4\omega^{4,7}-1/3x_5\omega^{5,7}+x_6\omega^{6,7}\rangle+e^{2\beta_2-2\beta_3}\ \langle x_7\omega^{1,3}\rangle+e^{-\beta_1+\beta_2}\ \langle x_6\omega^{1,3}\rangle+e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}+x_7\omega^{1,5}\rangle+2e^{\beta_2-2\beta_3}\ \langle x_4\omega^{1,3}+x_7\omega^{1,6},x_4\omega^{1,3}+x_7\omega^{3,5}\rangle+6e^{-\beta_3}\ \langle -x_1\omega^{1,3}+x_4\omega^{1,2},-x_1\omega^{1,3}+x_2\omega^{2,3}+x_4\omega^{3,4},-x_1\omega^{1,3}+x_5\omega^{3,5},x_1\omega^{1,3}-x_2\omega^{2,3}+x_7\omega^{3,7},-x_1\omega^{1,3}+x_5\omega^{1,6},x_6\omega^{3,6}\rangle+2e^{\beta_1-2\beta_3}\ \langle x_4\omega^{2,3},x_5\omega^{3,6}\rangle+e^{\beta_1+\beta_2-3\beta_3}\ \langle x_7\omega^{3,6}\rangle+e^{-\beta_2-\beta_3}\ \langle x_1(\omega^{1,6}-\omega^{3,5})-2/3x_2\omega^{2,6}+2/3x_3\omega^{3,6}+4/3x_4(\omega^{2,5}+3/4\omega^{3,7}-1/4\omega^{4,6})+1/3x_5\omega^{5,6}+x_7\omega^{6,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-3\beta_3}+e^{\beta_1-2\beta_2-2\beta_3}+e^{-\beta_1+\beta_2-2\beta_3}+6e^{-\beta_1-\beta_2-\beta_3}+3e^{-\beta_1-\beta_2}+10e^{-\beta_1-2\beta_2}+6e^{-\beta_1-3\beta_2+\beta_3}+e^{-\beta_1-4\beta_2+2\beta_3}+e^{-\beta_1-4\beta_2+\beta_3}+4e^{-\beta_1-\beta_3}+e^{-\beta_1-2\beta_3}+4e^{-2\beta_1-\beta_2+\beta_3}+2e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+2\beta_3}+6e^{-2\beta_1-2\beta_2+\beta_3}+5e^{-2\beta_1-3\beta_2+2\beta_3}+e^{-2\beta_1-4\beta_2+3\beta_3}+2e^{-2\beta_1}+e^{-3\beta_1-\beta_2+2\beta_3}+e^{-3\beta_1-2\beta_2+3\beta_3}+e^{-3\beta_1-2\beta_2+2\beta_3}+e^{-3\beta_1-3\beta_2+3\beta_3}+e^{\beta_2-3\beta_3}+5e^{-\beta_2-2\beta_3}+6e^{-2\beta_2-\beta_3}+e^{-3\beta_2-\beta_3}+2e^{-3\beta_2}+e^{-4\beta_2}+e^{-2\beta_3}+e^{-3\beta_3}$.

$\mathrm{ch}(H^4_{\mathcal{G}})=4e^{-\beta_1-\beta_2-2\beta_3}+6e^{-\beta_1-2\beta_2-\beta_3}+2e^{-\beta_1-2\beta_2-2\beta_3}+6e^{-\beta_1-3\beta_2-\beta_3}+2e^{-\beta_1-3\beta_2}+5e^{-\beta_1-4\beta_2}+e^{-\beta_1-5\beta_2+\beta_3}+e^{-\beta_1-3\beta_3}+4e^{-2\beta_1-\beta_2-\beta_3}+e^{-2\beta_1-2\beta_2-\beta_3}+8e^{-2\beta_1-2\beta_2}+6e^{-2\beta_1-3\beta_2+\beta_3}+4e^{-2\beta_1-3\beta_2}+e^{-2\beta_1-4\beta_2+2\beta_3}+6e^{-2\beta_1-4\beta_2+\beta_3}+2e^{-2\beta_1-5\beta_2+2\beta_3}+e^{-3\beta_1-\beta_2}+4e^{-3\beta_1-2\beta_2+\beta_3}+4e^{-3\beta_1-3\beta_2+2\beta_3}+e^{-3\beta_1-3\beta_2+\beta_3}+e^{-3\beta_1-4\beta_2+3\beta_3}+2e^{-3\beta_1-4\beta_2+2\beta_3}+e^{-3\beta_1-5\beta_2+3\beta_3}+e^{-4\beta_1-3\beta_2+3\beta_3}+e^{-\beta_2-3\beta_3}+e^{-2\beta_2-2\beta_3}+e^{-2\beta_2-3\beta_3}+2e^{-3\beta_2-2\beta_3}+e^{-4\beta_2-\beta_3}$.

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-\beta_1-2\beta_2-3\beta_3}+3e^{-\beta_1-3\beta_2-2\beta_3}+2e^{-\beta_1-4\beta_2-\beta_3}+e^{-\beta_1-4\beta_2-2\beta_3}+$

$e^{-\beta_1-5\beta_2-\beta_3}+e^{-2\beta_1-2\beta_2-\beta_3}+2e^{-2\beta_1-2\beta_2-2\beta_3}+6e^{-2\beta_1-3\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2-\beta_3}+7e^{-2\beta_1-4\beta_2}+2e^{-2\beta_1-5\beta_2+\beta_3}+2e^{-2\beta_1-5\beta_2}+e^{-2\beta_1-6\beta_2+\beta_3}+e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2}+e^{-3\beta_1-3\beta_2+\beta_3}+4e^{-3\beta_1-3\beta_2}+6e^{-3\beta_1-4\beta_2+\beta_3}+3e^{-3\beta_1-5\beta_2+2\beta_3}+e^{-3\beta_1-5\beta_2+\beta_3}+e^{-3\beta_1-6\beta_2+2\beta_3}+e^{-4\beta_1-3\beta_2+\beta_3}+2e^{-4\beta_1-4\beta_2+2\beta_3}+e^{-4\beta_1-5\beta_2+3\beta_3}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-2\beta_1-3\beta_2-2\beta_3}+e^{-2\beta_1-4\beta_2-\beta_3}+e^{-2\beta_1-4\beta_2-2\beta_3}+2e^{-2\beta_1-5\beta_2-\beta_3}+e^{-2\beta_1-6\beta_2}+e^{-3\beta_1-3\beta_2-\beta_3}+e^{-3\beta_1-4\beta_2-\beta_3}+2e^{-3\beta_1-4\beta_2}+e^{-3\beta_1-5\beta_2+\beta_3}+3e^{-3\beta_1-5\beta_2}+2e^{-3\beta_1-6\beta_2+\beta_3}+e^{-4\beta_1-4\beta_2+\beta_3}+e^{-4\beta_1-4\beta_2}+e^{-4\beta_1-5\beta_2+2\beta_3}+e^{-4\beta_1-5\beta_2+\beta_3}+e^{-4\beta_1-6\beta_2+2\beta_3}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-3\beta_1-5\beta_2-\beta_3}+e^{-3\beta_1-6\beta_2}+e^{-4\beta_1-5\beta_2}+e^{-4\beta_1-6\beta_2+\beta_3}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(3,22,75,150,175,122,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(3,18,48,78,80,52,21,4)$

S=$2(2\beta_1+3\beta_2)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,14,24,28,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,17,17,11,4,1)$

$d\omega^5=-\omega^{1,2}-\omega^{3,4}$; $d\omega^6=-\omega^{2,3}$; $d\omega^7=-\omega^{2,4}$; $d\omega^{1,5}=\omega^{1,3,4}$; $d\omega^{1,6}=\omega^{1,2,3}$; $d\omega^{1,7}=\omega^{1,2,4}$; $d\omega^{2,5}=\omega^{2,3,4}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{3,7}=-\omega^{2,3,4}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=\omega^{2,3,4}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{2,3,5}-\omega^{3,4,6}$; $d\omega^{5,7}=-\omega^{1,2,7}+\omega^{2,4,5}-\omega^{3,4,7}$; $d\omega^{6,7}=-\omega^{2,3,7}+\omega^{2,4,6}$; $d\omega^{1,2,5}=-\omega^{1,2,3,4}$; $d\omega^{1,3,7}=\omega^{1,2,3,4}$; $d\omega^{1,4,6}=-\omega^{1,2,3,4}$; $d\omega^{1,5,6}=-\omega^{1,2,3,5}+\omega^{1,3,4,6}$; $d\omega^{1,5,7}=-\omega^{1,2,4,5}+\omega^{1,3,4,7}$; $d\omega^{1,6,7}=\omega^{1,2,3,7}-\omega^{1,2,4,6}$; $d\omega^{2,5,6}=\omega^{2,3,4,6}$; $d\omega^{2,5,7}=\omega^{2,3,4,7}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,7}=\omega^{1,2,3,7}+\omega^{2,3,4,5}$; $d\omega^{3,6,7}=\omega^{2,3,4,6}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}-\omega^{2,3,4,5}$; $d\omega^{4,5,7}=\omega^{1,2,4,7}$; $d\omega^{4,6,7}=\omega^{2,3,4,7}$; $d\omega^{5,6,7}=-\omega^{1,2,6,7}+\omega^{2,3,5,7}-\omega^{2,4,5,6}-\omega^{3,4,6,7}$; $d\omega^{1,2,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,3,5,7}=-\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,4,5,6}=\omega^{1,2,3,4,5}$; $d\omega^{1,4,6,7}=-\omega^{1,2,3,4,7}$; $d\omega^{1,5,6,7}=-\omega^{1,2,3,5,7}+\omega^{1,2,4,5,6}+\omega^{1,3,4,6,7}$; $d\omega^{2,5,6,7}=\omega^{2,3,4,6,7}$; $d\omega^{3,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7}=\omega^{1,2,3,6,7}-\omega^{2,3,4,5,6}$; $d\omega^{4,5,6,7}=\omega^{1,2,4,6,7}-\omega^{2,3,4,5,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,6,7}$; $d\omega^{1,3,5,6,7}=\omega^{1,2,3,4,5,6}$; $d\omega^{1,4,5,6,7}=\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\langle\omega^1\rangle+e^{-\beta_2}\langle\omega^2\rangle+e^{-\beta_3}\langle\omega^3\rangle+e^{-\beta_1-\beta_2+\beta_3}\langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-\beta_1-\beta_3}\langle\omega^{1,3}\rangle+e^{-2\beta_1-\beta_2+\beta_3}\langle\omega^{1,4}\rangle+e^{-2\beta_2-\beta_3}\langle\omega^{2,6}\rangle+e^{-\beta_1-3\beta_2+\beta_3}\langle\omega^{2,7}\rangle+e^{-\beta_1-\beta_2}\langle\omega^{1,2}\rangle+e^{-\beta_1-\beta_2-\beta_3}\langle\omega^{1,6}-\omega^{3,5}\rangle+e^{-\beta_2-2\beta_3}\langle\omega^{3,6}\rangle+e^{-2\beta_1-2\beta_2+\beta_3}\langle\omega^{1,7}-\omega^{4,5}\rangle+2e^{-\beta_1-2\beta_2}\langle\omega^{2,5}-\omega^{4,6},\omega^{3,7}+\omega^{4,6}\rangle+e^{-2\beta_1-3\beta_2+2\beta_3}\langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=2e^{-\beta_1-2\beta_2-\beta_3}\langle\omega^{1,2,6},\omega^{2,3,5}\rangle+2e^{-2\beta_1-3\beta_2+\beta_3}\langle\omega^{1,2,7},\omega^{2,4,5}\rangle+e^{-2\beta_1-\beta_2-\beta_3}\langle\omega^{1,3,5}\rangle+e^{-\beta_1-\beta_2-2\beta_3}\langle\omega^{1,3,6}\rangle+e^{-3\beta_1-2\beta_2+\beta_3}\langle\omega^{1,4,5}\rangle+$

$e^{-3\beta_1-3\beta_2+2\beta_3}$ $\langle\omega^{1,4,7}\rangle$ + $e^{-2\beta_2-2\beta_3}$ $\langle\omega^{2,3,6}\rangle$ + $e^{-\beta_1-3\beta_2}$ $\langle\omega^{2,3,7}\rangle$ + $e^{-2\beta_1-4\beta_2+2\beta_3}$ $\langle\omega^{2,4,7}\rangle$ + $e^{-\beta_1-4\beta_2}$ $\langle\omega^{2,6,7}\rangle$ + $3e^{-2\beta_1-2\beta_2}$ $\langle\omega^{1,2,5}-\omega^{3,4,5},\omega^{1,3,7}+\omega^{3,4,5},\omega^{1,4,6}-\omega^{3,4,5}\rangle$ + $e^{-\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{2,5,6}-\omega^{3,6,7}\rangle$ + $e^{-2\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{2,5,7}-\omega^{4,6,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})=$ $2e^{-2\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,2,5,6}-\omega^{3,4,5,6},\omega^{1,3,6,7}-\omega^{3,4,5,6}\rangle$ + $2e^{-3\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{1,2,5,7}-\omega^{3,4,5,7},\omega^{1,4,6,7}-\omega^{3,4,5,7}\rangle$ + $3e^{-2\beta_1-4\beta_2}$ $\langle\omega^{1,2,6,7},\omega^{2,3,5,7},\omega^{2,4,5,6}\rangle$ + $e^{-3\beta_1-2\beta_2}$ $\langle\omega^{1,3,4,5}\rangle$ + $e^{-2\beta_1-2\beta_2-\beta_3}$ $\langle\omega^{1,2,3,5}\rangle$ + $e^{-3\beta_1-3\beta_2+\beta_3}$ $\langle\omega^{1,2,4,5}\rangle$ + $e^{-2\beta_1-2\beta_2-2\beta_3}$ $\langle\omega^{1,3,5,6}\rangle$ + $e^{-3\beta_1-3\beta_2}$ $\langle\omega^{1,3,5,7}+\omega^{1,4,5,6}\rangle$ + $e^{-4\beta_1-4\beta_2+2\beta_3}$ $\langle\omega^{1,4,5,7}\rangle$ + $e^{-\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{2,3,5,6}\rangle$ + $e^{-\beta_1-4\beta_2-\beta_3}$ $\langle\omega^{2,3,6,7}\rangle$ + $e^{-3\beta_1-5\beta_2+2\beta_3}$ $\langle\omega^{2,4,5,7}\rangle$ + $e^{-2\beta_1-5\beta_2+\beta_3}$ $\langle\omega^{2,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=$ $e^{-2\beta_1-3\beta_2-2\beta_3}$ $\langle\omega^{1,2,3,5,6}\rangle$ + $2e^{-3\beta_1-4\beta_2}$ $\langle\omega^{1,2,3,5,7},\omega^{1,2,4,5,6}\rangle$ + $e^{-4\beta_1-5\beta_2+2\beta_3}$ $\langle\omega^{1,2,4,5,7}\rangle$ + $e^{-3\beta_1-3\beta_2-\beta_3}$ $\langle\omega^{1,3,4,5,6}\rangle$ + $e^{-4\beta_1-4\beta_2+\beta_3}$ $\langle\omega^{1,3,4,5,7}\rangle$ + $e^{-2\beta_1-4\beta_2-\beta_3}$ $\langle\omega^{1,2,3,6,7}\rangle$ + $e^{-3\beta_1-5\beta_2+\beta_3}$ $\langle\omega^{1,2,4,6,7}\rangle$ + $e^{-2\beta_1-5\beta_2-\beta_3}$ $\langle\omega^{2,3,5,6,7}\rangle$ + $e^{-3\beta_1-6\beta_2+\beta_3}$ $\langle\omega^{2,4,5,6,7}\rangle$ + $e^{-3\beta_1-5\beta_2}$ $\langle\omega^{1,2,5,6,7}-\omega^{3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=$ $e^{-3\beta_1-5\beta_2-\beta_3}$ $\langle\omega^{1,2,3,5,6,7}\rangle$ + $e^{-4\beta_1-6\beta_2+\beta_3}$ $\langle\omega^{1,2,4,5,6,7}\rangle$ + $e^{-4\beta_1-5\beta_2}$ $\langle\omega^{1,3,4,5,6,7}\rangle$ + $e^{-3\beta_1-6\beta_2}$ $\langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})=$ $e^{-4\beta_1-6\beta_2}$ $\langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+2Y+5Y^2+2Y^3+Y^4)(1+Y+Y^2)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,14,24,28,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,17,17,11,4,1)$

Total dimension of trivial cohomology: 66.

Length associated polynomial: 66.

Riemann Hyp.: yes

$d^\star\omega^{1,2}=-\omega^5$; $d^\star\omega^{2,3}=-\omega^6$; $d^\star\omega^{2,4}=-\omega^7$; $d^\star\omega^{3,4}=-\omega^5$; $d^\star\omega^{1,2,3}=\omega^{1,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4}=\omega^{1,7}+\omega^{4,5}$; $d^\star\omega^{1,2,6}=-\omega^{5,6}$; $d^\star\omega^{1,2,7}=-\omega^{5,7}$; $d^\star\omega^{1,3,4}=\omega^{1,5}$; $d^\star\omega^{2,3,4}=\omega^{2,5}-\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{2,3,5}=\omega^{5,6}$; $d^\star\omega^{2,3,7}=-\omega^{6,7}$; $d^\star\omega^{2,4,5}=\omega^{5,7}$; $d^\star\omega^{2,4,6}=\omega^{6,7}$; $d^\star\omega^{3,4,6}=-\omega^{5,6}$; $d^\star\omega^{3,4,7}=-\omega^{5,7}$; $d^\star\omega^{1,2,3,4}=-\omega^{1,2,5}+\omega^{1,3,7}-\omega^{1,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5}=-\omega^{1,5,6}$; $d^\star\omega^{1,2,3,6}=\omega^{3,5,6}$; $d^\star\omega^{1,2,3,7}=\omega^{1,6,7}+\omega^{3,5,7}$; $d^\star\omega^{1,2,4,5}=-\omega^{1,5,7}$; $d^\star\omega^{1,2,4,6}=-\omega^{1,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,2,4,7}=\omega^{4,5,7}$; $d^\star\omega^{1,2,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,3,4,6}=\omega^{1,5,6}$; $d^\star\omega^{1,3,4,7}=\omega^{1,5,7}$; $d^\star\omega^{2,3,4,5}=\omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{2,3,4,6}=\omega^{2,5,6}+\omega^{3,6,7}$; $d^\star\omega^{2,3,4,7}=\omega^{2,5,7}+\omega^{4,6,7}$; $d^\star\omega^{2,3,5,7}=\omega^{5,6,7}$; $d^\star\omega^{2,4,5,6}=-\omega^{5,6,7}$; $d^\star\omega^{3,4,6,7}=-\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5}=-\omega^{1,3,5,7}+\omega^{1,4,5,6}$; $d^\star\omega^{1,2,3,4,6}=-\omega^{1,2,5,6}-\omega^{1,3,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7}=-\omega^{1,2,5,7}-\omega^{1,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,7}=$

$-\omega^{1,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,6,7} = \omega^{1,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,6,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = \omega^{1,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = \omega^{1,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{1,2,5,6,7} - \omega^{3,4,5,6,7}$;

$\Delta\omega^{5} = 2\omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2} + \omega^{3,4}$; $\Delta\omega^{1,5} = \omega^{1,5}$; $\Delta\omega^{1,6} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{2,3} = \omega^{2,3}$; $\Delta\omega^{2,4} = \omega^{2,4}$; $\Delta\omega^{2,5} = \omega^{2,5} - \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{3,4} = \omega^{1,2} + \omega^{3,4}$; $\Delta\omega^{3,5} = \omega^{1,6} + \omega^{3,5}$; $\Delta\omega^{3,7} = -\omega^{2,5} + \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{1,7} + \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{2,5} - \omega^{3,7} + \omega^{4,6}$; $\Delta\omega^{5,6} = 3\omega^{5,6}$; $\Delta\omega^{5,7} = 3\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5} - \omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{2,3,5} + \omega^{3,4,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7} - \omega^{2,4,5} + \omega^{3,4,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4}$; $\Delta\omega^{1,3,7} = -\omega^{1,2,5} + \omega^{1,3,7} - \omega^{1,4,6} - \omega^{3,4,5}$; $\Delta\omega^{1,4,6} = \omega^{1,2,5} - \omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{1,5,6} = 2\omega^{1,5,6}$; $\Delta\omega^{1,5,7} = 2\omega^{1,5,7}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{2,3,4} = 3\omega^{2,3,4}$; $\Delta\omega^{2,3,5} = -\omega^{1,2,6} + \omega^{2,3,5} - \omega^{3,4,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = -\omega^{1,2,7} + \omega^{2,4,5} - \omega^{3,4,7}$; $\Delta\omega^{2,4,6} = -\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6} + \omega^{3,6,7}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7} + \omega^{4,6,7}$; $\Delta\omega^{3,4,5} = \omega^{1,2,5} - \omega^{1,3,7} + \omega^{1,4,6} + \omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{1,2,6} - \omega^{2,3,5} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{1,2,7} - \omega^{2,4,5} + \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,6,7} + 2\omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = \omega^{2,5,6} + \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{1,6,7} - \omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{2,5,7} + \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 4\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 4\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5} - \omega^{1,3,4,6}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5} - \omega^{1,3,4,7}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + 2\omega^{1,2,4,6} - \omega^{2,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6} + \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} + \omega^{3,4,6,7}$; $\Delta\omega^{1,3,4,6} = -\omega^{1,2,3,5} + \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = -\omega^{1,2,4,5} + \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,5,6} + \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,2,5,7} + \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{1,5,6,7} = 3\omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = 2\omega^{2,3,4,6}$; $\Delta\omega^{2,3,4,7} = 2\omega^{2,3,4,7}$; $\Delta\omega^{2,3,5,7} = -\omega^{1,2,6,7} + \omega^{2,3,5,7} - \omega^{2,4,5,6} - \omega^{3,4,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} + \omega^{3,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{1,2,5,6} + \omega^{1,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = \omega^{1,2,5,7} + \omega^{1,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 3\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 3\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,5,6} - \omega^{1,3,4,6,7}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{2,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7} + \omega^{3,4,5,6,7}$; $\Delta\omega^{1,3,4,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6} + \omega^{1,3,4,6,7}$; $\Delta\omega^{1,3,5,6,7} = \omega^{1,3,5,6,7}$; $\Delta\omega^{1,4,5,6,7} = \omega^{1,4,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7}$; $\Delta\omega^{2,3,4,6,7} = \omega^{2,3,4,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{1,2,5,6,7} + \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} =$

$\omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = 2\omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathcal{C}})= e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathcal{C}})= e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle\omega^{1,4}\rangle + e^{-2\beta_2-\beta_3}\ \langle\omega^{2,6}\rangle + e^{-\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,7}\rangle + e^{-\beta_1-\beta_2}\ \langle\omega^{1,2} - \omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,6} - \omega^{3,5}\rangle + e^{-\beta_2-2\beta_3}\ \langle\omega^{3,6}\rangle + e^{-2\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,7} - \omega^{4,5}\rangle + 2e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5} - \omega^{4,6}, \omega^{3,7} + \omega^{4,6}\rangle + e^{-2\beta_1-3\beta_2+2\beta_3}\ \langle\omega^{4,7}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-3\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,4,5}\rangle + e^{-3\beta_1-3\beta_2+2\beta_3}\ \langle\omega^{1,4,7}\rangle + e^{-2\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-3\beta_2}\ \langle\omega^{2,3,7} + \omega^{2,4,6}\rangle + e^{-2\beta_1-4\beta_2+2\beta_3}\ \langle\omega^{2,4,7}\rangle + e^{-\beta_1-4\beta_2}\ \langle\omega^{2,6,7}\rangle + 3e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5} - \omega^{3,4,5}, \omega^{1,3,7} + \omega^{3,4,5}, \omega^{1,4,6} - \omega^{3,4,5}\rangle + 2e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,6} - \omega^{3,4,6}, \omega^{2,3,5} + \omega^{3,4,6}\rangle + 2e^{-2\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,2,7} - \omega^{3,4,7}, \omega^{2,4,5} + \omega^{3,4,7}\rangle + e^{-\beta_1-3\beta_2-\beta_3}\ \langle\omega^{2,5,6} - \omega^{3,6,7}\rangle + e^{-2\beta_1-4\beta_2+\beta_3}\ \langle\omega^{2,5,7} - \omega^{4,6,7}\rangle.$

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-2\beta_1-5\beta_2+\beta_3}\ \langle -\omega^{2,4,6,7}\rangle + e^{-3\beta_1-5\beta_2+2\beta_3}\ \langle\omega^{2,4,5,7}\rangle + e^{-\beta_1-4\beta_2-\beta_3}\ \langle\omega^{2,3,6,7}\rangle + e^{-\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-4\beta_1-4\beta_2+2\beta_3}\ \langle -\omega^{1,4,5,7}\rangle + e^{-3\beta_1-3\beta_2}\ \langle(\omega^{1,3,5,7} + \omega^{1,4,5,6})/2\rangle + e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle -\omega^{1,3,5,6}\rangle + e^{-3\beta_1-2\beta_2}\ \langle -\omega^{1,3,4,5}\rangle + 3e^{-2\beta_1-4\beta_2}\ \langle(-(\omega^{1,2,6,7} - \omega^{2,3,5,7} + \omega^{2,4,5,6} - 3\omega^{3,4,6,7}))/4, (\omega^{1,2,6,7} - \omega^{2,3,5,7} - 3\omega^{2,4,5,6} + \omega^{3,4,6,7})/4, (-(\omega^{1,2,6,7} + 3\omega^{2,3,5,7} + \omega^{2,4,5,6} + \omega^{3,4,6,7}))/4\rangle + 2e^{-3\beta_1-4\beta_2+\beta_3}\ \langle(\omega^{1,2,5,7} + \omega^{1,4,6,7} - 2\omega^{3,4,5,7})/3, (-(\omega^{1,2,5,7} - 2\omega^{1,4,6,7} + \omega^{3,4,5,7}))/3\rangle + 2e^{-2\beta_1-3\beta_2-\beta_3}\ \langle(-(\omega^{1,2,5,6} + \omega^{1,3,6,7} - 2\omega^{3,4,5,6}))/3, (\omega^{1,2,5,6} - 2\omega^{1,3,6,7} + \omega^{3,4,5,6})/3\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle(-(\omega^{1,2,4,5} + \omega^{1,3,4,7}))/2\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle(\omega^{1,2,3,5} + \omega^{1,3,4,6})/2\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-3\beta_1-6\beta_2+\beta_3}\ \langle -\omega^{2,4,5,6,7}\rangle + e^{-2\beta_1-5\beta_2-\beta_3}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-4\beta_1-4\beta_2+\beta_3}\ \langle -\omega^{1,3,4,5,7}\rangle + e^{-3\beta_1-3\beta_2-\beta_3}\ \langle\omega^{1,3,4,5,6}\rangle + e^{-3\beta_1-5\beta_2}\ \langle(-\omega^{1,2,5,6,7} + \omega^{3,4,5,6,7})/2\rangle + e^{-3\beta_1-5\beta_2+\beta_3}\ \langle(\omega^{1,2,4,6,7} + \omega^{2,3,4,5,7})/2\rangle + e^{-4\beta_1-5\beta_2+2\beta_3}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-2\beta_1-4\beta_2-\beta_3}\ \langle(-\omega^{1,2,3,6,7} - \omega^{2,3,4,5,6})/2\rangle + 2e^{-3\beta_1-4\beta_2}\ \langle(\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6} + 2\omega^{1,3,4,6,7})/3, (-\omega^{1,2,3,5,7} - 2\omega^{1,2,4,5,6} + \omega^{1,3,4,6,7})/3\rangle + e^{-2\beta_1-3\beta_2-2\beta_3}\ \langle\omega^{1,2,3,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-3\beta_1-6\beta_2}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-5\beta_2}\ \langle -\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-6\beta_2+\beta_3}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-3\beta_1-5\beta_2-\beta_3}\ \langle -\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-4\beta_1-6\beta_2}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y)\ =\ (1 + 2Y + 5Y^2 + 2Y^3 + Y^4)(1 + Y + Y^2)(1 + Y).$

5.6 rank 4 Lie Algebras

5.6.1 Algebra $\mathcal{G}_{7,4.1}$ †

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_3, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi_3^3 & 0 & 0 & 0 & 0 \\ \xi_1^4 & 0 & \xi_3^4 & \xi_4^4 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1+\xi_2^2 & \xi_3^2 & 0 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & 0 & \xi_1^1+\xi_3^3 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & 0 & -\xi_1^4 & \xi_3^3+\xi_4^4 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6, ad\ x_2 = -\delta_1^5, ad\ x_3 = -\delta_1^6 + \delta_4^7, ad\ x_4 = -\delta_3^7, ad\ x_5 = 0, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T = \langle t_1, t_2, t_3, t_4 \rangle$.

$\mathrm{ch}(\mathcal{G}) = e^{\beta_1}\ \langle x_1 \rangle + e^{\beta_2}\ \langle x_2 \rangle + e^{\beta_3}\ \langle x_3 \rangle + e^{\beta_4}\ \langle x_4 \rangle + e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_3+\beta_4}\ \langle x_7 \rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}}) = e^{\beta_1+\beta_2}\ \langle x_5 \rangle + e^{\beta_1+\beta_3}\ \langle x_6 \rangle + e^{\beta_3+\beta_4}\ \langle x_7 \rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{-\beta_3+\beta_4}\ \langle \delta_3^4 \rangle + e^{\beta_2}\ \langle \delta_1^5 \rangle + 2e^{\beta_1}\ \langle \delta_2^5, \delta_3^6 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{\beta_1+\beta_2-\beta_4}\ \langle \delta_4^5 \rangle + 4e^0\ \langle t_2, t_1, t_3, t_4 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_3^2 \rangle + 2e^{\beta_3}\ \langle \delta_1^6, \delta_4^7 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{\beta_1+\beta_3-\beta_4}\ \langle \delta_4^6 \rangle + e^{-\beta_1+\beta_3+\beta_4}\ \langle \delta_1^7 \rangle + e^{-\beta_2+\beta_3+\beta_4}\ \langle \delta_2^7 \rangle + e^{\beta_4}\ \langle \delta_3^7 \rangle + e^{-\beta_1+\beta_4}\ \langle \delta_1^4 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_3^4, \delta_4^5] = -\delta_3^5$; $[\delta_3^4, \delta_4^6] = -\delta_3^6$; $[\delta_3^4, \delta_4^7] = -\delta_3^7$; $[\delta_2^5, \delta_3^2] = \delta_3^5$; $[\delta_4^5, \delta_1^4] = \delta_1^5$; $[\delta_3^2, \delta_1^6] = \delta_1^5$; $[\delta_3^2, \delta_2^6] = -\delta_3^6 + \delta_2^5$; $[\delta_3^2, \delta_3^6] = \delta_3^5$; $[\delta_3^2, \delta_4^6] = \delta_4^5$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_1^6, \delta_1^4] = \delta_1^7$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_3^6, \delta_1^4] = \delta_3^7$; $[\delta_4^6, \delta_1^4] = \delta_4^7 + \delta_1^6$; $[\delta_4^7, \delta_1^4] = \delta_1^7$.

$\mathrm{ch}(H^1_{\mathcal{G}}) = e^{-\beta_1+\beta_2}\ \langle \delta_1^2 \rangle + e^{-\beta_3+\beta_4}\ \langle \delta_3^4 \rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle \delta_3^5 \rangle + e^{\beta_1+\beta_2-\beta_4}\ \langle \delta_4^5 \rangle + 4e^0\ \langle t_2, t_1, t_3, t_4 \rangle + e^{\beta_2-\beta_3}\ \langle \delta_3^2 \rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle \delta_2^6 \rangle + e^{\beta_1}\ \langle \delta_2^5 \rangle + e^{\beta_1+\beta_3-\beta_4}\ \langle \delta_4^6 \rangle + e^{-\beta_1+\beta_3+\beta_4}\ \langle \delta_1^7 \rangle + e^{-\beta_2+\beta_3+\beta_4}\ \langle \delta_2^7 \rangle + e^{\beta_3}\ \langle \delta_1^6 \rangle + e^{-\beta_1+\beta_4}\ \langle \delta_1^4 \rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_3^4, \delta_4^5] = -\delta_3^5$; $[\delta_3^4, \delta_4^6] = \delta_2^5$; $[\delta_3^2, \delta_2^6] = 2\delta_2^5$; $[\delta_3^2, \delta_2^5] = -\delta_3^5$; $[\delta_3^2, \delta_4^6] = \delta_4^5$; $[\delta_2^6, \delta_1^4] = \delta_2^7$; $[\delta_4^6, \delta_1^4] = 2\delta_1^6$; $[\delta_1^6, \delta_1^4] = \delta_1^7$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3}\rangle + e^{-\beta_1+\beta_2-\beta_4}\ \langle x_2\omega^{1,4}\rangle + e^{-2\beta_1}\ \langle x_2\omega^{1,5}\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,6}\rangle + 4e^{-\beta_3}\ \langle -x_1\omega^{1,3} + x_2\omega^{2,3}, -x_1\omega^{1,3} + x_5\omega^{3,5}, x_1\omega^{1,3} + x_5\omega^{2,6}, x_4\omega^{3,4}\rangle + e^{-\beta_1+\beta_3-\beta_4}\ \langle x_3\omega^{1,4}\rangle + 4e^{-\beta_1}\ \langle -x_3\omega^{1,3} + x_4\omega^{1,4}, x_2\omega^{1,2}, x_3\omega^{1,3} + x_7\omega^{1,7}, x_6\omega^{1,6}\rangle + e^{-\beta_2-\beta_3+\beta_4}\ \langle x_4\omega^{2,3}\rangle + 4e^{-\beta_2}\ \langle -x_1\omega^{1,2} - x_3\omega^{2,3} + x_4\omega^{2,4}, -x_1\omega^{1,2}+x_6\omega^{2,6}, x_5\omega^{2,5}, x_1\omega^{1,2}+x_3\omega^{2,3}+x_7\omega^{2,7}\rangle + e^{-\beta_1-2\beta_3+\beta_4}\ \langle x_4\omega^{3,6}\rangle + e^{-2\beta_3}\ \langle x_4\omega^{3,7}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + e^{\beta_2-2\beta_3}\ \langle x_5\omega^{3,6}\rangle + e^{\beta_1+\beta_2-2\beta_3-\beta_4}\ \langle x_5\omega^{3,7}\rangle + 4e^{-\beta_4}\ \langle -x_2\omega^{2,4} + x_5\omega^{4,5}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_6\omega^{4,6}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_3\omega^{3,4}, -x_1\omega^{1,4} + x_2\omega^{2,4} + x_6\omega^{1,7}\rangle + e^{\beta_2-\beta_3-\beta_4}\ \langle -x_2\omega^{3,4} + x_5\omega^{4,6}\rangle + e^{\beta_1+\beta_2-\beta_3-2\beta_4}\ \langle x_5\omega^{4,7}\rangle + e^{\beta_1-\beta_2+\beta_3-\beta_4}\ \langle x_6\omega^{2,4}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_6\omega^{2,5}\rangle + e^{\beta_1-\beta_2-\beta_4}\ \langle -x_1\omega^{2,4} + x_6\omega^{2,7}\rangle + e^{\beta_1-\beta_3-\beta_4}\ \langle x_1\omega^{3,4} + x_5\omega^{2,7}\rangle + e^{-\beta_2+\beta_3-\beta_4}\ \langle -x_3\omega^{2,4} + x_6\omega^{4,5}\rangle + e^{\beta_1-2\beta_4}\ \langle x_6\omega^{4,7}\rangle + e^{-2\beta_1-\beta_2+\beta_3+\beta_4}\ \langle x_7\omega^{1,5}\rangle + e^{-2\beta_1+\beta_4}\ \langle x_7\omega^{1,6}\rangle + e^{-\beta_1-2\beta_2+\beta_3+\beta_4}\ \langle x_7\omega^{2,5}\rangle + e^{-\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,2} + x_7\omega^{2,6}\rangle + e^{-\beta_1-\beta_3+\beta_4}\ \langle x_4\omega^{1,3}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_6\omega^{1,5}\rangle + e^{-\beta_1-\beta_3}\ \langle 2x_1\omega^{1,6} - x_2(\omega^{2,6}+\omega^{3,5}) - 2x_3\omega^{3,6} - x_4(\omega^{1,7} - \omega^{4,6}) + x_5\omega^{5,6} + x_7\omega^{6,7}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1+\beta_2-2\beta_3-2\beta_4} + e^{\beta_1-\beta_2-\beta_3-\beta_4} + e^{\beta_1-\beta_2-2\beta_3-\beta_4} + e^{\beta_1-\beta_2-2\beta_4} + e^{\beta_1-\beta_3-2\beta_4} + e^{-\beta_1+\beta_2-\beta_3-2\beta_4} + e^{-\beta_1+\beta_2-2\beta_3-\beta_4} + e^{-\beta_1-\beta_2+\beta_3-\beta_4} + 4e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3+\beta_4} + 2e^{-\beta_1-\beta_2-\beta_4} + e^{-\beta_1-2\beta_2+\beta_3} + e^{-\beta_1-2\beta_2-\beta_3+\beta_4} + 3e^{-\beta_1-2\beta_2} + 4e^{-\beta_1-\beta_3-\beta_4} + 4e^{-\beta_1-2\beta_3} + e^{-\beta_1-3\beta_3} + 2e^{-\beta_1-\beta_4} + e^{-\beta_1-2\beta_4} + e^{-2\beta_1+\beta_2-\beta_3-\beta_4} + e^{-2\beta_1+\beta_2-2\beta_3} + e^{-2\beta_1-\beta_2+\beta_3-\beta_4} + e^{-2\beta_1-\beta_2-\beta_3+\beta_4} + 4e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+\beta_3+\beta_4} + e^{-2\beta_1-2\beta_2+\beta_4} + 4e^{-2\beta_1-\beta_3} + e^{-2\beta_1-2\beta_3+\beta_4} + 2e^{-2\beta_1-\beta_4} + e^{-3\beta_1-\beta_2+\beta_4} + e^{-3\beta_1-\beta_3} + e^{\beta_2-2\beta_3-2\beta_4} + e^{\beta_2-3\beta_3-\beta_4} + 2e^{-\beta_2-\beta_3-\beta_4} + 2e^{-\beta_2-\beta_3} + 2e^{-\beta_2-2\beta_3} + 3e^{-\beta_2-\beta_4} + e^{-\beta_2-2\beta_4} + e^{-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_2-\beta_3} + e^{-2\beta_2-\beta_4} + 3e^{-\beta_3-2\beta_4} + 4e^{-2\beta_3-\beta_4}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=e^{\beta_1-\beta_2-2\beta_3-2\beta_4} + e^{-\beta_1+\beta_2-2\beta_3-2\beta_4} + 3e^{-\beta_1-\beta_2-\beta_3-\beta_4} + e^{-\beta_1-\beta_2-\beta_3-2\beta_4} + 2e^{-\beta_1-\beta_2-2\beta_3-\beta_4} + 2e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-\beta_2-3\beta_3} + e^{-\beta_1-\beta_2-2\beta_4} + e^{-\beta_1-2\beta_2-\beta_3-\beta_4} + 2e^{-\beta_1-2\beta_2-\beta_3} + 2e^{-\beta_1-2\beta_2-2\beta_3} + 3e^{-\beta_1-2\beta_2-\beta_4} + 2e^{-\beta_1-\beta_3-2\beta_4} + 2e^{-\beta_1-2\beta_3-\beta_4} + e^{-\beta_1-2\beta_3-2\beta_4} + 3e^{-\beta_1-3\beta_3-\beta_4} + e^{-2\beta_1+\beta_2-2\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-3\beta_3-\beta_4} + 2e^{-2\beta_1-\beta_2-\beta_3-\beta_4} + 2e^{-2\beta_1-\beta_2-\beta_3} + 2e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-\beta_2-\beta_4} + e^{-2\beta_1-\beta_2-2\beta_4} + e^{-2\beta_1-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3+\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2-2\beta_3+\beta_4} + 3e^{-2\beta_1-2\beta_2} + 2e^{-2\beta_1-\beta_3-\beta_4} + 2e^{-2\beta_1-\beta_3-2\beta_4} + 2e^{-2\beta_1-2\beta_3-\beta_4} + 2e^{-2\beta_1-2\beta_3} + e^{-2\beta_1-3\beta_3} + 3e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3+\beta_4} + e^{-3\beta_1-\beta_2-\beta_4} + e^{-3\beta_1-2\beta_2+\beta_4} + e^{-3\beta_1-\beta_3-\beta_4} + e^{-3\beta_1-2\beta_3} + e^{\beta_2-3\beta_3-2\beta_4} + 3e^{-\beta_2-\beta_3-2\beta_4} + 3e^{-\beta_2-2\beta_3-\beta_4} + e^{-\beta_2-3\beta_3-\beta_4} + e^{-2\beta_2-\beta_3-\beta_4} + e^{-2\beta_2-2\beta_3-\beta_4} + e^{-2\beta_2-2\beta_4} + 3e^{-2\beta_3-2\beta_4}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=3e^{-\beta_1-\beta_2-2\beta_3-2\beta_4} + 2e^{-\beta_1-\beta_2-3\beta_3-\beta_4} + 2e^{-\beta_1-2\beta_2-\beta_3-2\beta_4} + 3e^{-\beta_1-2\beta_2-2\beta_3-\beta_4} + e^{-\beta_1-2\beta_2-3\beta_3-\beta_4} + 2e^{-\beta_1-3\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-3\beta_3-2\beta_4} + 3e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} + 3e^{-2\beta_1-\beta_2-2\beta_3-\beta_4} + e^{-2\beta_1-\beta_2-3\beta_3-\beta_4} + 3e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} + 3e^{-2\beta_1-2\beta_2-2\beta_3} + e^{-2\beta_1-2\beta_2-3\beta_3} + e^{-2\beta_1-2\beta_2-\beta_4} + e^{-2\beta_1-2\beta_2-2\beta_4} + 3e^{-2\beta_1-2\beta_3-2\beta_4} + 2e^{-2\beta_1-3\beta_3-\beta_4} + 2e^{-3\beta_1-\beta_2-\beta_3-\beta_4} + e^{-3\beta_1-\beta_2-\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2-2\beta_3-\beta_4} +$

$2e^{-3\beta_1-\beta_2-2\beta_3}+e^{-3\beta_1-\beta_2-3\beta_3}+2e^{-3\beta_1-2\beta_2-\beta_3}+e^{-3\beta_1-2\beta_2-2\beta_3+\beta_4}+e^{-3\beta_1-2\beta_2-\beta_4}+e^{-3\beta_1-2\beta_3-2\beta_4}+e^{-3\beta_1-3\beta_3-\beta_4}+e^{-\beta_2-2\beta_3-2\beta_4}+e^{-\beta_2-3\beta_3-2\beta_4}+e^{-2\beta_2-2\beta_3-2\beta_4}$.

$\mathrm{ch}(H^6_{\mathcal{G}})=e^{-\beta_1-\beta_2-3\beta_3-2\beta_4}+e^{-\beta_1-2\beta_2-2\beta_3-2\beta_4}+e^{-\beta_1-2\beta_2-3\beta_3-2\beta_4}+e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4}+e^{-2\beta_1-\beta_2-3\beta_3-2\beta_4}+e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4}+e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4}+e^{-2\beta_1-2\beta_2-2\beta_3-2\beta_4}+2e^{-2\beta_1-2\beta_2-3\beta_3-\beta_4}+e^{-2\beta_1-3\beta_3-2\beta_4}+2e^{-3\beta_1-\beta_2-2\beta_3-2\beta_4}+2e^{-3\beta_1-\beta_2-3\beta_3-\beta_4}+e^{-3\beta_1-2\beta_2-\beta_3-\beta_4}+e^{-3\beta_1-2\beta_2-\beta_3-2\beta_4}+e^{-3\beta_1-2\beta_2-2\beta_3-\beta_4}+e^{-3\beta_1-2\beta_2-2\beta_3}+e^{-3\beta_1-2\beta_2-3\beta_3}+e^{-3\beta_1-3\beta_3-2\beta_4}$.

$\mathrm{ch}(H^7_{\mathcal{G}})=e^{-2\beta_1-2\beta_2-3\beta_3-2\beta_4}+e^{-3\beta_1-\beta_2-3\beta_3-2\beta_4}+e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4}+e^{-3\beta_1-2\beta_2-3\beta_3-\beta_4}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(3,20,71,149,174,122,46,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(3,16,42,73,78,51,21,4)$

S$=3\beta_1+2\beta_2+3\beta_3+2\beta_4$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,14,23,28,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,4,11,16,16,11,4,1)$

$d\omega^5=-\omega^{1,2}$; $d\omega^6=-\omega^{1,3}$; $d\omega^7=-\omega^{3,4}$; $d\omega^{1,7}=\omega^{1,3,4}$; $d\omega^{2,6}=-\omega^{1,2,3}$; $d\omega^{2,7}=\omega^{2,3,4}$; $d\omega^{3,5}=\omega^{1,2,3}$; $d\omega^{4,5}=\omega^{1,2,4}$; $d\omega^{4,6}=\omega^{1,3,4}$; $d\omega^{5,6}=-\omega^{1,2,6}+\omega^{1,3,5}$; $d\omega^{5,7}=-\omega^{1,2,7}+\omega^{3,4,5}$; $d\omega^{6,7}=-\omega^{1,3,7}+\omega^{3,4,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,4}$; $d\omega^{1,5,7}=-\omega^{1,3,4,5}$; $d\omega^{1,6,7}=-\omega^{1,3,4,6}$; $d\omega^{2,4,6}=\omega^{1,2,3,4}$; $d\omega^{2,5,6}=\omega^{1,2,3,5}$; $d\omega^{2,5,7}=-\omega^{2,3,4,5}$; $d\omega^{2,6,7}=-\omega^{1,2,3,7}-\omega^{2,3,4,6}$; $d\omega^{3,4,5}=-\omega^{1,2,3,4}$; $d\omega^{3,5,6}=\omega^{1,2,3,6}$; $d\omega^{3,5,7}=\omega^{1,2,3,7}$; $d\omega^{4,5,6}=\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{4,5,7}=\omega^{1,2,4,7}$; $d\omega^{4,6,7}=\omega^{1,3,4,7}$; $d\omega^{5,6,7}=-\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{3,4,5,6}$; $d\omega^{1,2,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{1,5,6,7}=\omega^{1,3,4,5,6}$; $d\omega^{2,4,5,6}=-\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7}=\omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7}=\omega^{1,2,3,5,7}+\omega^{2,3,4,5,6}$; $d\omega^{3,4,5,6}=-\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7}=-\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7}=\omega^{1,2,3,6,7}$; $d\omega^{4,5,6,7}=\omega^{1,2,4,6,7}-\omega^{1,3,4,5,7}$; $d\omega^{1,2,5,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7}=-\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7}=-\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})=e^{-\beta_1}\langle\omega^1\rangle+e^{-\beta_2}\langle\omega^2\rangle+e^{-\beta_3}\langle\omega^3\rangle+e^{-\beta_4}\langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})=e^{-\beta_1-\beta_4}\langle\omega^{1,4}\rangle+e^{-2\beta_1-\beta_2}\langle\omega^{1,5}\rangle+e^{-2\beta_1-\beta_3}\langle\omega^{1,6}\rangle+e^{-\beta_2-\beta_3}\langle\omega^{2,3}\rangle+e^{-\beta_2-\beta_4}\langle\omega^{2,4}\rangle+e^{-\beta_1-2\beta_2}\langle\omega^{2,5}\rangle+e^{-\beta_1-\beta_2-\beta_3}\langle\omega^{2,6}+\omega^{3,5}\rangle+e^{-\beta_1-2\beta_3}\langle\omega^{3,6}\rangle+e^{-2\beta_3-\beta_4}\langle\omega^{3,7}\rangle+e^{-\beta_1-\beta_3-\beta_4}\langle\omega^{1,7}-\omega^{4,6}\rangle+e^{-\beta_3-2\beta_4}\langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})=e^{-2\beta_1-2\beta_2}\langle\omega^{1,2,5}\rangle+e^{-2\beta_1-\beta_2-\beta_3}\langle\omega^{1,2,6}\rangle+e^{-2\beta_1-2\beta_3}\langle\omega^{1,3,6}\rangle+e^{-2\beta_1-\beta_2-\beta_4}\langle\omega^{1,4,5}\rangle+e^{-2\beta_1-\beta_3-\beta_4}\langle\omega^{1,4,6}\rangle+e^{-\beta_1-\beta_3-2\beta_4}\langle\omega^{1,4,7}\rangle+e^{-3\beta_1-\beta_2-\beta_3}\langle\omega^{1,5,6}\rangle+e^{-\beta_1-2\beta_2-\beta_3}\langle\omega^{2,3,5}\rangle+e^{-\beta_1-\beta_2-2\beta_3}\langle\omega^{2,3,6}\rangle+e^{-\beta_2-2\beta_3-\beta_4}\langle\omega^{2,3,7}\rangle+e^{-\beta_1-2\beta_2-\beta_4}\langle\omega^{2,4,5}\rangle+e^{-\beta_2-\beta_3-2\beta_4}\langle\omega^{2,4,7}\rangle+e^{-\beta_1-\beta_2-\beta_3-\beta_4}\langle\omega^{2,4,6}+\omega^{3,4,5}\rangle+e^{-\beta_1-2\beta_3-\beta_4}\langle\omega^{1,3,7}\rangle+e^{-2\beta_3-2\beta_4}\langle\omega^{3,4,7}\rangle+e^{-\beta_1-3\beta_3-\beta_4}\langle\omega^{3,6,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-2\beta_1-2\beta_2-\beta_4} \langle\omega^{1,2,4,5}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-2\beta_1-3\beta_3-\beta_4} \langle\omega^{1,3,6,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3-\beta_4} \langle\omega^{1,4,5,6}\rangle + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{1,4,5,7}\rangle + e^{-2\beta_1-2\beta_3-2\beta_4} \langle\omega^{1,4,6,7}\rangle + e^{-\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_3-\beta_4} \langle\omega^{2,3,5,7}\rangle + e^{-\beta_1-\beta_2-3\beta_3-\beta_4} \langle\omega^{2,3,6,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,5,7} + \omega^{2,4,5,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-\beta_4} \langle\omega^{1,2,6,7} + \omega^{3,4,5,6}\rangle + e^{-\beta_1-\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,4,6,7} + \omega^{3,4,5,7}\rangle + e^{-\beta_1-3\beta_3-2\beta_4} \langle\omega^{3,4,6,7}\rangle.$

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,4,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{1,2,4,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4} \langle\omega^{1,2,4,6,7}\rangle + e^{-2\beta_1-3\beta_3-2\beta_4} \langle\omega^{1,3,4,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3-\beta_4} \langle\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3-2\beta_4} \langle\omega^{1,4,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4} \langle\omega^{1,2,3,5,7}\rangle + e^{-\beta_1-2\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,3,4,5,7}\rangle + e^{-\beta_1-\beta_2-3\beta_3-2\beta_4} \langle\omega^{2,3,4,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3-\beta_4} \langle\omega^{2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-3\beta_3-\beta_4} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3-2\beta_4} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-3\beta_3-2\beta_4} \langle\omega^{2,3,4,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-3\beta_3-2\beta_4} \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) = (1+3Y+8Y^2+8Y^3+8Y^4+3Y^5+Y^6)(1+Y).$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,14,23,28,18,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1,4,11,16,16,11,4,1)$

Total dimension of trivial cohomology: 64.

Length associated polynomial: 64.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{3,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6} + \omega^{3,5}$; $d^\star\omega^{1,2,4} = \omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = \omega^{1,7} + \omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{2,3,4} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = \omega^{5,7}$; $d^\star\omega^{3,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,5,7} - \omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,5,7}$; $d^\star\omega^{2,3,4,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,5,7} - \omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{1,2,6,7} - \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^{5} = \omega^{5}$; $\Delta\omega^{6} = \omega^{6}$; $\Delta\omega^{7} = \omega^{7}$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,7} = \omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{2,6} = \omega^{2,6} - \omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,4} = \omega^{3,4}$; $\Delta\omega^{3,5} = -\omega^{2,6} + \omega^{3,5}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = \omega^{1,7} + \omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} - \omega^{1,3,5}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} - \omega^{2,4,6}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6} + \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{3,4,6}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7} + \omega^{4,5,6}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,4} = \omega^{2,3,4}$; $\Delta\omega^{2,4,6} = -\omega^{1,2,7} + \omega^{2,4,6} - \omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7} - \omega^{3,5,7}$; $\Delta\omega^{3,4,5} = -\omega^{2,4,6} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = -\omega^{1,3,7} + \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7} + \omega^{3,5,7}$; $\Delta\omega^{4,5,6} = \omega^{1,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6} - \omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} - \omega^{1,3,5,7}$; $\Delta\omega^{1,3,4,5} = -\omega^{1,2,4,6} + 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{3,4,5,6}$; $\Delta\omega^{1,5,6,7} = \omega^{1,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{1,2,3,7} + \omega^{2,3,4,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{1,2,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,3,5,7} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,2,5,6,7} = \omega^{1,2,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{1,2,3,5,7} + \omega^{2,3,4,5,6}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathfrak{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_4}\ \langle\omega^4\rangle.$

$\mathrm{ch}(H^2_{\mathfrak{C}}) = e^{-\beta_1-\beta_4}\ \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_3}\ \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_2-\beta_4}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2}\ \langle\omega^{2,5}\rangle + e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3}\ \langle\omega^{3,6}\rangle + e^{-2\beta_3-\beta_4}\ \langle\omega^{3,7}\rangle + e^{-\beta_1-\beta_3-\beta_4}\ \langle\omega^{1,7} - \omega^{4,6}\rangle + e^{-\beta_3-2\beta_4}\ \langle\omega^{4,7}\rangle.$

$\mathrm{ch}(H^3_{\mathfrak{C}}) = e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,6} + \omega^{1,3,5}\rangle + e^{-2\beta_1-2\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,4,5}\rangle + e^{-2\beta_1-\beta_3-\beta_4}\ \langle\omega^{1,4,6}\rangle + e^{-\beta_1-\beta_3-2\beta_4}\ \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,5,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_2-2\beta_3-\beta_4}\ \langle\omega^{2,3,7}\rangle + e^{-\beta_1-2\beta_2-\beta_4}\ \langle\omega^{2,4,5}\rangle + e^{-\beta_2-\beta_3-2\beta_4}\ \langle\omega^{2,4,7}\rangle + e^{-\beta_1-\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,2,7} + 2\omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_3-\beta_4}\ \langle\omega^{1,3,7} + \omega^{3,4,6}\rangle + e^{-2\beta_3-2\beta_4}\ \langle\omega^{3,4,7}\rangle + e^{-\beta_1-3\beta_3-\beta_4}\ \langle\omega^{3,6,7}\rangle.$

$\mathrm{ch}(H^4_{\mathfrak{C}}) = e^{-\beta_1-3\beta_3-2\beta_4}\ \langle\omega^{3,4,6,7}\rangle + e^{-\beta_1-\beta_2-2\beta_3-2\beta_4}\ \langle(-(\omega^{2,4,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-\beta_1-2\beta_2-\beta_3-2\beta_4}\ \langle\omega^{2,4,5,7}\rangle + e^{-\beta_1-\beta_2-3\beta_3-\beta_4}\ \langle\omega^{2,3,6,7}\rangle + e^{-\beta_1-2\beta_2-2\beta_3-\beta_4}\ \langle-\omega^{2,3,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{2,3,5,6}\rangle + e^{-\beta_2-2\beta_3-2\beta_4}\ \langle\omega^{2,3,4,7}\rangle +$

$e^{-2\beta_1-2\beta_3-2\beta_4}\ \langle\omega^{1,4,6,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4}\ \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,4,5,6}\rangle + e^{-2\beta_1-3\beta_3-\beta_4}\ \langle-\omega^{1,3,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3}\ \langle-\omega^{1,3,5,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-\beta_4}\ \langle(\omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{3,4,5,6})/6\rangle + e^{-2\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle(-(\omega^{1,2,5,7} + \omega^{2,4,5,6}))/2\rangle + e^{-3\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,2,4,5}\rangle.$

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-2\beta_1-2\beta_2-3\beta_3-\beta_4}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-\beta_1-\beta_2-3\beta_3-2\beta_4}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-\beta_1-2\beta_2-2\beta_3-2\beta_4}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3-2\beta_4}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3-\beta_4}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-2\beta_1-3\beta_3-2\beta_4}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4}\ \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4}\ \langle\omega^{1,2,4,5,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle-\omega^{1,2,4,5,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4}\ \langle(\omega^{1,2,3,5,7} - \omega^{2,3,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2-2\beta_3}\ \langle\omega^{1,2,3,5,6}\rangle.$

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-2\beta_1-2\beta_2-3\beta_3-2\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_3-2\beta_4}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-3\beta_3-\beta_4}\ \langle-\omega^{1,2,3,5,6,7}\rangle.$

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-3\beta_1-2\beta_2-3\beta_3-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$

$P(Y) \ = \ (1 + 3Y + 8Y^2 + 8Y^3 + 8Y^4 + 3Y^5 + Y^6)(1 + Y).$

5.6.2 Algebra $\mathcal{G}_{7,4.2}$ †

Adjoint cohomology

$[x_1, x_2] = x_5$; $[x_1, x_3] = x_6$; $[x_1, x_4] = x_7$.

$$\delta = \begin{pmatrix} \xi_1^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & \xi_4^2 & 0 & 0 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & \xi_4^3 & 0 & 0 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_4^4 & 0 & 0 & 0 \\ \xi_1^5 & \xi_2^5 & \xi_3^5 & \xi_4^5 & \xi_1^1+\xi_2^2 & \xi_3^2 & \xi_4^2 \\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_2^3 & \xi_1^1+\xi_3^3 & \xi_4^3 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_2^4 & \xi_3^4 & \xi_1^1+\xi_4^4 \end{pmatrix}$$

$ad\ x_1 = \delta_2^5 + \delta_3^6 + \delta_4^7, ad\ x_2 = -\delta_1^5, ad\ x_3 = -\delta_1^6, ad\ x_4 = -\delta_1^7, ad\ x_5 = 0, ad\ x_6 = 0, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3, t_4\rangle$.

ch($\mathcal{G}$)= $e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_4}\ \langle x_4\rangle + e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

ch($H^0_{\mathcal{G}}$)= $e^{\beta_1+\beta_2}\ \langle x_5\rangle + e^{\beta_1+\beta_3}\ \langle x_6\rangle + e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

ch($Z^1_{\mathcal{G}}$)= $e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_4}\ \langle\delta_1^4\rangle + e^{\beta_2}\ \langle\delta_1^5\rangle + 3e^{\beta_1}\ \langle\delta_2^5, \delta_3^6, \delta_4^7\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{\beta_1+\beta_2-\beta_4}\ \langle\delta_4^5\rangle + 4e^0\ \langle t_2, t_3, t_1, t_4\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{\beta_2-\beta_4}\ \langle\delta_4^2\rangle + e^{\beta_3}\ \langle\delta_1^6\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^6\rangle + e^{\beta_1+\beta_3-\beta_4}\ \langle\delta_4^6\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_3-\beta_4}\ \langle\delta_4^3\rangle + e^{\beta_4}\ \langle\delta_1^7\rangle + e^{\beta_1-\beta_2+\beta_4}\ \langle\delta_2^7\rangle + e^{\beta_1-\beta_3+\beta_4}\ \langle\delta_3^7\rangle + e^{-\beta_2+\beta_4}\ \langle\delta_2^4\rangle + e^{-\beta_3+\beta_4}\ \langle\delta_3^4\rangle$.

Comm. relations among the δ_i^j's $i \neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_1^2, \delta_2^5] = -\delta_1^5$; $[\delta_1^2, \delta_2^6] = -\delta_1^6$; $[\delta_1^2, \delta_2^3] = -\delta_1^3$; $[\delta_1^2, \delta_2^7] = -\delta_1^7$; $[\delta_1^2, \delta_2^4] = -\delta_1^4$; $[\delta_1^3, \delta_3^5] = -\delta_1^5$; $[\delta_1^3, \delta_3^2] = -\delta_1^2$; $[\delta_1^3, \delta_3^6] = -\delta_1^6$; $[\delta_1^3, \delta_3^7] = -\delta_1^7$; $[\delta_1^3, \delta_3^4] = -\delta_1^4$; $[\delta_1^4, \delta_4^5] = -\delta_1^5$; $[\delta_1^4, \delta_4^2] = -\delta_1^2$; $[\delta_1^4, \delta_4^6] = -\delta_1^6$; $[\delta_1^4, \delta_4^3] = -\delta_1^3$; $[\delta_1^4, \delta_4^7] = -\delta_1^7$; $[\delta_1^5, \delta_2^3] = -\delta_1^6$; $[\delta_1^5, \delta_2^4] = -\delta_1^7$; $[\delta_2^5, \delta_3^2] = \delta_3^5$; $[\delta_2^5, \delta_4^2] = \delta_4^5$; $[\delta_2^5, \delta_2^3] = -\delta_2^6$; $[\delta_2^5, \delta_2^4] = -\delta_2^7$; $[\delta_3^5, \delta_2^3] = -\delta_3^6 + \delta_2^5$; $[\delta_3^5, \delta_4^3] = \delta_4^5$; $[\delta_3^5, \delta_2^4] = -\delta_3^7$; $[\delta_4^5, \delta_2^3] = -\delta_4^6$; $[\delta_4^5, \delta_2^4] = -\delta_4^7 + \delta_2^5$; $[\delta_4^5, \delta_3^4] = \delta_3^5$; $[\delta_3^2, \delta_1^6] = \delta_1^5$; $[\delta_3^2, \delta_2^6] = -\delta_3^6 + \delta_2^5$; $[\delta_3^2, \delta_3^6] = \delta_3^5$; $[\delta_3^2, \delta_4^6] = \delta_4^5$; $[\delta_3^2, \delta_2^3] = -t_3 + t_2$; $[\delta_3^2, \delta_4^3] = \delta_4^2$; $[\delta_3^2, \delta_2^7] = -\delta_3^7$; $[\delta_3^2, \delta_2^4] = -\delta_3^4$; $[\delta_4^2, \delta_2^6] = -\delta_4^6$; $[\delta_4^2, \delta_2^3] = -\delta_4^3$; $[\delta_4^2, \delta_1^7] = \delta_1^5$; $[\delta_4^2, \delta_2^7] = -\delta_4^7 + \delta_2^5$; $[\delta_4^2, \delta_3^7] = \delta_3^5$; $[\delta_4^2, \delta_4^7] = \delta_4^5$; $[\delta_4^2, \delta_2^4] = -t_4 + t_2$; $[\delta_4^2, \delta_3^4] = \delta_3^2$; $[\delta_1^6, \delta_3^4] = -\delta_1^7$; $[\delta_2^6, \delta_3^4] = -\delta_2^7$; $[\delta_3^6, \delta_2^3] = \delta_2^6$; $[\delta_3^6, \delta_4^3] = \delta_4^6$; $[\delta_3^6, \delta_3^4] = -\delta_3^7$; $[\delta_4^6, \delta_2^4] = \delta_2^6$; $[\delta_4^6, \delta_3^4] = -\delta_4^7 + \delta_3^6$; $[\delta_2^3, \delta_3^7] = -\delta_2^7$; $[\delta_2^3, \delta_3^4] = -\delta_2^4$; $[\delta_4^3, \delta_1^7] = \delta_1^6$; $[\delta_4^3, \delta_2^7] = \delta_2^6$; $[\delta_4^3, \delta_3^7] = -\delta_4^7 + \delta_3^6$; $[\delta_4^3, \delta_4^7] = \delta_4^6$; $[\delta_4^3, \delta_2^4] = \delta_2^3$; $[\delta_4^3, \delta_3^4] = -t_4 + t_3$; $[\delta_4^7, \delta_2^4] = \delta_2^7$; $[\delta_4^7, \delta_3^4] = \delta_3^7$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle\delta_1^3\rangle + e^{-\beta_1+\beta_4}\ \langle\delta_1^4\rangle + e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^5\rangle + e^{\beta_1+\beta_2-\beta_4}\ \langle\delta_4^5\rangle + 4e^0\ \langle t_2,t_3,t_1,t_4\rangle + e^{\beta_2-\beta_3}\ \langle\delta_3^2\rangle + e^{\beta_2-\beta_4}\ \langle\delta_4^2\rangle + e^{\beta_1-\beta_2+\beta_3}\ \langle\delta_2^6\rangle + e^{\beta_1+\beta_3-\beta_4}\ \langle\delta_4^6\rangle + e^{-\beta_2+\beta_3}\ \langle\delta_2^3\rangle + e^{\beta_3-\beta_4}\ \langle\delta_4^3\rangle + e^{\beta_1-\beta_2+\beta_4}\ \langle\delta_2^7\rangle + e^{\beta_1-\beta_3+\beta_4}\ \langle\delta_3^7\rangle + 2e^{\beta_1}\ \langle\delta_2^5,\delta_3^6\rangle + e^{-\beta_2+\beta_4}\ \langle\delta_2^4\rangle + e^{-\beta_3+\beta_4}\ \langle\delta_3^4\rangle.$

Comm. relations among the δ_i^j's $i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_1^2,\delta_2^3]=-\delta_1^3$; $[\delta_1^2,\delta_2^4]=-\delta_1^4$; $[\delta_1^3,\delta_3^2]=-\delta_1^2$; $[\delta_1^3,\delta_3^4]=-\delta_1^4$; $[\delta_1^4,\delta_4^2]=-\delta_1^2$; $[\delta_1^4,\delta_4^3]=-\delta_1^3$; $[\delta_2^5,\delta_3^2]=\delta_3^5$; $[\delta_2^5,\delta_4^2]=\delta_4^5$; $[\delta_2^5,\delta_2^3]=-\delta_2^6$; $[\delta_2^5,\delta_2^4]=-\delta_2^7$; $[\delta_3^5,\delta_2^3]=-\delta_3^6+\delta_2^5$; $[\delta_3^5,\delta_4^3]=\delta_4^5$; $[\delta_3^5,\delta_2^4]=-\delta_3^7$; $[\delta_4^5,\delta_2^3]=-\delta_4^6$; $[\delta_4^5,\delta_2^4]=\delta_3^6+2\delta_2^5$; $[\delta_4^5,\delta_3^4]=\delta_3^5$; $[\delta_3^2,\delta_2^6]=-\delta_3^6+\delta_2^5$; $[\delta_3^2,\delta_3^6]=\delta_3^5$; $[\delta_3^2,\delta_4^6]=\delta_4^5$; $[\delta_3^2,\delta_2^3]=-t_3+t_2$; $[\delta_3^2,\delta_4^3]=\delta_4^2$; $[\delta_3^2,\delta_2^7]=-\delta_3^7$; $[\delta_3^2,\delta_2^4]=-\delta_3^4$; $[\delta_4^2,\delta_2^6]=-\delta_4^6$; $[\delta_4^2,\delta_2^3]=-\delta_4^3$; $[\delta_4^2,\delta_2^7]=\delta_3^6+2\delta_2^5$; $[\delta_4^2,\delta_3^7]=\delta_3^5$; $[\delta_4^2,\delta_2^4]=-t_4+t_2$; $[\delta_4^2,\delta_3^4]=\delta_3^2$; $[\delta_2^6,\delta_3^4]=-\delta_2^7$; $[\delta_3^6,\delta_2^3]=\delta_2^6$; $[\delta_3^6,\delta_4^3]=\delta_4^6$; $[\delta_3^6,\delta_3^4]=-\delta_3^7$; $[\delta_4^6,\delta_2^4]=\delta_2^6$; $[\delta_4^6,\delta_3^4]=2\delta_3^6+\delta_2^5$; $[\delta_2^3,\delta_3^7]=-\delta_2^7$; $[\delta_2^3,\delta_3^4]=-\delta_2^4$; $[\delta_4^3,\delta_2^7]=\delta_2^6$; $[\delta_4^3,\delta_3^7]=2\delta_3^6+\delta_2^5$; $[\delta_4^3,\delta_2^4]=\delta_2^3$; $[\delta_4^3,\delta_3^4]=-t_4+t_3$.

$\mathrm{ch}(H^2_{\mathcal{G}})= 3e^{-2\beta_1}\ \langle x_2\omega^{1,5},x_3\omega^{1,6},x_4\omega^{1,7}\rangle + e^{-2\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,6}\rangle + e^{-2\beta_1+\beta_2-\beta_4}\ \langle x_2\omega^{1,7}\rangle + e^{-2\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,5}\rangle + e^{-2\beta_1+\beta_3-\beta_4}\ \langle x_3\omega^{1,7}\rangle + 5e^{-\beta_4}\ \langle -x_1\omega^{1,4}+x_2\omega^{2,4}+x_3\omega^{3,4}, -x_2\omega^{2,4}+x_5\omega^{4,5}, -x_1\omega^{1,4}+x_2\omega^{2,4}+x_6\omega^{4,6}, x_2\omega^{2,4}+x_5\omega^{2,7}, x_1\omega^{1,4}-x_2\omega^{2,4}+x_6\omega^{3,7}\rangle + e^{-2\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,5}\rangle + e^{-2\beta_1-\beta_3+\beta_4}\ \langle x_4\omega^{1,6}\rangle + 5e^{-\beta_2}\ \langle x_1\omega^{1,2}+x_3\omega^{2,3}+x_4\omega^{2,4}, x_3\omega^{2,3}+x_6\omega^{2,6}, -x_1\omega^{1,2}-x_3\omega^{2,3}+x_7\omega^{2,7}, x_5\omega^{2,5}, -x_3\omega^{2,3}+x_6\omega^{3,5}\rangle + 5e^{-\beta_3}\ \langle x_1\omega^{1,3}-x_2\omega^{2,3}+x_4\omega^{3,4}, -x_2\omega^{2,3}+x_5\omega^{3,5}, -x_1\omega^{1,3}+x_2\omega^{2,3}+x_7\omega^{3,7}, x_2\omega^{2,3}+x_5\omega^{2,6}, x_6\omega^{3,6}\rangle + 3e^{-\beta_1}\ \langle x_2\omega^{1,2},x_3\omega^{1,3},x_4\omega^{1,4}\rangle + e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3}\rangle + e^{-\beta_1+\beta_2-\beta_4}\ \langle x_2\omega^{1,4}\rangle + e^{\beta_1+\beta_2-\beta_3-\beta_4}\ \langle x_5\omega^{3,4}\rangle + e^{\beta_2-2\beta_3}\ \langle x_5\omega^{3,6}\rangle + 2e^{\beta_2-\beta_3-\beta_4}\ \langle x_2\omega^{3,4}+x_5\omega^{3,7}, -x_2\omega^{3,4}+x_5\omega^{4,6}\rangle + e^{\beta_2-2\beta_4}\ \langle x_5\omega^{4,7}\rangle + e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2}\rangle + e^{-\beta_1+\beta_3-\beta_4}\ \langle x_3\omega^{1,4}\rangle + e^{\beta_1-\beta_2+\beta_3-\beta_4}\ \langle x_6\omega^{2,4}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_6\omega^{2,5}\rangle + 2e^{-\beta_2+\beta_3-\beta_4}\ \langle x_3\omega^{2,4}+x_6\omega^{2,7}, -x_3\omega^{2,4}+x_6\omega^{4,5}\rangle + e^{\beta_1-\beta_4}\ \langle x_5\omega^{2,4}\rangle + e^{\beta_3-2\beta_4}\ \langle x_6\omega^{4,7}\rangle + e^{-\beta_1-\beta_4}\ \langle 2x_1\omega^{1,7}-x_2(\omega^{2,7}+\omega^{4,5})-x_3(\omega^{3,7}+\omega^{4,6})-2x_4\omega^{4,7}+x_5\omega^{5,7}+x_6\omega^{6,7}\rangle + e^{-\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,2}\rangle + e^{-\beta_1-\beta_3+\beta_4}\ \langle x_4\omega^{1,3}\rangle + e^{\beta_1-\beta_2-\beta_3+\beta_4}\ \langle x_7\omega^{2,3}\rangle + e^{\beta_1-\beta_2}\ \langle x_6\omega^{2,3}\rangle + e^{-2\beta_2+\beta_4}\ \langle x_7\omega^{2,5}\rangle + 2e^{-\beta_2-\beta_3+\beta_4}\ \langle x_4\omega^{2,3}+x_7\omega^{2,6}, -x_4\omega^{2,3}+x_7\omega^{3,5}\rangle + e^{\beta_1-\beta_3}\ \langle x_5\omega^{2,3}\rangle + e^{-2\beta_3+\beta_4}\ \langle x_7\omega^{3,6}\rangle + e^{-\beta_1-\beta_2}\ \langle -2x_1\omega^{1,5}+2x_2\omega^{2,5}+x_3(\omega^{2,6}+\omega^{3,5})+x_4(\omega^{2,7}+\omega^{4,5})+x_6\omega^{5,6}+x_7\omega^{5,7}\rangle + e^{-\beta_1-\beta_3}\ \langle -2x_1\omega^{1,6}+x_2(\omega^{2,6}+\omega^{3,5})+2x_3\omega^{3,6}+x_4(\omega^{3,7}+\omega^{4,6})-x_5\omega^{5,6}+x_7\omega^{6,7}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-\beta_3-\beta_4}+e^{-\beta_1+\beta_2-\beta_3-2\beta_4}+e^{-\beta_1+\beta_2-2\beta_3-\beta_4}+e^{-\beta_1-\beta_2+\beta_3-2\beta_4}+5e^{-\beta_1-\beta_2-\beta_3}+e^{-\beta_1-\beta_2-2\beta_3+\beta_4}+5e^{-\beta_1-\beta_2-\beta_4}+e^{-\beta_1-2\beta_2+\beta_3-\beta_4}+e^{-\beta_1-2\beta_2-\beta_3+\beta_4}+3e^{-\beta_1-2\beta_2}+5e^{-\beta_1-\beta_3-\beta_4}+3e^{-\beta_1-2\beta_3}+3e^{-\beta_1-2\beta_4}+2e^{-2\beta_1+\beta_2-\beta_3-\beta_4}+e^{-2\beta_1+\beta_2-2\beta_3}+e^{-2\beta_1+\beta_2-2\beta_4}+2e^{-2\beta_1-\beta_2+\beta_3-\beta_4}+2e^{-2\beta_1-\beta_2-\beta_3+\beta_4}+5e^{-2\beta_1-\beta_2}+e^{-2\beta_1-2\beta_2+\beta_3}+e^{-2\beta_1-2\beta_2+\beta_4}+e^{-2\beta_1+\beta_3-2\beta_4}+5e^{-2\beta_1-\beta_3}+e^{-2\beta_1-2\beta_3+\beta_4}+5e^{-2\beta_1-\beta_4}+e^{-3\beta_1+\beta_2-\beta_3-\beta_4}+e^{-3\beta_1-\beta_2+\beta_3-\beta_4}+e^{-3\beta_1-\beta_2-\beta_3+\beta_4}+2e^{-3\beta_1-\beta_2}+$

$2e^{-3\beta_1-\beta_3} + 2e^{-3\beta_1-\beta_4} + e^{\beta_2-\beta_3-2\beta_4} + e^{\beta_2-2\beta_3-\beta_4} + e^{-\beta_2+\beta_3-2\beta_4} + 4e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3+\beta_4} + 4e^{-\beta_2-\beta_4} + e^{-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_2-\beta_3+\beta_4} + e^{-2\beta_2} + 4e^{-\beta_3-\beta_4} + e^{-2\beta_3} + e^{-2\beta_4}$.

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-\beta_1+\beta_2-2\beta_3-2\beta_4} + 6e^{-\beta_1-\beta_2-\beta_3-\beta_4} + 3e^{-\beta_1-\beta_2-2\beta_3} + 3e^{-\beta_1-\beta_2-2\beta_4} + e^{-\beta_1-2\beta_2+\beta_3-2\beta_4} + 3e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-2\beta_3+\beta_4} + 3e^{-\beta_1-2\beta_2-\beta_4} + 3e^{-\beta_1-\beta_3-2\beta_4} + 3e^{-\beta_1-2\beta_3-\beta_4} + e^{-2\beta_1+\beta_2-\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-2\beta_3-\beta_4} + e^{-2\beta_1-\beta_2+\beta_3-2\beta_4} + 2e^{-2\beta_1-\beta_2-\beta_3-\beta_4} + 3e^{-2\beta_1-\beta_2-\beta_3} + e^{-2\beta_1-\beta_2-2\beta_3+\beta_4} + e^{-2\beta_1-\beta_2-2\beta_3} + 3e^{-2\beta_1-\beta_2-\beta_4} + e^{-2\beta_1-\beta_2-2\beta_4} + e^{-2\beta_1-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3+\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3} + e^{-2\beta_1-2\beta_2-\beta_4} + 2e^{-2\beta_1-2\beta_2} + 3e^{-2\beta_1-\beta_3-\beta_4} + e^{-2\beta_1-\beta_3-2\beta_4} + e^{-2\beta_1-2\beta_3-\beta_4} + 2e^{-2\beta_1-2\beta_3} + 2e^{-2\beta_1-2\beta_4} + e^{-3\beta_1+\beta_2-\beta_3-2\beta_4} + e^{-3\beta_1+\beta_2-2\beta_3-\beta_4} + e^{-3\beta_1-\beta_2+\beta_3-2\beta_4} + 5e^{-3\beta_1-\beta_2-\beta_3} + e^{-3\beta_1-\beta_2-2\beta_3+\beta_4} + 5e^{-3\beta_1-\beta_2-\beta_4} + e^{-3\beta_1-2\beta_2+\beta_3-\beta_4} + e^{-3\beta_1-2\beta_2-\beta_3+\beta_4} + 2e^{-3\beta_1-2\beta_2} + 5e^{-3\beta_1-\beta_3-\beta_4} + 2e^{-3\beta_1-2\beta_3} + 2e^{-3\beta_1-2\beta_4} + e^{-4\beta_1-\beta_2-\beta_3} + e^{-4\beta_1-\beta_2-\beta_4} + e^{-4\beta_1-\beta_3-\beta_4} + e^{-\beta_2-\beta_3-\beta_4} + e^{-\beta_2-\beta_3-2\beta_4} + e^{-\beta_2-2\beta_3-\beta_4} + e^{-2\beta_2-\beta_3-\beta_4}$.

$\mathrm{ch}(H^5_{\mathcal{G}}) = e^{-\beta_1-\beta_2-\beta_3-2\beta_4} + e^{-\beta_1-\beta_2-2\beta_3-\beta_4} + e^{-\beta_1-\beta_2-2\beta_3-2\beta_4} + e^{-\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-\beta_1-2\beta_2-\beta_3-2\beta_4} + e^{-\beta_1-2\beta_2-2\beta_3-\beta_4} + 3e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} + 3e^{-2\beta_1-\beta_2-2\beta_3-\beta_4} + 3e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} + 2e^{-2\beta_1-2\beta_2-2\beta_3} + 2e^{-2\beta_1-2\beta_2-2\beta_4} + 2e^{-2\beta_1-2\beta_3-2\beta_4} + e^{-3\beta_1+\beta_2-2\beta_3-2\beta_4} + 5e^{-3\beta_1-\beta_2-\beta_3-\beta_4} + 3e^{-3\beta_1-\beta_2-2\beta_3} + 3e^{-3\beta_1-\beta_2-2\beta_4} + e^{-3\beta_1-2\beta_2+\beta_3-2\beta_4} + 3e^{-3\beta_1-2\beta_2-\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_3+\beta_4} + 3e^{-3\beta_1-2\beta_2-\beta_4} + 3e^{-3\beta_1-\beta_3-2\beta_4} + 3e^{-3\beta_1-2\beta_3-\beta_4} + 3e^{-4\beta_1-\beta_2-\beta_3-\beta_4} + e^{-4\beta_1-\beta_2-2\beta_3} + e^{-4\beta_1-\beta_2-2\beta_4} + e^{-4\beta_1-2\beta_2-\beta_3} + e^{-4\beta_1-2\beta_2-\beta_4} + e^{-4\beta_1-\beta_3-2\beta_4} + e^{-4\beta_1-2\beta_3-\beta_4}$.

$\mathrm{ch}(H^6_{\mathcal{G}}) = e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4} + e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4} + e^{-2\beta_1-2\beta_2-2\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2-\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2-2\beta_3-\beta_4} + e^{-3\beta_1-\beta_2-2\beta_3-2\beta_4} + e^{-3\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-3\beta_1-2\beta_2-\beta_3-2\beta_4} + e^{-3\beta_1-2\beta_2-2\beta_3-\beta_4} + e^{-3\beta_1-2\beta_2-2\beta_3} + e^{-3\beta_1-2\beta_2-2\beta_4} + e^{-3\beta_1-2\beta_3-2\beta_4} + 2e^{-4\beta_1-\beta_2-\beta_3-2\beta_4} + 2e^{-4\beta_1-\beta_2-2\beta_3-\beta_4} + 2e^{-4\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-4\beta_1-2\beta_2-2\beta_3} + e^{-4\beta_1-2\beta_2-2\beta_4} + e^{-4\beta_1-2\beta_3-2\beta_4}$.

$\mathrm{ch}(H^7_{\mathcal{G}}) = e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4} + e^{-4\beta_1-\beta_2-2\beta_3-2\beta_4} + e^{-4\beta_1-2\beta_2-\beta_3-2\beta_4} + e^{-4\beta_1-2\beta_2-2\beta_3-\beta_4}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 25, 78, 157, 178, 123, 46, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(3, 21, 54, 88, 90, 56, 22, 4)$

S=$2(2\beta_1 + \beta_2 + \beta_3 + \beta_4)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 15, 24, 29, 18, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 4, 12, 18, 18, 12, 4, 1)$

$d\omega^5 = -\omega^{1,2}$; $d\omega^6 = -\omega^{1,3}$; $d\omega^7 = -\omega^{1,4}$; $d\omega^{2,6} = -\omega^{1,2,3}$; $d\omega^{2,7} = -\omega^{1,2,4}$; $d\omega^{3,5} = \omega^{1,2,3}$; $d\omega^{3,7} = -\omega^{1,3,4}$; $d\omega^{4,5} = \omega^{1,2,4}$; $d\omega^{4,6} = \omega^{1,3,4}$;

$d\omega^{5,6} = -\omega^{1,2,6} + \omega^{1,3,5}$; $d\omega^{5,7} = -\omega^{1,2,7} + \omega^{1,4,5}$; $d\omega^{6,7} = -\omega^{1,3,7} + \omega^{1,4,6}$; $d\omega^{2,3,7} = -\omega^{1,2,3,4}$; $d\omega^{2,4,6} = \omega^{1,2,3,4}$; $d\omega^{2,5,6} = \omega^{1,2,3,5}$; $d\omega^{2,5,7} = \omega^{1,2,4,5}$; $d\omega^{2,6,7} = -\omega^{1,2,3,7} + \omega^{1,2,4,6}$; $d\omega^{3,4,5} = -\omega^{1,2,3,4}$; $d\omega^{3,5,6} = \omega^{1,2,3,6}$; $d\omega^{3,5,7} = \omega^{1,2,3,7}+\omega^{1,3,4,5}$; $d\omega^{3,6,7} = \omega^{1,3,4,6}$; $d\omega^{4,5,6} = \omega^{1,2,4,6}-\omega^{1,3,4,5}$; $d\omega^{4,5,7} = \omega^{1,2,4,7}$; $d\omega^{4,6,7} = \omega^{1,3,4,7}$; $d\omega^{5,6,7} = -\omega^{1,2,6,7} + \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $d\omega^{2,3,5,7} = \omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7} = \omega^{1,2,3,4,6}$; $d\omega^{2,4,5,6} = -\omega^{1,2,3,4,5}$; $d\omega^{2,4,6,7} = \omega^{1,2,3,4,7}$; $d\omega^{2,5,6,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $d\omega^{3,4,5,6} = -\omega^{1,2,3,4,6}$; $d\omega^{3,4,5,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $d\omega^{4,5,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $d\omega^{2,3,5,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{2,4,5,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = -\omega^{1,2,3,4,6,7}$.

$\mathrm{ch}(H^1_{\mathbb{C}})= e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_4} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})= e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle+e^{-2\beta_1-\beta_3} \langle\omega^{1,6}\rangle+e^{-2\beta_1-\beta_4} \langle\omega^{1,7}\rangle+e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle+ e^{-\beta_2-\beta_4} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_3-\beta_4} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6}+\omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle + e^{-\beta_1-\beta_2-\beta_4} \langle\omega^{2,7}+\omega^{4,5}\rangle + e^{-\beta_1-\beta_3-\beta_4} \langle\omega^{3,7}+\omega^{4,6}\rangle + e^{-\beta_1-2\beta_4} \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})= e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_4} \langle\omega^{1,2,7}\rangle + e^{-2\beta_1-\beta_3-\beta_4} \langle\omega^{1,3,7}\rangle + e^{-2\beta_1-2\beta_4} \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-\beta_2-\beta_4} \langle\omega^{1,5,7}\rangle + e^{-3\beta_1-\beta_3-\beta_4} \langle\omega^{1,6,7}\rangle + e^{-\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-\beta_4} \langle\omega^{2,4,5}\rangle+e^{-\beta_1-\beta_2-2\beta_4} \langle\omega^{2,4,7}\rangle+2e^{-\beta_1-\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,7}-\omega^{3,4,5}, \omega^{2,4,6}+ \omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_3-\beta_4} \langle\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_3-2\beta_4} \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_4} \langle\omega^{1,2,5,7}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,6,7}, \omega^{1,3,5,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle\omega^{1,3,5,6}\rangle + e^{-3\beta_1-2\beta_3-\beta_4} \langle\omega^{1,3,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_4} \langle\omega^{1,4,5,7}\rangle + e^{-3\beta_1-\beta_3-2\beta_4} \langle\omega^{1,4,6,7}\rangle + e^{-4\beta_1-\beta_2-\beta_3-\beta_4} \langle\omega^{1,5,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,4,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3-\beta_4} \langle\omega^{2,3,4,6}\rangle + e^{-\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,5,7}+ \omega^{2,4,5,6}\rangle + e^{-2\beta_1-2\beta_2-2\beta_4} \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-\beta_4} \langle\omega^{2,3,6,7}+\omega^{3,4,5,6}\rangle + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{2,4,6,7}+\omega^{3,4,5,7}\rangle + e^{-2\beta_1-2\beta_3-2\beta_4} \langle\omega^{3,4,6,7}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})= e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6}\rangle + e^{-3\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,3,5,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_4} \langle\omega^{1,2,4,5,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3-\beta_4} \langle\omega^{1,2,3,6,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{1,2,4,6,7}\rangle + e^{-3\beta_1-2\beta_3-2\beta_4} \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3-\beta_4} \langle\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{1,4,5,6,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4} \langle\omega^{2,3,4,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{2,3,4,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})= e^{-4\beta_1-2\beta_2-2\beta_3-\beta_4} \langle\omega^{1,2,3,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3-2\beta_4} \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})= e^{-4\beta_1-2\beta_2-2\beta_3-2\beta_4} \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$$P(Y) \;=\; (1+3Y+9Y^2+9Y^3+9Y^4+3Y^5+Y^6)(1+Y).$$

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathfrak{A}}$'s $0 \le j \le 7$: $(1,4,15,24,29,18,7,1)$

Dimensions of $H^j_{\mathfrak{A}}$'s $0 \le j \le 7$: $(1,4,12,18,18,12,4,1)$

Total dimension of trivial cohomology: 70.

Length associated polynomial: 70.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^5$; $d^\star\omega^{1,3} = -\omega^6$; $d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{1,2,3} = -\omega^{2,6}+\omega^{3,5}$; $d^\star\omega^{1,2,4} = -\omega^{2,7}+\omega^{4,5}$; $d^\star\omega^{1,2,6} = -\omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{5,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}+\omega^{4,6}$; $d^\star\omega^{1,3,5} = \omega^{5,6}$; $d^\star\omega^{1,3,7} = -\omega^{6,7}$; $d^\star\omega^{1,4,5} = \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,7}+\omega^{2,4,6}-\omega^{3,4,5}$; $d^\star\omega^{1,2,3,5} = \omega^{2,5,6}$; $d^\star\omega^{1,2,3,6} = \omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = -\omega^{2,6,7}+\omega^{3,5,7}$; $d^\star\omega^{1,2,4,5} = \omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{2,6,7}+\omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,5,7}$; $d^\star\omega^{1,2,6,7} = -\omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,7}-\omega^{4,5,6}$; $d^\star\omega^{1,3,4,6} = \omega^{3,6,7}$; $d^\star\omega^{1,3,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,3,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{2,3,5,7}-\omega^{2,4,5,6}$; $d^\star\omega^{1,2,3,4,6} = \omega^{2,3,6,7}-\omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = \omega^{2,4,6,7}-\omega^{3,4,5,7}$; $d^\star\omega^{1,2,3,5,7} = \omega^{2,5,6,7}$; $d^\star\omega^{1,2,3,6,7} = \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{2,5,6,7}$; $d^\star\omega^{1,2,4,6,7} = \omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = -\omega^{3,5,6,7}$; $d^\star\omega^{1,3,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{2,3,5,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{2,4,5,6,7}$; $d^\star\omega^{1,2,3,4,6,7} = -\omega^{3,4,5,6,7}$;

$\Delta\omega^5 = \omega^5$; $\Delta\omega^6 = \omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2}$; $\Delta\omega^{1,3} = \omega^{1,3}$; $\Delta\omega^{1,4} = \omega^{1,4}$; $\Delta\omega^{2,6} = \omega^{2,6}-\omega^{3,5}$; $\Delta\omega^{2,7} = \omega^{2,7}-\omega^{4,5}$; $\Delta\omega^{3,5} = -\omega^{2,6}+\omega^{3,5}$; $\Delta\omega^{3,7} = \omega^{3,7}-\omega^{4,6}$; $\Delta\omega^{4,5} = -\omega^{2,7}+\omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7}+\omega^{4,6}$; $\Delta\omega^{5,6} = 2\omega^{5,6}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,3} = 2\omega^{1,2,3}$; $\Delta\omega^{1,2,4} = 2\omega^{1,2,4}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6}-\omega^{1,3,5}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}-\omega^{1,4,5}$; $\Delta\omega^{1,3,4} = 2\omega^{1,3,4}$; $\Delta\omega^{1,3,5} = -\omega^{1,2,6}+\omega^{1,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}-\omega^{1,4,6}$; $\Delta\omega^{1,4,5} = -\omega^{1,2,7}+\omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7}+\omega^{1,4,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}-\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{2,3,7}+\omega^{2,4,6}-\omega^{3,4,5}$; $\Delta\omega^{2,5,6} = \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{2,5,7}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}-\omega^{3,5,7}+\omega^{4,5,6}$; $\Delta\omega^{3,4,5} = \omega^{2,3,7}-\omega^{2,4,6}+\omega^{3,4,5}$; $\Delta\omega^{3,5,6} = \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = -\omega^{2,6,7}+2\omega^{3,5,7}-\omega^{4,5,6}$; $\Delta\omega^{3,6,7} = \omega^{3,6,7}$; $\Delta\omega^{4,5,6} = \omega^{2,6,7}-\omega^{3,5,7}+2\omega^{4,5,6}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = 3\omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = 3\omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7}-\omega^{1,2,4,6}+\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,5} = \omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7}+2\omega^{1,2,4,6}-\omega^{1,3,4,5}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7}$; $\Delta\omega^{1,2,6,7} = \omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6}$; $\Delta\omega^{1,3,4,5} = \omega^{1,2,3,7}-\omega^{1,2,4,6}+2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,3,4,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7}$; $\Delta\omega^{1,3,5,7} = -\omega^{1,2,6,7}+\omega^{1,3,5,7}-\omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,6} = \omega^{1,2,6,7}-\omega^{1,3,5,7}+\omega^{1,4,5,6}$;

$\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7} - \omega^{3,4,5,6}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7} - \omega^{3,4,5,7}$; $\Delta\omega^{2,5,6,7} = 2\omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{2,3,6,7} + \omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{2,4,6,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,5,6,7} = 2\omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = 2\omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,4,7} = 2\omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = \omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7} - \omega^{1,3,4,5,6}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{1,3,4,5,7}$; $\Delta\omega^{1,3,4,5,6} = -\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,5,6,7} = \omega^{2,3,5,6,7}$; $\Delta\omega^{2,4,5,6,7} = \omega^{2,4,5,6,7}$; $\Delta\omega^{3,4,5,6,7} = \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = \omega^{1,2,3,4,5,7}$; $\Delta\omega^{1,2,3,4,6,7} = \omega^{1,2,3,4,6,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_4} \langle\omega^4\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-2\beta_1-\beta_2} \langle\omega^{1,5}\rangle + e^{-2\beta_1-\beta_3} \langle\omega^{1,6}\rangle + e^{-2\beta_1-\beta_4} \langle\omega^{1,7}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_2-\beta_4} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2} \langle\omega^{2,5}\rangle + e^{-\beta_3-\beta_4} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2-\beta_3} \langle\omega^{2,6} + \omega^{3,5}\rangle + e^{-\beta_1-2\beta_3} \langle\omega^{3,6}\rangle + e^{-\beta_1-\beta_2-\beta_4} \langle\omega^{2,7} + \omega^{4,5}\rangle + e^{-\beta_1-\beta_3-\beta_4} \langle\omega^{3,7} + \omega^{4,6}\rangle + e^{-\beta_1-2\beta_4} \langle\omega^{4,7}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-2\beta_1-2\beta_2} \langle\omega^{1,2,5}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,2,6} + \omega^{1,3,5}\rangle + e^{-2\beta_1-2\beta_3} \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_4} \langle\omega^{1,2,7} + \omega^{1,4,5}\rangle + e^{-2\beta_1-\beta_3-\beta_4} \langle\omega^{1,3,7} + \omega^{1,4,6}\rangle + e^{-2\beta_1-2\beta_4} \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-\beta_2-\beta_4} \langle\omega^{1,5,7}\rangle + e^{-3\beta_1-\beta_3-\beta_4} \langle\omega^{1,6,7}\rangle + e^{-\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-2\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-\beta_4} \langle\omega^{2,4,5}\rangle + e^{-\beta_1-\beta_2-2\beta_4} \langle\omega^{2,4,7}\rangle + 2e^{-\beta_1-\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,7} - \omega^{3,4,5}, \omega^{2,4,6} + \omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_3-\beta_4} \langle\omega^{3,4,6}\rangle + e^{-\beta_1-\beta_3-2\beta_4} \langle\omega^{3,4,7}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}}) = e^{-2\beta_1-2\beta_3-2\beta_4} \langle\omega^{3,4,6,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} \langle(-(\omega^{2,4,6,7} + \omega^{3,4,5,7}))/2\rangle + e^{-2\beta_1-2\beta_2-2\beta_4} \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-\beta_2-2\beta_3-\beta_4} \langle(\omega^{2,3,6,7} + \omega^{3,4,5,6})/2\rangle + e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} \langle(-(\omega^{2,3,5,7} + \omega^{2,4,5,6}))/2\rangle + e^{-2\beta_1-2\beta_2-2\beta_3} \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{2,3,4,7}\rangle + e^{-\beta_1-\beta_2-2\beta_3-\beta_4} \langle-\omega^{2,3,4,6}\rangle + e^{-\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,4,5}\rangle + e^{-4\beta_1-\beta_2-\beta_3-\beta_4} \langle-\omega^{1,5,6,7}\rangle + e^{-3\beta_1-\beta_3-2\beta_4} \langle\omega^{1,4,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_4} \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-2\beta_3-\beta_4} \langle-\omega^{1,3,6,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3} \langle-\omega^{1,3,5,6}\rangle + 2e^{-3\beta_1-\beta_2-\beta_3-\beta_4} \langle(-(\omega^{1,2,6,7} - \omega^{1,3,5,7} - 2\omega^{1,4,5,6}))/3, (\omega^{1,2,6,7} + 2\omega^{1,3,5,7} + \omega^{1,4,5,6})/3\rangle + e^{-3\beta_1-2\beta_2-\beta_4} \langle-\omega^{1,2,5,7}\rangle + e^{-3\beta_1-2\beta_2-\beta_3} \langle\omega^{1,2,5,6}\rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}}) = e^{-2\beta_1-\beta_2-2\beta_3-2\beta_4} \langle-\omega^{2,3,4,6,7}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{2,3,4,5,7}\rangle + e^{-2\beta_1-2\beta_2-2\beta_3-\beta_4} \langle-\omega^{2,3,4,5,6}\rangle + e^{-4\beta_1-\beta_2-\beta_3-2\beta_4} \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3-\beta_4} \langle-\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-2\beta_3-2\beta_4} \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3-\beta_4} \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-\beta_2-\beta_3-2\beta_4} \langle(-\omega^{1,2,4,6,7} - \omega^{1,3,4,5,7})/2\rangle + e^{-3\beta_1-2\beta_2-2\beta_4} \langle\omega^{1,2,4,5,7}\rangle + e^{-3\beta_1-\beta_2-2\beta_3-\beta_4} \langle(\omega^{1,2,3,6,7} + \omega^{1,3,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2-\beta_3-\beta_4} \langle(-\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2-2\beta_3} \langle\omega^{1,2,3,5,6}\rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}}) = e^{-3\beta_1-2\beta_2-2\beta_3-2\beta_4} \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-\beta_2-2\beta_3-2\beta_4} \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-\beta_3-2\beta_4} \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-2\beta_2-2\beta_3-\beta_4} \langle-\omega^{1,2,3,5,6,7}\rangle$.

$$\mathrm{ch}(H^7_{\mathfrak{C}}) = e^{-4\beta_1-2\beta_2-2\beta_3-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle.$$
$$P(Y) = (1+3Y+9Y^2+9Y^3+9Y^4+3Y^5+Y^6)(1+Y).$$

5.6.3 Algebra $\mathcal{G}_{7,4.3}$

Adjoint cohomology

$[x_1,x_2]=x_6$; $[x_3,x_5]=x_6$; $[x_4,x_5]=x_7$.

$$\delta=\begin{pmatrix} \xi_1^1 & \xi_2^1 & 0 & 0 & \xi_5^1 & 0 & 0\\ \xi_1^2 & \xi_2^2 & 0 & 0 & \xi_5^2 & 0 & 0\\ -\xi_5^2 & \xi_5^1 & \xi_3^3 & \xi_4^3 & \xi_5^3 & 0 & 0\\ 0 & 0 & 0 & \xi_4^4 & \xi_5^4 & 0 & 0\\ 0 & 0 & 0 & 0 & \xi_1^1+\xi_2^2-\xi_3^3 & 0 & 0\\ \xi_1^6 & \xi_2^6 & \xi_3^6 & \xi_4^6 & \xi_5^6 & \xi_1^1+\xi_2^2 & \xi_4^3\\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_5^7 & 0 & \xi_1^1+\xi_2^2-\xi_3^3+\xi_4^4 \end{pmatrix}$$

$ad\ x_1=\delta_2^6, ad\ x_2=-\delta_1^6, ad\ x_3=\delta_5^6, ad\ x_4=\delta_5^7, ad\ x_5=-(\delta_3^6+\delta_4^7), ad\ x_6=0, ad\ x_7=0.$

Maximal torus : $T=\langle t_1,t_2,t_3,t_4\rangle$.

$\mathrm{ch}(\mathcal{G})=e^{\beta_1}\ \langle x_1\rangle+e^{\beta_2}\ \langle x_2\rangle+e^{\beta_3}\ \langle x_3\rangle+e^{\beta_4}\ \langle x_4\rangle+e^{\beta_1+\beta_2-\beta_3}\ \langle x_5\rangle+e^{\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_2-\beta_3+\beta_4}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})=e^{\beta_1+\beta_2}\ \langle x_6\rangle+e^{\beta_1+\beta_2-\beta_3+\beta_4}\ \langle x_7\rangle$.

$\mathrm{ch}(Z^1_{\mathcal{G}})=e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_5^2\rangle+e^{-\beta_2+\beta_3}\ \langle\delta_5^1\rangle+e^{-\beta_1-\beta_2+2\beta_3}\ \langle\delta_5^3\rangle+e^{-\beta_1-\beta_2+\beta_3+\beta_4}\ \langle\delta_5^4\rangle+e^{\beta_2}\ \langle\delta_1^6\rangle+e^{\beta_1}\ \langle\delta_2^6\rangle+2e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6,\delta_4^7\rangle+e^{\beta_1+\beta_2-\beta_4}\ \langle\delta_4^6\rangle+e^{\beta_3}\ \langle\delta_5^6\rangle+e^{\beta_3-\beta_4}\ \langle\delta_4^3\rangle+e^{\beta_2-\beta_3+\beta_4}\ \langle\delta_1^7\rangle+e^{\beta_1-\beta_3+\beta_4}\ \langle\delta_2^7\rangle+e^{\beta_1+\beta_2-2\beta_3+\beta_4}\ \langle\delta_3^7\rangle+e^{\beta_4}\ \langle\delta_5^7\rangle+4e^0\ \langle t_1,t_2,t_3,t_4\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $Z^1_{\mathcal{G}}$: $[\delta_2^1,\delta_1^2]=-t_2+t_1$; $[\delta_2^1,\delta_5^2]=\delta_5^1$; $[\delta_2^1,\delta_1^6]=-\delta_2^6$; $[\delta_2^1,\delta_1^7]=-\delta_2^7$; $[\delta_1^2,\delta_5^1]=\delta_5^2$; $[\delta_1^2,\delta_2^6]=-\delta_1^6$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_5^2,\delta_5^1]=-2\delta_5^3$; $[\delta_5^2,\delta_2^6]=-\delta_5^6$; $[\delta_5^2,\delta_3^6]=\delta_1^6$; $[\delta_5^2,\delta_2^7]=-\delta_5^7$; $[\delta_5^2,\delta_3^7]=\delta_1^7$; $[\delta_5^1,\delta_1^6]=-\delta_5^6$; $[\delta_5^1,\delta_3^6]=-\delta_2^6$; $[\delta_5^1,\delta_1^7]=-\delta_5^7$; $[\delta_5^1,\delta_3^7]=-\delta_2^7$; $[\delta_5^3,\delta_3^6]=-\delta_5^6$; $[\delta_5^3,\delta_3^7]=-\delta_5^7$; $[\delta_5^4,\delta_4^6]=-\delta_5^6$; $[\delta_5^4,\delta_4^3]=-\delta_5^3$; $[\delta_5^4,\delta_4^7]=-\delta_5^7$; $[\delta_3^6,\delta_4^3]=\delta_4^6$; $[\delta_4^3,\delta_1^7]=\delta_1^6$; $[\delta_4^3,\delta_2^7]=\delta_2^6$; $[\delta_4^3,\delta_3^7]=-\delta_4^7+\delta_3^6$; $[\delta_4^3,\delta_4^7]=\delta_4^6$; $[\delta_4^3,\delta_5^7]=\delta_5^6$.

$\mathrm{ch}(H^1_{\mathcal{G}})=e^{\beta_1-\beta_2}\ \langle\delta_2^1\rangle+e^{-\beta_1+\beta_2}\ \langle\delta_1^2\rangle+e^{-\beta_1+\beta_3}\ \langle\delta_5^2\rangle+e^{-\beta_2+\beta_3}\ \langle\delta_5^1\rangle+e^{-\beta_1-\beta_2+2\beta_3}\ \langle\delta_5^3\rangle+e^{-\beta_1-\beta_2+\beta_3+\beta_4}\ \langle\delta_5^4\rangle+e^{\beta_1+\beta_2-\beta_4}\ \langle\delta_4^6\rangle+e^{\beta_3-\beta_4}\ \langle\delta_4^3\rangle+e^{\beta_2-\beta_3+\beta_4}\ \langle\delta_1^7\rangle+e^{\beta_1-\beta_3+\beta_4}\ \langle\delta_2^7\rangle+e^{\beta_1+\beta_2-2\beta_3+\beta_4}\ \langle\delta_3^7\rangle+e^{\beta_1+\beta_2-\beta_3}\ \langle\delta_3^6\rangle+4e^0\ \langle t_1,t_2,t_3,t_4\rangle$.

Comm. relations among the δ_i^j's $\ i\neq j$ in $H^1_{\mathcal{G}}$: $[\delta_2^1,\delta_1^2]=-t_2+t_1$; $[\delta_2^1,\delta_5^2]=\delta_5^1$; $[\delta_2^1,\delta_1^7]=-\delta_2^7$; $[\delta_1^2,\delta_5^1]=\delta_5^2$; $[\delta_1^2,\delta_2^7]=-\delta_1^7$; $[\delta_5^2,\delta_5^1]=-2\delta_5^3$; $[\delta_5^2,\delta_3^7]=\delta_1^7$; $[\delta_5^1,\delta_3^7]=-\delta_2^7$; $[\delta_5^4,\delta_4^3]=-\delta_5^3$; $[\delta_4^3,\delta_3^7]=2\delta_3^6$; $[\delta_4^3,\delta_3^6]=-\delta_4^6$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_3} \langle x_1\omega^{2,3}\rangle + e^{\beta_1-\beta_2-\beta_4} \langle x_1\omega^{2,4}\rangle + e^{-2\beta_2+\beta_3} \langle x_1\omega^{2,5}\rangle + e^{-\beta_1+\beta_2-\beta_3} \langle x_2\omega^{1,3}\rangle + e^{-\beta_1+\beta_2-\beta_4} \langle x_2\omega^{1,4}\rangle + e^{-2\beta_1+\beta_3} \langle x_2\omega^{1,5}\rangle + 4e^{-\beta_3} \langle -x_1\omega^{1,3}+x_2\omega^{2,3}, x_4\omega^{3,4}+x_7\omega^{3,7}, -x_4\omega^{3,4}+x_5\omega^{3,5}, -x_1\omega^{1,3}+x_6\omega^{3,6}\rangle + 4e^{-\beta_4} \langle -x_1\omega^{1,4}+x_2\omega^{2,4}, -x_1\omega^{1,4} - x_3\omega^{3,4} + x_6\omega^{4,6}, x_3\omega^{3,4} + x_5\omega^{4,5}, x_3\omega^{3,4} + x_6\omega^{3,7}\rangle + 4e^{-\beta_1-\beta_2+\beta_3} \langle -x_1\omega^{1,5}+x_2\omega^{2,5}, x_1\omega^{1,5}+x_3\omega^{1,2}, x_3\omega^{3,5}, x_4\omega^{4,5}\rangle + 4e^{-\beta_1} \langle x_2\omega^{3,5}+x_3\omega^{1,3}, x_2\omega^{3,5}-x_4\omega^{1,4}+x_5\omega^{1,5}, x_2\omega^{1,2}, x_4\omega^{1,4}+x_7\omega^{1,7}\rangle + e^{-\beta_1+\beta_3-\beta_4} \langle x_2\omega^{4,5}+x_3\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2+2\beta_3} \langle x_3\omega^{1,5}\rangle + 4e^{-\beta_2} \langle -x_1\omega^{3,5} + x_3\omega^{2,3}, -x_1\omega^{3,5} - x_4\omega^{2,4} + x_5\omega^{2,5}, x_1\omega^{1,2}, x_4\omega^{2,4} + x_7\omega^{2,7}\rangle + e^{-\beta_2+\beta_3-\beta_4} \langle -x_1\omega^{4,5} + x_3\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2+2\beta_3} \langle x_3\omega^{2,5}\rangle + e^{-2\beta_1-2\beta_2+3\beta_3-\beta_4} \langle x_3\omega^{5,7}\rangle + e^{-2\beta_1-\beta_2+\beta_3+\beta_4} \langle x_4\omega^{1,5}\rangle + e^{-\beta_1-2\beta_2+\beta_3+\beta_4} \langle x_4\omega^{2,5}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3} \langle x_4\omega^{5,7}\rangle + e^{\beta_2-\beta_3-\beta_4} \langle x_2\omega^{3,4}+x_5\omega^{1,4}\rangle + e^{\beta_1-\beta_3-\beta_4} \langle -x_1\omega^{3,4} + x_5\omega^{2,4}\rangle + e^{\beta_1+\beta_2-2\beta_3-\beta_4} \langle x_5\omega^{3,4}\rangle + e^{\beta_3-2\beta_4} \langle x_6\omega^{4,7}\rangle + e^{-\beta_1-\beta_2+2\beta_3-\beta_4} \langle x_3\omega^{4,5}\rangle + e^{\beta_2-2\beta_3+\beta_4} \langle x_7\omega^{1,3}\rangle + e^{-\beta_1-\beta_3+\beta_4} \langle x_4\omega^{1,3} + x_7\omega^{1,6}\rangle + e^{\beta_1-2\beta_3+\beta_4} \langle x_7\omega^{2,3}\rangle + e^{-\beta_2-\beta_3+\beta_4} \langle x_4\omega^{2,3} + x_7\omega^{2,6}\rangle + e^{-\beta_1-\beta_2+\beta_4} \langle x_4\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}})=e^{\beta_1-\beta_2-\beta_3-\beta_4} + e^{\beta_1-2\beta_3-\beta_4} + e^{-\beta_1+\beta_2-\beta_3-\beta_4} + 3e^{-\beta_1-\beta_2+\beta_3-2\beta_4} + e^{-\beta_1-\beta_2-\beta_3+\beta_4} + e^{-\beta_1-\beta_2-\beta_3} + 2e^{-\beta_1-\beta_2-\beta_4} + 2e^{-\beta_1-\beta_2} + e^{-\beta_1-2\beta_2+2\beta_3-2\beta_4} + 2e^{-\beta_1-2\beta_2+\beta_3-\beta_4} + 2e^{-\beta_1-2\beta_2+\beta_3} + e^{-\beta_1-2\beta_2+\beta_4} + 2e^{-\beta_1-2\beta_2} + e^{-\beta_1-3\beta_2+2\beta_3-\beta_4} + e^{-\beta_1-3\beta_2+\beta_3} + 3e^{-\beta_1-\beta_3} + e^{-\beta_1-2\beta_3+\beta_4} + 3e^{-\beta_1-\beta_4} + e^{-\beta_1-2\beta_4} + e^{-2\beta_1-\beta_2+2\beta_3-2\beta_4} + 2e^{-2\beta_1-\beta_2+\beta_3-\beta_4} + 2e^{-2\beta_1-\beta_2+\beta_3} + e^{-2\beta_1-\beta_2+\beta_4} + 2e^{-2\beta_1-\beta_2} + e^{-2\beta_1-2\beta_2+3\beta_3-2\beta_4} + 4e^{-2\beta_1-2\beta_2+2\beta_3-\beta_4} + 3e^{-2\beta_1-2\beta_2+\beta_3} + e^{-2\beta_1-3\beta_2+3\beta_3-\beta_4} + 2e^{-2\beta_1-3\beta_2+2\beta_3} + e^{-2\beta_1-3\beta_2+\beta_3+\beta_4} + e^{-2\beta_1+\beta_3-2\beta_4} + e^{-2\beta_1-\beta_3} + e^{-2\beta_1-\beta_4} + e^{-3\beta_1-\beta_2+2\beta_3-\beta_4} + e^{-3\beta_1-\beta_2+\beta_3} + e^{-3\beta_1-2\beta_2+3\beta_3-\beta_4} + 2e^{-3\beta_1-2\beta_2+2\beta_3} + e^{-3\beta_1-2\beta_2+\beta_3+\beta_4} + e^{\beta_2-2\beta_3-\beta_4} + 3e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3+\beta_4} + 3e^{-\beta_2-\beta_4} + e^{-\beta_2-2\beta_4} + e^{-2\beta_2+\beta_3-2\beta_4} + e^{-2\beta_2-\beta_3} + e^{-2\beta_2-\beta_4} + 3e^{-\beta_3-\beta_4} + e^{-\beta_3-2\beta_4}.$

$\mathrm{ch}(H^4_{\mathcal{G}})=2e^{-\beta_1-\beta_2-\beta_3-\beta_4} + e^{-\beta_1-\beta_2-2\beta_3+\beta_4} + 3e^{-\beta_1-\beta_2-2\beta_4} + 3e^{-\beta_1-2\beta_2+\beta_3-2\beta_4} + 3e^{-\beta_1-2\beta_2-\beta_3} + 4e^{-\beta_1-2\beta_2-\beta_4} + e^{-\beta_1-3\beta_2+2\beta_3-2\beta_4} + e^{-\beta_1-3\beta_2+\beta_3-\beta_4} + e^{-\beta_1-3\beta_2} + e^{-\beta_1-\beta_3-\beta_4} + e^{-\beta_1-\beta_3-2\beta_4} + e^{-\beta_1-2\beta_3-\beta_4} + 3e^{-2\beta_1-\beta_2+\beta_3-2\beta_4} + 3e^{-2\beta_1-\beta_2-\beta_3} + 4e^{-2\beta_1-\beta_2-\beta_4} + 3e^{-2\beta_1-2\beta_2+2\beta_3-2\beta_4} + 4e^{-2\beta_1-2\beta_2+\beta_3-\beta_4} + 4e^{-2\beta_1-2\beta_2} + e^{-2\beta_1-3\beta_2+3\beta_3-2\beta_4} + 3e^{-2\beta_1-3\beta_2+2\beta_3-\beta_4} + 4e^{-2\beta_1-3\beta_2+\beta_3} + e^{-2\beta_1-3\beta_2+\beta_4} + e^{-2\beta_1-4\beta_2+2\beta_3-\beta_4} + e^{-2\beta_1-\beta_3-\beta_4} + e^{-2\beta_1-2\beta_4} + e^{-3\beta_1-\beta_2+2\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2+\beta_3-\beta_4} + e^{-3\beta_1-\beta_2} + e^{-3\beta_1-2\beta_2+3\beta_3-2\beta_4} + 3e^{-3\beta_1-2\beta_2+2\beta_3-\beta_4} + 4e^{-3\beta_1-2\beta_2+\beta_3} + e^{-3\beta_1-2\beta_2+\beta_4} + e^{-3\beta_1-3\beta_2+2\beta_3-\beta_4} + e^{-3\beta_1-3\beta_2+2\beta_3} + e^{-3\beta_1-3\beta_2+\beta_3+\beta_4} + e^{-3\beta_1-4\beta_2+3\beta_3-\beta_4} + e^{-3\beta_1-4\beta_2+2\beta_3} + e^{-4\beta_1-2\beta_2+2\beta_3-\beta_4} + e^{-4\beta_1-3\beta_2+3\beta_3-\beta_4} + e^{-4\beta_1-3\beta_2+2\beta_3} + e^{-\beta_2-\beta_3-\beta_4} + e^{-\beta_2-\beta_3-2\beta_4} + e^{-\beta_2-2\beta_3-\beta_4} + e^{-2\beta_2-\beta_3-\beta_4} + e^{-2\beta_2-2\beta_4}.$

$\mathrm{ch}(H^5_{\mathcal{G}})=e^{-\beta_1-\beta_2-2\beta_3-\beta_4} + 2e^{-\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-\beta_1-2\beta_2-\beta_3-2\beta_4} + e^{-\beta_1-2\beta_2-2\beta_4} + e^{-\beta_1-3\beta_2-2\beta_4} + 2e^{-2\beta_1-\beta_2-\beta_3-\beta_4} + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4} + e^{-2\beta_1-\beta_2-2\beta_4} + e^{-2\beta_1-2\beta_2+\beta_3-2\beta_4} + 2e^{-2\beta_1-2\beta_2-\beta_3} + 2e^{-2\beta_1-2\beta_2-\beta_4} + 2e^{-2\beta_1-2\beta_2-2\beta_4} +$

$e^{-2\beta_1-3\beta_2+2\beta_3-2\beta_4} + e^{-2\beta_1-3\beta_2+\beta_3-\beta_4} + 2e^{-2\beta_1-3\beta_2+\beta_3-2\beta_4} + e^{-2\beta_1-3\beta_2-\beta_4} + 2e^{-2\beta_1-3\beta_2} + e^{-2\beta_1-4\beta_2+2\beta_3-2\beta_4} + e^{-2\beta_1-4\beta_2+\beta_3-\beta_4} + e^{-3\beta_1-\beta_2-2\beta_4} + e^{-3\beta_1-2\beta_2+2\beta_3-2\beta_4} + e^{-3\beta_1-2\beta_2+\beta_3-\beta_4} + 2e^{-3\beta_1-2\beta_2+\beta_3-2\beta_4} + e^{-3\beta_1-2\beta_2-\beta_4} + 2e^{-3\beta_1-2\beta_2} + 2e^{-3\beta_1-3\beta_2+2\beta_3-2\beta_4} + 2e^{-3\beta_1-3\beta_2+\beta_3-\beta_4} + 2e^{-3\beta_1-3\beta_2+\beta_3} + e^{-3\beta_1-3\beta_2+\beta_4} + e^{-3\beta_1-4\beta_2+3\beta_3-2\beta_4} + 3e^{-3\beta_1-4\beta_2+2\beta_3-\beta_4} + e^{-3\beta_1-4\beta_2+\beta_3} + e^{-4\beta_1-2\beta_2+2\beta_3-2\beta_4} + e^{-4\beta_1-2\beta_2+\beta_3-\beta_4} + e^{-4\beta_1-3\beta_2+3\beta_3-2\beta_4} + 3e^{-4\beta_1-3\beta_2+2\beta_3-\beta_4} + e^{-4\beta_1-3\beta_2+\beta_3} + e^{-4\beta_1-4\beta_2+3\beta_3-\beta_4} + e^{-4\beta_1-4\beta_2+2\beta_3}$.

ch($H^6_{\mathcal{G}}$)=$e^{-2\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-2\beta_1-2\beta_2-\beta_3-2\beta_4} + 2e^{-2\beta_1-3\beta_2-2\beta_4} + e^{-2\beta_1-4\beta_2+\beta_3-2\beta_4} + 2e^{-3\beta_1-2\beta_2-2\beta_4} + 3e^{-3\beta_1-3\beta_2+\beta_3-2\beta_4} + e^{-3\beta_1-3\beta_2-\beta_4} + e^{-3\beta_1-3\beta_2} + 2e^{-3\beta_1-4\beta_2+2\beta_3-2\beta_4} + 2e^{-3\beta_1-4\beta_2+\beta_3-\beta_4} + e^{-4\beta_1-2\beta_2+\beta_3-2\beta_4} + 2e^{-4\beta_1-3\beta_2+2\beta_3-2\beta_4} + 2e^{-4\beta_1-3\beta_2+\beta_3-\beta_4} + e^{-4\beta_1-4\beta_2+3\beta_3-2\beta_4} + 2e^{-4\beta_1-4\beta_2+2\beta_3-\beta_4} + e^{-4\beta_1-4\beta_2+\beta_3}$.

ch($H^7_{\mathcal{G}}$)=$e^{-3\beta_1-3\beta_2-2\beta_4} + e^{-3\beta_1-4\beta_2+\beta_3-2\beta_4} + e^{-4\beta_1-3\beta_2+\beta_3-2\beta_4} + e^{-4\beta_1-4\beta_2+2\beta_3-2\beta_4} + e^{-4\beta_1-4\beta_2+\beta_3-\beta_4}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 21, 72, 150, 175, 125, 47, 7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0 \le j \le 7$: $(2, 16, 44, 75, 80, 55, 25, 5)$

S=$4\beta_1 + 4\beta_2 - \beta_3 + 2\beta_4$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 5, 12, 25, 26, 19, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 5, 10, 16, 16, 10, 5, 1)$

$d\omega^6 = -\omega^{1,2} - \omega^{3,5}$; $d\omega^7 = -\omega^{4,5}$; $d\omega^{1,6} = \omega^{1,3,5}$; $d\omega^{1,7} = \omega^{1,4,5}$; $d\omega^{2,6} = \omega^{2,3,5}$; $d\omega^{2,7} = \omega^{2,4,5}$; $d\omega^{3,6} = \omega^{1,2,3}$; $d\omega^{3,7} = \omega^{3,4,5}$; $d\omega^{4,6} = \omega^{1,2,4} - \omega^{3,4,5}$; $d\omega^{5,6} = \omega^{1,2,5}$; $d\omega^{6,7} = -\omega^{1,2,7} - \omega^{3,5,7} + \omega^{4,5,6}$; $d\omega^{1,2,6} = -\omega^{1,2,3,5}$; $d\omega^{1,2,7} = -\omega^{1,2,4,5}$; $d\omega^{1,3,7} = -\omega^{1,3,4,5}$; $d\omega^{1,4,6} = \omega^{1,3,4,5}$; $d\omega^{1,6,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $d\omega^{2,3,7} = -\omega^{2,3,4,5}$; $d\omega^{2,4,6} = \omega^{2,3,4,5}$; $d\omega^{2,6,7} = \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $d\omega^{3,4,6} = -\omega^{1,2,3,4}$; $d\omega^{3,5,6} = -\omega^{1,2,3,5}$; $d\omega^{3,6,7} = \omega^{1,2,3,7} - \omega^{3,4,5,6}$; $d\omega^{4,5,6} = -\omega^{1,2,4,5}$; $d\omega^{4,6,7} = \omega^{1,2,4,7} - \omega^{3,4,5,7}$; $d\omega^{5,6,7} = \omega^{1,2,5,7}$; $d\omega^{1,2,3,7} = \omega^{1,2,3,4,5}$; $d\omega^{1,2,4,6} = -\omega^{1,2,3,4,5}$; $d\omega^{1,2,6,7} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $d\omega^{1,3,6,7} = \omega^{1,3,4,5,6}$; $d\omega^{1,4,6,7} = \omega^{1,3,4,5,7}$; $d\omega^{2,3,6,7} = \omega^{2,3,4,5,6}$; $d\omega^{2,4,6,7} = \omega^{2,3,4,5,7}$; $d\omega^{3,4,5,6} = \omega^{1,2,3,4,5}$; $d\omega^{3,4,6,7} = -\omega^{1,2,3,4,7}$; $d\omega^{3,5,6,7} = -\omega^{1,2,3,5,7}$; $d\omega^{4,5,6,7} = -\omega^{1,2,4,5,7}$; $d\omega^{1,2,3,6,7} = -\omega^{1,2,3,4,5,6}$; $d\omega^{1,2,4,6,7} = -\omega^{1,2,3,4,5,7}$; $d\omega^{3,4,5,6,7} = \omega^{1,2,3,4,5,7}$.

ch($H^1_{\mathbb{C}}$)= $e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_4} \langle\omega^4\rangle + e^{-\beta_1-\beta_2+\beta_3} \langle\omega^5\rangle$.

ch($H^2_{\mathbb{C}}$)= $e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-\beta_1-\beta_4} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2+\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_2-\beta_4} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle\omega^{2,5}\rangle + e^{-\beta_3-\beta_4} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2} \langle\omega^{1,2}\rangle + e^{-\beta_1-\beta_2+\beta_3-2\beta_4} \langle\omega^{4,7}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3-\beta_4} \langle\omega^{5,7}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}})= e^{-\beta_1-\beta_3-\beta_4}\ \langle\omega^{1,3,4}\rangle + e^{-2\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,3,7} + \omega^{1,4,6}\rangle + e^{-2\beta_1-\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-2\beta_2+\beta_3}\ \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-2\beta_2+2\beta_3-\beta_4}\ \langle\omega^{1,5,7}\rangle + e^{-\beta_2-\beta_3-\beta_4}\ \langle\omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3}\ \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-\beta_4}\ \langle\omega^{2,3,7}+\omega^{2,4,6}\rangle + e^{-\beta_1-2\beta_2+\beta_3-2\beta_4}\ \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-3\beta_2+\beta_3}\ \langle\omega^{2,5,6}\rangle + e^{-2\beta_1-3\beta_2+2\beta_3-\beta_4}\ \langle\omega^{2,5,7}\rangle + e^{-\beta_1-\beta_2-2\beta_4}\ \langle\omega^{3,4,7}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,6}-\omega^{3,5,6}\rangle + e^{-2\beta_1-2\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,2,7}-\omega^{4,5,6}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{4,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}})= e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,3,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_4}\ \langle\omega^{1,2,4,6}+\omega^{3,4,5,6}\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,2,5,6}\rangle + e^{-2\beta_1-\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,3,4,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_4}\ \langle\omega^{1,3,4,7}\rangle + e^{-3\beta_1-2\beta_2}\ \langle\omega^{1,3,5,6}\rangle + e^{-3\beta_1-2\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,3,5,7}\rangle + e^{-3\beta_1-2\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{1,4,5,7}\rangle + e^{-4\beta_1-3\beta_2+2\beta_3-\beta_4}\ \langle\omega^{1,5,6,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle\omega^{2,3,4,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{2,3,4,7}\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-2\beta_1-3\beta_2+\beta_3-\beta_4}\ \langle\omega^{2,3,5,7}\rangle + e^{-2\beta_1-3\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{2,4,5,7}\rangle + e^{-3\beta_1-4\beta_2+2\beta_3-\beta_4}\ \langle\omega^{2,5,6,7}\rangle + e^{-2\beta_1-2\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,4,7}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{C}})= e^{-2\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,2,3,4,6}\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle + e^{-4\beta_1-4\beta_2+2\beta_3-\beta_4}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,3,5,6,7}\rangle + e^{-4\beta_1-3\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_4}\ \langle\omega^{2,3,4,6,7}\rangle + e^{-3\beta_1-4\beta_2+\beta_3-\beta_4}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-3\beta_1-4\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{2,4,5,6,7}\rangle + e^{-3\beta_1-3\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,4,6,7}+\omega^{3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{C}})= e^{-3\beta_1-3\beta_2-2\beta_4}\ \langle\omega^{1,2,3,4,6,7}\rangle + e^{-4\beta_1-4\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,2,3,5,6,7}\rangle + e^{-4\beta_1-4\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,3,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2+\beta_3-2\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{C}})= e^{-4\beta_1-4\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y+6Y^2+10Y^3+6Y^4+4Y^5+Y^6)(1+Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1,5,12,25,26,19,7,1)$

Dimensions of $H^j_{\mathcal{C}}$'s $0 \le j \le 7$: $(1,5,10,16,16,10,5,1)$

Total dimension of trivial cohomology: 64.

Length associated polynomial: 64.

Riemann Hyp.: yes

$d^\star\omega^{1,2} = -\omega^6$; $d^\star\omega^{3,5} = -\omega^6$; $d^\star\omega^{4,5} = -\omega^7$; $d^\star\omega^{1,2,3} = \omega^{3,6}$; $d^\star\omega^{1,2,4} = \omega^{4,6}$; $d^\star\omega^{1,2,5} = \omega^{5,6}$; $d^\star\omega^{1,2,7} = -\omega^{6,7}$; $d^\star\omega^{1,3,5} = \omega^{1,6}$; $d^\star\omega^{1,4,5} = \omega^{1,7}$; $d^\star\omega^{2,3,5} = \omega^{2,6}$; $d^\star\omega^{2,4,5} = \omega^{2,7}$; $d^\star\omega^{3,4,5} = \omega^{3,7}-\omega^{4,6}$; $d^\star\omega^{3,5,7} = -\omega^{6,7}$; $d^\star\omega^{4,5,6} = \omega^{6,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{3,4,6}$; $d^\star\omega^{1,2,3,5} = -\omega^{1,2,6}-\omega^{3,5,6}$; $d^\star\omega^{1,2,3,7} = \omega^{3,6,7}$; $d^\star\omega^{1,2,4,5} = -\omega^{1,2,7}-\omega^{4,5,6}$; $d^\star\omega^{1,2,4,7} = \omega^{4,6,7}$; $d^\star\omega^{1,2,5,7} = \omega^{5,6,7}$; $d^\star\omega^{1,3,4,5} = -\omega^{1,3,7}+\omega^{1,4,6}$; $d^\star\omega^{1,3,5,7} = \omega^{1,6,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{2,3,7}+\omega^{2,4,6}$; $d^\star\omega^{2,3,5,7} = \omega^{2,6,7}$; $d^\star\omega^{2,4,5,6} = -\omega^{2,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{3,6,7}$;

$d^\star\omega^{3,4,5,7} = -\omega^{4,6,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,2,3,7} - \omega^{1,2,4,6} + \omega^{3,4,5,6}$; $d^\star\omega^{1,2,3,4,7} = -\omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,5,7} = -\omega^{1,2,6,7} - \omega^{3,5,6,7}$; $d^\star\omega^{1,2,4,5,6} = \omega^{1,2,6,7}$; $d^\star\omega^{1,2,4,5,7} = -\omega^{4,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,3,6,7}$; $d^\star\omega^{1,3,4,5,7} = \omega^{1,4,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,3,6,7}$; $d^\star\omega^{2,3,4,5,7} = \omega^{2,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,3,6,7}$; $d^\star\omega^{1,2,3,4,5,7} = -\omega^{1,2,4,6,7} + \omega^{3,4,5,6,7}$;

$\Delta\omega^6 = 2\omega^6$; $\Delta\omega^7 = \omega^7$; $\Delta\omega^{1,2} = \omega^{1,2} + \omega^{3,5}$; $\Delta\omega^{1,6} = \omega^{1,6}$; $\Delta\omega^{1,7} = \omega^{1,7}$; $\Delta\omega^{2,6} = \omega^{2,6}$; $\Delta\omega^{2,7} = \omega^{2,7}$; $\Delta\omega^{3,5} = \omega^{1,2} + \omega^{3,5}$; $\Delta\omega^{3,6} = \omega^{3,6}$; $\Delta\omega^{3,7} = \omega^{3,7} - \omega^{4,6}$; $\Delta\omega^{4,5} = \omega^{4,5}$; $\Delta\omega^{4,6} = -\omega^{3,7} + 2\omega^{4,6}$; $\Delta\omega^{5,6} = \omega^{5,6}$; $\Delta\omega^{6,7} = 3\omega^{6,7}$; $\Delta\omega^{1,2,3} = \omega^{1,2,3}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4} - \omega^{3,4,5}$; $\Delta\omega^{1,2,5} = \omega^{1,2,5}$; $\Delta\omega^{1,2,6} = \omega^{1,2,6} + \omega^{3,5,6}$; $\Delta\omega^{1,2,7} = 2\omega^{1,2,7} + \omega^{3,5,7}$; $\Delta\omega^{1,3,5} = \omega^{1,3,5}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7} - \omega^{1,4,6}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5}$; $\Delta\omega^{1,4,6} = -\omega^{1,3,7} + \omega^{1,4,6}$; $\Delta\omega^{1,6,7} = 2\omega^{1,6,7}$; $\Delta\omega^{2,3,5} = \omega^{2,3,5}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7} - \omega^{2,4,6}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5}$; $\Delta\omega^{2,4,6} = -\omega^{2,3,7} + \omega^{2,4,6}$; $\Delta\omega^{2,6,7} = 2\omega^{2,6,7}$; $\Delta\omega^{3,4,5} = -\omega^{1,2,4} + 2\omega^{3,4,5}$; $\Delta\omega^{3,4,6} = \omega^{3,4,6}$; $\Delta\omega^{3,5,6} = \omega^{1,2,6} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{1,2,7} + \omega^{3,5,7} - \omega^{4,5,6}$; $\Delta\omega^{3,6,7} = 2\omega^{3,6,7}$; $\Delta\omega^{4,5,6} = -\omega^{3,5,7} + 2\omega^{4,5,6}$; $\Delta\omega^{4,6,7} = 2\omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = 2\omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,7} = 2\omega^{1,2,3,7} - \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5}$; $\Delta\omega^{1,2,4,6} = -\omega^{1,2,3,7} + \omega^{1,2,4,6} - \omega^{3,4,5,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{3,4,5,7}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7}$; $\Delta\omega^{1,2,6,7} = 2\omega^{1,2,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,3,4,5} = 2\omega^{1,3,4,5}$; $\Delta\omega^{1,3,5,7} = \omega^{1,3,5,7} - \omega^{1,4,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = -\omega^{1,3,5,7} + \omega^{1,4,5,6}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7}$; $\Delta\omega^{2,3,4,5} = 2\omega^{2,3,4,5}$; $\Delta\omega^{2,3,5,7} = \omega^{2,3,5,7} - \omega^{2,4,5,6}$; $\Delta\omega^{2,3,6,7} = \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = -\omega^{2,3,5,7} + \omega^{2,4,5,6}$; $\Delta\omega^{2,4,6,7} = \omega^{2,4,6,7}$; $\Delta\omega^{3,4,5,6} = -\omega^{1,2,4,6} + 2\omega^{3,4,5,6}$; $\Delta\omega^{3,4,5,7} = -\omega^{1,2,4,7} + \omega^{3,4,5,7}$; $\Delta\omega^{3,4,6,7} = \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = \omega^{1,2,6,7} + \omega^{3,5,6,7}$; $\Delta\omega^{4,5,6,7} = \omega^{4,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 3\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,7} = \omega^{1,2,3,4,7}$; $\Delta\omega^{1,2,3,5,7} = 2\omega^{1,2,3,5,7} - \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,3,6,7} = \omega^{1,2,3,6,7}$; $\Delta\omega^{1,2,4,5,6} = -\omega^{1,2,3,5,7} + \omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7}$; $\Delta\omega^{1,2,4,6,7} = \omega^{1,2,4,6,7} - \omega^{3,4,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = \omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,5,7} = \omega^{1,3,4,5,7}$; $\Delta\omega^{2,3,4,5,6} = \omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,4,5,7} = \omega^{2,3,4,5,7}$; $\Delta\omega^{3,4,5,6,7} = -\omega^{1,2,4,6,7} + \omega^{3,4,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = \omega^{1,2,3,4,5,6}$; $\Delta\omega^{1,2,3,4,5,7} = 2\omega^{1,2,3,4,5,7}$;

$\mathrm{ch}(H^1_{\mathbb{C}}) = e^{-\beta_1} \langle\omega^1\rangle + e^{-\beta_2} \langle\omega^2\rangle + e^{-\beta_3} \langle\omega^3\rangle + e^{-\beta_4} \langle\omega^4\rangle + e^{-\beta_1-\beta_2+\beta_3} \langle\omega^5\rangle.$

$\mathrm{ch}(H^2_{\mathbb{C}}) = e^{-\beta_1-\beta_3} \langle\omega^{1,3}\rangle + e^{-\beta_1-\beta_4} \langle\omega^{1,4}\rangle + e^{-2\beta_1-\beta_2+\beta_3} \langle\omega^{1,5}\rangle + e^{-\beta_2-\beta_3} \langle\omega^{2,3}\rangle + e^{-\beta_2-\beta_4} \langle\omega^{2,4}\rangle + e^{-\beta_1-2\beta_2+\beta_3} \langle\omega^{2,5}\rangle + e^{-\beta_3-\beta_4} \langle\omega^{3,4}\rangle + e^{-\beta_1-\beta_2} \langle\omega^{1,2} - \omega^{3,5}\rangle + e^{-\beta_1-\beta_2+\beta_3-2\beta_4} \langle\omega^{4,7}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3-\beta_4} \langle\omega^{5,7}\rangle.$

$\mathrm{ch}(H^3_{\mathbb{C}}) = e^{-\beta_1-\beta_3-\beta_4} \langle\omega^{1,3,4}\rangle + e^{-2\beta_1-\beta_2-\beta_3} \langle\omega^{1,3,6}\rangle + e^{-2\beta_1-\beta_2-\beta_4} \langle\omega^{1,3,7} + \omega^{1,4,6}\rangle + e^{-2\beta_1-\beta_2+\beta_3-2\beta_4} \langle\omega^{1,4,7}\rangle + e^{-3\beta_1-2\beta_2+\beta_3} \langle\omega^{1,5,6}\rangle + e^{-3\beta_1-2\beta_2+2\beta_3-\beta_4} \langle\omega^{1,5,7}\rangle + e^{-\beta_2-\beta_3-\beta_4} \langle\omega^{2,3,4}\rangle + e^{-\beta_1-2\beta_2-\beta_3} \langle\omega^{2,3,6}\rangle + e^{-\beta_1-2\beta_2-\beta_4} \langle\omega^{2,3,7} + \omega^{2,4,6}\rangle + e^{-\beta_1-2\beta_2+\beta_3-2\beta_4} \langle\omega^{2,4,7}\rangle + e^{-2\beta_1-3\beta_2+\beta_3} \langle\omega^{2,5,6}\rangle +$

$e^{-2\beta_1-3\beta_2+2\beta_3-\beta_4}\ \langle\omega^{2,5,7}\rangle + e^{-\beta_1-\beta_2-2\beta_4}\ \langle\omega^{3,4,7}\rangle + e^{-2\beta_1-2\beta_2}\ \langle\omega^{1,2,6}-\omega^{3,5,6}\rangle + e^{-2\beta_1-2\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,2,7}-2\omega^{3,5,7}-\omega^{4,5,6}\rangle + e^{-2\beta_1-2\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{4,5,7}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{A}})= e^{-3\beta_1-4\beta_2+2\beta_3-\beta_4}\ \langle\omega^{2,5,6,7}\rangle + e^{-2\beta_1-3\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{2,4,5,7}\rangle + e^{-2\beta_1-3\beta_2+\beta_3-\beta_4}\ \langle(-(\omega^{2,3,5,7}+\omega^{2,4,5,6}))/2\rangle + e^{-2\beta_1-3\beta_2}\ \langle\omega^{2,3,5,6}\rangle + e^{-\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{2,3,4,7}\rangle + e^{-\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle-\omega^{2,3,4,6}\rangle + e^{-4\beta_1-3\beta_2+2\beta_3-\beta_4}\ \langle-\omega^{1,5,6,7}\rangle + e^{-3\beta_1-2\beta_2+2\beta_3-2\beta_4}\ \langle-\omega^{1,4,5,7}\rangle + e^{-3\beta_1-2\beta_2+\beta_3-\beta_4}\ \langle(\omega^{1,3,5,7}+\omega^{1,4,5,6})/2\rangle + e^{-3\beta_1-2\beta_2}\ \langle-\omega^{1,3,5,6}\rangle + e^{-2\beta_1-\beta_2-2\beta_4}\ \langle-\omega^{1,3,4,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,3,4,6}\rangle + e^{-3\beta_1-3\beta_2+\beta_3}\ \langle\omega^{1,2,5,6}\rangle + e^{-2\beta_1-2\beta_2+\beta_3-2\beta_4}\ \langle(-(\omega^{1,2,4,7}+\omega^{3,4,5,7}))/2\rangle + e^{-2\beta_1-2\beta_2-\beta_4}\ \langle(\omega^{1,2,3,7}+2\omega^{1,2,4,6}+\omega^{3,4,5,6})/6\rangle + e^{-2\beta_1-2\beta_2-\beta_3}\ \langle\omega^{1,2,3,6}\rangle$.

$\mathrm{ch}(H^5_{\mathcal{A}})= e^{-3\beta_1-4\beta_2+2\beta_3-2\beta_4}\ \langle-\omega^{2,4,5,6,7}\rangle + e^{-3\beta_1-4\beta_2+\beta_3-\beta_4}\ \langle\omega^{2,3,5,6,7}\rangle + e^{-2\beta_1-3\beta_2-2\beta_4}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-4\beta_1-3\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2+\beta_3-\beta_4}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1-2\beta_2-2\beta_4}\ \langle\omega^{1,3,4,6,7}\rangle + e^{-4\beta_1-4\beta_2+2\beta_3-\beta_4}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-3\beta_2+\beta_3-2\beta_4}\ \langle(\omega^{1,2,4,6,7}+\omega^{3,4,5,6,7})/2\rangle + e^{-3\beta_1-3\beta_2}\ \langle\omega^{1,2,3,5,6}\rangle + e^{-2\beta_1-2\beta_2-\beta_3-\beta_4}\ \langle-\omega^{1,2,3,4,6}\rangle$.

$\mathrm{ch}(H^6_{\mathcal{A}})= e^{-3\beta_1-4\beta_2+\beta_3-2\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_2+\beta_3-2\beta_4}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2+2\beta_3-2\beta_4}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-4\beta_2+\beta_3-\beta_4}\ \langle-\omega^{1,2,3,5,6,7}\rangle + e^{-3\beta_1-3\beta_2-2\beta_4}\ \langle\omega^{1,2,3,4,6,7}\rangle$.

$\mathrm{ch}(H^7_{\mathcal{A}})= e^{-4\beta_1-4\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1+4Y+6Y^2+10Y^3+6Y^4+4Y^5+Y^6)(1+Y)$.

5.6.4 Algebra $\mathcal{G}_{7,4.4}$ (7-dim Heisenberg Lie Algebra)

Adjoint cohomology $[x_1, x_4] = x_7$; $[x_2, x_5] = x_7$; $[x_3, x_6] = x_7$;

$$\delta = \begin{pmatrix} \xi_1^1 & \xi_2^1 & \xi_3^1 & \xi_4^1 & \xi_5^1 & \xi_6^1 & 0 \\ \xi_1^2 & \xi_2^2 & \xi_3^2 & \xi_5^1 & \xi_5^2 & \xi_6^2 & 0 \\ \xi_1^3 & \xi_2^3 & \xi_3^3 & \xi_6^1 & \xi_6^2 & \xi_6^3 & 0 \\ \xi_1^4 & \xi_2^4 & \xi_3^4 & \xi_4^4 & -\xi_1^2 & -\xi_1^3 & 0 \\ \xi_2^4 & \xi_2^5 & \xi_3^5 & -\xi_2^1 & \xi_1^1 - \xi_2^2 + \xi_4^4 & -\xi_2^3 & 0 \\ \xi_3^4 & \xi_3^5 & \xi_3^6 & -\xi_3^1 & -\xi_3^2 & \xi_1^1 - \xi_3^3 + \xi_4^4 & 0 \\ \xi_1^7 & \xi_2^7 & \xi_3^7 & \xi_4^7 & \xi_5^7 & \xi_6^7 & \xi_1^1 + \xi_4^4 \end{pmatrix}$$

$ad\ x_1 = \delta_4^7, ad\ x_2 = \delta_5^7, ad\ x_3 = \delta_6^7, ad\ x_4 = -\delta_1^7, ad\ x_5 = -\delta_2^7, ad\ x_6 = -\delta_3^7, ad\ x_7 = 0.$

Maximal torus : $T=\langle t_1, t_2, t_3, t_4\rangle$.

$\mathrm{ch}(\mathcal{G})= e^{\beta_1}\ \langle x_1\rangle + e^{\beta_2}\ \langle x_2\rangle + e^{\beta_3}\ \langle x_3\rangle + e^{\beta_4}\ \langle x_4\rangle + e^{\beta_1-\beta_2+\beta_4}\ \langle x_5\rangle + e^{\beta_1-\beta_3+\beta_4}\ \langle x_6\rangle + e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

$\mathrm{ch}(H^0_{\mathcal{G}})= e^{\beta_1+\beta_4}\ \langle x_7\rangle$.

$\mathrm{ch}(H^1_{\mathcal{G}})= e^{\beta_1-\beta_4}\ \langle \delta_4^1\rangle + e^{\beta_2-\beta_4}\ \langle \delta_5^1\rangle + e^{-\beta_1+2\beta_2-\beta_4}\ \langle \delta_5^2\rangle + e^{\beta_3-\beta_4}\ \langle \delta_6^1\rangle + e^{-\beta_1+\beta_2+\beta_3-\beta_4}\ \langle \delta_6^2\rangle + e^{-\beta_1+2\beta_3-\beta_4}\ \langle \delta_6^3\rangle + e^{-\beta_1+\beta_4}\ \langle \delta_1^4\rangle + e^{-\beta_1+\beta_2}\ \langle \delta_1^2\rangle + e^{-\beta_1+\beta_3}\ \langle \delta_1^3\rangle + e^{-\beta_2+\beta_4}\ \langle \delta_2^4\rangle + e^{\beta_1-2\beta_2+\beta_4}\ \langle \delta_2^5\rangle + e^{\beta_1-\beta_2}\ \langle \delta_2^1\rangle + 4e^0\ \langle t_2, t_3, t_1, t_4\rangle + e^{-\beta_2+\beta_3}\ \langle \delta_2^3\rangle + e^{-\beta_3+\beta_4}\ \langle \delta_3^4\rangle + e^{\beta_1-\beta_2-\beta_3+\beta_4}\ \langle \delta_3^5\rangle + e^{\beta_1-2\beta_3+\beta_4}\ \langle \delta_3^6\rangle + e^{\beta_1-\beta_3}\ \langle \delta_3^1\rangle + e^{\beta_2-\beta_3}\ \langle \delta_3^2\rangle$.

$\mathrm{ch}(H^2_{\mathcal{G}})= e^{\beta_1-\beta_2-\beta_4}\ \langle x_1\omega^{2,4}\rangle + e^{\beta_1-\beta_3-\beta_4}\ \langle x_1\omega^{3,4}\rangle + e^{\beta_2-2\beta_4}\ \langle x_1\omega^{4,5}\rangle + e^{\beta_3-2\beta_4}\ \langle x_1\omega^{4,6}\rangle + e^{-2\beta_1+2\beta_2-\beta_4}\ \langle x_2\omega^{1,5}\rangle + 5e^{-\beta_4}\ \langle x_1\omega^{2,5} + x_2\omega^{2,4}, x_1\omega^{3,6} + x_3\omega^{3,4}, -x_1\omega^{2,5} + x_5\omega^{4,5}, -x_1\omega^{3,6} + x_6\omega^{4,6}, x_1\omega^{1,4}\rangle + 2e^{\beta_2-\beta_3-\beta_4}\ \langle x_1\omega^{3,5} + x_2\omega^{3,4}, -x_1\omega^{3,5} + x_6\omega^{4,5}\rangle + e^{-\beta_1+2\beta_2-\beta_3-\beta_4}\ \langle x_2\omega^{3,5}\rangle + e^{-\beta_1+2\beta_2-2\beta_4}\ \langle x_2\omega^{4,5}\rangle + 2e^{-\beta_1+\beta_2+\beta_3-2\beta_4}\ \langle x_1\omega^{5,6}+x_2\omega^{4,6}, -x_1\omega^{5,6}+x_3\omega^{4,5}\rangle+e^{-2\beta_1+2\beta_2+\beta_3-2\beta_4}\ \langle x_2\omega^{5,6}\rangle+ 2e^{-2\beta_1+\beta_2+\beta_3-\beta_4}\ \langle x_2\omega^{1,6} + x_3\omega^{1,5}, -x_2\omega^{1,6} + x_4\omega^{5,6}\rangle + e^{-2\beta_1+2\beta_3-\beta_4}\ \langle x_3\omega^{1,6}\rangle + 2e^{-\beta_2+\beta_3-\beta_4}\ \langle x_1\omega^{2,6} + x_3\omega^{2,4}, -x_1\omega^{2,6} + x_5\omega^{4,6}\rangle + e^{-\beta_1-\beta_2+2\beta_3-\beta_4}\ \langle x_3\omega^{2,6}\rangle + 5e^{-\beta_1+\beta_2-\beta_4}\ \langle x_2\omega^{3,6} + x_3\omega^{3,5}, -x_1\omega^{1,5} + x_4\omega^{4,5}, -x_2\omega^{3,6} + x_6\omega^{5,6}, x_1\omega^{1,5} + x_2\omega^{1,4}, x_2\omega^{2,5}\rangle + e^{-\beta_1+2\beta_3-2\beta_4}\ \langle x_3\omega^{4,6}\rangle + e^{-2\beta_1+\beta_2+2\beta_3-2\beta_4}\ \langle x_3\omega^{5,6}\rangle + e^{-\beta_1-\beta_2+\beta_4}\ \langle x_4\omega^{1,2}\rangle + e^{-\beta_1-\beta_3+\beta_4}\ \langle x_4\omega^{1,3}\rangle + e^{-2\beta_1+\beta_2}\ \langle x_4\omega^{1,5}\rangle + e^{-2\beta_1+\beta_3}\ \langle x_4\omega^{1,6}\rangle + 5e^{-\beta_2}\ \langle x_1\omega^{1,2} + x_4\omega^{2,4}, -x_3\omega^{2,3} + x_6\omega^{2,6}, -x_1\omega^{1,2} + x_5\omega^{1,4}, x_5\omega^{2,5}, x_3\omega^{2,3} + x_5\omega^{3,6}\rangle + 2e^{-\beta_1-\beta_2+\beta_3}\ \langle x_3\omega^{1,2} + x_4\omega^{2,6}, -x_3\omega^{1,2} + x_5\omega^{1,6}\rangle + 5e^{-\beta_3}\ \langle x_1\omega^{1,3} + x_4\omega^{3,4}, x_2\omega^{2,3} + x_5\omega^{3,5}, -x_1\omega^{1,3} + x_6\omega^{1,4}, -x_2\omega^{2,3} + x_6\omega^{2,5}, x_6\omega^{3,6}\rangle + 2e^{-\beta_1+\beta_2-\beta_3}\ \langle x_2\omega^{1,3} + x_4\omega^{3,5}, -x_2\omega^{1,3} + x_6\omega^{1,5}\rangle + 5e^{-\beta_1+\beta_3-\beta_4}\ \langle -x_1\omega^{1,6} + x_4\omega^{4,6}, -x_2\omega^{2,6} + x_5\omega^{5,6}, x_1\omega^{1,6} + x_3\omega^{1,4}, x_2\omega^{2,6} +$

$x_3\omega^{2,5}, x_3\omega^{3,6}\rangle + e^{-2\beta_2+\beta_4}\ \langle x_5\omega^{1,2}\rangle + 2e^{-\beta_2-\beta_3+\beta_4}\ \langle x_4\omega^{2,3} + x_5\omega^{1,3}, -x_4\omega^{2,3} + x_6\omega^{1,2}\rangle + 5e^{-\beta_1}\ \langle -x_2\omega^{1,2} + x_5\omega^{1,5}, -x_3\omega^{1,3} + x_6\omega^{1,6}, x_4\omega^{1,4}, x_2\omega^{1,2} + x_4\omega^{2,5}, x_3\omega^{1,3} + x_4\omega^{3,6}\rangle + e^{\beta_1-2\beta_2-\beta_3+\beta_4}\ \langle x_5\omega^{2,3}\rangle + e^{\beta_1-2\beta_2}\ \langle x_5\omega^{2,4}\rangle + e^{-2\beta_2+\beta_3}\ \langle x_5\omega^{2,6}\rangle + 2e^{\beta_1-\beta_2-\beta_3}\ \langle x_1\omega^{2,3} + x_5\omega^{3,4}, -x_1\omega^{2,3} + x_6\omega^{2,4}\rangle + e^{-2\beta_3+\beta_4}\ \langle x_6\omega^{1,3}\rangle + e^{\beta_1-\beta_2-2\beta_3+\beta_4}\ \langle x_6\omega^{2,3}\rangle + e^{\beta_1-2\beta_3}\ \langle x_6\omega^{3,4}\rangle + e^{\beta_2-2\beta_3}\ \langle x_6\omega^{3,5}\rangle.$

$\mathrm{ch}(H^3_{\mathcal{G}}) = e^{\beta_1-\beta_2-\beta_3-\beta_4} + e^{\beta_1-\beta_2-2\beta_3} + e^{\beta_1-2\beta_2-\beta_3} + e^{-\beta_1+2\beta_2-\beta_3-2\beta_4} + e^{-\beta_1+\beta_2+\beta_3-3\beta_4} + 4e^{-\beta_1+\beta_2-\beta_3-\beta_4} + e^{-\beta_1+\beta_2-2\beta_3} + 4e^{-\beta_1+\beta_2-2\beta_4} + e^{-\beta_1-\beta_2+2\beta_3-2\beta_4} + 4e^{-\beta_1-\beta_2+\beta_3-\beta_4} + e^{-\beta_1-\beta_2-\beta_3+\beta_4} + 4e^{-\beta_1-\beta_2} + e^{-\beta_1-2\beta_2+\beta_3} + 4e^{-\beta_1+\beta_3-2\beta_4} + 4e^{-\beta_1-\beta_3} + 6e^{-\beta_1-\beta_4} + e^{-2\beta_1+2\beta_2+\beta_3-3\beta_4} + e^{-2\beta_1+2\beta_2-\beta_3-\beta_4} + e^{-2\beta_1+2\beta_2-2\beta_4} + e^{-2\beta_1+\beta_2+2\beta_3-3\beta_4} + 4e^{-2\beta_1+\beta_2+\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-\beta_3} + 4e^{-2\beta_1+\beta_2-\beta_4} + e^{-2\beta_1-\beta_2+2\beta_3-\beta_4} + e^{-2\beta_1-\beta_2+\beta_3} + e^{-2\beta_1+2\beta_3-2\beta_4} + 4e^{-2\beta_1+\beta_3-\beta_4} + e^{-2\beta_1} + e^{-3\beta_1+2\beta_2+\beta_3-2\beta_4} + e^{-3\beta_1+\beta_2+2\beta_3-2\beta_4} + e^{-3\beta_1+\beta_2+\beta_3-\beta_4} + e^{\beta_2-\beta_3-2\beta_4} + e^{\beta_2-2\beta_3-\beta_4} + e^{-\beta_2+\beta_3-2\beta_4} + 4e^{-\beta_2-\beta_3} + e^{-\beta_2-2\beta_3+\beta_4} + 4e^{-\beta_2-\beta_4} + e^{-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_2-\beta_3+\beta_4} + e^{-2\beta_2} + 4e^{-\beta_3-\beta_4} + e^{-2\beta_3} + e^{-2\beta_4}.$

$\mathrm{ch}(H^4_{\mathcal{G}}) = e^{-\beta_1+\beta_2-\beta_3-2\beta_4} + e^{-\beta_1+\beta_2-\beta_3-3\beta_4} + e^{-\beta_1+\beta_2-2\beta_3-2\beta_4} + e^{-\beta_1-\beta_2+\beta_3-2\beta_4} + e^{-\beta_1-\beta_2+\beta_3-3\beta_4} + 3e^{-\beta_1-\beta_2-\beta_3-\beta_4} + e^{-\beta_1-\beta_2-\beta_3} + e^{-\beta_1-\beta_2-2\beta_3} + e^{-\beta_1-\beta_2-\beta_4} + 3e^{-\beta_1-\beta_2-2\beta_4} + e^{-\beta_1-2\beta_2+\beta_3-2\beta_4} + e^{-\beta_1-2\beta_2-\beta_3} + e^{-\beta_1-2\beta_2-\beta_4} + e^{-\beta_1-\beta_3-\beta_4} + 3e^{-\beta_1-\beta_3-2\beta_4} + e^{-\beta_1-2\beta_3-\beta_4} + e^{-\beta_1-2\beta_4} + e^{-\beta_1-3\beta_4} + e^{-2\beta_1+2\beta_2-\beta_3-3\beta_4} + e^{-2\beta_1+\beta_2+\beta_3-3\beta_4} + e^{-2\beta_1+\beta_2+\beta_3-4\beta_4} + e^{-2\beta_1+\beta_2-\beta_3-\beta_4} + 3e^{-2\beta_1+\beta_2-\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-2\beta_3-\beta_4} + e^{-2\beta_1+\beta_2-2\beta_4} + 3e^{-2\beta_1+\beta_2-3\beta_4} + e^{-2\beta_1-\beta_2+2\beta_3-3\beta_4} + e^{-2\beta_1-\beta_2+\beta_3-\beta_4} + 3e^{-2\beta_1-\beta_2+\beta_3-2\beta_4} + e^{-2\beta_1-\beta_2-\beta_3} + 3e^{-2\beta_1-\beta_2-\beta_4} + e^{-2\beta_1-2\beta_2+\beta_3-\beta_4} + e^{-2\beta_1+\beta_3-2\beta_4} + 3e^{-2\beta_1+\beta_3-3\beta_4} + 3e^{-2\beta_1-\beta_3-\beta_4} + e^{-2\beta_1-\beta_4} + 4e^{-2\beta_1-2\beta_4} + e^{-3\beta_1+2\beta_2+\beta_3-4\beta_4} + e^{-3\beta_1+2\beta_2-\beta_3-2\beta_4} + e^{-3\beta_1+2\beta_2-3\beta_4} + e^{-3\beta_1+\beta_2+2\beta_3-4\beta_4} + e^{-3\beta_1+\beta_2+\beta_3-2\beta_4} + 3e^{-3\beta_1+\beta_2+\beta_3-3\beta_4} + e^{-3\beta_1+\beta_2-\beta_3-\beta_4} + 3e^{-3\beta_1+\beta_2-2\beta_4} + e^{-3\beta_1-\beta_2+2\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2+\beta_3-\beta_4} + e^{-3\beta_1+2\beta_3-3\beta_4} + 3e^{-3\beta_1+\beta_3-2\beta_4} + e^{-3\beta_1-\beta_4} + e^{-4\beta_1+2\beta_2+\beta_3-3\beta_4} + e^{-4\beta_1+\beta_2+2\beta_3-3\beta_4} + e^{-4\beta_1+\beta_2+\beta_3-2\beta_4} + e^{-\beta_2-\beta_3-\beta_4} + e^{-\beta_2-\beta_3-2\beta_4} + e^{-\beta_2-2\beta_3-\beta_4} + e^{-2\beta_2-\beta_3-\beta_4}.$

$\mathrm{ch}(H^5_{\mathcal{G}}) = 3e^{-\beta_1-\beta_2-\beta_3-2\beta_4} + e^{-\beta_1-\beta_2-2\beta_3-\beta_4} + e^{-\beta_1-\beta_2-3\beta_4} + e^{-\beta_1-2\beta_2-\beta_3-\beta_4} + e^{-\beta_1-2\beta_2-2\beta_4} + e^{-\beta_1-\beta_3-3\beta_4} + e^{-\beta_1-2\beta_3-2\beta_4} + 3e^{-2\beta_1+\beta_2-\beta_3-3\beta_4} + e^{-2\beta_1+\beta_2-2\beta_3-2\beta_4} + e^{-2\beta_1+\beta_2-4\beta_4} + 3e^{-2\beta_1-\beta_2+\beta_3-3\beta_4} + 3e^{-2\beta_1-\beta_2-\beta_3-\beta_4} + 5e^{-2\beta_1-\beta_2-2\beta_4} + e^{-2\beta_1-2\beta_2+\beta_3-2\beta_4} + e^{-2\beta_1-2\beta_2-\beta_4} + e^{-2\beta_1+\beta_3-4\beta_4} + 5e^{-2\beta_1-\beta_3-2\beta_4} + e^{-2\beta_1-2\beta_3-\beta_4} + 5e^{-2\beta_1-3\beta_4} + e^{-3\beta_1+2\beta_2-\beta_3-3\beta_4} + e^{-3\beta_1+2\beta_2-4\beta_4} + 3e^{-3\beta_1+\beta_2+\beta_3-4\beta_4} + 3e^{-3\beta_1+\beta_2-\beta_3-2\beta_4} + 5e^{-3\beta_1+\beta_2-3\beta_4} + e^{-3\beta_1-\beta_2+2\beta_3-3\beta_4} + 3e^{-3\beta_1-\beta_2+\beta_3-2\beta_4} + e^{-3\beta_1-\beta_2-\beta_4} + e^{-3\beta_1+2\beta_3-4\beta_4} + 5e^{-3\beta_1+\beta_3-3\beta_4} + e^{-3\beta_1-\beta_3-\beta_4} + 5e^{-3\beta_1-2\beta_4} + e^{-4\beta_1+2\beta_2+\beta_3-4\beta_4} + e^{-4\beta_1+2\beta_2-3\beta_4} + e^{-4\beta_1+\beta_2+2\beta_3-4\beta_4} + 3e^{-4\beta_1+\beta_2+\beta_3-3\beta_4} + e^{-4\beta_1+\beta_2-2\beta_4} + e^{-4\beta_1+2\beta_3-3\beta_4} + e^{-4\beta_1+\beta_3-2\beta_4}.$

$\mathrm{ch}(H^6_{\mathcal{G}})$=$2e^{-2\beta_1-\beta_2-\beta_3-2\beta_4}+2e^{-2\beta_1-\beta_2-3\beta_4}+e^{-2\beta_1-2\beta_2-2\beta_4}+2e^{-2\beta_1-\beta_3-3\beta_4}+e^{-2\beta_1-2\beta_3-2\beta_4}+e^{-2\beta_1-4\beta_4}+2e^{-3\beta_1+\beta_2-\beta_3-3\beta_4}+2e^{-3\beta_1+\beta_2-4\beta_4}+2e^{-3\beta_1-\beta_2+\beta_3-3\beta_4}+2e^{-3\beta_1-\beta_2-2\beta_4}+2e^{-3\beta_1+\beta_3-4\beta_4}+2e^{-3\beta_1-\beta_3-2\beta_4}+5e^{-3\beta_1-3\beta_4}+e^{-4\beta_1+2\beta_2-4\beta_4}+2e^{-4\beta_1+\beta_2+\beta_3-4\beta_4}+2e^{-4\beta_1+\beta_2-3\beta_4}+e^{-4\beta_1+2\beta_3-4\beta_4}+2e^{-4\beta_1+\beta_3-3\beta_4}+e^{-4\beta_1-2\beta_4}$.

$\mathrm{ch}(H^7_{\mathcal{G}})$=$e^{-3\beta_1-\beta_2-3\beta_4}+e^{-3\beta_1-\beta_3-3\beta_4}+e^{-3\beta_1-4\beta_4}+e^{-4\beta_1+\beta_2-4\beta_4}+e^{-4\beta_1+\beta_3-4\beta_4}+e^{-4\beta_1-3\beta_4}$.

Dimensions of $Z^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,28,91,140,189,134,48,7)$

Dimensions of $H^j_{\mathcal{G}}$'s $0\le j\le 7$: $(1,22,70,84,84,78,35,6)$

S=$4(\beta_1+\beta_4)$.

Trivial cohomology:

Dimensions of $Z^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,6,15,20,29,20,7,1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0\le j\le 7$: $(1,6,14,14,14,14,6,1)$

$d\omega^7=-\omega^{1,4}-\omega^{2,5}-\omega^{3,6}$; $d\omega^{1,7}=\omega^{1,2,5}+\omega^{1,3,6}$; $d\omega^{2,7}=-\omega^{1,2,4}+\omega^{2,3,6}$; $d\omega^{3,7}=-\omega^{1,3,4}-\omega^{2,3,5}$; $d\omega^{4,7}=-\omega^{2,4,5}-\omega^{3,4,6}$; $d\omega^{5,7}=\omega^{1,4,5}-\omega^{3,5,6}$; $d\omega^{6,7}=\omega^{1,4,6}+\omega^{2,5,6}$; $d\omega^{1,2,7}=-\omega^{1,2,3,6}$; $d\omega^{1,3,7}=\omega^{1,2,3,5}$; $d\omega^{1,4,7}=\omega^{1,2,4,5}+\omega^{1,3,4,6}$; $d\omega^{1,5,7}=\omega^{1,3,5,6}$; $d\omega^{1,6,7}=-\omega^{1,2,5,6}$; $d\omega^{2,3,7}=-\omega^{1,2,3,4}$; $d\omega^{2,4,7}=\omega^{2,3,4,6}$; $d\omega^{2,5,7}=\omega^{1,2,4,5}+\omega^{2,3,5,6}$; $d\omega^{2,6,7}=\omega^{1,2,4,6}$; $d\omega^{3,4,7}=-\omega^{2,3,4,5}$; $d\omega^{3,5,7}=\omega^{1,3,4,5}$; $d\omega^{3,6,7}=\omega^{1,3,4,6}+\omega^{2,3,5,6}$; $d\omega^{4,5,7}=-\omega^{3,4,5,6}$; $d\omega^{4,6,7}=\omega^{2,4,5,6}$; $d\omega^{5,6,7}=-\omega^{1,4,5,6}$; $d\omega^{1,2,4,7}=-\omega^{1,2,3,4,6}$; $d\omega^{1,2,5,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,3,4,7}=\omega^{1,2,3,4,5}$; $d\omega^{1,3,6,7}=-\omega^{1,2,3,5,6}$; $d\omega^{1,4,5,7}=\omega^{1,3,4,5,6}$; $d\omega^{1,4,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{2,3,5,7}=\omega^{1,2,3,4,5}$; $d\omega^{2,3,6,7}=\omega^{1,2,3,4,6}$; $d\omega^{2,4,5,7}=\omega^{2,3,4,5,6}$; $d\omega^{2,5,6,7}=-\omega^{1,2,4,5,6}$; $d\omega^{3,4,6,7}=\omega^{2,3,4,5,6}$; $d\omega^{3,5,6,7}=-\omega^{1,3,4,5,6}$; $d\omega^{1,2,4,5,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{1,3,4,6,7}=-\omega^{1,2,3,4,5,6}$; $d\omega^{2,3,5,6,7}=-\omega^{1,2,3,4,5,6}$;

$\mathrm{ch}(H^1_{\mathbb{C}})$= $e^{-\beta_1}\ \langle\omega^1\rangle+e^{-\beta_2}\ \langle\omega^2\rangle+e^{-\beta_3}\ \langle\omega^3\rangle+e^{-\beta_4}\ \langle\omega^4\rangle+e^{-\beta_1+\beta_2-\beta_4}\ \langle\omega^5\rangle+e^{-\beta_1+\beta_3-\beta_4}\ \langle\omega^6\rangle$.

$\mathrm{ch}(H^2_{\mathbb{C}})$= $e^{-\beta_1-\beta_2}\ \langle\omega^{1,2}\rangle+e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle+2e^{-\beta_1-\beta_4}\ \langle\omega^{1,4},\omega^{2,5}\rangle+e^{-2\beta_1+\beta_2-\beta_4}\ \langle\omega^{1,5}\rangle+e^{-2\beta_1+\beta_3-\beta_4}\ \langle\omega^{1,6}\rangle+e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle+e^{-\beta_2-\beta_4}\ \langle\omega^{2,4}\rangle+e^{-\beta_1-\beta_2+\beta_3-\beta_4}\ \langle\omega^{2,6}\rangle+e^{-\beta_3-\beta_4}\ \langle\omega^{3,4}\rangle+e^{-\beta_1+\beta_2-\beta_3-\beta_4}\ \langle\omega^{3,5}\rangle+e^{-\beta_1+\beta_2-2\beta_4}\ \langle\omega^{4,5}\rangle+e^{-\beta_1+\beta_3-2\beta_4}\ \langle\omega^{4,6}\rangle+e^{-2\beta_1+\beta_2+\beta_3-2\beta_4}\ \langle\omega^{5,6}\rangle$.

$\mathrm{ch}(H^3_{\mathbb{C}})$= $e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,3}\rangle+e^{-2\beta_1-\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,2,6}\rangle+e^{-2\beta_1+\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,3,5}\rangle+e^{-2\beta_1-\beta_4}\ \langle\omega^{1,2,5}\rangle+e^{-3\beta_1+\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,5,6}\rangle+e^{-\beta_2-\beta_3-\beta_4}\ \langle\omega^{2,3,4}\rangle+e^{-\beta_1-\beta_3-\beta_4}\ \langle\omega^{1,3,4}\rangle+e^{-\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,2,4}\rangle+e^{-\beta_1-\beta_2+\beta_3-2\beta_4}\ \langle\omega^{2,4,6}\rangle+e^{-2\beta_1+\beta_3-2\beta_4}\ \langle\omega^{1,4,6}\rangle+e^{-\beta_1+\beta_2-\beta_3-2\beta_4}\ \langle\omega^{3,4,5}\rangle+e^{-\beta_1-2\beta_4}\ \langle\omega^{2,4,5}\rangle+e^{-2\beta_1+\beta_2-2\beta_4}\ \langle\omega^{1,4,5}\rangle+e^{-2\beta_1+\beta_2+\beta_3-3\beta_4}\ \langle\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathbb{C}})$= $e^{-2\beta_1-\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,2,3,7}\rangle+e^{-3\beta_1-\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,6,7}\rangle+e^{-3\beta_1+\beta_2-\beta_3-2\beta_4}\ \langle\omega^{1,3,5,7}\rangle+e^{-3\beta_1-2\beta_4}\ \langle\omega^{1,2,5,7}-\omega^{1,3,6,7}\rangle+$

$e^{-4\beta_1+\beta_2+\beta_3-3\beta_4}$ $\langle \omega^{1,5,6,7} \rangle$ + $e^{-\beta_1-\beta_2-\beta_3-2\beta_4}$ $\langle \omega^{2,3,4,7} \rangle$ + $e^{-2\beta_1-\beta_3-2\beta_4}$ $\langle \omega^{1,3,4,7} - \omega^{2,3,5,7} \rangle$ + $e^{-2\beta_1-\beta_2-2\beta_4}$ $\langle \omega^{1,2,4,7} + \omega^{2,3,6,7} \rangle$ + $e^{-2\beta_1-\beta_2+\beta_3-3\beta_4}$ $\langle \omega^{2,4,6,7} \rangle$ + $e^{-3\beta_1+\beta_3-3\beta_4}$ $\langle \omega^{1,4,6,7} - \omega^{2,5,6,7} \rangle$ + $e^{-2\beta_1+\beta_2-\beta_3-3\beta_4}$ $\langle \omega^{3,4,5,7} \rangle$ + $e^{-2\beta_1-3\beta_4}$ $\langle \omega^{2,4,5,7} - \omega^{3,4,6,7} \rangle$ + $e^{-3\beta_1+\beta_2-3\beta_4}$ $\langle \omega^{1,4,5,7} + \omega^{3,5,6,7} \rangle$ + $e^{-3\beta_1+\beta_2+\beta_3-4\beta_4}$ $\langle \omega^{4,5,6,7} \rangle$.

$\mathrm{ch}(H^5_{\mathbb{C}})=$ $e^{-2\beta_1-\beta_2-\beta_3-2\beta_4}$ $\langle \omega^{1,2,3,4,7} \rangle$ + $e^{-3\beta_1-\beta_3-2\beta_4}$ $\langle \omega^{1,2,3,5,7} \rangle$ + $e^{-3\beta_1-\beta_2-2\beta_4}$ $\langle \omega^{1,2,3,6,7} \rangle$ + $2e^{-3\beta_1-3\beta_4}$ $\langle \omega^{1,2,4,5,7} - \omega^{2,3,5,6,7}, \omega^{1,3,4,6,7} - \omega^{2,3,5,6,7} \rangle$ + $e^{-3\beta_1-\beta_2+\beta_3-3\beta_4}$ $\langle \omega^{1,2,4,6,7} \rangle$ + $e^{-4\beta_1+\beta_3-3\beta_4}$ $\langle \omega^{1,2,5,6,7} \rangle$ + $e^{-3\beta_1+\beta_2-\beta_3-3\beta_4}$ $\langle \omega^{1,3,4,5,7} \rangle$ + $e^{-4\beta_1+\beta_2-3\beta_4}$ $\langle \omega^{1,3,5,6,7} \rangle$ + $e^{-4\beta_1+\beta_2+\beta_3-4\beta_4}$ $\langle \omega^{1,4,5,6,7} \rangle$ + $e^{-2\beta_1-\beta_3-3\beta_4}$ $\langle \omega^{2,3,4,5,7} \rangle$ + $e^{-2\beta_1-\beta_2-3\beta_4}$ $\langle \omega^{2,3,4,6,7} \rangle$ + $e^{-3\beta_1+\beta_3-4\beta_4}$ $\langle \omega^{2,4,5,6,7} \rangle$ + $e^{-3\beta_1+\beta_2-4\beta_4}$ $\langle \omega^{3,4,5,6,7} \rangle$.

$\mathrm{ch}(H^6_{\mathbb{C}})=$ $e^{-3\beta_1-\beta_3-3\beta_4}$ $\langle \omega^{1,2,3,4,5,7} \rangle$ + $e^{-3\beta_1-\beta_2-3\beta_4}$ $\langle \omega^{1,2,3,4,6,7} \rangle$ + $e^{-4\beta_1-3\beta_4}$ $\langle \omega^{1,2,3,5,6,7} \rangle$ + $e^{-4\beta_1+\beta_3-4\beta_4}$ $\langle \omega^{1,2,4,5,6,7} \rangle$ + $e^{-4\beta_1+\beta_2-4\beta_4}$ $\langle \omega^{1,3,4,5,6,7} \rangle$ + $e^{-3\beta_1-4\beta_4}$ $\langle \omega^{2,3,4,5,6,7} \rangle$.

$\mathrm{ch}(H^7_{\mathbb{C}})=$ $e^{-4\beta_1-4\beta_4}$ $\langle \omega^{1,2,3,4,5,6,7} \rangle$.

$P(Y) = (1 + 5Y + 9Y^2 + 5Y^3 + 9Y^4 + 5Y^5 + Y^6)(1 + Y)$.

Trivial cohomology (via harmonic cocycles and Poincaré Duality):

Dimensions of $Z^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 6, 15, 20, 29, 20, 7, 1)$

Dimensions of $H^j_{\mathbb{C}}$'s $0 \le j \le 7$: $(1, 6, 14, 14, 14, 14, 6, 1)$

Total dimension of trivial cohomology: 70.

Length associated polynomial: 70.

Riemann Hyp.: yes

$d^\star\omega^{1,4} = -\omega^7$; $d^\star\omega^{2,5} = -\omega^7$; $d^\star\omega^{3,6} = -\omega^7$; $d^\star\omega^{1,2,4} = -\omega^{2,7}$; $d^\star\omega^{1,2,5} = \omega^{1,7}$; $d^\star\omega^{1,3,4} = -\omega^{3,7}$; $d^\star\omega^{1,3,6} = \omega^{1,7}$; $d^\star\omega^{1,4,5} = \omega^{5,7}$; $d^\star\omega^{1,4,6} = \omega^{6,7}$; $d^\star\omega^{2,3,5} = -\omega^{3,7}$; $d^\star\omega^{2,3,6} = \omega^{2,7}$; $d^\star\omega^{2,4,5} = -\omega^{4,7}$; $d^\star\omega^{2,5,6} = \omega^{6,7}$; $d^\star\omega^{3,4,6} = -\omega^{4,7}$; $d^\star\omega^{3,5,6} = -\omega^{5,7}$; $d^\star\omega^{1,2,3,4} = -\omega^{2,3,7}$; $d^\star\omega^{1,2,3,5} = \omega^{1,3,7}$; $d^\star\omega^{1,2,3,6} = -\omega^{1,2,7}$; $d^\star\omega^{1,2,4,5} = \omega^{1,4,7} + \omega^{2,5,7}$; $d^\star\omega^{1,2,4,6} = \omega^{2,6,7}$; $d^\star\omega^{1,2,5,6} = -\omega^{1,6,7}$; $d^\star\omega^{1,3,4,5} = \omega^{3,5,7}$; $d^\star\omega^{1,3,4,6} = \omega^{1,4,7} + \omega^{3,6,7}$; $d^\star\omega^{1,3,5,6} = \omega^{1,5,7}$; $d^\star\omega^{1,4,5,6} = -\omega^{5,6,7}$; $d^\star\omega^{2,3,4,5} = -\omega^{3,4,7}$; $d^\star\omega^{2,3,4,6} = \omega^{2,4,7}$; $d^\star\omega^{2,3,5,6} = \omega^{2,5,7} + \omega^{3,6,7}$; $d^\star\omega^{2,4,5,6} = \omega^{4,6,7}$; $d^\star\omega^{3,4,5,6} = -\omega^{4,5,7}$; $d^\star\omega^{1,2,3,4,5} = \omega^{1,3,4,7} + \omega^{2,3,5,7}$; $d^\star\omega^{1,2,3,4,6} = -\omega^{1,2,4,7} + \omega^{2,3,6,7}$; $d^\star\omega^{1,2,3,5,6} = -\omega^{1,2,5,7} - \omega^{1,3,6,7}$; $d^\star\omega^{1,2,4,5,6} = -\omega^{1,4,6,7} - \omega^{2,5,6,7}$; $d^\star\omega^{1,3,4,5,6} = \omega^{1,4,5,7} - \omega^{3,5,6,7}$; $d^\star\omega^{2,3,4,5,6} = \omega^{2,4,5,7} + \omega^{3,4,6,7}$; $d^\star\omega^{1,2,3,4,5,6} = -\omega^{1,2,4,5,7} - \omega^{1,3,4,6,7} - \omega^{2,3,5,6,7}$;

$\Delta\omega^7 = 3\omega^7$; $\Delta\omega^{1,4} = \omega^{1,4} + \omega^{2,5} + \omega^{3,6}$; $\Delta\omega^{1,7} = 2\omega^{1,7}$; $\Delta\omega^{2,5} = \omega^{1,4} + \omega^{2,5} + \omega^{3,6}$; $\Delta\omega^{2,7} = 2\omega^{2,7}$; $\Delta\omega^{3,6} = \omega^{1,4} + \omega^{2,5} + \omega^{3,6}$; $\Delta\omega^{3,7} = 2\omega^{3,7}$; $\Delta\omega^{4,7} = 2\omega^{4,7}$; $\Delta\omega^{5,7} = 2\omega^{5,7}$; $\Delta\omega^{6,7} = 2\omega^{6,7}$; $\Delta\omega^{1,2,4} = \omega^{1,2,4} - \omega^{2,3,6}$;

$\Delta\omega^{1,2,5} = \omega^{1,2,5} + \omega^{1,3,6}$; $\Delta\omega^{1,2,7} = \omega^{1,2,7}$; $\Delta\omega^{1,3,4} = \omega^{1,3,4} + \omega^{2,3,5}$; $\Delta\omega^{1,3,6} = \omega^{1,2,5} + \omega^{1,3,6}$; $\Delta\omega^{1,3,7} = \omega^{1,3,7}$; $\Delta\omega^{1,4,5} = \omega^{1,4,5} - \omega^{3,5,6}$; $\Delta\omega^{1,4,6} = \omega^{1,4,6} + \omega^{2,5,6}$; $\Delta\omega^{1,4,7} = 2\omega^{1,4,7} + \omega^{2,5,7} + \omega^{3,6,7}$; $\Delta\omega^{1,5,7} = \omega^{1,5,7}$; $\Delta\omega^{1,6,7} = \omega^{1,6,7}$; $\Delta\omega^{2,3,5} = \omega^{1,3,4} + \omega^{2,3,5}$; $\Delta\omega^{2,3,6} = -\omega^{1,2,4} + \omega^{2,3,6}$; $\Delta\omega^{2,3,7} = \omega^{2,3,7}$; $\Delta\omega^{2,4,5} = \omega^{2,4,5} + \omega^{3,4,6}$; $\Delta\omega^{2,4,7} = \omega^{2,4,7}$; $\Delta\omega^{2,5,6} = \omega^{1,4,6} + \omega^{2,5,6}$; $\Delta\omega^{2,5,7} = \omega^{1,4,7} + 2\omega^{2,5,7} + \omega^{3,6,7}$; $\Delta\omega^{2,6,7} = \omega^{2,6,7}$; $\Delta\omega^{3,4,6} = \omega^{2,4,5} + \omega^{3,4,6}$; $\Delta\omega^{3,4,7} = \omega^{3,4,7}$; $\Delta\omega^{3,5,6} = -\omega^{1,4,5} + \omega^{3,5,6}$; $\Delta\omega^{3,5,7} = \omega^{3,5,7}$; $\Delta\omega^{3,6,7} = \omega^{1,4,7} + \omega^{2,5,7} + 2\omega^{3,6,7}$; $\Delta\omega^{4,5,7} = \omega^{4,5,7}$; $\Delta\omega^{4,6,7} = \omega^{4,6,7}$; $\Delta\omega^{5,6,7} = \omega^{5,6,7}$; $\Delta\omega^{1,2,3,4} = \omega^{1,2,3,4}$; $\Delta\omega^{1,2,3,5} = \omega^{1,2,3,5}$; $\Delta\omega^{1,2,3,6} = \omega^{1,2,3,6}$; $\Delta\omega^{1,2,4,5} = 2\omega^{1,2,4,5} + \omega^{1,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,2,4,6} = \omega^{1,2,4,6}$; $\Delta\omega^{1,2,4,7} = \omega^{1,2,4,7} - \omega^{2,3,6,7}$; $\Delta\omega^{1,2,5,6} = \omega^{1,2,5,6}$; $\Delta\omega^{1,2,5,7} = \omega^{1,2,5,7} + \omega^{1,3,6,7}$; $\Delta\omega^{1,3,4,5} = \omega^{1,3,4,5}$; $\Delta\omega^{1,3,4,6} = \omega^{1,2,4,5} + 2\omega^{1,3,4,6} + \omega^{2,3,5,6}$; $\Delta\omega^{1,3,4,7} = \omega^{1,3,4,7} + \omega^{2,3,5,7}$; $\Delta\omega^{1,3,5,6} = \omega^{1,3,5,6}$; $\Delta\omega^{1,3,6,7} = \omega^{1,2,5,7} + \omega^{1,3,6,7}$; $\Delta\omega^{1,4,5,6} = \omega^{1,4,5,6}$; $\Delta\omega^{1,4,5,7} = \omega^{1,4,5,7} - \omega^{3,5,6,7}$; $\Delta\omega^{1,4,6,7} = \omega^{1,4,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{2,3,4,5} = \omega^{2,3,4,5}$; $\Delta\omega^{2,3,4,6} = \omega^{2,3,4,6}$; $\Delta\omega^{2,3,5,6} = \omega^{1,2,4,5} + \omega^{1,3,4,6} + 2\omega^{2,3,5,6}$; $\Delta\omega^{2,3,5,7} = \omega^{1,3,4,7} + \omega^{2,3,5,7}$; $\Delta\omega^{2,3,6,7} = -\omega^{1,2,4,7} + \omega^{2,3,6,7}$; $\Delta\omega^{2,4,5,6} = \omega^{2,4,5,6}$; $\Delta\omega^{2,4,5,7} = \omega^{2,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{2,5,6,7} = \omega^{1,4,6,7} + \omega^{2,5,6,7}$; $\Delta\omega^{3,4,5,6} = \omega^{3,4,5,6}$; $\Delta\omega^{3,4,6,7} = \omega^{2,4,5,7} + \omega^{3,4,6,7}$; $\Delta\omega^{3,5,6,7} = -\omega^{1,4,5,7} + \omega^{3,5,6,7}$; $\Delta\omega^{1,2,3,4,5} = 2\omega^{1,2,3,4,5}$; $\Delta\omega^{1,2,3,4,6} = 2\omega^{1,2,3,4,6}$; $\Delta\omega^{1,2,3,5,6} = 2\omega^{1,2,3,5,6}$; $\Delta\omega^{1,2,4,5,6} = 2\omega^{1,2,4,5,6}$; $\Delta\omega^{1,2,4,5,7} = \omega^{1,2,4,5,7} + \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,3,4,5,6} = 2\omega^{1,3,4,5,6}$; $\Delta\omega^{1,3,4,6,7} = \omega^{1,2,4,5,7} + \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{2,3,4,5,6} = 2\omega^{2,3,4,5,6}$; $\Delta\omega^{2,3,5,6,7} = \omega^{1,2,4,5,7} + \omega^{1,3,4,6,7} + \omega^{2,3,5,6,7}$; $\Delta\omega^{1,2,3,4,5,6} = 3\omega^{1,2,3,4,5,6}$;

$\mathrm{ch}(H^1_{\mathcal{C}}) = e^{-\beta_1}\ \langle\omega^1\rangle + e^{-\beta_2}\ \langle\omega^2\rangle + e^{-\beta_3}\ \langle\omega^3\rangle + e^{-\beta_4}\ \langle\omega^4\rangle + e^{-\beta_1+\beta_2-\beta_4}\ \langle\omega^5\rangle + e^{-\beta_1+\beta_3-\beta_4}\ \langle\omega^6\rangle$.

$\mathrm{ch}(H^2_{\mathcal{C}}) = e^{-\beta_1-\beta_2}\ \langle\omega^{1,2}\rangle + e^{-\beta_1-\beta_3}\ \langle\omega^{1,3}\rangle + e^{-2\beta_1+\beta_2-\beta_4}\ \langle\omega^{1,5}\rangle + e^{-2\beta_1+\beta_3-\beta_4}\ \langle\omega^{1,6}\rangle + e^{-\beta_2-\beta_3}\ \langle\omega^{2,3}\rangle + e^{-\beta_2-\beta_4}\ \langle\omega^{2,4}\rangle + e^{-\beta_1-\beta_2+\beta_3-\beta_4}\ \langle\omega^{2,6}\rangle + e^{-\beta_3-\beta_4}\ \langle\omega^{3,4}\rangle + e^{-\beta_1+\beta_2-\beta_3-\beta_4}\ \langle\omega^{3,5}\rangle + 2e^{-\beta_1-\beta_4}\ \langle\omega^{1,4} - \omega^{3,6}, \omega^{2,5} - \omega^{3,6}\rangle + e^{-\beta_1+\beta_2-2\beta_4}\ \langle\omega^{4,5}\rangle + e^{-\beta_1+\beta_3-2\beta_4}\ \langle\omega^{4,6}\rangle + e^{-2\beta_1+\beta_2+\beta_3-2\beta_4}\ \langle\omega^{5,6}\rangle$.

$\mathrm{ch}(H^3_{\mathcal{C}}) = e^{-\beta_1-\beta_2-\beta_3}\ \langle\omega^{1,2,3}\rangle + e^{-2\beta_1-\beta_2+\beta_3-\beta_4}\ \langle\omega^{1,2,6}\rangle + e^{-2\beta_1+\beta_2-\beta_3-\beta_4}\ \langle\omega^{1,3,5}\rangle + e^{-2\beta_1-\beta_4}\ \langle\omega^{1,2,5} - \omega^{1,3,6}\rangle + e^{-3\beta_1+\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,5,6}\rangle + e^{-\beta_2-\beta_3-\beta_4}\ \langle\omega^{2,3,4}\rangle + e^{-\beta_1-\beta_3-\beta_4}\ \langle\omega^{1,3,4} - \omega^{2,3,5}\rangle + e^{-\beta_1-\beta_2-\beta_4}\ \langle\omega^{1,2,4} + \omega^{2,3,6}\rangle + e^{-\beta_1-\beta_2+\beta_3-2\beta_4}\ \langle\omega^{2,4,6}\rangle + e^{-2\beta_1+\beta_3-2\beta_4}\ \langle\omega^{1,4,6} - \omega^{2,5,6}\rangle + e^{-\beta_1+\beta_2-\beta_3-2\beta_4}\ \langle\omega^{3,4,5}\rangle + e^{-\beta_1-2\beta_4}\ \langle\omega^{2,4,5} - \omega^{3,4,6}\rangle + e^{-2\beta_1+\beta_2-2\beta_4}\ \langle\omega^{1,4,5} + \omega^{3,5,6}\rangle + e^{-2\beta_1+\beta_2+\beta_3-3\beta_4}\ \langle\omega^{4,5,6}\rangle$.

$\mathrm{ch}(H^4_{\mathcal{C}}) = e^{-3\beta_1+\beta_2+\beta_3-4\beta_4}\ \langle\omega^{4,5,6,7}\rangle + e^{-2\beta_1+\beta_2-\beta_3-3\beta_4}\ \langle-\omega^{3,4,5,7}\rangle + e^{-2\beta_1-\beta_2+\beta_3-3\beta_4}\ \langle-\omega^{2,4,6,7}\rangle + e^{-2\beta_1-3\beta_4}\ \langle(-(\omega^{2,4,5,7} - \omega^{3,4,6,7}))/2\rangle + e^{-\beta_1-\beta_2-\beta_3-2\beta_4}\ \langle\omega^{2,3,4,7}\rangle + e^{-4\beta_1+\beta_2+\beta_3-3\beta_4}\ \langle-\omega^{1,5,6,7}\rangle +$

$e^{-3\beta_1+\beta_3-3\beta_4}\ \langle(-(\omega^{1,4,6,7} - \omega^{2,5,6,7}))/2\rangle + e^{-3\beta_1+\beta_2-3\beta_4}\ \langle(-(\omega^{1,4,5,7} + \omega^{3,5,6,7}))/2\rangle + e^{-3\beta_1+\beta_2-\beta_3-2\beta_4}\ \langle\omega^{1,3,5,7}\rangle + e^{-2\beta_1-\beta_3-2\beta_4}\ \langle(\omega^{1,3,4,7} - \omega^{2,3,5,7})/2\rangle + e^{-3\beta_1-\beta_2+\beta_3-2\beta_4}\ \langle\omega^{1,2,6,7}\rangle + e^{-3\beta_1-2\beta_4}\ \langle(\omega^{1,2,5,7} - \omega^{1,3,6,7})/2\rangle + e^{-2\beta_1-\beta_2-2\beta_4}\ \langle(\omega^{1,2,4,7} + \omega^{2,3,6,7})/2\rangle + e^{-2\beta_1-\beta_2-\beta_3-\beta_4}\ \langle-\omega^{1,2,3,7}\rangle$.

$\mathrm{ch}(H^5_{\mathfrak{X}})= e^{-3\beta_1+\beta_2-4\beta_4}\ \langle\omega^{3,4,5,6,7}\rangle + e^{-3\beta_1+\beta_3-4\beta_4}\ \langle-\omega^{2,4,5,6,7}\rangle + e^{-2\beta_1-\beta_2-3\beta_4}\ \langle-\omega^{2,3,4,6,7}\rangle + e^{-2\beta_1-\beta_3-3\beta_4}\ \langle\omega^{2,3,4,5,7}\rangle + e^{-4\beta_1+\beta_2+\beta_3-4\beta_4}\ \langle\omega^{1,4,5,6,7}\rangle + e^{-4\beta_1+\beta_2-3\beta_4}\ \langle-\omega^{1,3,5,6,7}\rangle + e^{-3\beta_1+\beta_2-\beta_3-3\beta_4}\ \langle-\omega^{1,3,4,5,7}\rangle + e^{-4\beta_1+\beta_3-3\beta_4}\ \langle\omega^{1,2,5,6,7}\rangle + e^{-3\beta_1-\beta_2+\beta_3-3\beta_4}\ \langle-\omega^{1,2,4,6,7}\rangle + 2e^{-3\beta_1-3\beta_4}\ \langle(-\omega^{1,2,4,5,7} - \omega^{1,3,4,6,7} + 2\omega^{2,3,5,6,7})/3, (-\omega^{1,2,4,5,7} + 2\omega^{1,3,4,6,7} - \omega^{2,3,5,6,7})/3\rangle + e^{-3\beta_1-\beta_2-2\beta_4}\ \langle\omega^{1,2,3,6,7}\rangle + e^{-3\beta_1-\beta_3-2\beta_4}\ \langle-\omega^{1,2,3,5,7}\rangle + e^{-2\beta_1-\beta_2-\beta_3-2\beta_4}\ \langle\omega^{1,2,3,4,7}\rangle$.

$\mathrm{ch}(H^6_{\mathfrak{X}})= e^{-3\beta_1-4\beta_4}\ \langle\omega^{2,3,4,5,6,7}\rangle + e^{-4\beta_1+\beta_2-4\beta_4}\ \langle-\omega^{1,3,4,5,6,7}\rangle + e^{-4\beta_1+\beta_3-4\beta_4}\ \langle\omega^{1,2,4,5,6,7}\rangle + e^{-4\beta_1-3\beta_4}\ \langle-\omega^{1,2,3,5,6,7}\rangle + e^{-3\beta_1-\beta_2-3\beta_4}\ \langle\omega^{1,2,3,4,6,7}\rangle + e^{-3\beta_1-\beta_3-3\beta_4}\ \langle-\omega^{1,2,3,4,5,7}\rangle$.

$\mathrm{ch}(H^7_{\mathfrak{X}})= e^{-4\beta_1-4\beta_4}\ \langle\omega^{1,2,3,4,5,6,7}\rangle$.

$P(Y) = (1 + 5Y + 9Y^2 + 5Y^3 + 9Y^4 + 5Y^5 + Y^6)(1 + Y)$.

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